Revenue Management with Bundles

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Today’s firms face the challenge of managing large assortments of differentiated items, coupled with large heterogeneity in consumers’ tastes. Multi-item firms adopt different selling mechanisms in an effort to extract consumer surplus. One common selling mechanism is component pricing where the firm sets prices for each item separately. However, component pricing is known to be inefficient in extracting a large consumer surplus (Adams and Yellen 1976). For this reason, some firms adopt bundling mechanisms. The most general bundling mechanism is mixed bundling where the firm offers every possible combination of its items. However, mixed bundling is known to be complicated for both sellers and customers, since it involves selling an exponential number of bundles. In order to simplify their selling mechanism, some firms use pure bundling where they sell all their items as one comprehensive bundle. Other firms use bundle size pricing where they price each bundle based on the number of items in the bundle.

There is an extensive literature on bundling that dates back to Adams and Yellen (1976). However, most this research ignores the effect of two important operational aspects of the bundling selling mechanism, which is limited inventory and finite selling horizons. The seminal paper by Gallego and Van Ryzin (1997) was among the first to study the dynamic multi-item pricing problem for a firm that sells multiple items with limited inventory and a finite selling horizon. This problem is commonly referred to as the network revenue management problem.

In the classical network revenue management literature, the firm has multiple items, also referred to as resources, which can be combined to sell multiple different products. The products are linked to the resources by a network design matrix that links each product to the number of resources it uses. It is often assumed that the network design matrix is fixed and the firm needs to find only the optimal dynamic pricing policy. However, the firm can extract a larger profit by considering the
problem of jointly finding the optimal bundling mechanism along with the corresponding dynamic pricing policy.

In order to incorporate the bundling mechanism design into classical revenue management theory, we can think of this problem as jointly deciding on the network design matrix along with the optimal dynamic pricing problem. But formulating this joint problem can be complicated. Nevertheless, by allowing the firm to set prices at infinity, this problem can be equivalently formulated as finding the optimal dynamic pricing policy under the mixed bundling mechanism. The advantage of this modeling approach is that it allows us to formulate the problem as a stochastic control problem using the classical network revenue management framework. However, this comes at the expense of having an exponential number of pricing policies for the exponential number of possible bundles.

The most common solution approach for the classical network revenue management problem is to solve the corresponding deterministic problem under the fluid regime proposed by Gallego and Van Ryzin (1997), where both the arrival rate and the amount of available inventory are scaled simultaneously to infinity. This fluid regime provides an approximation to the original problem where there are a large number of arrivals and a large amount of inventory. However, in our problem the resulting deterministic problem is not tractable since it is a variant of the static mixed bundling problem, which is known to be hard to solve due to the exponential number of pricing policies required for the bundles.

In this paper, we go beyond the classical fluid regime of Gallego and Van Ryzin (1997). In particular, in our problem we have three parameters that we can scale to obtain different approximations for the original stochastic problem: (i) the arrival rate $\lambda$, (ii) the inventory $C$, and (iii) the number of item types $N$. Therefore, by considering the different ways these three parameters can be scaled to infinity, we can capture different dynamics of the problem that corresponds to different market properties. More interestingly, for some of these regimes we show that the limiting problem is tractable and we can provide closed form asymptotically optimal solutions.

We summarize our results in Figure 1 which reveals the interplay between important operational parameters and the optimal selling strategy. For example, in a market with high demand (i.e. large
arrival rate), the firm should avoid selling bundles and should consider only component pricing. On the other hand, when the firm has a large number of item types, it should consider bundle size pricing with sizes that are inversely proportional to the remaining selling time. Hence, the firm has to balance the incentive to extract high customer valuations and the incentive to liquidate its available inventory. Meanwhile, when the firm has a large amount of inventory, it should consider the static mixed bundling problem. The caveat in this fluid regime is that the static mixed bundling problem is in general hard to solve. However, when also the number of item types is large, the firm can adopt the simple pure bundling mechanism.

Figure 1  Asymptotically optimal solutions under different fluid regimes.

References
