Jumping the Line, Charitably: Analysis and Remedy of Donor-Priority Rule

The ongoing shortage of organs for transplantation has generated an expanding literature on efficient and equitable allocation of the donated cadaveric organs. By contrast, organ donation has been little explored. In this paper, we develop a parsimonious model of organ donation to analyze the social-welfare consequences of introducing the donor-priority rule, which grants registered organ donors priority in receiving organs, should they need transplants in the future. We model an individual’s decision to join the donor registry, which entails a tradeoff between abundance of supply, exclusivity of priority, and cost of donating (e.g., psychological burden). Assuming heterogeneity in the cost of donating only, we find the introduction of the donor-priority rule leads to improved social welfare. By incorporating heterogeneity in the probability of requiring an organ transplant and in organ quality, we show that, in contrast to the literature, introducing the donor-priority rule can lower social welfare due to unbalanced incentives across different types of individuals. In view of the potentially undesirable social-welfare consequences, we propose a freeze-period remedy, under which an individual is not entitled to a higher queueing priority until after having been on the organ-donor registry for a specified period of time. We show that, echoing the theory of the second best (Lipsey and Lancaster 1956), this additional market friction helps rebalance the incentive structure, and in conjunction with the donor-priority rule, can guarantee an increase in social welfare by boosting organ supply without compromising organ quality or inducing excessively high costs of donating.

Key words: Organ donation, donor-priority rule, health policy, queueing games

1. Introduction

The United States, as much of the rest of the world, is experiencing an organ-shortage crisis, with about 18 people per day dying while waiting for transplants, and a new candidate being added to the transplant waiting list every 10 minutes (Organdonor.gov 2016). In the decade leading up to 2014, the number of people wait-listed for organ transplants increased by 3.5 times, but the number of people who pledged to donate their organs following death grew by merely 1.7 times. As cadaveric organs remain a major source of organs for transplantation, the organ-shortage crisis can be largely attributed to a low share of registered organ donors. The shortage is particularly alarming in populated states such as Texas, New York, and California, where only 7%, 15%, and 28%, respectively, of adults are registered donors (Donate Life America 2011).
Numerous initiatives, as showcased in a White House summit on organ donation in June 2016, have been proposed to encourage more people to add their names to the organ-donor registry (Bernstein 2016). The contemporary public discourse, for the most part, focuses on educational measures to enhance public awareness of the benefits of organ donation. In May 2012, Facebook unveiled a new sharing function that enables its users to advertise their donor status on their timelines, in the hope that this move will exert peer pressure on people who have not registered as organ donors (Cameron et al. 2013). In July 2016, Apple Inc. announced that iPhone users will be able to become nationally registered donors using the Health app under iOS 10 (Olivarez-Giles 2016). In addition, various US and UK regions have experimented with “nudge” strategies to encourage minority ethnic groups to register as organ donors (Morgan et al. 2015).

Health economists, on the other hand, have long been at the forefront in advocating the usage of monetary incentives to individuals for registering as potential cadaveric organ donors. Although few would contest that doing so would boost organ donation, critics have asserted this market-based approach allegedly “fosters class distinctions (and exploitation), infringes on the inalienable values of life and liberty, and is therefore ethically unacceptable” (Delmonico et al. 2002).

In addition to enhancing public awareness and providing monetary incentives, two noteworthy policy initiatives have been proposed: (i) The donor-priority rule, which provides priority status to individuals registering to become potential organ donors. Under the rule, should registered donors need organ transplants, they are given priority over non-donors in receiving cadaveric organs. (ii) The presumed-consent (a.k.a. “opt-out”) policy, which, in contrast to the current US practice, automatically registers adults as organ donors (e.g., when they apply for a driver’s license) unless they follow the required procedures to opt out of the organ-donation program. Various studies have endorsed legislation of the presumed-consent policy (see, e.g., Abadie and Gay 2006), but it faces numerous hurdles, including the public’s fear of misrepresentation of individuals’ willingness to donate (Johnson and Goldstein 2003; Teresi 2012).

Our paper focuses on analyzing the donor-priority rule. We develop a strategic queueing theoretic model of donor registration and organ allocation. In modeling the tradeoffs behind each individual’s decision to register as a potential organ donor, we follow Kessler and Roth’s (2012) modeling approach by assuming each individual incurs a cost of donating associated with registering as an organ donor, which may be either positive or negative; a positive cost of donating represents an internal loss (e.g., fear and discomfort) from becoming a registered organ donor, whereas a negative cost of donating represents an internal reward. Different from Kessler and Roth (2012), we capture each individual’s utility from organ transplantation using the expected total quality-adjusted life.

Because we focus on the broad implication of adopting the rule, we do not restrict ourselves to a specific type of organ (e.g., kidney, liver, or tissue).
expectancy (QALE) and applying the approximation results from the queueing literature (e.g., Zenios 1999).

A commonly felt concern about the donor-priority rule is that it may be perceived as “oppressive” in that it may induce individuals to register as organ donors against their inherent fear of doing so. In other words, this concern means the priority stemming from becoming a registered organ donor can be so valuable that certain individuals may opt to register as organ donors despite their excessively high costs of donating. To operationalize this line of reasoning, consider three groups of individuals: (1) those who register as organ donors, regardless of whether the donor-priority rule is introduced, and will benefit from the donor-priority rule; (2) those who do not register as organ donors, regardless of whether the donor-priority rule is introduced, and will suffer from the donor-priority rule because, under the rule, they rank lower in priority for organ transplants; and (3) those who register as organ donors only under the donor-priority rule, and will enjoy higher utility than those who do not register before or after the introduction of the rule, but may still be either better or worse off, depending on their costs of donating. Taken together, the social-welfare consequences of the donor-priority rule is not immediately clear. Our analysis helps elucidate the social-welfare consequences of the donor-priority rule under different assumptions.

First, we show that when individuals are heterogeneous in their costs of donating only, the introduction of the donor-priority rule will expand the size of the donor registry, increase the overall availability of obtaining an organ, and unequivocally result in increased social welfare. This result is consistent with the findings by Kessler and Roth (2012).

Second, when the individuals are heterogeneous in health status as well as costs of donating, we show that, different from what Kessler and Roth (2012) would predict, the introduction of the donor-priority rule may indeed lower social welfare. The intuition is that under this rule, even individuals with the same cost of donating would respond differently in their registry-joining decisions, because they can have different probabilities of requiring organ transplants in the future. Specifically, we show that ceteris paribus, the donor-priority rule, by providing a stronger incentive to high-risk individuals (i.e., those with a high probability of requiring organ transplants in the future) than to low-risk individuals, results in (1) a pool of organs with an average quality lower than that of the overall population, and (2) a proportion of new organ donors with excessively high costs of donating. When this incentive structure becomes sufficiently disproportional, the resultant social-welfare loss—due to the reduction in the average organ quality and the increase in costs of donating—can outweigh the social-welfare gain from the expanded organ-donor registry. Furthermore, even when organ quality is homogeneous and the average organ quality does not decrease after introducing the donor-priority rule, the donor-priority rule can result in a lower
social welfare because it attracts those high-risk individuals with high cost of donating. Those high-risk individuals with excessively high costs of donating are “pressured” into becoming registered organ donors because of their high risk of getting sick and the slim chance of receiving an organ transplant if they remain unregistered under the donor-priority rule. Without internalizing the negative externality to the non-donors, those individuals’ decisions to donate are privately optimal under the donor-priority rule, but not socially optimal.

Last and perhaps most interesting, we propose a simple-to-implement remedy, under which an individual is not entitled to a higher queueing priority until after having been on the organ-donor registry for a specified period of time, which we refer to as a “freeze period.” We prove the freeze-period remedy, if well calibrated, can overcome the aforementioned effect of quality decrease due to the introduction of the donor-priority rule: When the freeze-period remedy is imposed in conjunction with the donor-priority rule, the average quality of the donated organs can be restored to the level of the population average. The intuition behind this improvement is that the freeze-period remedy adds a friction to the organ-donation system, which, in some sense, provides a disincentive to all individuals for becoming organ donors. The strength of the disincentive differs across risk types such that high-risk individuals are discouraged to a greater extent than low-risk ones. Thus, a well-calibrated freeze-period remedy may be able to restore the incentive structure from distortions induced by the donor-priority rule. We analytically prove, the optimal freeze-period remedy, when imposed in conjunction with the donor-priority rule, guarantees higher social welfare than before the introduction of donor-priority rule—the remedy helps improves social welfare through boosting the supply of organs without compromising the quality or inducing excessively high costs of donating. The theory of the second best (Lipsey and Lancaster 1956) applies here: in the presence of asymmetric incentives, introducing a second market friction partially counteracts the asymmetric incentives, and leads to a more efficient outcome.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. In Section 3, we describe our analytical framework. In Section 4, we consider the case in which all the individuals have homogeneous health status. Section 5 models population heterogeneity in health status. Section 6 proposes a freeze-period remedy. Section 7 presents an extension of our modeling framework in Section 5 and 6 to a continuous-type setup. Section 8 concludes the paper with a summary of our key insights. All the technical proofs are in the appendix.

2. Literature
Our paper contributes to a growing body of (broadly defined) operations management literature on organ-transplant services, most of which focuses on organ allocation (see, e.g., Akan et al. 2012; Ata et al. 2016; Bertsimas et al. 2013; Gentry et al. 2015; Kong et al. 2010; Sandikçi et al. 2013;
Su and Zenios 2004, 2006) and surgical decisions (e.g., Alagoz et al. 2004; Howard 2002; Zhang 2010). To the best of our knowledge, our paper presents the first analytical model in the field of operations management to examine an organ-donation policy. Our paper is relevant to several papers examining the queueing discipline for organ allocation. Su and Zenios (2004) develop and analyze a queueing model to study the role of patient choice in the kidney-transplant waiting system, and highlight the conflict between equity and efficiency in kidney allocation. They show that under an LCFS priority discipline, the competitive equilibrium is socially optimal. Our paper, with a specific focus on organ donation, also considers a priority-queueing discipline, but the queueing priority is principally tied to each individual’s organ-donor status. Su and Zenios (2006) propose an organ-allocation method whereby heterogeneous patients have to declare which types of kidneys they would be willing to accept at the time they join the waiting list (rather than at the time they are offered the kidney). This scheme eliminates the need for a lengthy search at the time of the kidney transportation to the transplantation site, because each candidate’s private information can be revealed during the allocation process. Our paper highlights the impact of individual heterogeneity, and our analysis builds on the intermediate result that individuals’ heterogeneous health status can influence their decision to join the donor registry; these decisions collectively determine the quality of donated organs. Ata et al. (2016) develop a fluid queueing model to study the social-welfare implication of OrganJet, a transshipment system that enables organ-transplant candidates to list at multiple transplant centers and receive transplants in a wider geographic range. Our paper also adopts a fluid-approximation approach, but jointly considers organ donation and organ transplantation, albeit with an emphasis on organ donation.

The economics literature has examined the practice of charitable fund-raising and giving; see, for example, Andreoni (1989), Eckel and Grossman (2003), and Landry et al. (2006). Yet theoretic modeling effort of the US organ-donation system under the donor-priority rule remains scant. The only paper that analytically models organ donation is Kessler and Roth (2012). Our work differs from theirs along several dimensions. First, Kessler and Roth (2012) address the impact of the donor-priority rule mainly through behavioral experiments. Second, their analytical model, built to illustrate their subsequent behavioral investigation, focuses solely on how the donor-priority rule affects an individual’s probability of receiving an organ. Our paper, by contrast, builds a queueing model of the organ-allocation process and makes different assumptions regarding an individual’s utility. This different setup allows us to generate rich and interesting insights into the social-welfare consequences of the donor-priority rule. Under Kessler and Roth’s original framework, social welfare will always increase after introduction of the donor-priority rule. We use an individual’s QALE, rather than the probability of receiving organs, as the measure of an individual’s utility, which allows us to generate very different policy implications. As a followup to their 2012 paper, Kessler
and Roth (2014) show through laboratory experiments that in the presence of a loophole such that an individual may enjoy the priority of receiving donated organs without incurring the cost of donating, the positive incentive effect, as characterized in their earlier work, would disappear. Although our paper does not explicitly model the loophole Kessler and Roth (2014) address, it shares the spirit of theirs in that we characterize unintended incentive distortions due to the priority status associated with registered organ donors.

Our paper, by building a strategic queueing model of individuals’ decisions to become registered organ donors, bridges the operations management and economics literature on organ transplantation. To the best of our knowledge, this paper is the first to jointly incorporate queueing considerations and the cost of donating. Our paper focuses on both the quantity and quality of the pool of donated organs, and proposes an operational approach to address a complex yet crucial social problem.

Broadly speaking, our paper is relevant to the growing rational queueing literature (see, e.g., Afeche 2013; Anand et al. 2011; Dai et al. 2016; Debo et al. 2008; Guo et al. 2016; Kostami and Rajagopalan 2013; Paç and Veeraraghavan 2015; Wang et al. 2010; Zhan and Ward 2014) in that individuals jointly determine (1) the service rate and (2) the arrival rate of individuals with priority. Without enough donors, the benefit of joining the registry is low due to a limited organ supply; when the number of donors reaches a critical point, the value of the priority would be minimal. Our paper enriches the rational queueing literature by characterizing this new tradeoff between abundance of supply, exclusivity of priority, and cost of donating.

### 3. Modeling Framework

In this section, we introduce our modeling environment, laying out the foundation for equilibrium characterization under different organ-donation policies.

For simplicity of analysis, following Kessler and Roth (2012), we assume each individual may be at one of three states: healthy (in a condition not requiring an organ transplant), sick and in need of an organ transplant, or deceased from premature death. A healthy individual makes the donor-registry-joining decision without certainty as to the need for an organ transplant in the future.\(^2\)

Two types of stochastic events are central to our modeling of the organ-allocation system: (1) a healthy individual may fall sick and be listed for an organ transplant, and (2) a healthy individual

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\(^2\)Countries implementing the donor-priority rule often require individuals to stay in the organ-donor registry for a freeze period before receiving donor priority (Lavee et al. 2010). We consider the effect of imposing a freeze period for donor priority in Section 6. For now, we assume the following: (i) no sick individuals requiring an organ will sign up as an organ donor, (ii) any individual, having become a donor, will not renege from the donor registry, and (iii) brain deaths of registered organ donors will lead to the harvest of organs. These assumptions are fairly reasonable, and ensure individuals signing up for the organ-donor registry will actually become suppliers of organs in the case of premature brain death.
may suffer from premature brain death and become a potential deceased organ donor. We model these two events as two separate, independent stochastic processes. We denote by $\theta$ the arrival rate of individuals in need of organs, $\Theta$ an individual’s lifetime probability of requiring an organ transplant, and $\phi$ the rate of all brain deaths; we assume, without loss of generality, both processes are Poisson. Letting $n$ denote the number of organs each potential donor can provide, the arrival rate of cadaveric organs is $\phi n$ if all the individuals sign up for the donor registry. We assume $\phi n > \theta$, meaning the organs from all the brain deaths are adequate to support those individuals in need of organs, largely in line with the status of the US organ-transplantation system: according to the Organ Procurement & Transplantation Network’s (OPTN) Deceased Donor Potential Study (Klassen et al. 2016), the potential number of deceased organ donors is approximately 37,258; in the case of kidney transplant, because, for instance, each donor can provide up to two kidneys, the supply would be sufficient to serve the approximately 35,000 new patients added to the waiting list each year (Organdonor.gov 2016).

Each individual incurs a burden from registering as an organ donor, which we—following Kessler and Roth (2012)—refer to as the cost of donating and denote by $c$. The cost of donating $c$ has a support of $(-\infty, \infty)$, a probability density function of $f(\cdot)$, and a cumulative distribution function of $F(\cdot)$. When an individual has a cost of donating $c > 0$, he or she must overcome certain burdens to register as a potential organ donor. For example, some individuals fear physicians might not try their best to save registered organ donors’ lives (Teresi 2012). As another example, certain religious beliefs disfavor the practice of organ donation (Bruzzone 2008). When an individual has a cost of donating $c < 0$, he or she earns a positive non-monetary gain—for example, social recognition and self-fulfilment (Prottas 1983)—from registering to be an organ donor.

Following the convention in the organ-transplantation literature (e.g., Su and Zenios 2006), we use QALE to measure the utility of an individual who is in need of an organ transplant. An individual’s QALE is written as

$$u = \alpha D + \beta \pi T,$$

where $\alpha$ is the quality-of-life score while on the waiting list, $D$ is the individual’s life expectancy from the time he is put on the waiting list to the time he dies or receives an organ (whichever comes first), $\beta$ is the quality-of-life score after transplantation such that $\beta > \alpha$, $\pi$ is the probability of receiving an organ, and $T$ is the individual’s post-transplantation life expectancy.

For a healthy individual, the tradeoff behind the decision to join the donor registry is between the cost of donating and the expected benefit from organ donation (if any). Ceteris paribus, an individual with a higher cost of donating will have a lower incentive to join the donor registry. Thus, given any organ donation policy, a threshold $x \in (-\infty, \infty)$ exists such that all the individuals
with $c \leq x$ will become organ donors, and all the individuals with $c > x$ will not. Given $x$, the corresponding donation rate (i.e., the proportion of the population who are on the organ-donor registry) is $F(x)$. Because $F(x)$ increases in $x$, a larger $x$ corresponds to a higher share of organ donors. We define a constant $\hat{x} = F^{-1}(\theta/(\phi n))$ such that $\phi n F(\hat{x}) = \theta$ at $x = \hat{x}$. In other words, $F(\hat{x})$ is the share of organ donors at which the supply rate of donated organs is equal to the demand rate for organ transplants. We assume $\hat{x} > 0$, meaning that in equilibrium, the supply of donated organs would not be able to meet all the demand if the cutoff cost of donating is zero (i.e., only individuals with negative costs of donating join the donor registry).

Let $d$ denote a sick individual’s life expectancy without organ transplantation. When the threshold is $x$, we represent, approximately, the individual’s pre-transplantation life expectancy while on the waiting list as

$$D(x) = \left( \frac{\theta - F(x) \phi n}{\theta} \right)^+ \cdot d = \begin{cases} \frac{\theta - F(x) \phi n}{\theta} \cdot d & \text{if } x \leq \hat{x} \\ 0 & \text{otherwise.} \end{cases}$$

(1)

The probability that an individual with cost of donating $x$ receives an organ, denoted by $\pi(x)$, is, approximately,

$$\pi(x) = \min \left\{ 1, \frac{F(x) \phi n}{\theta} \right\} = \begin{cases} \frac{F(x) \phi n}{\theta} & \text{if } x \leq \hat{x} \\ 1 & \text{otherwise.} \end{cases}$$

(2)

The above approximations (1)–(2) are due to Zenios (1999) and have been used in the organ-donation literature (e.g., Su and Zenios 2006).

4. Preliminary: Heterogeneity in Cost of Donating Only

In this section, we consider the case in which individuals are heterogeneous in their costs of donating but homogeneous in their health status (i.e., their probability of falling sick and requiring an organ transplant). We first present the social optimum as a benchmark. Then we characterize the equilibria for the cases before and after the introduction of the donor-priority rule, respectively. We compare social welfare across both cases and show that introducing the donor-priority rule leads to improved social welfare.

4.1. Benchmark: Social Optimum

As a benchmark, we characterize the social optimum. Because all individuals possess the same ex-ante expected utility from receiving organ transplants, social welfare—irrespective of the allocation scheme—varies across individuals according to their cost of donating. In the social optimum, the donors must be the ones whose costs of donating are sufficiently low. In other words, the social optimum is dictated by a threshold, denoted by $x^{SO}$, such that all individuals with costs of donating
lower than \( x^{SO} \) will be on the organ-donor registry, and those with costs of donating higher than \( x^{SO} \) will not.

Social welfare \( W_s \) as a function of \( x \) can be expressed as the aggregated QALE of all the listed individuals less the aggregated costs of donating incurred by all the registered organ donors (i.e., those whose cost of donating is lower than \( x \)):

\[
W_s(x) = \Theta(\alpha D(x) + \beta \pi(x) T) - E[(c | c \leq x)] F(x) \\
= \begin{cases} 
\Theta(\alpha d(1 - F(x) \phi n / \theta) + \beta T F(x) \phi n / \theta) - \int_{-\infty}^x cf(c) dc & \text{if } x < \hat{x}, \\
\Theta \beta T - \int_{-\infty}^x cf(c) dc & \text{otherwise.} 
\end{cases}
\]

Maximizing social welfare gives the socially efficient cutoff cost.

**Lemma 1.** The socially efficient cutoff cost is \( x^{SO} = \min \{ \Theta(\beta T - \alpha d) \phi n / \theta, \hat{x} \} \).

Intuitively, the socially efficient cutoff cost cannot be above \( \hat{x} \); otherwise, any donor whose cost of donating is above \( \hat{x} \) would contribute to an increase in the total cost of donating but would make no difference to the welfare of sick individuals who need organs. Whether \( \hat{x} \) is equal to the socially efficient threshold depends on the magnitude of \( \hat{x} \): under a large \( \hat{x} \), implying that the marginal donor bears a high cost of donating in order to fulfill all the demand for organs, the socially efficient cost is lower than \( \hat{x} \). In this case, the socially efficient threshold increases in \( \beta T - \alpha d \), which reflects the marginal benefit of organ transplantation.

### 4.2. Before Introduction of Donor-Priority Rule

We begin with the case in which the donor-priority rule is not introduced and registered organ donors are not granted queueing priority. Because each individual gains no benefit from becoming an organ donor, only those with negative costs of donating (i.e., \( c < 0 \)) have the incentive to join the organ-donor registry. Thus, in equilibrium, the threshold cost, denoted by \( x^{*}_{np} \), is zero; the share of organ donors (i.e., the proportion of the population registering as donors) is \( F(0) \). The resultant arrival rate for the supply process of organs is therefore \( n \phi F(0) \). We assume \( n \phi F(0) < \theta \), meaning the supply rate of organs is lower than the demand rate before introducing the donor-priority rule, which is in line with the current US organ-transplantation system (Organdonor.gov 2016).

Each transplant candidate’s pre-transplantation life expectancy can be approximated as

\[
D(x^{*}_{np}) = \left( \frac{\theta - F(0) \phi n}{\theta} \right)^+ \cdot d = \frac{\theta - F(0) \phi n}{\theta} \cdot d.
\]

The probability of a transplant candidate receiving an organ can be approximated as

\[
\pi(x^{*}_{np}) = \frac{F(0) \phi n}{\theta} < 1.
\]
Social welfare with no donor-priority rule can thus be represented as the aggregated QALE of all the transplant candidates less the costs of donating of all those who choose to register as organ donors:

\[
W_{np} = \Theta(\alpha D + \beta \pi T) - \mathbb{E}[c|c \leq 0]F(0) \\
= \Theta(\alpha d \cdot \frac{\theta - F(0) \phi n}{\theta} + \beta T \cdot \frac{F(0) \phi n}{\theta}) - \mathbb{E}[c|c \leq 0]F(0).
\]

The first term in (4) represents social welfare deriving from patients on the waiting list for organ transplantation; the second term represents the marginal improvement in social welfare due to patients who have received organ transplantation; the third term represents the total cost of donating of all the registered organ donors.

**Comparison with Social Optimum.** In the social optimum characterized in Section 4.1, we have \( x^{SO} = \min \{ \Theta(\beta T - \alpha d) \phi n/\theta, \hat{x} \} \). Note from Section 3 that \( \hat{x} > 0 \). In addition, the condition \( \beta T > \alpha d \) gives \( \Theta(\beta T - \alpha d) \phi n/\theta > 0 = x_{np}^* \). Therefore, \( x^{SO} = \min \{ \Theta(\beta T - \alpha d) \phi n/\theta, \hat{x} \} > 0 \). In other words, before introducing the donor-priority rule, the donation rate is strictly below the social-optimum level. From the social planner’s perspective, the marginal benefit to sick individuals from organs donated by certain individuals with a positive cost of donating is higher than the cost of donating itself. Nonetheless, without any external incentives, those individuals would choose not to join the donor registry, because they are unable to internalize the positive externality of their becoming registered organ donors.

4.3. **After Introduction of Donor-Priority Rule**

Now we consider the case in which the donor-priority rule has been introduced; that is, all the registered donors are granted priority over non-donors if and when they need cadaveric organs. Due to the introduction of the policy, two queueing classes waiting for donated organs exist: the priority queue with organ donors, and the regular queue with non-donors. Two competing effects are behind the characterization of the equilibrium: first, when an individual decides to be an organ donor, the individual is, in essence, acquiring an option of joining a priority queue in the future should the individual need an organ transplant. Therefore, the individual would benefit from a larger organ pool provided by more organ donors. Second, the value of becoming a donor is diminishing as more people become donors, even a non-donor would be able to benefit from the increased organ supply. Together with the cost of donating, each individual’s decision to join the organ-donor registry is ultimately ruled by the tradeoff between abundance of supply, exclusivity of priority, and cost of donating.

To characterize the equilibrium, we first derive an organ donor’s utility. We use \( x_p^* \) to denote the cutoff cost at which an individual is indifferent between being a donor and not. The supply rate
of organs is now \( \phi n F(x_p^*) \); the demand rate for organs remains \( \theta \), of which \( F(x_p^*) \theta \) is the arrival rate of donors, and the remaining \( (1 - F(x_p^*)) \theta \) is the arrival rate of non-donors. Consequentially, two waiting lists—donor list and non-donor list—exist for organs. Because a donor has a higher priority than a non-donor, they have different utility values:

(i) A donor’s pre-transplantation life cannot be computed using (1)–(2), because we have from \( \theta < \phi n \) that the arrival rate of donors, \( \lambda_d = F(x_p^*) \phi n \), is always lower than the supply rate of organs, \( \mu_d = F(x_p^*) \). Following the fluid-approximation formulations developed by Zenios (1999), a donor’s probability of receiving an organ is 1, and his pre-transplantation life expectancy is 0. Therefore, a donor’s net utility is the QALE from a potential organ transplant, less the cost of donating; that is, \( u_d = \beta \theta T - c \).

(ii) A non-donor, by comparison, faces a rationed organ supply. The total arrival rate of sick non-donors is \( \lambda_n = (1 - F(x_p^*)) \theta \), and the supply rate of organs for non-donors is \( \mu_n = F(x_p^*) (\phi n - \theta) \). A non-donor’s pre-transplantation life expectancy and probability of receiving an organ transplant can be computed using (1)–(2):

\[
D_n = \frac{\lambda_n - \mu_n}{\lambda_n} \cdot d = \frac{(1 - F(x_p^*)) \theta - F(x_p^*) (\phi n - \theta)}{(1 - F(x_p^*)) \theta} \cdot d = \frac{\theta - \phi n F(x_p^*)}{(1 - F(x_p^*)) \theta} \cdot d,
\]

and the non-donor’s probability of receiving an organ transplant is

\[
\pi_n = \frac{\mu_n}{\lambda_n} = \frac{F(x_p^*) (\phi n - \theta)}{(1 - F(x_p^*)) \theta}.
\]

Thus, a non-donor’s expected utility is

\[
\Theta \left( \alpha d \cdot \frac{\lambda_n - \mu_n}{\lambda_n} + \beta T \cdot \frac{\mu_n}{\lambda_n} \right) = \Theta \left( \alpha d \cdot \frac{\frac{\theta - F(x_p^*) (\phi n - \theta)}{\theta (1 - F(x_p^*))} + \beta T \cdot \frac{F(x_p^*) (\phi n - \theta)}{\theta (1 - F(x_p^*))}} \right).
\]

An individual is willing to join the organ-donor registry if and only if the individual achieves a non-negative marginal benefit from doing so, which alludes to the existence of a cutoff cost of donating below which the individual will join the registry. The following proposition characterizes the cutoff cost in the equilibrium.

**Proposition 1.** In the case in which individuals differ only in their costs of donating, under the donor-priority rule, in equilibrium, only those individuals with donation costs \( c \) below the cutoff cost \( x_p^* \) will elect to join the organ-donor registry, where \( x_p^* \) satisfies

\[
x_p^* = \frac{\phi n F(x_p^*)}{\theta (1 - F(x_p^*))} \cdot \Theta \cdot (\beta T - \alpha d).
\]

Such an equilibrium exists and is unique.
In equilibrium, an individual’s expected utility from registering as a donor is decreasing in the fraction of donors, whereas that from not registering is increasing in the fraction of donors. Hence, a unique cutoff cost exists.

Based on Proposition 1, we can write the social-welfare representation under the donor-priority rule as

\[ W_p = \begin{align*}
\int_{-\infty}^{x^*_p} (\Theta \beta T - c) f(c) dc + \int_{x^*_p}^{\infty} \Theta \left( \alpha d \cdot \frac{\lambda_n - \mu_n}{\lambda_n} + \beta T \cdot \frac{\mu_n}{\lambda_n} \right) f(c) dc \\
\int_{-\infty}^{x^*_p} (\Theta \beta T - c) f(c) dc + \int_{x^*_p}^{\infty} (\Theta \beta T - x^*_p) f(c) dc
\end{align*} \]

Proposition 1 gives the following corollary:

**Corollary 1.** \( x^*_p > x^*_{np} = 0 \).

Corollary 1 implies the donor-priority rule helps expand the size of the donor registry: prior to the introduction of the donor-priority rule, individuals with positive costs of donating would not register as donors; after the introduction of the donor-priority rule, however, some individuals with positive costs of donating are incentivized to become donors due to the priority of receiving organs should they become sick and need organ transplants.

**Corollary 2.** \( x^*_p \) increases in \( \theta \) and \( \Theta \), and decreases in \( \phi \).

The above corollary suggests an individual is more likely to join the organ-donor registry if the individual’s chance of requiring an organ transplant increases, and vice versa. In addition, all else being the same, an individual is less likely to become an organ donor when brain deaths occur more frequently, providing a more abundant supply of organs.

**Comparison with Social Optimum.** Corollary 1 implies introducing the donor-priority rule will lead to a higher donation rate. Nevertheless, we can show the equilibrium donation rate still trails the socially optimal donation rate:

**Corollary 3.** \( x^*_p < x^{SO} \).

Intuitively, under the donor-priority rule, an individual’s decision to become an organ donor is self-enriching; it also enriches others by increasing the potential organ supply. Therefore, the donor-priority rule only partially internalizes the marginal benefit to social welfare. In other words, for the marginal donor with a cost of donating \( x^*_p \), although the cost of donating is equal to the individual’s benefit from the donor-priority rule, that cost is still below the marginal benefit to social welfare. Hence, the donation rate in the market equilibrium is below that in the social optimum.
4.4. Social-Welfare Implications

Consider three categories of individuals: (1) those who register as organ donors regardless of whether the donor-priority rule is introduced, (2) those who do not register as organ donors regardless of whether the donor-priority rule is in place, and (3) those who register as organ donors only under the donor-priority rule. It is straightforward that individuals belonging to the first category would benefit from the introduction of the donor-priority rule, because they would acquire priority status in accessing the expanded pool of donated organs. Individuals belonging to the second category would be worse off under the introduction of the donor-priority rule, because they would rank lower in priority for organ transplants under the rule. Individuals belonging to the third category, however, will have higher utility than those belonging to the second category—otherwise, they would not register as organ donors and thus belong to the second category instead—but they may still be either better or worse off, depending on their costs of donating. Thus, the social-welfare consequences of the donor-priority rule are not immediately clear.

Our analytical framework provides a venue for elucidating the social-welfare consequences of introducing the donor-priority rule. We compare social welfare before and after the introduction of the donor-priority rule, and summarize the result in the following proposition:

**Proposition 2.** Under heterogeneity in costs of donating but not in health status, the introduction of the donor-priority rule always increases social welfare.

Proposition 2 states that introducing the donor-priority rule increases social welfare. This improvement is principally achieved through an enlarged donor pool. Because the demand for organs does not change, the increased supply of donated organs leads to improved QALEs of the overall population. Note that granting donors priority in receiving organs has two effects: it increases the total costs of donating, because a proportion of donors with positive costs are incentivized to register as donors; it also increases the supply of donors and allows for more abundant opportunities for organ transplants. Proposition 2 suggests the second effect outweighs the first, leading to increased social welfare: intuitively, if an individual with a low albeit positive cost of donating chooses to switch to donating due to the donor-priority rule, it must be the case that the benefit from joining the organ-donor registry outweighs the cost of donating. Moreover, the increased organ supply from those donors also engenders a positive externality and thus enhances social welfare.

The following corollary further refines Proposition 2.

**Corollary 4.** The change in social welfare due to the introduction of the donor-priority rule, \( W_p - W_{np} \), increases in \( \theta \) and \( \Theta \), and decreases in \( \phi \).
As $\theta$ or $\Theta$ increase or $\phi$ decreases, obtaining an organ becomes increasingly difficult for a candidate. The donor-priority rule provides a stronger incentive for individuals to become registered donors, and leads to a more substantial increase in social welfare.

5. Heterogeneity in Both Health Risk and Cost of Donating

In the previous section, we show that introducing the donor-priority rule increases social welfare under heterogeneity in the cost of donating. In this section, we broaden the scope of our model by incorporating another dimension of heterogeneity among individuals: their chance of becoming sick and requiring an organ. For simplicity of analysis, we focus on the case with two risk types; later, in Section 7, we will generalize our model and analysis to the case with a continuum of risk types.

Suppose an individual can have either a high or low risk level. A high-risk individual is more likely to need an organ transplant, and is additionally more likely to have low-quality organs. Specifically, with probability $0 < p_H < 1$, an individual is high risk; with probability $p_L = 1 - p_H$, an individual is low risk. Furthermore, we consider two separate, independent random processes, respectively, corresponding to high- and low-risk individuals becoming sick and requiring organ transplants, with arrival rates of $\theta_H$ and $\theta_L$, and the probability of requiring organ transplants in their lifetime being $\Theta_H$ and $\Theta_L$. The total arrival rate of individuals requiring organ transplants is $\sum_{i \in \{H, L\}} p_i \theta_i$.

Because the two types differ in their potential needs for organs due to their different risk types, we can reasonably assume that they also differ in their organ quality. The quality of an organ is measured in terms of the post-transplantation life expectancy of the individual who receives it. Letting $G_H(t)$ and $G_L(t)$ denote the respective c.d.f.’s of the post-transplantation life expectancy due to an organ donated by a high-risk and low-risk individual, we assume the life expectancy of a person who receives an organ from a high-risk donor is lower than that from a low-risk donor; that is, $\mathbb{E}[G_H(t)] = T_H < \mathbb{E}[G_L(t)] = T_L$. Furthermore, consistent with the organ-transplantation practice, we assume the total demand rate for donated organs is lower than the maximum possible organ-supply rate; that is, $p_H \theta_H + p_L \theta_L < \phi n$. In addition, we assume the arrival rate (the probability) of a high-risk individual becoming sick and requiring an organ transplant is higher than that for a low-risk individual; that is, $\theta_H > \theta_L$ ($\Theta_H > \Theta_L$).

Later, through Corollary 7, we show that our insights hold qualitatively even if the two types share the same organ quality.
5.1. Before Introduction of Donor-Priority Rule

We first consider the case before the donor-priority rule is introduced. As in Section 4.2, only those with negative costs of donating have the incentive to register as organ donors. In other words, the cutoff cost of donating in equilibrium is $x^*_i = 0$. Hence, the aggregated post-transplantation life expectancy is $T_a = \sum_{i=L,H} p_i T_i$. We can thus represent social welfare as

$$W_{hnp}^h = \sum_{i=H,L} p_i \Theta_i \left( \alpha d \frac{\sum_{j=H,L} p_j \theta_j - F(0) \phi n}{\sum_{j=H,L} p_j \theta_j} + \beta T_a \frac{F(0) \phi n}{\sum_{j=H,L} p_j \theta_j} \right) - \mathbb{E}[\{(c|c \leq 0]\}F(0).$$

In (6), the first term represents the expected QALEs without transplants, the second term represents the increased expected QALEs due to transplants, and the third term represents the registered organ donors’ aggregated cost of donating.

5.2. After Introduction of Donor-Priority Rule

We now consider the case in which the donor-priority rule has been introduced. In this case, different from our previous analysis with heterogeneity in the cost of donating only, we now have two types of individuals with different risk levels, low and high. These two types would respond to the donor-priority rule differently by setting different cutoff costs of donating, denoted by $x^*_H$ and $x^*_L$, respectively, for high- and low-risk individuals. As a result, the arrival rate of type $i$ donor patients is $\lambda_d^i = p_i F(x^*_i) \theta_i$, and the arrival rate of type $i$ non-donor patients is $\lambda_n^i = p_i (1 - F(x^*_i)) \theta_i$, for $i = H, L$. In addition, the total arrival rate of organs from both types of organ donors is $\mu = \sum_{i=H,L} p_i F(x^*_i) \phi n$.

**Donor Utility.** The total arrival rate of organ donors is

$$\lambda_d = \sum_{i=H,L} \lambda_d^i = \sum_{i=H,L} p_i F(x^*_i) \theta_i.$$

The donors are granted priority in receiving all available organs; that is, the total supply rate of organs available to donors is $\mu_d = \mu$. Hence, a type $i$ organ donor with a cost $c$ has a net utility of

$$u_d^i(c) = \Theta_i \beta T_p(x^*_H, x^*_L) - c,$$

where $T_p(x^*_H, x^*_L)$ is the aggregated post-transplantation life expectancy such that

$$T_p(x^*_H, x^*_L) = \frac{\sum_{i=H,L} p_i F(x^*_i) T_i}{\sum_{i=H,L} p_i F(x^*_i)}.$$
Non-donor Utility. The total non-donor arrival rate is \( \lambda_n = \sum_{i=H,L} \lambda_i^H = \sum_{i=H,L} p_i (1 - F(x_i^*)) \theta_i \), and the total supply rate of organs available to non-donors is \( \mu_n = \mu - \lambda_d = \sum_{i=H,L} p_i F(x_i^*) (\phi_n - \theta_i) \). Hence, a type \( i \) non-donor with a cost \( c \) has a net utility of
\[
u_i^*(c) = \Theta_i \left( \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x_H^*, x_L^*) \right).
\]

We characterize the equilibrium in the following proposition.

**Proposition 3.** In equilibrium, the cutoff costs of donating, \( x_H^* \) and \( x_L^* \), satisfy
\[
x_i^* = \Theta_i \cdot (\beta T_p(\Theta_H y, \Theta_L y) - \alpha d) \cdot \frac{\sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi_n)}{\sum_{j=H,L} p_j (1 - F(\Theta_j y)) \theta_j}, \text{ for } i = H, L,
\]
and the above equilibrium exists and is unique.

We provide some intuition for the existence and uniqueness of the equilibrium. Define \( y^* = x_i^*/\Theta_i \), which can be interpreted as a measurement of the overall percentage of organ donors in equilibrium. Proposition 3 implies finding \( (x_H^*, x_L^*) \) is equivalent to finding \( y^* \) satisfying
\[
y = (\beta T_p(\Theta_H y, \Theta_L y) - \alpha d) \cdot \frac{\sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi_n)}{\sum_{j=H,L} p_j (1 - F(\Theta_j y)) \theta_j}, \text{ at } y = y^*,
\]
which means an individual of either risk type (with cutoff cost \( x_H^* \) or \( x_L^* \)) is indifferent between joining the organ-donor registry and not. Specifically, the left-hand side can be interpreted as a marginal donor’s cost of donating, which is increasing in the overall organ supply \( y \). The right-hand side can be interpreted as the benefits from a higher chance of getting organ transplantation due to the donor-priority rule, which is decreasing in the overall organ supply \( y \). Intuitively, the post-transplantation life expectancy \( T_p(\Theta_H y, \Theta_L y) \) is decreasing in \( y \) as a higher \( y \) asymmetrically increases more organ supply from the high-risk type. The term \( \frac{\sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi_n)}{\sum_{j=H,L} p_j (1 - F(\Theta_j y)) \theta_j} \) measures the probability that non-donors cannot receive organ transplantation, which decreases in \( y \). Hence, the solution \( y^* \) uniquely exists, and so does the equilibrium \( (x_H^*, x_L^*) \).

The following corollary immediately follows from Proposition 3:

**Corollary 5.** \( x_H^*/x_L^* = \Theta_H/\Theta_L > 1 \).

The ex-post benefit from registering as an organ donor due to the donor-priority rule is the same regardless of an individual’s risk type; that is, the individual’s benefit from a potential organ transplant in the future is independent of the individual’s type. Thus, the marginal benefit of registering as a donor increases in the likelihood that the individual needs an organ transplant in the future. In equilibrium, the cost of donating of a marginal donor in each type is equal to the marginal benefit from being a registered organ donor. Hence, the result follows. Corollary 5, in turn, gives the following corollary:
Corollary 6.  $T_p(x_H^*, x_L^*) < T_a = \sum_{i=H,L} p_i T_i$.

Corollaries 5 and 6 show that individuals of different risk types perceive the appeal of becoming registered organ donors differently. More specifically, high-risk individuals are more likely to become organ donors. As a result, the average quality of the donated organs is lower than the average quality of organs from the overall population.

Note that under the donor-priority rule, a marginal donor with the cost of donating $x_i^*$ is indifferent between joining the organ-donor registry and not. Moreover, the utility from not joining is independent of the cost of donating. Hence, the utility of any type $i$ non-donor (regardless of the cost of donating) must be exactly the same as the utility of the type $i$ marginal donor. Under the donor-priority rule, social welfare is thus

$$W_p^h = \sum_{i=H,L} p_i \int_{-\infty}^{x_i^*} (\Theta_i \beta T_p^* - c) f(c) dc + \sum_{i=H,L} p_i \int_{x_i^*}^{\infty} (\Theta_i \beta T_p^* - x_i^*) f(c) dc$$

$$= \sum_{i=H,L} p_i \Theta_i \beta T_p^* - \sum_{i=H,L} p_i \int_{-\infty}^{x_i^*} cf(c) dc - \sum_{j=H,L} p_j (1 - F(x_j^*)) x_j^*.$$

Next, we show the introduction of the donor-priority rule can lead to lower social welfare.

Proposition 4. Introducing the donor-priority rule leads to a reduction in social welfare if and only if

$$\sum_{i=H,L} p_i \Theta_i \left( \beta T_p^* - \alpha d \right) - \frac{F(0)}{\sum_{j=H,L} p_j \theta_j} \left( \beta T_a - \alpha d \right)$$

$$< \sum_{i=H,L} p_i \left( \int_{0}^{x_i^*} cf(c) dc + (1 - F(x_i^*)) x_i^* \right).$$

(7)

Proposition 4 reveals an unexpected consequence of introducing the donor-priority rule, which, albeit conducive to expanding the organ-donor registry, may lead to lower social welfare when the additional cost of new donors and the welfare loss of non-donors outweighs the benefit provided for donors. Specifically, the left-hand side of (7) can be interpreted as the benefit from the donor-priority rule for donors. This benefit is low if the post-transplant life expectancy under the donor-priority rule, $T_p^*$, is low, which is true if (i) more type-$H$ individuals sign up as organ donors, that is, $x_H^* > x_L^*$; and (ii) the post-transplant life expectancy of an individual receiving an organ from a high-risk donor, $T_H$, is lower compared to that from a low-risk donor, $T_L$. The right-hand side of (7) is the sum of the increased costs of donating and the utility reduction of non-donors after introducing the donor-priority rule. This sum is larger when more individuals—particularly high-risk individuals who have higher cutoff costs of donating—register as organ donors. Overall,
it implies the condition holds if the donor-priority rule asymmetrically attracts too many high-risk or high-cost individuals to become organ donors; that is, the resultant \( x_H^* \) is high, given a large enough life expectancy gap between \( T_H \) and \( T_L \). Those conditions hold when \( \Theta_H - \Theta_L, \theta_H - \theta_L, \) and \( T_L - T_H \) are large enough.

So far, in this section, we have assumed organ quality differs across risk types, that is, \( T_H < T_L \). We now depart from this assumption, and isolate the effect of the heterogeneity of the probability of requiring organ transplants. We show in the following proposition that social welfare may still decrease:

**Proposition 5.** Even when \( T_H = T_L \), social welfare may still decrease after introducing the donor-priority rule.

Proposition 5 is rather surprising; it states that social welfare can decrease even if the asymmetric incentives do not reduce the average quality of the organ supply. The reason social welfare can decrease after introducing the donor-priority rule when \( T_H = T_L \) is that when high-risk individuals are much more likely to get sick and need organ transplants, they might respond to the donor-priority rule by registering as donors even when their costs of donating are excessively high. Indeed, those high-risk individuals perceive the benefit of becoming registered organ donors under the donor-priority rule without considering the negative externality on non-donors. Hence, their high costs of donating can outweigh the social-welfare from additional registered organ donors. The decisions to donate by high-risk individuals with excessively high costs of donating are privately optimal under the donor-priority rule but not socially optimal.

Proposition 5 highlights a commonly overlooked aspect in analyzing the social-welfare consequences of organ-donation policies: certain individuals may be “pressured” into becoming registered organ donors despite their excessively high costs of donating; as a result, more organ donation—even under the same organ quality—may not necessarily translate into higher social welfare.

One implication from this proposition is that policies such as the donor-priority rule should be complemented with initiatives to help individuals overcome their burdens associated with registering as organ donors. Efforts to enhance public awareness (e.g., Facebook’s sharing function or nudging; see Section 1 for details) can play a complementary role in this area.

### 6. Freeze-Period Remedy

We have shown the introduction of the donor-priority rule can lead to a reduction in social welfare due to the imbalanced incentive structure formed among individuals of heterogeneous risk types. In this section, we propose a simple and easy-to-implement freeze-period remedy. We show the remedy can effectively mitigate the quality-distorting effect as a result of the donor-priority rule. We also
prove that when used in conjunction with the donor-priority rule, this remedy can ensure social-welfare improvement by expanding the size of the donor registry without reducing the average quality of donated organs or inducing unnecessarily high costs of donating.

Under our proposed freeze-period remedy, registered organ donors do not enjoy priority in receiving organ transplants until they have been on the registry for a specified period of time, which we refer to as a freeze period and denote by $S$. During the freeze period, each type-$i$ healthy individual has a positive flow probability (assumed to be Poisson rate) $\theta_i$ of requiring an organ transplant; the time the individual remains healthy until requiring an organ transplant has a mean of $1/\theta_i$.

Thus, during the freeze period, the individual remains healthy with chance $e^{-\theta_i S}$. We can prove our results hold under a general distribution; the detailed proof is available upon request.

Under the freeze-period remedy, we can show each individual uses a threshold policy in determining whether to sign up for organ-donor registry, such that individuals of type $i$ choose to register as organ donors if and only if their cost of donating is no more than a cutoff cost, denoted by $x_i^\#$. Thus, under the donor-priority rule, the arrival rate of the patients is $\lambda_p = \sum_{i=H,L} p_i F(x_i^H) \theta_i e^{-\theta_i S}$, the arrival rate of the patients without queueing priority is $\lambda_n = \sum_{i=H,L} p_i \theta_i (1 - F(x_i^H) e^{-\theta_i S})$, and the total arrival rate of organs is $\mu = \sum_{i=H,L} p_i F(x_i^H) \phi n$. The patients with priority are granted priority in receiving all available organs; that is, the total supply rate of organs available to donors $\mu_p$ is the same as $\mu$, whereas the total supply rate of organs available to individuals without queueing priority is

$$\mu_n = \mu - \lambda_p = \sum_{i=H,L} p_i F(x_i^H) (\phi n - \theta_i e^{-\theta_i S}).$$

(8)

The aggregated post-transplantation life expectancy for the patients receiving organs is

$$T_p(x_H^x, x_L^x) = \frac{\sum_{i=H,L} p_i F(x_i^H) T_i}{\sum_{i=H,L} p_i F(x_i^H)}.$$  

By choosing to become a registered organ donor, a type $i$ individual with a cost $c$ has an expected utility of

$$u_p^i(c) = e^{-\theta_i S} \beta \Theta_i T_p(x_H^x, x_L^x) + \Theta_i (1 - e^{-\theta_i S}) \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x_H^x, x_L^x) \right) - c,$$

whereas by choosing not to become a registered organ donor, a type $i$ individual with a cost $c$ has an expected utility of

$$u_n^i(c) = \Theta_i \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x_H^x, x_L^x) \right).$$

We characterize the equilibrium in the following proposition.
Proposition 6. In equilibrium, the cutoff costs $x^*_i, i = H, L$ satisfy:

$$x^*_i = \Theta_i e^{-\theta_i S} \left( \beta T_p(x^*_H, x^*_L) - \alpha d \right) \sum_{i=H,L} p_i \left( \theta_i - F(x^*_i) + \alpha d \right) e^{-\theta_i S} > 0.$$ 

The following corollary immediately follows from Proposition 6:

Corollary 7. Under the donor-priority rule complemented by a freeze period $S$, in equilibrium, the cutoff costs $x^*_i, i = H, L$ satisfy

$$\frac{x^*_H}{x^*_L} = \frac{\Theta_H e^{-\theta_H S}}{\Theta_L e^{-\theta_L S}}.$$ 

Next, we characterize the freeze period under which the average quality of donated organs is the same as the population average, which we refer to as “quality-restoring freeze period” and denote by $x^*_QR$.

Corollary 8. If $S = S_{QR} = \ln(\Theta_H/\Theta_L) - (\theta_H - \theta_L)$, the average quality of the pool of donated organs in the equilibrium is the same as the population average; that is, $x^*_H/x^*_L = 1$ and $T_p(x^*_H, x^*_L) = \sum_{i=H,L} p_i T_i$.

The above corollaries imply a unique quality-restoring freeze period exists, which eliminates the distorted incentives due to the donor-priority rule. However, it is still unclear why imposing such a quality-restoring freeze period along with the donor-priority rule would indeed improve social welfare, because social welfare depends on both the benefits from organ transplants and the costs of donating.

Note that a type $i$ individual with the cutoff cost $x^*_i, i = H, L$ is indifferent between joining the organ-donor registry or not. Following the proof of Proposition 6, the type $i$ non-donor’s expected welfare can be rewritten as $\Theta_i \beta T^*_p - \frac{x^*_i}{e^{-\theta_i S}}$. Thus, we can represent social welfare under the donor-priority rule with a freeze-period remedy as

$$W^*_p = \sum_{i=H,L} p_i \int_{-\infty}^{x^*_i} \left( \Theta_i \beta T^*_p - \frac{x^*_i}{e^{-\theta_i S}} - c \right) f(c) dc + \sum_{i=H,L} \int_{x^*_i}^{\infty} \left( \Theta_i \beta T^*_p - \frac{x^*_i}{e^{-\theta_i S}} - c \right) f(c) dc$$

$$= \sum_{i=H,L} p_i \left( \Theta_i \beta T^*_p - \frac{x^*_i}{e^{-\theta_i S}} \right) - \sum_{i=H,L} p_i \int_{-\infty}^{x^*_i} cf(c) dc + \sum_{j=H,L} p_j F(x^*_j) x^*_j.$$

Plugging in the quality-restoring freeze period to the above formula and comparing it with social welfare before the introduction of the donor-priority rule, we can generate implications as to whether the quality-restoring freeze period improves social welfare.

Proposition 7. The quality-restoring freeze period, when enforced along with the donor-priority rule, leads to better social welfare than before introducing the donor-priority rule.
Proposition 7 shows that the donor-priority rule, complemented by a quality-restoring freeze period, always leads to better social welfare. The intuition is that the quality-restoring freeze period counteracts the asymmetric incentives induced by the donor-priority rule. Hence, it increases the total organ supply without reducing the average quality of the organ supply. Moreover, it also discourages individuals with both high risks and high costs of donating from inefficiently registering as an organ donor.

**Proposition 8.** A finite optimal freeze period exists, under which introducing the donor-priority rule always leads to an increase in social welfare.

Proposition 8 shows the existence of a finite optimal freeze period, and more importantly, that such a freeze period can guarantee an improvement in social welfare if the freeze-period remedy is implemented in conjunction with the donor-priority rule.

To connect our findings with the economics literature, we would like to point out that our proposed freeze-period remedy may be viewed as an operational application of the theory of the second best (Lipsey and Lancaster 1956). The imbalanced incentive structure due to the heterogeneous health status puts the donor-priority rule in a second-best situation. Moreover, the freeze-period remedy appears to be another distortion that disincentives all individuals to register as organ donors. Therefore, introducing it counteracts the asymmetric incentives and results in higher social welfare.

**7. Continuous Risk Types**

Until now, for simplicity of analysis, we assume there are a discrete number of risk types. In this section, we generalize our analysis to the case in which there is a continuum of risk types, and show our key insights carry over.

Each individual is characterized by a cost of donating, denoted by $c$, and a risk type $i \in I$, with a larger $i$ corresponding to a riskier type. We denote by $p_i$ the probability of an individual being type $i$ such that $\int p_i di = 1$. The arrival rate for individuals with type $i$ listed for organ transplants is $\theta_i$; thus, the total arrival rate is $\int_{i \in I} p_i \theta_i di$. The life-time probability of an individual requiring an organ transplant is denoted by $\Theta_i$. The post-transplantation life expectancy of a transplant candidate who receives an organ from an individual with risk type $i$ is $T_i$. Following the assumptions in the discrete case, we assume the life expectancy $T_i$ is decreasing in risk type $i$, the arrival rate $\theta_i$ is increasing in risk type $i$, and the total demand rate is lower than the maximum possible organ supply rate, that is, $\int_{i \in I} p_i \theta_i di < \phi n$. 
7.1. Effect of Donor-Priority Rule

Before the introduction of the donor-priority rule, only those with negative costs of donating choose to register as organ donors, which implies the cutoff cost in equilibrium for each type \( i \) is \( x_i^* = 0 \). Hence, the aggregated post-transplantation life expectancy is \( T_\alpha = \int_{c \in 1} p_i T_i di \). We can thus represent social welfare as

\[
W^h_n = \int_{c \in 1} p_i \Theta_i \left( \alpha d \int_{j \in 1} p_j \theta_jdj - F(0) \phi n + \beta T_a \frac{F(0) \phi n}{\int_{j \in 1} p_j \theta_j dj} \right) di - E[(c|c \leq 0)] F(0).
\]

After the introduction of the donor-priority rule, different risk types respond by choosing different cutoff costs of donating, denoted by \( x_i^* \), \( \forall i \in I \). As a result, the arrival rate of type \( i \) donor patients is \( \lambda_i = p_i F(x_i^*) \theta_i \), and the arrival rate of type \( i \) non-donor patients is \( \lambda_n = p_i (1 - F(x_i^*)) \theta_i \), \( \forall i \in I \).

In equilibrium, the total arrival rate of organs is \( \mu = \int_{i \in 1} p_i F(x_i^*) \phi ndi \). The total arrival rate of organ donors is \( \lambda_d = \int_{i \in 1} \lambda_p di = \int_{i \in 1} p_i F(x_i^*) \theta_p di \). The donors are granted priority in receiving organ transplants; that is, the total supply rate of organs available to donors is \( \mu_d = \mu \). Hence, a type \( i \) organ donor with a cost \( c \) has a net utility of \( u_i^d(c) = \Theta_i \beta T_p^* - c \), where \( T_p^* \) is the aggregated post-transplantation life expectancy; that is,

\[
T_p^* = \frac{\int_{i \in 1} p_i F(x_i^*) T_i di}{\int_{i \in 1} p_i F(x_i^*) di}.
\]

The total arrival rate of non-donors is \( \lambda_n = \int_{i \in 1} \lambda_n^i di = \int_{i \in 1} p_i (1 - F(x_i^*)) \theta_n di \), and the total supply rate of donated organs available to non-donors is \( \mu_n = \mu - \lambda_d = \int_{i \in 1} p_i F(x_i^*) (\phi n - \theta_i) di \).

Hence, a type \( i \) non-donor with a cost \( c \) has a net utility of

\[
u_i^n(c) = \Theta_i \left( \alpha \frac{\lambda_n - \mu_n d}{\lambda_n} + \beta \frac{\mu_n}{\lambda_n} T_p^* \right).
\]

We characterize the equilibrium in the following proposition.

**Proposition 9.** In equilibrium, the cutoff cost \( x_i^* \) satisfies:

\[
x_i^* = \Theta_i \cdot (\beta T_p^* - \alpha d) \cdot \frac{\int_{j \in 1} p_j (\theta_j - F(x_j^*) \phi n)}{\int_{j \in 1} p_j (1 - F(x_j^*)) \theta_j}, \forall i \in I,
\]

where \( T_p^* = \frac{\int_{i \in 1} p_i F(x_i^*) T_i di}{\int_{i \in 1} p_i F(x_i^*) di} \).

The following corollary immediately follows from Proposition 9:

**Corollary 9.** \( \forall i, j \in I, x_i^*/x_j^* = \Theta_i/\Theta_j \).
Once an individual is listed for an organ transplant, the benefit from having registered as a donor is the same regardless of the individual’s type. Thus, the marginal benefit of registering as a donor only depends on the likelihood that an individual becomes sick and needs an organ transplant. In equilibrium, for each type of marginal donor, the cost of donating has to be equal to the marginal benefit. Corollary 9, in turn, gives the following corollary:

**Corollary 10.** \( T_p^* < T_n = \int_{i \in I} p_i T_i di \). 

Consistent with the results in the discrete-type case, Corollaries 9 and 10 reveal high-risk individuals are more likely to become organ donors. The distributions of donors with and without the donor priority rule bear the familiar monotone likelihood ratio property (see, e.g., Levin 2001) in the post-transplantation life expectancy. As a result, the average quality of the donated organs is lower than the average quality of organs from the overall population.

Under the donor-priority rule, social welfare is thus

\[
W_p^h = \int_{i \in I} p_i \left( F(x_i^*) \Theta_i \beta T_p + (1 - F(x_i^*)) \Theta_i \left( \alpha d \frac{\int_{j \in I} p_j (\theta_j - F(x_j^*)) \phi n d j}{\int_{j \in I} p_j (1 - F(x_j^*)) \theta_j d j} \right) + \beta T_p \frac{\int_{j \in I} p_j F(x_j^*) (\phi n - \theta_j) d j}{\int_{j \in I} p_j (1 - F(x_j^*)) \theta_j d j} \right) - E [c | c \leq x_i^*] F(x_i^*) \right) di.
\]

The above formulation allows us to compare social welfare before and after introducing the donor-priority rule. We can proceed to show the results from Section 4.4 qualitatively hold.

### 7.2. Freeze-Period Remedy

We now analyze the case in which a freeze period is imposed along with the donor-priority rule, corresponding to Section 6 except that now we draw individual types from a continuum. The total arrival rate of organs is \( \mu = \int_{i \in I} p_i F(x_i^*) \phi n d i \). The patients with priority are granted priority in receiving all available organs; that is, the total supply rate of organs available to donors is \( \mu_p = \mu \), whereas the total supply rate of donors available to the patients without queueing priority is

\[
\mu_n = \mu - \lambda_p = \int_{i \in I} p_i F(x_i^*) (\phi n - \theta_i e^{-\theta_i S}) di.
\]

The aggregated post-transplantation life expectancy for the patients receiving organ transplants is

\[
T_p(x^*) = \frac{\int_{i \in I} p_i F(x_i^*) T_i d i}{\int_{i \in I} p_i F(x_i^*) d i},
\]

where \( x^* \equiv \{ x_i^* | i \in I \} \). Therefore, a type \( i \) organ donor with a cost \( c \) has a net utility of

\[
u_i^d(c) = e^{-\theta_i S} \Theta_i \beta T_p(x^*) + \Theta_i \left( 1 - e^{-\theta_i S} \right) \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x^*) \right) - c,
\]
whereas a type $i$ non-donor with a cost $c$ has a net utility of

$$u_i(c) = \Theta_i \cdot \left( \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x^i) \right).$$

We characterize the equilibrium in the following proposition.

**Proposition 10.** In equilibrium, the cutoff costs $x^i_\star$, $i \in I$ satisfy:

$$x^i_\star = \Theta_i e^{-\theta_i S} \left( \beta T_p(x^i_\star) - \alpha d \right) \frac{\int_{j \in I} p_j \left( \frac{\theta_j - F(x^i_\star)}{\phi n} \right) dj}{\int_{j \in I} \theta_j \left( 1 - F(x^i_\star) \right) e^{-\theta_j S} dj} > 0.$$

The following corollaries follow from Proposition 10:

**Corollary 11.** $\forall i, j \in I, x^i_\star x^j_\star = \frac{\Theta_i e^{-\theta_i S}}{\Theta_j e^{-\theta_j S}}$.

**Corollary 12.** A finite quality-restoring freeze period $S^eq$ exists such that $T_p(x^i_\star) = T_a$.

With these essential intermediate results at hand, we can proceed to show our key findings from Section 6 hold qualitatively—a freeze period adds a friction that may rebalance the quality of donated organs, and social welfare is guaranteed to improve under the optimal freeze period.

**8. Concluding Remarks**

In this paper, we model and analyze the donor-priority rule, an initiative aiming to expand the organ-donor registry. Under the donor-priority rule, registered organ donors enjoy queueing priority over non-donors, should they need organ transplants in the future. The inner workings of the donor-priority rule present a compelling venue for queueing theoretic analysis, because in the steady state, more registered organ donors not only imply an increased supply of organs, but also translate to more organ-transplant candidates with priority status. This situation presents a three-way trade-off between abundance of supply, exclusivity of priority, and cost of donating. We adopt a strategic queueing approach and QALE criteria to model each individual’s decision to become a registered organ donor. To the best of our knowledge, the queueing literature has not examined these types of queueing problems before.

Our research leads to several interesting findings. One might expect society to be better off under the donor-priority rule, because more people would sign up for organ donation. Yet the welfare consequences are not immediately clear, because social welfare also crucially depends on the “cost of donating” associated with an average registered organ donor—when such a cost is excessively high, it indicates the rule may be perceived by the public as overly “oppressive.” Our model helps elucidate the social-welfare consequences of the donor-priority rule. Our analysis shows society would indeed be better off when individuals differ only in terms of their costs of donating (Section 4). Furthermore, our analysis of the heterogeneous-population case in Section 5 reveals an
unbalanced incentive structure induced by the donor-priority rule. Specifically, high-risk individuals are better motivated than low-risk ones to become registered organ donors. As a result, although the initiative induces a more sizable organ-donor registry, the average quality of the donated organs can be lower due to the distorted incentives. Our analysis uncovers conditions under which social welfare is likely to decrease due to introduction of the donor-priority rule.

Furthermore, social welfare may decrease even if all donors provide organs of equal quality such that the average quality of the donated organs is held constant. This result, by isolating the effect of the heterogeneity in the probability of requiring organ transplants, highlights a little explored aspect in analyzing organ-donation policies—individuals may be pressured into becoming registered organ donors despite their excessively high costs of donating. As a result, increased organ donation, despite organ quality remaining unchanged, does not necessarily translate into improved social welfare. Thus, one would draw from this finding that the public will continue to benefit from efforts to enhance public awareness and ultimately reduce individuals’ burdens associated with registering as organ donors.

To address the unbalanced incentive structure induced by the donor-priority rule, we propose an operational remedy that entails enforcing a freeze period, that is, a specified delay in granting individuals priority on waiting lists for organ transplants. We show the freeze-period remedy, if well calibrated, can effectively restore the efficiency of the organ donation and allocation system. The reason is that the freeze period provides a disincentive for both types of individuals to become organ donors, but the disincentive is stronger for high-risk individuals than for low-risk individuals. Thus, appropriately choosing the length of the freeze period can mitigate the quality-distorting effect introduced by the donor-priority rule. Echoing the theory of the second best (Lipsey and Lancaster 1956), we show this second market distortion, in conjunction with donor-priority rule, can ensure an increase in social welfare by boosting the supply of organs without sacrificing the quality.

References


**Appendix: Proofs**

*Proof of Lemma 1.* When $x < \hat{x}$, we have from (3) that $dW_s(x)/dx = (\Theta(\beta T - \alpha d) \phi n/\theta - x) f(x)$. Thus, the regional maximum for $x \in (-\infty, \hat{x})$ is attained at $\Theta(\beta T - \alpha d) \phi n/\theta$ or $\hat{x}$, whichever is lower. If $x > \hat{x}$, we observe from (3) that $W_s(x)$ decreases in $x$, indicating the regional maximum for $x \in (\hat{x}, \infty)$ is achieved at $\hat{x}$. Taken together, the socially optimal threshold $x^{SO}$ is $\Theta(\beta T - \alpha d) \phi n/\theta$ or $\hat{x}$, whichever is lower. Q.E.D.

*Proof of Proposition 1.* An individual with the cutoff cost $x^*_p$ is indifferent between joining the donor registry or not, that is,

$$\Theta \beta T - x^*_p = \Theta \left( \frac{\theta - \theta F(x^*_p)}{1 - F(x^*_p)} \phi n + \frac{\theta - \phi n}{\theta} \right),$$

or, equivalently,

$$\Theta \beta T - x^*_p = \Theta \left( \alpha d \cdot \frac{\theta - F(x^*_p) \phi n}{\theta (1 - F(x^*_p))} + \frac{\beta T \cdot F(x^*_p) (\phi n - \theta)}{\theta (1 - F(x^*_p))} \right).$$

The above equation, after rearrangements of its terms, gives (5).

The left-hand side of equation (5) is increasing in $x^*_p$ while its right-hand side can be rewritten as

$$\frac{\theta - \phi n F(x^*_p)}{\theta (1 - F(x^*_p))} \cdot \Theta \cdot (\beta T - \alpha d) = \left( \frac{\phi n}{\theta} + \frac{\theta - \phi n}{\theta (1 - F(x^*_p))} \right) \cdot \Theta \cdot (\beta T - \alpha d),$$

which is decreasing in $x^*_p$ because $\theta - \phi n < 0$. Moreover, when $x_p = 0$, $LHS = 0 < RHS$, and

$$\lim_{x_p \to \infty} LHS = \infty > -\infty = \lim_{x_p \to \infty} RHS.$$
Hence, a unique solution \( x_p^* > 0 \) to (10) exists. \( Q.E.D. \)

**Proof of Corollary 2.** We have from (5) that

\[
\frac{\phi_n}{\theta} - 1 = \left(1 - \frac{x_p^*}{\Theta(\beta T - \alpha d)}\right) \left(\frac{1}{F(x_p^*)} - 1\right).
\]

As \( \theta \) increases, the left-hand side of (11) decreases, requiring a higher \( x_p^* \) to decrease the right-hand side of (11) and balance the equation. Similarly, we can show \( x_p^* \) increases in \( \Theta \) and decreases in \( \phi \). \( Q.E.D. \)

**Proof of Corollary 3.** Recall that \( x^{SO} = \min \{ \Theta (\beta T - \alpha d) \phi n/\theta, \hat{x} \} \). On the one hand, as \( \theta - n\phi F(x_p^*) > 0 \) (from (5)), we have \( x_p^* < \hat{x} = F^{-1}(\theta/\phi n) \). On the other hand, we have

\[
x_p^* = \Theta \frac{\theta - \phi n F(x_p^*)}{\theta (1 - F(x_p^*))} (\beta T - \alpha d)
\]

\[
= \Theta \left(\frac{\theta - \phi n}{\theta (1 - F(x_p^*))} + \frac{\phi n}{\theta}\right) (\beta T - \alpha d)
\]

\[
< \Theta (\beta T - \alpha d) \phi n/\theta \text{ (since } \theta < \phi n) .
\]

Hence, we have \( x_p^* < \min \{ \Theta (\beta T - \alpha d) \phi n/\theta, \hat{x} \} = x^{SO} \). \( Q.E.D. \)

**Proof of Proposition 2.** We examine the difference in social welfare before and after the introduction of the donor-priority rule:

\[
W_p - W_{np} = \Theta \left(\frac{\theta - F(0) \phi n}{\theta}\right) (\beta T - \alpha d) + E(c|c \leq 0) F(0) - E(c|c \leq x_p^*) F(x_p^*) - x_p^* (1 - F(x_p^*))
\]

\[
= \Theta \left(\frac{\theta - F(0) \phi n}{\theta}\right) (\beta T - \alpha d) - \int_0^{x_p^*} c f(c) dc - x_p^* (1 - F(x_p^*))
\]

which, by Proposition 1, can be rewritten as

\[
W_p - W_{np} = (\theta - F(0) \phi n) \frac{1 - F(x_p^*)}{\theta - F(x_p^*)} \frac{x_p^*}{\phi n} - \int_0^{x_p^*} c f(c) dc - x_p^* (1 - F(x_p^*))
\]

\[
= x_p^* (1 - F(x_p^*)) \frac{F(x_p^*) - F(0)}{\theta / (\phi n) - F(x_p^*)} - \int_0^{x_p^*} c f(c) dc
\]

\[
= x_p^* (F(x_p^*) - F(0)) \frac{1 - F(x_p^*)}{\theta / (\phi n) - F(x_p^*)} - \int_0^{x_p^*} c f(c) dc .
\]

(12)

Now, because \( \theta < \phi n \), we have

\[
\frac{1 - F(x_p^*)}{\theta / (\phi n) - F(x_p^*)} > 1,
\]

which gives

\[
W_p - W_{np} > x_p^* (F(x_p^*) - F(0)) - \int_0^{x_p^*} c f(c) dc > 0.
\]

That is, social welfare improves after the introduction of the donor-priority rule. \( Q.E.D. \)

**Proof of Corollary 4.** The social-welfare difference, by (12), can be rewritten as

\[
W_p - W_{np} = x_p^* (F(x_p^*) - F(0)) \left(1 + \frac{1 - \theta / (\phi n)}{\theta / (\phi n) - F(x_p^*)}\right) - \int_0^{x_p^*} c f(c) dc,
\]
which is increasing in \( x_i^* \). As \( \theta \) increases, we have from Corollary 2 that \( x_i^* \) increases, and so does the social-welfare improvement. Similarly, we can show the social-welfare improvement increases in \( \Theta \) and decreases in \( \phi \).

**Q.E.D.**

**Proof of Proposition 3.** Consider a type 1 individual with the cutoff cost \( x_i^* \), \( i = H, L \). In equilibrium, the individual is indifferent between joining the donor registry or not; that is,

\[
\Theta_i \delta T_p(x_{iH}^*, x_{iL}^*) - x_i^* = \Theta_i \left( \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x_{iH}^*, x_{iL}^*) \right) \quad \text{for } i = H, L.
\]

The above equation can be rewritten as \( x_i^* = \Theta_i \left( \beta T_p(x_{iH}^*, x_{iL}^*) - \alpha d \right) \cdot (\lambda_n - \mu_n)/\lambda_n \) for \( i = H, L \), which can be further reorganized as

\[
x_i^* = \Theta_i \cdot (\beta T_p(x_{iH}^*, x_{iL}^*) - \alpha d) \cdot \frac{\sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi n)}{\sum_{j=H,L} p_j (1 - F(\Theta_j y) \phi n)} \quad \text{for } i = H, L.
\]

Denoting \( y^* = \frac{x_i^*}{\Theta_i} \), which can be interpreted as the overall donor-registry level in equilibrium. The original problem is equivalent to find a \( y^* \) satisfying

\[
y = (\beta T_p(\Theta_H y, \Theta_L y) - \alpha d) \cdot \frac{\sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi n)}{\sum_{j=H,L} p_j (1 - F(\Theta_j y) \phi n)},
\]

in which LHS is increasing in \( y \), and RHS is decreasing in \( y \) when \( y \) is not too large. Intuitively, the life expectancy post transplantation \( T_p(\Theta_H y, \Theta_L y) \) is decreasing in \( y \) as a higher \( y \) asymmetrically increases organ supply from the high-risk type. The term \( \frac{\sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi n)}{\sum_{j=H,L} p_j (1 - F(\Theta_j y) \phi n)} \) measures the probability that non-donors cannot receive organ transplantation, which decreases in \( y \). Moreover, \( 0 = LHS < RHS \) when \( y = 0 \), and \( LHS > RHS = 0 \) when \( y > 0 \) satisfies \( \sum_{j=H,L} p_j (\theta_j - F(\Theta_j y) \phi n) = 0 \). Hence, the solution \( y^* \) exists and is unique, so does \( (x_L^*, x_H^*) \).

**Q.E.D.**

**Proof of Corollary 6.** The result follows by using \( T_p(x_{iH}^*, x_{iL}^*) = \frac{\sum_{j=H,L} p_j F(\Theta_j y) \phi n}{\sum_{j=H,L} p_j (1 - F(\Theta_j y) \phi n)} \), \( x_H^* > x_L^* \) and \( T_H < T_L \). **Q.E.D.**

**Proof of Proposition 4.** We use \( \Delta U_i(c) \) to denote the expected utility change of a type 1 individual with a cost of \( c \) due to the introduction of the donor-priority rule. We derive \( \Delta U_i(c) \) as follows:

\[
\Delta U_i(c) = \begin{cases} 
\Theta_i \left( \beta T_p(x_{iH}^*, x_{iL}^*) - \alpha d \frac{\sum_{j=H,L} p_j \theta_j - F(0) \phi n}{\sum_{j=H,L} p_j (1 - F(0) \phi n)} - \beta T_p(0) \frac{F(0) \phi n}{\sum_{j=H,L} p_j (1 - F(0) \phi n)} \right) & \text{if } c \leq 0 \\
\Theta_i \left( \beta T_p(x_{iH}^*, x_{iL}^*) - \alpha d \frac{\sum_{j=H,L} p_j \theta_j - F(0) \phi n}{\sum_{j=H,L} p_j (1 - F(0) \phi n)} - \beta T_p(0) \frac{F(0) \phi n}{\sum_{j=H,L} p_j (1 - F(0) \phi n)} \right) - c & \text{if } 0 < c \leq x_i^* \\
\Theta_i \left( \beta T_p(x_{iH}^*, x_{iL}^*) - \alpha d \frac{\sum_{j=H,L} p_j \theta_j - F(0) \phi n}{\sum_{j=H,L} p_j (1 - F(0) \phi n)} - \beta T_p(0) \frac{F(0) \phi n}{\sum_{j=H,L} p_j (1 - F(0) \phi n)} \right) - x_i^* & \text{otherwise},
\end{cases}
\]

which implies \( \Delta U(c) \) is decreasing in \( c \). This condition, in turn, gives the necessary and sufficient condition for \( W_D^1 - W_L^1 < 0 \):

\[
\sum_{i=H,L} p_i \int_{-\infty}^{\infty} c \Delta U_i(c) dc < 0,
\]

which can be rewritten as (7).

**Q.E.D.**
Proof of Proposition 5. The proposition can be shown through an example. For example, when \( p_H = 0.05, p_L = 0.95, \theta_H = 0.205, \theta_L = 0.005, \Theta_H = 0.11, \Theta_L = 0.01, t_H = T_L = 15.6, \beta = 0.9, \alpha = 0.75, d = 7 \) and \( c \sim N(0, 1) \), we have \( x_H = 1.0048, x_L = 0.0913 \) and the resultant social welfare difference becomes \( W^k_p - W^h_n = -0.0137 < 0 \). Q.E.D.

Proof of Proposition 6. A type \( i \) individual with the cutoff cost \( c_i, i = H, L \) is indifferent between joining the donor registry and not. In other words, it is enough to show

\[
\Theta \left( e^{-\theta_i S} \beta T_p (x^H_i, x^L_i) + (1 - e^{-\theta_i S}) \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p (x^H_i, x^L_i) \right) \right) - x^L_i = 0
\]

which can be rewritten as

\[
x^L_i = \Theta e^{-\theta_i S} (\beta T_p (x^H_i, x^L_i) - \alpha d) \frac{\lambda_n - \mu_n}{\lambda_n}.
\]

Substituting (8) into the above equation gives

\[
x^L_i = \Theta e^{-\theta_i S} (\beta T_p (x^H_i, x^L_i) - \alpha d) \frac{\sum_{i=H,L} p_i (\theta_i - F(x^i_H)) \phi n}{\sum_{i=H,L} p_i (1 - F(x^i_H)) e^{-\theta_i S}} > 0.
\]

Q.E.D.

Proof of Corollary 8. The result follows by using \( T_p (x^H_i, x^L_i) = \frac{\sum_{i=H,L} n_i F(x^i_H)}{\sum_{i=H,L} n_i F(x^i_H)} \) and \( x^H_i = x^L_i \). Q.E.D.

Proof of Proposition 7. When \( S = \varnothing_Q \), we have \( x^L_i = x^H_i = x^*_{QR} \). Hence,

\[
W^k_p = \sum_{i=H,L} p_i \left( \Theta \beta T_p \frac{x^L_i}{e^{-\theta_i S}} - \frac{x^H_i}{e^{-\theta_i S}} \right) - \sum_{i=H,L} p_i \int_{-\infty}^{x^L_i} c f(c) dc + \sum_{j=H,L} p_j F(x^j_i) x^j_i
\]

where \( \frac{\partial W^k_p}{\partial x^L_i} = \sum_{i=H,L} p_i (F(x^*_{QR}) - e^{\theta_i \varnothing_Q n}) \), which is negative because \( e^{\theta_i \varnothing_Q n} > 1 > F(x^*_{QR}) \).

Moreover, \( W^k_p \) is greater than \( W^h_p \) if and only if

\[
\sum_{i=H,L} p_i \left( -\frac{x^L_i}{e^{-\theta_i \varnothing_Q n}} \right) - \sum_{i=H,L} p_i \int_{0}^{x^L_i} c f(c) dc + \sum_{j=H,L} p_j F(x^j_{QR}) x^j_{QR}
\]

where \( LHS > \sum_{i=H,L} p_i \left( F(0) - e^{\theta_i \varnothing_Q n} \right) x^L_{QR} \). Plugging in

\[
x^L_{QR} = \Theta_i e^{-\theta_i \varnothing_Q n} (\beta T_a - \alpha d) \frac{\sum_{i=H,L} p_i (\theta_i - F(x^*_{QR}) \phi n)}{\sum_{i=H,L} p_i (1 - F(x^*_{QR}) e^{-\theta_i \varnothing_Q n})},
\]

it is enough to show

\[
(F(0) e^{-\theta_i \varnothing_Q n} - 1) \frac{\sum_{i=H,L} p_i (\theta_i - F(x^*_{QR}) \phi n)}{\sum_{i=H,L} p_i (1 - F(x^*_{QR}) e^{-\theta_i \varnothing_Q n})} > \frac{F(0) \phi n - \sum_{i=H,L} p_i \theta_i}{\sum_{i=H,L} p_i \theta_i}
\]

\[
\Leftrightarrow \frac{\sum_{i=H,L} p_i (\theta_i - F(x^*_{QR}) \phi n)}{\sum_{i=H,L} p_i (1 - F(x^*_{QR}) e^{-\theta_i \varnothing_Q n})} < \frac{\sum_{i=H,L} p_i \theta_i - F(0) \phi n}{\sum_{i=H,L} p_i \theta_i (1 - F(0) e^{-\theta_i \varnothing_Q n})}.
\]
which is true as long as \( x^*_{QR} > 0 \). 

**Proof of Proposition 8.** The proof consists of two steps:

Step 1. As shown in Proposition 7, social welfare always increases under the quality-restoring freeze period \( S_{QR} = \frac{ln(\frac{\theta_i}{\theta_j})}{\theta_i - \theta_j} \). By definition, the optimal freeze period results in higher social welfare than the quality-restoring freeze period.

Step 2. Social welfare under the donor-priority rule with \( S = \infty \) is the same as that before the introduction of the donor-priority rule.

Thus, a finite optimal freeze period exists, which, when combined with the donor-priority rule, always leads to increased social welfare. 

**Q.E.D.**

**Proof of Proposition 9.** Consider a type \( i \) individual with the cutoff cost \( x^*_i, i \in I \). In equilibrium, the individual is indifferent between joining the donor registry and not; that is,

\[
\Theta_i \beta T^*_p - x^*_i = \Theta_i \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T^*_p \right).
\]

The above equation can be rewritten as

\[
x^*_i = \Theta_i \left( \beta T^*_p - \alpha d \right) \frac{\lambda_n - \mu_n}{\lambda_n},
\]

which can be further reorganized as (9).

**Q.E.D.**

**Proof of Corollary 10.** Recall that \( T^*_p = \frac{\int_{\Theta} \lambda_s T_s(p(x^*_i)) \, \Theta}{\int_{\Theta} \lambda_s F(x^*_i) \, \Theta} \). In addition, \( \forall i, j \in I, \Theta_i < \Theta_j \), it must be the case that \( x^*_i < x^*_j \) and, thus, \( \frac{p_i(s(x^*_i))}{p_j(s(x^*_j))} < \frac{p_i}{p_j} \), which implies \( \{p_i\} \) and \( \{\frac{p_i(s(x^*_i))}{p_j(s(x^*_j))} \} \) bear the monotone likelihood ratio property in \( T_s \). The result then follows using the monotone likelihood ratio property. 

**Q.E.D.**

**Proof of Proposition 10.** A type \( i \) individual with the cutoff cost \( c_i, i \in I \), is indifferent between joining the donor registry and not. In other words,

\[
\Theta_i \left( e^{-\theta_i S} \beta T^*_p(x^*_i) + (1 - e^{-\theta_i S}) \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T^*_p(x^*_i) \right) \right) - x^*_i = \Theta_i \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T^*_p(x^*_i) \right),
\]

which can be rearranged as

\[
x^*_i = \Theta_i e^{-\theta_i S} \left( \beta T^*_p(x^*_i) - \alpha d \right) \frac{\lambda_n - \mu_n}{\lambda_n},
\]

or, equivalently,

\[
x^*_i = \Theta_i e^{-\theta_i S} \left( \beta T^*_p(x^*_i) - \alpha d \right) \frac{\int_{\Theta} \lambda_s T_s(p(x^*_i)) \, \Theta}{\int_{\Theta} \lambda_s F(x^*_i) \, \Theta} \frac{\int_{\Theta} \lambda_s (1 - F(x^*_i) e^{-\theta_i S}) \, \Theta}{\int_{\Theta} \lambda_s (1 - F(x^*_i) e^{-\theta_i S}) \, \Theta},
\]

which completes the proof. 

**Q.E.D.**

**Proof of Corollary 12.** Notice \( T_p(x^*_i) = \frac{\int_{\Theta} \lambda_s T_s(p(x^*_i)) \, \Theta}{\int_{\Theta} \lambda_s F(x^*_i) \, \Theta} \) and \( \forall i, j \in I, \) if \( \theta_i < \theta_j, \Theta_i < \Theta_j, T_i > T_j \), then, \( \frac{\theta_i}{\theta_j} = \frac{\Theta_i}{\Theta_j} e^{-\theta_j S} = \frac{\Theta_i e^{(1-\theta_i) S}}{\Theta_j} \), which is equal to 1 if \( S = S^{eq}(i, j) \equiv \frac{ln(\frac{\theta_i}{\theta_j})}{\theta_i - \theta_j} \). Hence, when \( S = S^{sup} \equiv \sup_{i,j \in I} S^{eq}(i, j) \), \( \forall i, j \in I, \) if \( \theta_i < \theta_j, \Theta_i < \Theta_j, T_i > T_j \), then, \( \frac{\theta_i}{\theta_j} > 1 \) and, thus, \( \frac{p_i(s(x^*_i))}{p_j(s(x^*_j))} > \frac{p_i}{p_j} \), which implies \( \{\frac{p_i(s(x^*_i))}{p_j(s(x^*_j))} \} \) and \( \{p_i\} \) bear the monotone likelihood ratio property in \( T_s \). Hence, we have \( T_p(x^*_i) > T_n \) using the monotone likelihood ratio property. As shown in Corollary 10, \( T_p(x^*_i) = T^*_p > T_n \) when \( S = 0 \). Moreover, \( T_p(x^*_i) \) is continuous in \( S \), so we have that \( \exists S^{eq} \in (0, S^{sup}) \) s.t. \( T_p(x^*_i) = T_n \). 

**Q.E.D.**