

3D Printing vs. Traditional Flexible Technology: Implications for Manufacturing Strategy

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We study a firm's manufacturing strategy under two types of flexible production technologies: the traditional flexible technology and 3D printing. Under the traditional flexible technology, capacity becomes more expensive as it handles more product variants; under 3D printing, however, capacity cost is independent of the number of product variants processed. The firm adopts a dedicated technology and one type of flexible technology, either the traditional one or 3D printing. It needs to choose an assortment from a potential set of variants, assigns each chosen variant to a production technology, and finally invests in capacities. We first establish that the optimal assortment must contain a number of the most popular variants from the potential set. Based on the variants' popularity rankings, we find that the optimal technology assignment can follow an unexpected reversed structure under the traditional flexible technology, while the optimal assignment always follows an ordered structure under 3D printing. Surprisingly, we find that adopting the traditional flexible technology in addition to the dedicated one may reduce product variety chosen by the firm. 3D printing, by contrast, always enhances product variety. Furthermore, 3D printing allows the firm to choose a much larger assortment than optimal without significant profit loss. These results demonstrate that the rising 3D printing has significantly different implications for firms' assortment and production strategy than the traditional flexible technology.

Key words: 3D printing; technology management; assortment planning; manufacturing flexibility; product variety; multinomial logit model

1. Introduction

3D printing, also known as additive manufacturing, has attracted increasing media attention in recent years. The technology uses digital profiles generated by computers to create real-world

objects ranging from simple toy pieces to complex fighter jet parts. This cutting-edge technology has been moving from the research phase to day-to-day use over the past decade. In the 2013 State of the Union address, President Obama highlighted 3D printing as the innovation that could fuel new high-tech jobs in the United States (Gross, 2013). In fact, 3D printing has already been extensively adopted by many industries, including apparel, toy, construction, medical devices, and even human organs (Griggs, 2014). According to a PwC survey of over 100 manufacturing firms, 11% had switched to volume production with 3D printing (Earls and Baya, 2014). Meanwhile, reports on new breakthroughs in 3D printing have frequently made headlines. As reported by a recent article published in *Science*, researchers are proposing a new approach to 3D printing (Tumbleston et al., 2015). By playing with a trick of chemistry, they “have sped up, and smoothed, the process of three-dimensional (3D) printing, producing objects in minutes instead of hours” (Castelvecchi, 2015). Such new developments promise to widen and speed up the application of 3D printing in industry.

3D printing has several key advantages compared to traditional manufacturing technologies. First, 3D printing is greener or more sustainable. It applies the so-called “additive” process instead of a traditional “subtractive” process, which leads to much less material waste. Second, only a computer-aided design (CAD) file is required to prototype a 3D-printed product. Hence product design is much faster and cheaper with 3D printing. Third, 3D printers are able to build almost any geometric structure, so this technology can potentially drive more innovation and provide more freedom in product design (*The Economist*, 2011).

Moreover, from an operations management perspective, another crucial advantage is that 3D printing brings tremendous capacity flexibility to firms, which allows them to better match supply with demand. The notion of flexible capacity is not new. Flexible manufacturing systems have been utilized in industry for decades, and there is extensive literature on the value of manufacturing flexibility (see, e.g., Beach et al., 2000; Van Mieghem, 2003). However, the flexibility from the traditional flexible manufacturing systems and the flexibility gained through 3D printing are quite

different in nature. Traditionally, a flexible manufacturing system is usually designed to produce a specific set of product variants. In order to be more flexible, the system requires more versatile machines, more complex operating systems, and a better trained workforce. As a consequence, a higher degree of flexibility entails a higher capacity cost. Thus, flexibility gained through traditional technologies is limited from the perspectives of design and cost. By contrast, 3D printing is naturally flexible and is not designed for any specific set of product variants. A 3D printer is able to manufacture any standardized product variant in a certain category, provided the availability of the corresponding CAD file. Unit capacity cost, including costs of 3D printers and 3D printing materials, remains constant regardless of how many product variants are manufactured.

It is widely believed that 3D printing will revolutionize the manufacturing industry. In particular, the advantages brought by 3D printing may change firms' operations strategy, including decisions on product design, assortment, production scheduling, and capacity investment. With the rapid development of 3D printing, a natural question arises: How does such an innovative technology affect the above operations decisions, especially compared to the traditional flexible technology? The main purpose of this paper is to shed some light on this question, which, to our knowledge, has not been formally explored in the literature.

We consider a typical situation where a firm sells horizontally differentiated product variants in a market with uncertain demand. The firm adopts two types of production technologies to manufacture the product variants: dedicated and flexible. A resource based on the dedicated technology can produce only one product variant, whereas a resource based on the flexible technology can handle multiple product variants. Adopting the flexible technology in addition to the dedicated technology is a common practice (Koren, 2010; Ross et al., 2016) because the flexible technology, though expensive, complements the dedicated technology by helping the firm deal with demand fluctuations in the market. To investigate the implications of 3D printing, we consider two scenarios where the flexible technology can be either 3D printing or the traditional flexible technology. Within this problem setting, answers to the following three questions help us understand flexible

technologies' implications for manufacturing strategy: First, for a given assortment, how should the firm assign product variants to the dedicated and flexible technologies? Second, what is the structure of the optimal assortment? Finally, does the adoption of the flexible technology always increase product variety? The main findings are summarized as follows.

First, for a given assortment, the optimal technology assignment largely depends on the type of flexible technology adopted. One may intuit that product variants with higher demand variability should be assigned to the flexible technology because it can better deal with demand uncertainty through pooling of capacity. However, we show that with the traditional flexible technology, this intuition is not always true. In fact, the optimal assignment may present a *reversed* structure where the variants with more demand variability are assigned to the dedicated technology and the ones with less variability are assigned to the flexible technology. Additionally, a more uncertain market may lead to fewer variants assigned to the flexible technology. By contrast, if the flexible technology is 3D printing, the optimal technology assignment always follows the *ordered* structure, consistent with the intuition that variants with higher demand variability (normally associated with lower volume) are assigned to 3D printing. This finding corroborates industry observations that 3D printing is often used to produce low-volume product variants (Miller, 2013; BCTIM and MSCI, 2015). Moreover, as market uncertainty increases, the number of variants assigned to the flexible technology also increases due to the ordered structure.

Second, in both scenarios, the optimal assortment should consist of the most popular variants. This finding is consistent with the existing literature (e.g., van Ryzin and Mahajan, 1999; Cachon et al., 2005; Cachon and Kök, 2007). However, existing studies focus on retail assortment planning only and do not consider the technology assignment decision. We show that the optimality structure carries over to a more general setting where multiple types of production technologies exist and a manufacturer needs to make both assortment and technology-assignment decisions. By producing the most popular variants, the firm is able to gain sufficient market coverage and meanwhile mitigate supply-demand mismatch cost under all types of technology portfolios.

Finally, we study the impact of flexible technologies on the firm's choice of product variety, which is measured by the optimal assortment size. To this end, we compare the two-technology scenarios with a benchmark case where the firm only adopts the dedicated technology. Interestingly, we find that adoption of the flexible technology does not necessarily increase product variety. Specifically, when adopting the traditional flexible technology (in addition to the dedicated technology), the optimal assortment size for the firm may decrease. This is because the firm may wish to reduce cannibalization and thus centralize demand for the flexible resource by offering less variety when it requires a significantly higher unit capacity cost for the flexible resource to handle more variants. However, adopting 3D printing will always increase product variety. The constancy of unit capacity cost ensures stable performance of the 3D printing resource. As the assortment expands, the pooling effect enables the firm to gain increasingly more benefit with the adoption of 3D printing than in the benchmark case. Moreover, numerical analysis shows that, with 3D printing, the firm's profit remains near-optimal even if the firm chooses a much larger assortment than the optimal one. This implies that 3D printing allows the firm to expand assortment aggressively to gain more market coverage and promote brand image, without losing much on profit.

The above results suggest that the type of flexible technology adopted has important implications for firms' operations decisions such as assortment, capacity investment, and production planning. Managers need to understand the difference when adopting different flexible technologies. They should exercise caution with the traditional flexible technology because it may lead to counterintuitive assortment (i.e., the optimal assortment size may decrease after adopting the flexible technology) and technology assignment decisions (i.e., it may be optimal to assign variants with more variable demand to the dedicated technology and vice versa). In contrast, flexibility gained through 3D printing gives rise to more intuitive decisions on assortment and technology assignment. It allows the firm to expand the product portfolio beyond the optimal level without losing much profit. Given that product variety has become increasingly important to competing in today's marketplace, 3D printing is far more appealing than the traditional flexible technology in delivering values both to firms and to consumers.

The remainder of the paper is organized as follows: In §2, we review the related literature. In §3, we present the model and initial analysis. §4 and §5 characterize the firm’s optimal assortment and technology assignment decisions for the scenarios with the traditional flexible technology and with 3D printing, respectively. We study the impact of flexible technologies on product variety in §6. The paper concludes with §7.

2. Literature Review

This paper is mainly related to two areas of research in the literature: assortment planning and flexible manufacturing technology. In this section, we review the most related studies from each area.

Assortment planning has been extensively studied in the literature. Kök et al. (2009) provide a comprehensive review of the early studies. The most recent development on this topic can be found in Alptekinoglu and Semple (2016) and Wang and Wang (2016). Van Ryzin and Mahajan (1999) are among the first to study assortment planning with demand uncertainty and inventory considerations. By using a multinomial logit (MNL) model, they show that the optimal assortment must consist of the most popular variants, i.e., it must follow a threshold structure such that a variant should be included in the assortment if and only if its popularity ranking is above a threshold. This “most popular” property has been checked in other assortment studies under various demand environments. Cachon et al. (2005), in a study of consumer search behavior’s impact on the optimal assortment, find that the “most popular” property continues to hold under independent consumer search, but it may no longer hold under overlapping consumer search. Gaur and Honhon (2006) replace the MNL model with a location model and show that the most popular variants may not be included in the optimal assortment. Cachon and Kök (2007) prove the “most popular” property for the optimal assortment in a duopoly setting. All these studies consider assortment planning from a retailer’s perspective. Our paper is also concerned about assortment planning, but we study the problem from a manufacturer’s perspective. In addition to assortment, the firm also needs to

decide the means of manufacturing (e.g., capacity investment for different technologies and assignment of variants to technologies). Casting product assortment and technology assignment decisions as *set division* problems, we show that the optimal assortment must be ordered (i.e., the “most popular” property continues to hold). However, with respect to technology assignment, the optimal policy is not necessarily ordered. In particular, we find that 3D printing and the traditional flexible technology have different implications for the firm’s technology assignment.

There is a large body of literature on the flexible technology. Fine and Freund (1990) and Van Mieghem (1998) study the capacity investment problem under the flexible technology. Their studies are followed by Bish and Wang (2004), Chod and Rudi (2005), Chod et al. (2010), Boyabatlı and Toktay (2011), Chod and Zhou (2014), and Boyabatlı et al. (2015), where a various of factors including pricing, secondary market, and external financing are considered in addition to the capacity investment problem. Jordan and Graves (1995) study the design of flexible network and discover that the performance of a simple “long chain” structure can be close to that of full flexibility. Bassamboo et al. (2010) follow their work and characterize the optimal flexibility configuration in newsvendor networks. They assume that the unit capacity cost of a flexible resource increases linearly in the number of product variants handled, whereas we consider general increasing unit capacity cost under the traditional flexible technology. Röller and Tombak (1993), Goyal and Netessine (2007), and Alptekinoglu and Corbett (2008) study the flexible technology in the presence of competition, and investigate issues including market differentiation, demand substitution, and product variety. In Alptekinoglu and Corbett (2008), one mass-customization firm adopts the *perfectly flexible technology*, which is similar to 3D printing except that 3D printing incurs a per-variant fixed cost because each variant needs to be individually prototyped (e.g., the CAD file required for 3D printing); in their paper, there is no per-variant fixed cost because they assume product variants are custom-made and thus the custom design effort can be included in the variable cost. We extend the flexible-technology literature by endogenizing the product assortment, which is set to be exogenous in most of the existing studies. In addition, we focus on comparing two types

of flexible technologies, the traditional flexible technology and 3D printing, whereas only one type of flexible technology is usually considered in the literature.

3. Model

We present the model setting in this section. Section 3.1 describes the problem under study; Section 3.2 formulates the problem and provides some initial analysis.

3.1. Problem Description

We consider a single firm selling horizontally differentiated product variants in the market. The firm faces two major decisions: first, which product variants to include in the assortment it offers to the market; second, how to manufacture these product variants using different technologies. The detailed description of the problem is given in three parts: market structure, technologies, and sequence of events.

Market Structure Let $\mathbb{U} = \{1, 2, \dots, |\mathbb{U}|\}$ be a set of potential product variants from which the firm chooses its product assortment, where $|\cdot|$ stands for the cardinality of a set (the number of elements in the set). Let $\mathbb{S} \subseteq \mathbb{U}$ denote the assortment chosen by the firm, i.e., the set of product variants the firm decides to offer in the market. Since all variants are horizontally differentiated, we assume that they have an identical selling price, p , which is exogenously determined by market competition.

We adopt the multinomial logit (MNL) paradigm to model consumer choice. Consumers are infinitesimal and have different valuations for product variants in \mathbb{S} . Consumer j 's utility from purchasing variant i is given by $U_{ij} = V_{ij} - p = V - p + \xi_i + \epsilon_{ij}$, where V represents the common value delivered by all product variants, ξ_i is the mean value of variant i to all consumers, and ϵ_{ij} represents product i 's specific value to consumer j , following Gumbel distribution with mean 0 and variance $\pi^2/6$ across the consumer population. The outside choice has an index of 0, and yields a utility of $U_0 = 0$. We assume that all variants are equally dissimilar, which is consistent with the independence of irrelevant alternatives (IIA) property of MNL model (a similar assumption can

be found in Dong et al. (2009)). Hence, the probability that a consumer chooses variant i (or the market share of variant i) is given by $s_i = \frac{v_i}{1 + \sum_{j \in \mathbb{S}} v_j}$, where $v_i := e^{V-p+\xi_i}$ represents the *popularity* of variant i , i.e., the relative magnitude of how different variants are liked by consumers. Following the literature (van Ryzin and Mahajan, 1999; Chong et al., 2001; Cachon et al., 2005), we assume that a consumer will not make any purchase if her first choice is out of stock. This assumption is appropriate in situations where consumers have strong preferences for certain variants (e.g., toy action figures), so they are reluctant to accept the second choice when the first choice is not available.

Consumer arrivals follow a Poisson process with arrival rate λ during a selling season of length L . Upon arrival, each consumer decides her favorite variant based on the probabilities derived from the MNL model and purchases I units of the variant, where I is a random value independently drawn from a given distribution. By decomposition, the demand generating process for variant i is a compound Poisson process with arrival rate λs_i , independent of the demand generating processes of other variants in \mathbb{S} . It follows that the aggregated demand of variant i , Y_i , is a random variable following compound Poisson distribution with mean $\lambda s_i LE[I]$ and standard deviation $\sqrt{\lambda s_i LE[I^2]}$, where $E[\cdot]$ is the expectation operator. Note that demands of different variants are independently distributed. For a large population of consumers, the aggregated demands of all variants can be approximated by normal distributions. We therefore assume $Y_i \sim N(\Lambda s_i, \sigma \sqrt{s_i})$, where $\Lambda = \lambda LE[I]$ measures the market size and $\sigma = \sqrt{\lambda LE[I^2]}$ measures the market uncertainty. Similar demand models are commonly used in the assortment planning literature (see van Ryzin and Mahajan, 1999; Gaur and Honhon, 2006). Note that the standard deviation of variant i 's demand, $\sigma \sqrt{s_i}$, increases more slowly as s_i increases. It reflects the fact that firms often have better forecasting accuracy on high-volume variants that have higher mean demands. In other words, it means that a variant's *demand variability*, measured by the coefficient of variation $\frac{\sigma}{\Lambda \sqrt{s_i}}$, decreases as the mean increases.

Technologies Once the firm has chosen the product variants to offer, it needs to decide the production technologies for these variants and invest in the corresponding resources. We consider

three types of technologies in this paper: the dedicated technology, the traditional flexible technology, and 3D printing. Although all three technologies deliver the same product quality, their cost structures, which consist of the fixed cost and the variable cost, are technology specific. The fixed cost of a resource includes costs for designing, prototyping, and setting up the production line. The variable cost is capacity dependent, and includes costs for materials, machines, toolings, and labor. We describe the cost structures of all three technologies below.

Under the dedicated technology, a resource is specialized to produce only one variant. The fixed cost K_D for the dedicated resource is a constant. Since it produces only one variant, the unit capacity cost c_D is also a constant. Consequently, for a dedicated resource with capacity level x , the total cost will be $K_D + c_D x$.

Under the traditional flexible technology, a resource is able to produce multiple variants. We measure the degree of flexibility by the number of variants that can be produced by the resource and refer to a resource that can handle n variants as *n-flexible*. Such a measurement is consistent with the notion of “mixed flexibility” defined in Suarez et al. (1995) since we assume that all variants are equally dissimilar. Because a more flexible resource often requires a higher initial investment such as designing more complex systems, we assume the fixed cost, $K_T(n)$, is (weakly) increasing in n with $K_T(0) = 0$. The unit capacity cost of an n -flexible resource, $c_T(n)$, is a strictly increasing function with $c_T(2) > c_D$. This reflects the fact that, for the traditional flexible technology, each unit of capacity becomes more expensive if the resource is more flexible: A resource capacity capable of producing more varieties would be more expensive (e.g., it may require more versatile machines and a better trained workforce). Consequently, for an n -flexible resource with capacity level x , the total cost will be $K_T(n) + c_T(n)x$.

3D printing represents a new type of flexible technology, whose cost structure is different from that of the traditional flexible technology. The fixed cost $K_P(n)$ of 3D printing is (weakly) increasing in n with $K_P(0) = 0$, because more varieties lead to more prototyping effort. However, because prototyping with 3D printing only requires a computer-aided design (CAD) file for each variant, the

marginal cost of prototyping decreases in the number of variants and we therefore assume $K_P(n)$ is concave in n . In addition, we also assume $K_P(n+1) - K_P(n) \leq K_D$ for any $n \in \mathbb{N}^+$, i.e., prototyping one more variant under 3D printing costs no more than investing in one more dedicated resource. Given the nature of 3D printing, the capacity-related cost should not depend on its degree of flexibility (e.g., the unit costs for 3D printers, 3D printing materials, and workforce do not change in the number of variants it handles)¹. Thus, we model the unit capacity cost $c_P(n) = c_P$ as a constant function independent of n . Nevertheless, we assume $c_P > c_D$, which captures two features of 3D printing: First, 3D printers and 3D printing materials are more expensive than those of the dedicated technology; second, the production rate of 3D printing is relatively slow (i.e., it usually takes a longer time to 3D-print a product), which, equivalently, may translate into a higher unit capacity cost. Consequently, for a 3D-printing resource handling n variants with capacity level x , the total cost will be $K_P(n) + c_P(n)x$.

For simplicity, we normalize the production costs to zero under all technologies. The qualitative results will remain unchanged under positive production costs. Note that the three technologies will give rise to a total of seven possible technology combinations (e.g., the firm may adopt one, two, or all three of the technologies). We are interested in the implications of different types of flexible technologies for manufacturing strategy, thus we focus on the comparison of two cases: Case *DT*, where both the dedicated and the traditional flexible technologies are used; and Case *DP*, where both the dedicated technology and 3D printing are used. To examine how the flexible technologies affect the product variety decisions, later we will also compare a benchmark case in which only the dedicated technology is adopted, Case *D*, with Case *DT* and Case *DP* respectively.

Sequence of Events Now we introduce the sequence of events, which applies to both Case *DT* and Case *DP*.

¹ The capacity-related cost may depend on the variety of materials it has to handle (e.g., handling both plastic and metal will be more expensive than handling just plastic). In this paper, we consider horizontally differentiated product variants and thus they should have the same material requirements.

Recall $\mathbb{U} = \{1, 2, \dots, |\mathbb{U}|\}$ is the set of all potential variants that can be possibly produced. Without loss of generality, we assume the popularities of variants in \mathbb{U} follows the ranking of $v_1 \geq v_2 \geq \dots \geq v_{|\mathbb{U}|}$. From market research, the firm has perfect information of the values $(v_1, \dots, v_{|\mathbb{U}|})$. The firm's first decision is to select the product assortment, $\mathbb{S} \subseteq \mathbb{U}$. Let $\mathbb{W} = \mathbb{U} \setminus \mathbb{S}$ be the set of variants not chosen by the firm. For example, \mathbb{U} can be the complete set of characters in the "Transformers" movie series, and the firm may produce toys based on the chosen characters in \mathbb{S} . Note that v_i represents the popularity of variant i in the consumer population and is independent of the firm's assortment decision.

After \mathbb{S} is chosen, the firm assigns each variant in \mathbb{S} to available technologies for production. Let \mathbb{D} denote the set of variants in \mathbb{S} assigned to the dedicated technology, and \mathbb{F} denote the set of variants in \mathbb{S} assigned to the flexible technology (that is, the traditional flexible technology for Case *DT*, 3D printing for Case *DP*). The pair of sets (\mathbb{D}, \mathbb{F}) is obtained by dividing \mathbb{S} into two disjoint subsets, to which we refer as the firm's technology assignment. For clarity of the analysis, we assume that the firm can invest in multiple dedicated resources but only one flexible resource in the base model. The main results of the paper remain valid when the firm considers investing in multiple flexible resources, each handling its own set of variants. The fact that \mathbb{D} and \mathbb{F} are disjoint implies that the firm cannot assign a variant to more than one technology. We provide two justifications for such a non-overlapping assumption. First, producing the same variant using both technologies will incur unnecessary additional fixed costs (i.e., it incurs an additional K_D and meanwhile increases $K_T(\cdot)$ or $K_P(\cdot)$). Second, with a large number of variants, a firm may want to reduce management complexity by avoiding duplicated assignment.

After determining the technology assignment, the firm invests in production resources accordingly. For Case *DT*, the firm invests in capacities for both the dedicated and the traditional flexible resources; for Case *DP*, the firm invests in capacities for both the dedicated and 3D-printing resources. We let x_{Di} represent the capacity dedicated to producing variant $i \in \mathbb{D}$, and x_F represent the flexible resource's capacity ($F = T, P$ depending on the technology).

Finally, demands of variants, Y_i ($i \in \mathbb{S}$), are realized, and the firm satisfies market demand by producing variants in \mathbb{S} using the available capacities of assigned technologies. Given the long lead time, the firm is not allowed to add capacity after observing the demand realization.

3.2. Problem Formulation

The firm's decisions take place in three stages. In the first stage, the firm selects the assortment \mathbb{S} from \mathbb{U} ; in the second stage, the firm decides the technology assignment, (\mathbb{D}, \mathbb{F}) ; in the third stage, the firm makes the capacity decisions for different resources. Solving for the optimal decisions by backward induction, we start with the third-stage capacity investment decision in the third stage. For a given assignment (\mathbb{D}, \mathbb{F}) under Case DF ($F = T, P$), the firm's expected profit can be written as

$$\begin{aligned} \pi_{DF}(\{x_{Di}\}_{i \in \mathbb{D}}, x_F) = & E \left[p \left(\sum_{i \in \mathbb{D}} \min\{Y_i, x_{Di}\} + \min\left\{ \sum_{j \in \mathbb{F}} Y_j, x_F \right\} \right) \right] \\ & - c_D \sum_{i \in \mathbb{D}} x_{Di} - c_F(|\mathbb{F}|)x_F - K_D \cdot |\mathbb{D}| - K_F(|\mathbb{F}|), \quad F = T, P, \end{aligned} \quad (1)$$

which is the expected revenue from selling variants in \mathbb{S} net the variable and fixed costs of capacity investment. By maximizing the expected profit we obtain the following optimal capacity levels:

$$x_{Di}^* = \Lambda s_i + z_D \sigma \sqrt{s_i}, \quad \forall i \in \mathbb{D}, \quad (2)$$

$$x_F^* = \Lambda \sum_{j \in \mathbb{F}} s_j + z_F(|\mathbb{F}|) \sigma \sqrt{\sum_{j \in \mathbb{F}} s_j}. \quad (3)$$

Throughout this paper, let $\Phi(\cdot)$ and $\phi(\cdot)$ respectively denote the c.d.f. and the p.d.f. of a standard normal distribution, and thus, in (2) and (3), $z_D = \Phi^{-1}(1 - \frac{c_D}{p})$ and $z_F(|\mathbb{F}|) = \Phi^{-1}(1 - \frac{c_F(|\mathbb{F}|)}{p})$ are the safety factors. Substituting x_{Di}^* and x_F^* back into the profit function (1), we obtain the profit function under the optimal capacity investment:

$$\begin{aligned} \pi_{DF}(\{x_{Di}^*\}_{i \in \mathbb{D}}, x_F^*) = & \Lambda \left[(p - c_D) \sum_{i \in \mathbb{D}} s_i + (p - c_F(|\mathbb{F}|)) \sum_{j \in \mathbb{F}} s_j \right] - \sigma p \left[\phi(z_D) \sum_{i \in \mathbb{D}} \sqrt{s_i} \right. \\ & \left. + \phi(z_F(|\mathbb{F}|)) \sqrt{\sum_{j \in \mathbb{F}} s_j} \right] - K_D \cdot |\mathbb{D}| - K_F(|\mathbb{F}|). \end{aligned} \quad (4)$$

Scaling the optimal profit function (4) by a constant $\frac{1}{\Lambda}$, we obtain a normalized profit as a function of the technology assignment (\mathbb{D}, \mathbb{F}) :

$$\Pi_{DF}(\mathbb{D}, \mathbb{F}) = \frac{1}{\Lambda} \pi_{DF}(\{x_{Di}^*\}_{i \in \mathbb{D}}, x_F^*) = \mathcal{P}_{DF}(\mathbb{D}, \mathbb{F}) - \mathcal{M}_{DF}(\mathbb{D}, \mathbb{F}) - \mathcal{K}_{DF}(\mathbb{D}, \mathbb{F}), \quad (5)$$

where

$$\mathcal{P}_{DF}(\mathbb{D}, \mathbb{F}) = (p - c_D) \sum_{i \in \mathbb{D}} s_i + (p - c_F(|\mathbb{F}|)) \sum_{j \in \mathbb{F}} s_j, \quad (6)$$

$$\mathcal{M}_{DF}(\mathbb{D}, \mathbb{F}) = \sigma \left[m_D \sum_{i \in \mathbb{D}} \sqrt{s_i} + m_F(|\mathbb{F}|) \sqrt{\sum_{j \in \mathbb{F}} s_j} \right], \quad (7)$$

and

$$\mathcal{K}_{DF}(\mathbb{D}, \mathbb{F}) = \frac{1}{\Lambda} \left[K_D \cdot |\mathbb{D}| + K_F(|\mathbb{F}|) \right], \quad (8)$$

where in the expression of $\mathcal{M}_{DF}(\mathbb{D}, \mathbb{F})$, $m_D = \frac{p\phi(z_D)}{\Lambda}$ and $m_F(|\mathbb{F}|) = \frac{p\phi(z_F(|\mathbb{F}|))}{\Lambda}$. $\Pi_{DF}(\mathbb{D}, \mathbb{F})$ consists of three parts: the gross profit $\mathcal{P}_{DF}(\mathbb{D}, \mathbb{F})$, the supply-demand mismatch cost due to the demand uncertainty of variants in the assortment $\mathcal{M}_{DF}(\mathbb{D}, \mathbb{F})$, and the total fixed cost $\mathcal{K}_{DF}(\mathbb{D}, \mathbb{F})$. Then, given \mathbb{S} , the firm's second-stage problem of finding the optimal technology assignment can be written as²

$$\Omega_{DF}(\mathbb{S}) := \max_{\mathbb{D} \cap \mathbb{F} = \emptyset, \mathbb{D} \cup \mathbb{F} = \mathbb{S}} \Pi_{DF}(\mathbb{D}, \mathbb{F}), \quad F = T, P. \quad (9)$$

Two observations on $\Pi_{DF}(\mathbb{D}, \mathbb{F})$ help us understand the trade-off involved in the technology assignment decision:

(i) For a given technology assignment (\mathbb{D}, \mathbb{F}) , moving one variant from \mathbb{D} to \mathbb{F} decreases the gross profit, $\mathcal{P}_{DF}(\mathbb{D}, \mathbb{F})$, because the dedicated technology has a lower unit capacity cost and the unit capacity cost of the traditional flexible technology increases in the number of variants handled. It implies that, in the absence of demand uncertainty and fixed cost, the flexible technology has no value to the firm and the optimal \mathbb{F} would be empty.

²Note that there might be multiple optimal assignments given an assortment \mathbb{S} . All the results will hold regardless of the optimal technology assignment chosen.

(ii) $\mathcal{M}_{DF}(\mathbb{D}, \mathbb{F})$, the supply-demand mismatch cost, consists of the mismatch cost associated with each individual variant in \mathbb{D} , $m_D \sigma \sqrt{s_i}$, and the “pooled” mismatch cost associated with the collective variants in \mathbb{F} , $m_F(|\mathbb{F}|) \sigma \sqrt{\sum_{j \in \mathbb{F}} s_j}$, where m_D and $m_F(|\mathbb{F}|)$ are the corresponding mismatch-cost coefficients. Note that the mismatch cost associated with each variant in \mathbb{D} is proportional to the standard deviation of its demand, and recall that the standard deviation increases more slowly as s_i increases. Consequently, economies of scale of mean demand applies to the mismatch cost: As mean demand increases, the mismatch cost associated with one variant in \mathbb{D} increases more slowly. Similar observations and arguments can be found in Cachon et al. (2005) and Gaur and Honhon (2006). The economies of scale of mean demand also applies to the flexible technology: As mean demand of any variant in \mathbb{F} increases, the total mismatch cost associated with the collective variants in \mathbb{F} increases more slowly. Moreover, the flexible technology also enjoys “statistical” economies of scale (Eppen, 1979), also known as the pooling effect: as the number of variants in \mathbb{F} increases, the standard deviation of the aggregated demand increase more slowly. This is the major advantage of the flexible technology over the dedicated technology, and the firm can utilize it to reduce $\mathcal{M}_{DF}(\mathbb{D}, \mathbb{F})$ ³.

The firm’s first-stage decision of the optimal assortment can be written as⁴

$$\max_{\mathbb{S} \subseteq \mathbb{U}} \Omega_{DF}(\mathbb{S}), F = T, P. \quad (10)$$

By examining the demand structure determined by the assortment, we make one additional observation, which helps us understand the trade-off involved in the assortment decision.

(iii) Adding a variant to \mathbb{S} requires the firm to revisit the technology assignment decision—variants originally in \mathbb{D} may be assigned to \mathbb{F} , or vice versa. Recall that $s_i = \frac{v_i}{1 + \sum_{j \in \mathbb{S}} v_j}$. When

³ The difference of the two types of economies of scale is that, the economies of scale of mean demand is connected to the fact that demand variability decreases in mean demand, whereas the “statistical” economies of scale associated with pooling exists regardless of whether individual variant’s demand variability decreases or increases in its mean demand.

⁴ There might be multiple optimal assortments for a given \mathbb{U} . Again, all the results will hold regardless of the optimal assortment chosen.

As \mathbb{S} expands, the aggregated mean demand, $\frac{\Lambda(\sum_{j \in \mathbb{S}} v_j)}{1 + \sum_{j \in \mathbb{S}} v_j}$, increases, whereas individual variant's mean demand, $\frac{\Lambda v_i}{1 + \sum_{j \in \mathbb{S}} v_j}$, decreases. Consequently, the variability (coefficient of variation) of each variant in the assortment, $\frac{\sigma}{\Lambda} \sqrt{\frac{1 + \sum_{j \in \mathbb{S}} v_j}{v_i}}$, increases. Hence, the basic trade-off in assortment expansion is between the higher aggregated mean demand (positive effect) and the more variable individual demands (negative effect).

4. Strategy Under the Traditional Flexible Technology

As equations (2) and (3) characterize the third-stage optimal capacity decisions, in this section, we characterize the firm's optimal assortment and assignment decisions when both the dedicated technology and the traditional flexible technology are adopted. Following the backward induction approach, we first characterize the optimal technology assignment decision given any assortment \mathbb{S} , and then characterize the optimal assortment decision.

4.1. Technology Assignment Decision

Assigning each variant in \mathbb{S} to either of the two production technologies is equivalent to a set division, i.e., dividing \mathbb{S} into two disjoint subsets, \mathbb{D} and \mathbb{F} . Finding the optimal technology assignment of a given assortment \mathbb{S} that maximizes the firm's expected profit in general is an NP-hard problem. The following Proposition 1 presents an important property that can simplify the search for the optimal division. Throughout this paper, we let $\min \emptyset =: +\infty$ and $\max \emptyset =: 0$ for expositional simplicity.

PROPOSITION 1. *For Case DT, let $(\mathbb{D}^*, \mathbb{F}^*)$ be the optimal technology assignment for a given assortment \mathbb{S} . For each variant i in \mathbb{D}^* , either*

- (i) *it is (weakly) less popular than all variants in \mathbb{F}^* , i.e., $v_i \leq \min\{v_j : j \in \mathbb{F}^*\}$; or*
- (ii) *it is (weakly) more popular than all variants in \mathbb{F}^* , i.e., $v_i \geq \max\{v_j : j \in \mathbb{F}^*\}$.*

Proposition 1 implies that, under the optimal technology assignment, \mathbb{F} comprises a *clustering* set of variants with adjacent popularities. To understand the rationale behind Proposition 1, let us consider a simple but representative example of $\mathbb{S} = \{1, 2, 3\}$ with $v_1 > v_2 > v_3$. We shall argue that

the assignment $(\{2\}, \{1, 3\})$, which is the only one that violates the clustering structure, must be dominated by another assignment, which, in this setting, can be either $(\{1\}, \{2, 3\})$ or $(\{3\}, \{1, 2\})$. To see the reason, we compare the profits of the three assignments, whose expressions are given by equations (5)-(8). Because $|\mathbb{F}| = 2$ in all three assignments, the flexible-technology related cost parameters $c_T(|\mathbb{F}|)$, $m_T(|\mathbb{F}|)$, and $\mathcal{K}_T = \frac{1}{\lambda}(K_D \cdot (|\mathbb{S}| - |\mathbb{F}|) + K_T(|\mathbb{F}|))$ are the same. Thus, it suffices to focus on their popularities, or, mean demands. Specifically, let us consider how the gross profit \mathcal{P}_{DT} and mismatch cost \mathcal{M}_{DT} change as the aggregated mean demand handled by the flexible resource, denoted as s_A , increases. Among the three assignments, assignments $(\{3\}, \{1, 2\})$ and $(\{1\}, \{2, 3\})$ have the highest and the lowest s_A respectively, with assignment $(\{2\}, \{1, 3\})$ falling in between. As s_A increases, \mathcal{P}_{DT} decreases linearly, the mismatch cost associated with the dedicated resources decrease more quickly, and the mismatch cost associated with the flexible resource increases more slowly. Recall $\Pi_{DT} = \mathcal{P}_{DT} - \mathcal{M}_{DT} - \mathcal{K}_{DT}$, it follows that the overall profit is convex in s_A . Consequently, to maximize Π_{DT} , s_A should take either the largest possible value, which corresponds to assignment $(\{3\}, \{1, 2\})$, or the smallest value, which corresponds to assignment $(\{1\}, \{2, 3\})$. Assignment $(\{2\}, \{1, 3\})$ is never a candidate for optimality. Intuitively, the assignment inducing moderately high \mathcal{P}_{DT} and moderately low \mathcal{M}_{DT} is not efficient due to the lack of scale economies. The firm would be better off by choosing either significantly high \mathcal{P}_{DT} or significantly low \mathcal{M}_{DT} . In general, for any three variants $i_1, i_2, i_3 \in \mathbb{S}$ with $v_{i_1} > v_{i_2} > v_{i_3}$, assigning i_2 to the dedicated technology and i_1, i_3 to the flexible technology is always dominated by some other assignment with the same size of \mathbb{F} , with either i_1, i_2 assigned to \mathbb{F} and i_3 assigned to \mathbb{D} or i_2, i_3 assigned to \mathbb{F} and i_1 assigned to \mathbb{D} . Such a micro structural property leads to the global clustering structure of the optimal technology assignment.

Depending on how the variants are assigned to \mathbb{D} and \mathbb{F} , all possible clustering structures can be summarized into three categories:

- (i) The ordered structure, in which all variants in \mathbb{D} are (weakly) more popular than all variants in \mathbb{F} ($\mathbb{D} = \mathbb{S}$ and $\mathbb{F} = \mathbb{S}$ can be considered as two trivial cases of the ordered structure);

(ii) The reversed structure, in which all variants in \mathbb{D} are (weakly) less popular than all variants in \mathbb{F} ;

(iii) The sandwiched structure, in which some variants in \mathbb{D} are (weakly) more popular than all variants in \mathbb{F} while other variants in \mathbb{D} are (weakly) less popular than all variants in \mathbb{F} .

Intuition may suggest that it is optimal to produce high-volume variants using the dedicated technology so as to take advantage of its low unit capacity cost, and to produce high-variability variants using the flexible technology so as to control the supply-demand mismatch through risk pooling. The ordered structure, which assigns the more popular variants to the dedicated technology, is consistent with this intuition. The sandwiched structure, which assigns medium popular variants to the flexible technology, and the reversed structure, which assigns the more popular variants to the flexible technology, are less intuitive. However, we find that all three structures are possible in the optimal assignment. In order to further understand the driving force behind these structures, we first derive one key result below.

PROPOSITION 2. *For Case DT, consider two market uncertainty levels $\sigma_1 < \sigma_2$. Given any assortment \mathbb{S} , let $(\mathbb{D}_i^*, \mathbb{F}_i^*)$ ($i = 1, 2$) be the optimal technology assignment associated with $\sigma = \sigma_i$. If either*

(A) $c_D + c_F(2) \geq p$, or

(B) $K_T(n+1) - K_T(n) \geq K_D$ for $\forall n \in \{2, 3, \dots\}$ and $K_T(2) \geq 2K_D$,

then one of the following must hold:

(i) $|\mathbb{F}_2^*| > |\mathbb{F}_1^*|$;

(ii) $|\mathbb{F}_2^*| \leq |\mathbb{F}_1^*|$, and $\max\{v_i : i \in \mathbb{F}_2^*\} \geq \max\{v_i : i \in \mathbb{F}_1^*\}$.

Proposition 2 characterizes the evolution of the optimal technology assignment when the market uncertainty increases. Conditions (A) and (B) eliminate some extreme and uninteresting cases⁵.

⁵In some cases, the traditional flexible resource is associated with a higher mismatch-cost coefficient ($m_T(\mathbb{F})$) but a much lower fixed cost than the dedicated technology. Thus, assigning variants to the flexible technology results in lower gross profit and higher mismatch cost given certain assortments, but the firm may still assign variants to the

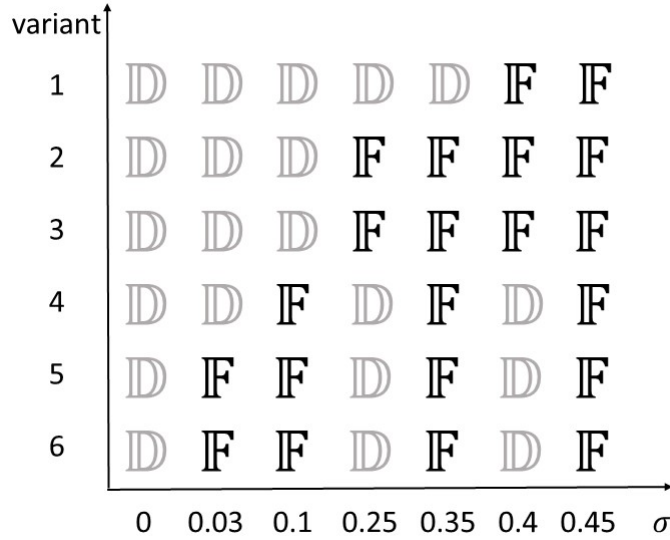
The proposition implies that, as the market becomes more uncertain, the firm has two options to further reduce the mismatch cost: (i) to assign more variants to the flexible technology (i.e., $|\mathbb{F}_2^*| > |\mathbb{F}_1^*|$); or (ii) to decrease or maintain the number of variants handled by the flexible technology (i.e., $|\mathbb{F}_2^*| \leq |\mathbb{F}_1^*|$), but increase the popularity of the most popular variant handled by the flexible technology (i.e., $\max\{v_i : i \in \mathbb{F}_2^*\} \geq \max\{v_i : i \in \mathbb{F}_1^*\}$). In fact, the firm's option (ii) is to generally assign variants with higher popularities to the flexible technology: By the clustering structure, the most popular variant in \mathbb{F}_2^* 's being more popular than the most popular variant in \mathbb{F}_1^* implies that the j th popular variant in \mathbb{F}_2^* is more popular than the j th popular variant in \mathbb{F}_1^* . Both options increase the demand volume handled by the flexible resource and thus counter the higher market uncertainty. Option (i) has a disadvantage of inducing higher unit capacity cost, and thus the firm may choose option (ii). Next, we use an example to illustrate how option (ii) may lead to a sandwiched or reversed structure.

Consider a setting in which $\mathbb{S} = \{1, 2, 3, 4\}$, and $c_T(4) > c_T(3) \geq p$, i.e., the 3-flexible resource and 4-flexible resource can be excluded due to high costs. Thus, the firm only considers the 2-flexible resource. In this example, the ordered structure corresponds to assignment $(\{1, 2, 3, 4\}, \emptyset)$ and $(\{1, 2\}, \{3, 4\})$, the sandwiched structure corresponds to assignment $(\{1, 4\}, \{2, 3\})$, and the reversed structure corresponds to assignment $(\{3, 4\}, \{1, 2\})$. By Proposition 2, as σ increases, the firm has to assign variants with higher popularities to the flexible technology (i.e., choose option (i)) if $|\mathbb{F}|$ reaches 2 because increasing $|\mathbb{F}|$ to 3 (i.e., choose option (ii)) is not profitable. Recall the expressions of the overall profit $\Pi_{DF}(\mathbb{D}, \mathbb{F})$ and the mismatch cost $\mathcal{M}_{DF}(\mathbb{D}, \mathbb{F})$ in equation (5) and equation (7), in which the value of σ determines the weight of \mathcal{M}_{DF} in the overall profit. When σ is small, all variants face less variable demands and the effect of \mathcal{M}_{DF} is small comparing to the effect of \mathcal{P}_{DF} . In this case, all variants are assigned to the dedicated technology, leading to the assignment $(\{1, 2, 3, 4\}, \emptyset)$. As σ increases, \mathcal{M}_{DF} becomes a more influential term in the overall flexible technology due to its fixed-cost advantage. It is possible that neither (i) or (ii) holds in these cases. If either (A) or (B) holds, these cases can be eliminated. They are sufficient conditions, and the result still holds in most cases even if both of them are violated.

profit. If σ is moderately large, the more popular variants have low demand variabilities and it is more beneficial to assign them to the dedicated technology, which maintains a sufficiently high value of \mathcal{P}_{DF} . Meanwhile, the less popular variants have high demand variabilities and thus the firm can reduce the mismatch cost associated with them through the pooling effect of the flexible technology, leading to the assignment $(\{1,2\}, \{3,4\})$. When σ becomes sufficiently large, \mathcal{M}_{DF} has a significant impact on Π_{DF} . Now even the more popular variants are facing high demand variabilities. Ideally, in order to reduce the mismatch cost, the firm should assign all variants to the flexible technology, which might be impractical due to the significantly high unit capacity costs (reflected in the assumption of $c_T(4) > c_T(3) \geq p$ in this example). The firm is better off by focusing on the reduction of the more popular variants' supply-demand mismatch because of their higher impacts on the overall profit. The less popular variants, though suffering from more demand variabilities, have to be "sacrificed," i.e., assigned to the dedicated technology, because of their relatively lower impacts on the overall profit. Such reasoning consequently leads to the assignment $(\{1,4\}, \{2,3\})$, which has the sandwiched structure, or even the assignment $(\{3,4\}, \{1,2\})$, which has the reversed structure. In summary, the sandwiched structure and the reversed structure emerge when the firm has to focus on reducing the mismatch cost associated with the less variable but more impactful high-mean variants under significant market uncertainty.

In general, as the market gradually becomes more uncertain, the firm may either increase $|\mathbb{F}|$, or not increase $|\mathbb{F}|$ but assign more popular variants to \mathbb{F} . In Figure 1, we provide an illustration of such a process given a 6-variant assortment. We can observe that the optimal assignment follows the sandwiched structure when $\sigma = 0.25$ and the reversed structure when $\sigma = 0.4$. Also, interestingly, increasing the market uncertainty may decrease the number of variants assigned to the flexible technology: When σ increases from 0.1 to 0.25, the number decreases from 3 to 2; and when σ increases from 0.35 to 0.4, the number decreases from 5 to 3. This is because when the firm assigns the more popular variants to the flexible technology, it may want to reduce the unit capacity cost by excluding some less popular variants originally handled by the flexible resource.

Figure 1 Evolution of the Optimal Technology Assignment as σ Increases



Note. $\mathbb{S} = \{1, 2, 3, 4, 5, 6\}$, $(v_1, v_2, v_3, v_4, v_5, v_6) = (0.8, 0.3, 0.3, 0.01, 0.01, 0.01)$, $p = 2$, $c_D = 0.95$, $\Lambda = 1$, $K_T(n) = n \cdot K_D$, $(c_T(2), c_T(3), c_T(4), c_T(5), c_T(6)) = (1.05, 1.15, 1.25, 1.26, 1.27)$.

4.2. Assortment Decision

Now we characterize the optimal assortment decision. In the assortment planning literature, it is shown that in many settings the firm's optimal assortment is in the “popular assortment set” defined as $\{\{1\}, \{1, 2\}, \dots, \{1, 2, \dots, |\mathbb{U}|\}\}$ (Kök et al., 2009). We can restate this characterization of the optimal assortment using the concept of set division. One can view an assortment decision as a division of \mathbb{U} , (\mathbb{S}, \mathbb{W}) , where \mathbb{S} contains the chosen variants and \mathbb{W} contains the rest. Thus, similar to the technology-assignment characterization, we say an assortment decision is ordered if one of the following is true: $\mathbb{S} = \mathbb{U}$, $\mathbb{S} = \emptyset$, or all variants in \mathbb{S} are (weakly) more popular than all variants in \mathbb{W} ⁶. For expositional simplicity, we use \mathbb{S} (omitting the specification of \mathbb{W}) to represent the firm's assortment decision. The following proposition characterizes the structure of the optimal assortment decision.

PROPOSITION 3. *For Case DT, the optimal assortment decision is always ordered.*

⁶ In fact, our characterization using the concept of set division may contain some optimal assortments omitted by the traditional characterization using the “popular assortment set”. For example, suppose $\mathbb{U} = \{1, 2, 3\}$, $v_2 = v_3$, and $\{1, 2\}$ is optimal, then $\{1, 3\}$ is also optimal. It is an ordered assortment, but not in the “popular assortment set.”

Proposition 3 confirms that the result in van Ryzin and Mahajan (1999) and many follow-up studies continues to hold in the presence of endogenous technology assignment. The rationale of our result is similar to that of the results in van Ryzin and Mahajan (1999) and Cachon et al. (2005), but we need to go further beyond their proofs due to the endogenous technology assignment. Specifically, we also need to consider which technology the “new variant” is assigned to, how the inclusion of the new variant impacts the technology assignment of existing variants in \mathbb{S} , and how the new assignment affects the unit capacity cost of the flexible resource. In fact, for any clustering-structured assignment of a given unordered assortment, we can find another assortment with an associated assignment, and let the latter dominate the former. Consequently, an unordered assortment under its optimal assignment is dominated by some other assortment. Next we provide a representative example to explain such a fact.

Consider a set $\mathbb{U} = \{1, 2, 3, 4, 5\}$ with $v_1 > v_2 > v_3 > v_4 > v_5$ and an unordered assortment $\mathbb{S} = \{1, 2, 3, 5\}$, where the unorderedness is due to the selection of variant 5. Further, let $1 \in \mathbb{D}$ and $2, 3 \in \mathbb{F}$ (variant 1 is assigned to the dedicated technology, variant 2 and variant 3 are assigned to the flexible technology). Regardless of the technology assignment of variant 5, we show that \mathbb{S} is never optimal:

(i) If $5 \in \mathbb{D}$, the corresponding technology assignment is $(\{1, 5\}, \{2, 3\})$. We show that it is dominated by either assortment $\{1, 2, 3, 4\}$ with assignment $(\{1, 4\}, \{2, 3\})$ or assortment $\{1, 2, 3\}$ with assignment $(\{1\}, \{2, 3\})$. Consider the process of adding one more variant to assortment $\{1, 2, 3\}$ with assignment $(\{1\}, \{2, 3\})$ and the variant is assigned to \mathbb{D} . Similar to the arguments made in van Ryzin and Mahajan (1999) and Cachon et al. (2005), if the firm adds one more variant to the assortment and produce it dedicatedly, then the profit of the new assortment is a quasi-convex function of this variant’s popularity. Hence, it is optimal to either add the variant with the highest popularity, leading to assortment $\{1, 2, 3, 4\}$, or add nothing, leading to assortment $\{1, 2, 3\}$. Consequently, the unordered assortment $\{1, 2, 3, 5\}$ is never a candidate for optimality.

(ii) If $5 \in \mathbb{F}$, the corresponding technology assignment is $(\{1\}, \{2, 3, 5\})$. We show that it is dominated by either assortment $\{1\}$ (with assignment $(\{1\}, \emptyset)$) or assortment $\{1, 2, 3, 4\}$ with assignment

$(\{1\}, \{2, 3, 4\})$. In this case, we cannot consider the process of adding one more variant to assortment $\{1, 2, 3\}$ with assignment $(\{1\}, \{2, 3\})$ and the variant is assigned to \mathbb{F} like what has been done in (i). That is because adding variant 4 or 5 results in $|\mathbb{F}| = 3$ but adding nothing results in $|\mathbb{F}| = 2$, and thus profits resulting from the three choices are not directly comparable due to nonidentical unit capacity costs of the flexible resource. Instead, we consider the process of adding three variants to assortment $\{1\}$ with assignment $(\{1\}, \emptyset)$ and the three variants must be assigned to \mathbb{F} . Similar to (i), the profit of the new assortment is a quasi-convex function of the three new variants' aggregated popularity. Hence, it is optimal to either add the variants with the highest aggregated popularity, leading to assortment $\{1, 2, 3, 4\}$, or add nothing, leading to assortment $\{1\}$, of which the profit can be written as

$$(p - c_D) \frac{v_1}{1 + v_1 + 0} + (p - c_T(3)) \frac{0}{1 + v_1 + 0} - \sigma \left(m_D \frac{\sqrt{v_1}}{\sqrt{1 + v_1 + 0}} + m_T(3) \frac{\sqrt{0}}{\sqrt{1 + v_1 + 0}} \right) - \mathcal{K}_{DT}(\{1\}, \emptyset),$$

i.e., we unify the unit capacity costs under the three strategies by treating assortment $\{1\}$ with assignment $(\{1\}, \emptyset)$ as investing in zero capacity of a 3-flexible resource. Consequently, $\{1, 2, 3, 5\}$ is always suboptimal as well if variant 5 is assigned to the flexible technology.

Generally, depending on which technology the variant that breaks the ordered structure is assigned to, we can exchange this variant, exclude this variant, or even exclude all variants assigned to the flexible technology to verify that any clustering-structured technology assignment of an unordered assortment is dominated. As a consequence, the optimal assortment must be ordered, i.e., it consists of the most popular variants from \mathbb{U} . Intuitively, the most efficient way to choose an assortment is to gain enough market coverage with the lowest supply-demand mismatch induced. Hence, it is better for the firm to choose the most popular variants with relatively higher mean demand and lower demand variability.

Finally, we point out that the process of searching for the optimal strategy can be significantly simplified due to the characterization of optimal assortment and technology assignment decisions. The original combinatorial problem becomes algebraic: The firm only needs to determine $|\mathbb{S}|$, $|\mathbb{F}| \leq |\mathbb{S}|$ and the number of variants in \mathbb{D} that are more popular than variants in \mathbb{F} . The search time

is reduced from $O(3^{|\mathbb{U}|})$ to $O(|\mathbb{U}|^3)$. This is a significant reduction, especially when $|\mathbb{U}|$ is large. In addition, the algebraic structure also facilitates further analytical study (as will be seen in Section 6).

5. Strategy Under 3D Printing

This section investigates Case *DP*, in which the firm adopts 3D printing in addition to the dedicated technology. Economically speaking, 3D printing differs from the traditional flexible technology mainly in the cost structure: Under 3D printing, the unit capacity cost is independent of the number of variants handled, whereas under the traditional flexible technology, the unit capacity cost increases in the number of variants handled. This difference, as we will show in this section, leads to distinct optimal technology assignment decisions.

5.1. Technology Assignment Decision

We first characterize the optimal technology assignment for a given \mathbb{S} .

PROPOSITION 4. *For Case DP, given any \mathbb{S} , the optimal technology assignment $(\mathbb{D}^*, \mathbb{F}^*)$*

(a) *is ordered, i.e., $\forall i \in \mathbb{D}, v_i \geq \max\{v_j : j \in \mathbb{F}\}$;*

(b) *assigns equally popular variants to the same technology.*

To understand the rationale of Proposition 4(a), recall the discussion following Proposition 1, where we consider a simple but representative case of $\mathbb{S} = \{1, 2, 3\}$ with $v_1 > v_2 > v_3$. We have established that the assignment $(\{2\}, \{1, 3\})$ must be dominated by $(\{1\}, \{2, 3\})$, which leads to a lower aggregated mean demand handled by the flexible resource, or $(\{3\}, \{1, 2\})$, which leads to a higher aggregated mean demand handled by the flexible resource. In fact, assignment $(\emptyset, \{1, 2, 3\})$ leads to a even higher mean aggregated mean demand than $(\{3\}, \{1, 2\})$, but the profits of these two assignments are not directly comparable under the traditional flexible technology because of non-identical unit capacity costs of 2-flexible and 3-flexible resources. Such an obstacle is, however, eliminated when the flexible technology is 3D printing because unit capacity costs of all-degree flexible resources are identical. Following the same rationale of the discussion following Proposition

1, we can argue that either $(\{1\}, \{2, 3\})$ or $(\emptyset, \{1, 2, 3\})$, satisfying the ordered structure, dominates $(\{3\}, \{1, 2\})$ that violates the ordered structure, because the former two assignments respectively lead to lower and higher mean aggregated mean demands handled by the flexible resource than the latter assignment. Extending this fact to general cases establishes the ordered structure's optimality for technology assignment under 3D printing.

A similar rationale applies to Proposition 4(b). Consider an assortment \mathbb{S} in which T variants have an identical popularity v . We argue that splitting those T variants between the dedicated technology and 3D printing is not optimal. Suppose t variants are assigned to the flexible technology, and $T - t$ variants are assigned to the dedicated technology. As t increases, the gross profit and the mismatch cost associated with the dedicated resources decreases linearly, and the mismatch cost associated with the flexible resource increases concavely. These imply that Π_{DP} is convex in t . As a result, the firm should assign all T variants to the dedicated technology and thus achieve significantly high \mathcal{P}_{DP} , or assign all T variants to the flexible technology and thus achieve significantly low \mathcal{M}_{DP} . Either of these two assignments dominates any in-between assignment with $0 < t < T$. By contrast, for Case DT , the gross profit may be convex in t and the mismatch cost associated with the flexible resource may no longer be concave. Therefore, splitting equally popular variants to different technologies may be optimal.

Recall that, under the traditional flexible technology, we have shown that the number of variants assigned to the flexible technology may decrease in market uncertainty as is illustrated in Figure 1. By contrast, we show that the firm must assign more variants to the flexible technology as the market becomes more uncertain if 3D printing is adopted in the following proposition.

PROPOSITION 5. *For Case DP , consider two market uncertainty levels $\sigma_1 < \sigma_2$. Given any assortment \mathbb{S} , let $(\mathbb{D}_i^*, \mathbb{F}_i^*)$ ($i = 1, 2$) be the optimal technology assignment associated with $\sigma = \sigma_i$.*

$|\mathbb{F}_1^| \leq |\mathbb{F}_2^*|$ holds if either*

(A) $c_D + c_P \geq p$, or

(B) $K_P(n) = K_D n + \kappa$ for $\forall n \in \mathbb{N}^+$, where $\kappa \geq 0$.

Similar to conditions (A) and (B) in Proposition 2, conditions (A) and (B) in Proposition 5 eliminate some extreme and uninteresting cases. With these cases eliminated, the intuition that a more uncertain market leads to more variants assigned to the flexible technology always holds. Note that, $|\mathbb{F}^*|$ may not increase continuously as σ increases. For example, consider an assortment \mathbb{S} in which all variants are equally popular. Based on Proposition 4(b) and Proposition 5, $|\mathbb{F}^*|$ directly jumps from 0 to $|\mathbb{S}|$ once σ exceeds a threshold.

5.2. Assortment Decision

Similar to Proposition 3, we can characterize the optimal assortment decision as follows.

PROPOSITION 6. *For Case DP, the optimal assortment decision must be ordered.*

Combining Proposition 4(a) and Proposition 6, the optimal strategy for Case DP enjoys a simple two-fold ordered structure: Both the assortment decision and the technology assignment are ordered, which reduces the search time of the optimal strategy from $O(3^{|\mathbb{U}|})$ to $O(|\mathbb{U}|^2)$. In addition, by Proposition 4(b), if there are n_v different popularity values in \mathbb{U} , the search time can be further reduced to $O(n_v|\mathbb{U}|)$.

The optimal assortment is surprisingly simple when the fixed cost of 3D printing, $K_P(\cdot)$, is constant, as is shown in the following proposition.

PROPOSITION 7. *For Case DP, if $K_P(n)$ is constant for $\forall n \geq 2$, then the optimal assortment and technology assignment decisions must satisfy one of the following properties:*

- (i) *Assortment is full, i.e., $\mathbb{S} = \mathbb{U}$.*
- (ii) *Assortment is not full and the flexible technology is not utilized, i.e., $\mathbb{S} \subset \mathbb{U}$ and $\mathbb{F} = \emptyset$.*

Constant $K_P(\cdot)$ corresponds to the case in which adding more variants produced with 3D printing only requires minor changes to the CAD file of a basic model, and thus the effect on the fixed cost associated with prototyping is negligible (e.g., the “my little pony” toys produced by Hasbro; see, Kell, 2014). In this case, the optimal assortment is either the full potential set \mathbb{U} , or a subset of \mathbb{U} with variants in it all assigned to the dedicated technology. To understand the intuition, we can

consider the assortment decision involving two steps: deciding \mathbb{D} and then choose the optimal \mathbb{F} for that \mathbb{D} . Because the fixed cost associated with the flexible resource is not affected by the size of \mathbb{F} and demands for variants in \mathbb{F} are pooled together, these variants can be collectively treated as one combined “flexible variant.” Hence, choosing the optimal \mathbb{F} for a given \mathbb{D} is equivalent to adding the “flexible variant” to an assortment whose variants are all assigned to the dedicated technology. As mentioned before, it is optimal to either add the variant with the highest possible popularity or add nothing. The “flexible variant” with the highest possible popularity is formed by all variants in $\mathbb{U} \setminus \mathbb{D}$, leading to $\mathbb{S} = \mathbb{U}$, while adding nothing leads to $\mathbb{F} = \emptyset$.

6. Impact of Flexible Technologies on Product Variety

In this section, we investigate how the adoption of flexible technologies affects the firm’s choice of product variety. To answer this question, we compare Case DF ($F = T, P$) with a benchmark Case D , where the firm adopts the dedicated technology only. With only the dedicated technology, the firm’s decision consists of the assortment and the capacity investment decisions. Following the formulation in Section 3.2, we can write the firm’s expected profit function for a given assortment \mathbb{S} as

$$\Omega_D(\mathbb{S}) = \underbrace{(p - c_D) \frac{\sum_{i \in \mathbb{S}} v_i}{1 + \sum_{i \in \mathbb{S}} v_i}}_{\mathcal{P}_D(\mathbb{S})} - m_D \sigma \underbrace{\frac{\sum_{i \in \mathbb{S}} \sqrt{v_i}}{\sqrt{1 + \sum_{i \in \mathbb{S}} v_i}}}_{\mathcal{M}_D(\mathbb{S})} - \underbrace{\frac{1}{\Lambda} K_D \cdot |\mathbb{S}|}_{\mathcal{K}_D(\mathbb{S})}. \quad (11)$$

For all three cases, D , DT , and DP , we measure product variety using the optimal assortment size. In case of multiple optimal assortments, we set product variety to be the largest optimal assortment size, i.e.,

$$\mathcal{V}_X := \max\{|\mathbb{S}^*| : \mathbb{S}^* \in \arg \max_{\mathbb{S} \subseteq \mathbb{U}} \Omega_X(\mathbb{S})\}, \quad X = D, DT, DP. \quad (12)$$

In the following, we will compare \mathcal{V}_{DT} and \mathcal{V}_{DP} with \mathcal{V}_D .

6.1. Impact of the Traditional Flexible Technology

We first compare Case DT and Case D . One may conjecture that the firm will increase product variety after adopting the flexible technology. However, we show in the following proposition that the opposite may happen even in a 3-variant case when (i) the market uncertainty is moderate, (ii) the unit capacity cost of a 2-flexible resource is sufficiently low whereas the unit capacity cost of a 3-flexible resource is sufficiently high, and (iii) the fixed costs of both technologies are small.

PROPOSITION 8. *Consider a set $\mathbb{U} = \{1, 2, 3\}$ with $v_1 = v_2 > v_3$. There exist thresholds (i) $\underline{\sigma}, \bar{\sigma} \in (0, \infty)$ with $\underline{\sigma} < \bar{\sigma}$, (ii) $c_2, c_3 \in (c_D, p)$, (iii) $K \in [0, \infty)$, such that: if (i) $\underline{\sigma} < \sigma < \bar{\sigma}$, (ii) $c_T(2) < c_2$, $c_T(3) > c_3$, (iii) $K_D, K_T(3) < K$, then $\mathcal{V}_D = 3$ and $\mathcal{V}_{DT} = 2$.*

For both Case D and Case DT , we need to compare the profits of all four ordered assortments, \emptyset , $\{1\}$, $\{1, 2\}$, and $\{1, 2, 3\}$ to find the optimal assortment. \emptyset and $\{1\}$ can be excluded by the conditions given in the proposition, and we focus on comparing $\{1, 2\}$, and $\{1, 2, 3\}$ to explain the intuition. Recall that, when the assortment expands, the firm faces a trade-off between the higher aggregated mean demand and the more variable individual demands. Next, we examine the impacts of these driving forces in both cases.

For Case D , the mismatch cost under the assortment $\{1, 2\}$ is already sufficiently high because both variants are produced with the dedicated technology. Thus, when expanding $\{1, 2\}$ to $\{1, 2, 3\}$, the firm does not gain significant incremental mismatch cost under moderate market uncertainty ($\sigma < \bar{\sigma}$). As a consequence, the increase of mismatch cost is outweighed by the increase of gross profit, and the firm decides to produce all three variants.

This, however, is not necessarily true for Case DT . We first decide the optimal technology assignment for each assortment. For the assortment $\{1, 2\}$, the optimal assignment assigns both variants to the flexible technology, i.e., $(\emptyset, \{1, 2\})$, because the unit capacity cost of the 2-flexible resource is low ($c_T(2) < c_2$). For the assortment $\{1, 2, 3\}$, the optimal assignment is $(\{3\}, \{1, 2\})$, which has the reversed structure for two reasons: (i) the 3-flexible resource is very expensive ($c_T(3) > c_3$), and (ii) the firm can produce the two more popular variants with the 2-flexible resource

to reduce the mismatch cost and meanwhile maintains a sufficiently high gross profit (note that $c_T(2) < c_2$, i.e., the 2-flexible resource is inexpensive).

Now we are able to compare the profits of assortments $\{1, 2\}$ and $\{1, 2, 3\}$. Under the optimal technology assignments, expanding assortment $\{1, 2\}$ to assortment $\{1, 2, 3\}$ with assignment $(\{3\}, \{1, 2\})$ is equivalent to keeping variant 1 and 2 handled by the flexible resource and adding variant 3 handled by the dedicated resource. Though the expansion leads to a higher gross profit, it also leads to a significant increase in the mismatch cost under sufficiently high market uncertainty ($\sigma > \underline{\sigma}$): Originally in $\{1, 2\}$, the flexible resource handles all the demand, whereas in $\{1, 2, 3\}$, the flexible resource only handles a portion of the total demand; moreover, the mean demand handled by the flexible resource decreases due to the cannibalization effect. Consequently, the increase in gross profit is outweighed by the increase of mismatch cost, and the firm decides to produce only the two more popular variants.

The insights from this 3-variant example also carry over to more general settings. The firm may have the incentive to give up the less popular variants and thus focusing on serving the more popular variants with the flexible resource. As a result, the product variety can even be reduced when the traditional flexible technology is adopted in addition to the dedicated technology.

6.2. Impact of 3D Printing

We proceed to compare Case DP and Case D . Recall that 3D printing has a fixed-cost advantage over the dedicated technology, i.e., the condition $K_P(n+1) - K_P(n) \leq K_D$ means that the fixed cost of adding a variant to 3D printing is lower than the fixed cost of adding the variant to the dedicated technology. The following proposition shows that, even in the absence of such an advantage, i.e., when $K_P(n) = K_D \cdot n + \kappa$ (implying $K_P(n+1) - K_P(n) = K_D$), adopting 3D printing in addition to the dedicated technology always leads to a higher product variety.

PROPOSITION 9. *Suppose $K_P(n) = K_D n + \kappa$ for $\forall n \in \mathbb{N}^+$, where $\kappa \geq 0$. Then*

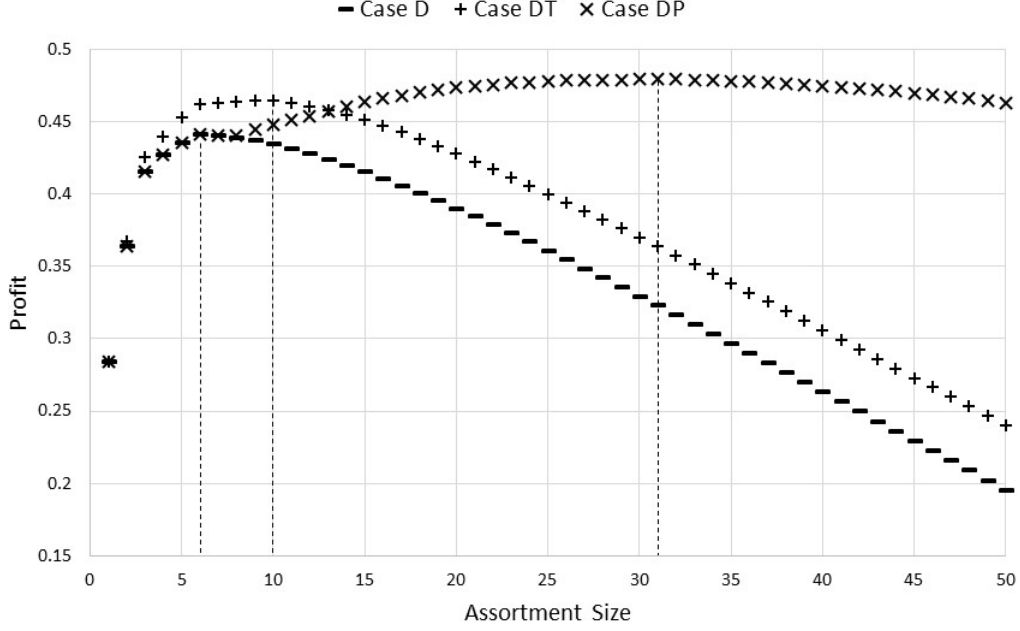
- (a) *For all ordered assortment decisions, $\Omega_{DP}(\mathbb{S}) - \Omega_D(\mathbb{S})$ (weakly) increases in $|\mathbb{S}|$;*
- (b) $\mathcal{V}_{DT} \geq \mathcal{V}_D$.

The main driving force of Proposition 9 is the effect of increasing $|\mathbb{S}|$ on the mismatch cost. To be specific, let us consider the assortment expansion processes in both Case D and Case DP . For Case D , the mismatch cost increases very fast as the assortment expands because each variant obtains more individual demand variability. For Case DP , the mismatch cost increases much more slowly because additional variants assigned to the flexible technology further enhances the pooling effect. The different paces of mismatch cost increase in the two cases lead to Proposition 9(a): The profit difference between the two cases increases as the assortment expands. This implies that, when the profit for Case D decreases, the profit for Case DP may still increase, or decrease at a slower pace. Consequently, Proposition 9(a) implies Proposition 9(b), i.e., the firm for Case DP always chooses a higher product variety than the firm for Case D .

Figure 2 illustrates how the profits for Case D and Case DP change as the assortment size increases. For Case D , the profit first increases, peaks at assortment size 6, and quickly decreases afterwards. For Case DP , the profit initially peaks at assortment size 6, then decreases; however, when the assortment size reaches 8 where some variants are assigned to 3D printing, the profit increases again and finally peaks at assortment size 31, which is the global optimum. Moreover, with 3D printing, the firm's profit performance is quite robust with respect to the assortment size: Even when the assortment size is as large as 50, the profit is still close to the optimal one. The robustness of the profit performance for Case DP is further confirmed in an extensive numerical study, in which the average profit loss is only 5% when the firm chooses the assortment size ranging from the optimal plus one to the full assortment size, whereas the average profit loss is as large as 31% for Case DT . Thus, another advantage of 3D printing is that, when the fixed cost of adding variants doesn't increase very quickly, near-optimal profit can be achieved even when the firm produces a much larger assortment than the optimal one. Thus, the firm is able to satisfy various consumers' needs and promote its brand name by making aggressive assortment expansion without compromising profit performance.

Finally, we present the following proposition characterizing how the optimal technology assignment changes as the assortment size goes beyond a threshold.

Figure 2 Firm's Profit as a Function of Assortment Size



Note. $(v_1, \dots, v_{50}) = (0.5, 0.3, 0.3, 0.15, 0.15, 0.15, 0.1, \dots, 0.1)$, $p = 2$, $c_D = 1$, $c_T(n) = 1 + 0.02n$, $c_P = 1.2$, $\Lambda = 1$, $\sigma = 0.1$, $K_D = 0.003$, and $K_T(n) = K_P(n) = 0.003n$.

PROPOSITION 10. For Case DP, define μ_{DP} as the size of the largest ordered assortment for which the optimal technology assignment is dedicated-only. Consider two ordered assortments \mathbb{S}_1 and \mathbb{S}_2 associated with the optimal technology assignments $(\mathbb{D}_1^*, \mathbb{F}_1^*)$ and $(\mathbb{D}_2^*, \mathbb{F}_2^*)$. If $\mu_{DP} < |\mathbb{S}_1| < |\mathbb{S}_2|$, then $|\mathbb{D}_1^*| \geq |\mathbb{D}_2^*|$.

Proposition 10 implies that, when 3D printing is actively used together with the dedicated technology, assortment expansion leads to less variants assigned to the dedicated technology. As the assortment expands, mean demands of the variants in the original assortment decrease and, consequently, these variants face higher demand variabilities. In order to mitigate the mismatch cost, some variants originally assigned to the dedicated technology are thus reallocated to the flexible technology. The result indicates that, not only the adoption of 3D printing drives more product variety, but also targeting at higher product variety drives more utilization of the technology itself. If the firm produces a very large assortment such that each variant gets little mean demand, a sensible strategy is to give up the dedicated technology and fully rely on 3D printing.

7. Conclusion

Motivated by the rapid development of flexible production technologies such as 3D printing, this paper studies the impact of different flexible technologies on a firm's manufacturing strategy. Two types of flexible technologies are compared: the traditional flexible technology and 3D printing. The firm adopts one of these flexible technologies in addition to the dedicated technology. There are three decisions in the firm's manufacturing strategy: assortment decision (i.e., which product variants to offer), technology assignment decision (i.e., how to assign the variants between the dedicated and the flexible technologies), and capacity investment decision (i.e., how much capacity to acquire for each resource). We find that 3D printing has different implications for the firm's manufacturing strategy than the traditional flexible technology.

First, for a given assortment, the firm's optimal assignment structure may differ under different flexible technologies. Under the traditional flexible technology, the firm may assign the variants with lower demand variabilities to the flexible technology and the variants with higher demand variabilities to the dedicated technology. That is, the optimal assignment may represent the unintuitive reversed or sandwiched structure. Further, under the traditional flexible technology, the number of product variants assigned to the flexible technology may decrease as the market becomes more uncertain. Under 3D printing, however, the firm should always assign the variants with less variable demands to the dedicated technology and the variants with more variable demands to the flexible technology, which corresponds to the intuitive ordered structure. Also, the number of variants assigned to 3D printing always increases as the market becomes more uncertain.

Second, we consider how the adoption of flexible technologies in addition to the dedicated technology may change the optimal assortment size for the firm. Interestingly, we find that the firm may reduce the assortment size (i.e., offer less product variety) when adding the traditional flexible technology on top of the dedicated technology. This is because the firm may want to "centralize" demand for the flexible resource by reducing cannibalization among the variants. In contrast, we

find that adding 3D printing always increases the firm’s optimal assortment size (i.e., the firm offers more product variety). Using numerical analysis, we also show that 3D printing allows the firm to aggressively expand its product assortment without compromising profitability much, which is generally not true for the traditional flexible technology. Through the above findings, our paper demonstrates the different managerial implications that 3D printing may have on a firm’s manufacturing strategy compared to the traditional flexible technology.

There has been a constant debate about the value of 3D printing since the advent of this innovative technology. Many believe that 3D printing has reached a tipping point and will revolutionize the manufacturing industry (D’Aveni, 2013, 2015a, 2015b; McCue, 2015). Others are more skeptical and think 3D printing will not dramatically change the manufacturing sector (Holweg, 2015; Ross et al., 2016). Our paper is among the first to study the role of 3D printing from an operations perspective. The results indicate that 3D printing provides several advantages for the firm, including simplified production strategy, wider product variety, and robust profitability performance. These findings corroborate industry observations that “3D printing’s greatest value is not as a technology, but as an enabler to derive greater business value” (Stratasys, 2015). Clearly, future research is needed to deepen our understanding of the potential impact of 3D printing. For example, 3D printing allows the creation of geometric shapes that traditional technologies cannot achieve (*The Economist*, 2011). It is worthwhile investigating how this property impacts firms’ manufacturing strategies. Additionally, 3D printing is more sustainable. As is estimated in Gebler et al. (2014), the energy consumption and CO₂ emissions from industrial manufacturing can be remarkably reduced through 3D printing. A promising research direction is thus to examine the value of 3D printing from the sustainability perspective.

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Appendix. Extensions and Technical Notes

A. Extensions

We provide two extensions of the base model in this section.

A.1. Multiple Flexible Resources

Now we extend our base model by allowing the firm to invest in multiple flexible resources. We first argue that, if the firm adopts 3D printing, it is always optimal to invest in at most one flexible resource. This is because it is always more profitable to have one flexible pool to enhance the pooling effect when the unit capacity cost is constant. However, with the traditional flexible technology, the firm may have the incentive to invest in multiple flexible resources to control the unit capacity costs.

Let Case DT^M denote the case in which the firm is able to invest in arbitrary number of traditional flexible resources. In this case, the firm still faces a three-stage decision. First, the firm chooses an assortment $\mathbb{S} \subseteq \mathbb{U}$. In the second stage, the firm needs to assign each variant to either the dedicated technology or the flexible technology, and also allocate all variants assigned to the flexible technology to different flexible resources. For both technical and expositional convenience, we define the dedicated technology as the 1-flexible technology, i.e., we expand the domain of $c_T(\cdot)$ and $K_T(\cdot)$ to \mathbb{N} , meanwhile let $c_T(1) = c_D$ and $K_T(1) = K_D$. The firm's second-stage decision thus reduces to a resource-allocation problem: it separates the assortment \mathbb{S} into several variant groups, where variants in the same group are to be handled by the same resource; if a group has only one variant, the corresponding resource is dedicated. We use $(\mathbb{R}_1, \dots, \mathbb{R}_M)$ (M is endogenously determined), where \mathbb{R}_m is the set of all variants assigned to the m th resource, to denote the resource allocation. In the third stage, the firm invests in the capacities of all resources.

For a given resource allocation $(\mathbb{R}_1, \dots, \mathbb{R}_M)$, we can solve the third-stage capacity investment problem and obtain the normalized profit function:

$$\Pi_{DT^M}(\mathbb{R}_1, \dots, \mathbb{R}_M) = \sum_{m=1}^M \left[\left(p - c_T(|\mathbb{R}_m|) \right) \sum_{j \in \mathbb{R}_m} s_j - m_T(|\mathbb{R}_m|) \sigma \sqrt{\sum_{j \in \mathbb{R}_m} s_j} - \frac{1}{\Lambda} K_T(|\mathbb{R}_m|) \right].$$

The firm's second-stage technology assignment decision is thus formulated as

$$\Omega_{DT^M}(\mathbb{S}) := \max_{\substack{\mathbb{R}_i \cap \mathbb{R}_j = \emptyset, \forall i, j \in \{1, \dots, M\}; \\ \bigcup_{i=1}^M \mathbb{R}_i = \mathbb{S}}} \Pi_{DT^M}(\mathbb{R}_1, \dots, \mathbb{R}_M).$$

The firm's first-stage assortment decision problem is formulated as

$$\max_{\mathbb{S} \subseteq \mathbb{U}} \Omega_{DT^M}(\mathbb{S}).$$

We have the following proposition characterizing the optimal strategy.

PROPOSITION 11. *Consider Case DT^M .*

(a) *Given any assortment, the optimal resource allocation $(\mathbb{R}_1^*, \dots, \mathbb{R}_M^*)$ must satisfy: for any two sets $\mathbb{R}_{m_1}^*$ and $\mathbb{R}_{m_2}^*$, either all variants in $\mathbb{R}_{m_1}^*$ are (weakly) more popular than all variants in $\mathbb{R}_{m_2}^*$ or all variants in $\mathbb{R}_{m_1}^*$ are (weakly) less popular than all variants in $\mathbb{R}_{m_2}^*$.*

(b) *The optimal assortment decision must be ordered.*

Proposition 11 establishes a more general “clustering” structure for Case DT^M . The rationale is similar to what has been discussed for Case DT . Generally, it is optimal for the firm to “cluster” variants with close popularity values into the same flexible resource, which ensures high efficiency of pooling.

Even when the firm is able to invest in multiple flexible resources, the adoption of the traditional flexible technology may still reduce product variety due to the same rationale behind Proposition 8. Consider a 5-variant set $\mathbb{U} = \{1, 2, 3, 4, 5\}$. When the market uncertainty is within a moderate range, there is a significant difference in unit capacity costs between the 2-flexible and 3-flexible resources, and the fixed costs of all technologies are small, it is possible that the firm in Case D has the optimal assortment of $\{1, 2, 3, 4, 5\}$ whereas the firm in Case DT^M has the optimal assortment of $\{1, 2, 3, 4\}$ with resource allocation $(\{1, 2\}, \{3, 4\})$, i.e., it is optimal for the firm to focus on the four more popular variants with two 2-flexible resources.

A.2. Overlapping Technology Assignment

In our base model, the firm is able to assign one variant to one technology. We now numerically study the situation in which one variant can be assigned to both the dedicated and the flexible technologies, which is denoted by Case $\tilde{D}F$ ($F = T, P$). Now the firm needs to make decisions in four stages: First, the firm chooses the assortment $\mathbb{S} \in \mathbb{U}$. Second, the firm assigns the variants in \mathbb{S} to different technologies. Because overlapping technology assignment is allowed, we let \mathbb{D} denote the set of variants assigned to the dedicated technology only, \mathbb{F} denote the set of variants assigned to the flexible technology only, and \mathbb{E} denote the set of variants assigned to both technologies. Third, the firm invests in resource capacities x_{Di} for $i \in \mathbb{D} \cup \mathbb{E}$ and x_F . Finally, after demand realization, the firm decides the production quantities of each variant handled by corresponding resources to maximize the ex post revenue. Let q_{Di} ($i \in \mathbb{D} \cup \mathbb{E}$) denote the production quantity of variant i handled by the dedicated resource, and q_{Fi} ($i \in \mathbb{E} \cup \mathbb{F}$) denote the production quantity of variant i handled by the flexible resource.

Given capacities $(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F)$ and realized demands y_i ($i \in \mathbb{S}$), the firm's fourth-stage revenue maximization problem is formulated as

$$\mathbf{R}_{\tilde{D}F}(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F, \{y_i\}_{i \in \mathbb{S}}) := \max_{q_{Di} \geq 0, q_{Fi} \geq 0} p\left(\sum_{i \in \mathbb{D} \cup \mathbb{E}} q_{Di} + \sum_{i \in \mathbb{E} \cup \mathbb{F}} q_{Fi}\right),$$

subject to

$$\begin{aligned} 0 \leq q_{Di} \leq x_{Di}, \quad i \in \mathbb{D} \cup \mathbb{E}; \quad 0 \leq q_{Fi}, \quad \sum_{i \in \mathbb{E} \cup \mathbb{F}} q_{Fi} \leq x_F, \quad i \in \mathbb{E} \cup \mathbb{F}, \\ q_{Di} \leq y_i, \quad i \in \mathbb{D}; \quad q_{Di} + q_{Fi} \leq y_i, \quad i \in \mathbb{E}; \quad q_{Fi} \leq y_i, \quad i \in \mathbb{F}, \end{aligned}$$

$F = T, P$ and $c_P(\cdot)$ is constant. Because the price is identical for all variants, it is always optimal for the firm to first exploit the dedicated resources and then use the flexible resource when producing variants in \mathbb{E} .

Thus, we can solve the fourth-stage production quantity problem and rewrite $\mathbf{R}_{\tilde{D}F}(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F, y)$ as

$$\mathbf{R}_{\tilde{D}F}(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F, \{y_i\}_{i \in \mathbb{S}}) = p\left[\sum_{i \in \mathbb{D} \cup \mathbb{E}} \min\{x_{Di}, y_i\} + \min\left\{\sum_{i \in \mathbb{F}} y_i + \sum_{i \in \mathbb{E}} (y_i - x_{Di})^+, x_F\right\}\right].$$

The third-stage capacity investment problem is formulated as

$$\Pi_{\tilde{D}F}(\mathbb{D}, \mathbb{E}, \mathbb{F}) := \max_{x_{Di} \geq 0, i \in \mathbb{D} \cup \mathbb{E}; x_F \geq 0} \pi_{\tilde{D}F}(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F),$$

where

$$\begin{aligned} \pi_{\tilde{D}F}(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F) = & E[\mathbf{R}(\{x_{Di}\}_{i \in \mathbb{D} \cup \mathbb{E}}, x_F, \{Y_i\}_{i \in \mathbb{S}})] - c_D \sum_{i \in \mathbb{D} \cup \mathbb{E}} x_{Di} \\ & - c_F(|\mathbb{E}| + |\mathbb{F}|)x_F - K_D \cdot (|\mathbb{D}| + |\mathbb{E}|) - K_F(|\mathbb{E}| + |\mathbb{F}|). \end{aligned}$$

The second-stage technology assignment problem is formulated as

$$\Omega_{\tilde{D}F}(\mathbb{S}) := \max_{\mathbb{D} \cap \mathbb{E} = \emptyset, \mathbb{E} \cap \mathbb{F} = \emptyset, \mathbb{F} \cap \mathbb{D} = \emptyset, \mathbb{D} \cup \mathbb{E} \cup \mathbb{F} = \mathbb{S}} \Pi_{\tilde{D}F}(\mathbb{D}, \mathbb{E}, \mathbb{F}).$$

Finally, the first-stage problem of deciding the optimal assortment is formulated as

$$\max_{\mathbb{S} \subseteq \mathbb{U}} \Omega_{\tilde{D}F}(\mathbb{S}).$$

We solve the problem by Monte Carlo simulation of sample size 10000. Due to the exponential time complexity, we focus on a case with $|\mathbb{U}| = 8$. Table 1 and Table 2 list the optimal strategies and the corresponding profits in both the base models and the extended models. For example, $\mathbb{DDEEFFWW}$ in the table means that variant 1 and variant 2 are assigned to the dedicated technology only, variant 3 and variant 4 are assigned to both technologies, variant 5 and variant 6 are assigned to the flexible technology only, and variant 7 and 8 are not produced.

Table 1 Strategies and Profits in both Models, under Traditional Flexible Technology

	$K_D = 0, K_T(n) = 0$	$K_D = 0.03, K_T(n) = 0.02n$	$K_D = 0.05, K_T(n) = 0.03n$
$\sigma = 0.1$	EEEEEDDD, 2.25	DDDDDDFFW, 1.99	DDDDFFFW, 1.87
	DDDDDDFF, 2.19	DDDDDDFFW, 1.99	DDDDFFFW, 1.87
$\sigma = 0.3$	EEFWWWWW, 1.25	EEFWWWWW, 1.10	EEFWWWWW, 1.07
	FFFWWWWW, 1.15	FFFWWWWW, 1.08	FFFWWWWW, 1.06

Note: $p = 10, c_D = 4, c_f(n) = 3.5 + 0.5n, (v_1, \dots, v_8) = (0.25, 0.2, 0.15, 0.1, 0.08, 0.07, 0.05, 0.04)$.

Table 2 Strategies and Profits in both Models, under 3D Printing

	$K_D = 0, K_P(n) = 0$	$K_D = 0.03, K_P(n) = 0.02n$	$K_D = 0.05, K_P(n) = 0.03n$
$\sigma = 0.1$	EEEEEEEE, 2.32	DDDDDDWW, 1.96	DDDDDDWW, 1.84
	DDDDDDDD, 2.18	DDDDDDWW, 1.96	DDDDDDWW, 1.84
$\sigma = 0.3$	EEFFFFFF, 1.28	EEFFFFFF, 1.04	EEFFFFFFW, 0.94
	FFFFFFF, 1.14	FFFFFFF, 0.96	DDDDWWWW, 0.90

Note: $p = 10, c_D = 4, c_f = 6, (v_1, \dots, v_8) = (0.25, 0.2, 0.15, 0.1, 0.08, 0.07, 0.05, 0.04)$.

There are several observations from the numerical study: (i) The extended model does not gain significantly more profit than the base model (around 5%). If the fixed costs are sufficiently large and the market uncertainty is moderate, it is usually sub-optimal to assign any variant to both technologies even if it is allowed. That is because variants assigned to both technologies induce fixed costs of two resources, which may be not worthwhile. Hence, our base model can be a good approximation of the extended model. (ii) The firm still chooses several most popular variants in the assortment. (iii) The most popular variants are assigned to both technologies thus they can take advantage of both low capacity cost of the dedicated technology and the pooling effect of the flexible technology. In addition, variants with adjacent popularities tend to have the same assignment.

B. Technical Notes

In this section, we provide technical details and proofs of all propositions.

B.1. Strategy under the Traditional Flexible Technology

Proof of Proposition 1 Given \mathbb{S} , consider an assignment (\mathbb{D}, \mathbb{F}) in which there is one variant $i' \in \mathbb{D}$ more popular than the least popular variant in \mathbb{F} but less popular than the most popular variant in \mathbb{F} . Let j'_1 be one least popular variant in \mathbb{F} and j'_2 be one most popular variant in \mathbb{F} , thus $v_{j'_1} < v_{i'} < v_{j'_2}$. Consider the following three assignments:

(1) (\mathbb{D}, \mathbb{F}) with

$$\begin{aligned} \Pi_{DT}(\mathbb{D}, \mathbb{F}) = & (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + v_{i'}}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{i'}}{1 + \sum_{k \in \mathbb{S}} v_k} \\ & - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{v_{i'}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{i'}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P(|\mathbb{F}|)), \end{aligned}$$

(2) $(\mathbb{D} \setminus \{i'\} \cup \{j'_1\}, \mathbb{F} \cup \{i'\} \setminus \{j'_1\})$ with

$$\begin{aligned} \Pi_{DT}(\mathbb{D}, \mathbb{F}) = & (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + v_{j'_1}}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{j'_1}}{1 + \sum_{k \in \mathbb{S}} v_k} \\ & - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{v_{j'_1}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{j'_1}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P(|\mathbb{F}|)), \end{aligned}$$

and (3) $(\mathbb{D} \setminus \{i'\} \cup \{j'_2\}, \mathbb{F} \cup \{i'\} \setminus \{j'_2\})$ with

$$\begin{aligned} \Pi_{DT}(\mathbb{D}, \mathbb{F}) = & (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + v_{j'_2}}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{j'_2}}{1 + \sum_{k \in \mathbb{S}} v_k} \\ & - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{v_{j'_2}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{j'_2}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P(|\mathbb{F}|)). \end{aligned}$$

Note that $|\mathbb{F}| = |\mathbb{F} \cup \{i'\} \setminus \{j'_1\}| = |\mathbb{F} \cup \{i'\} \setminus \{j'_2\}|$. Consider the function

$$\begin{aligned} f_{\text{Proposition 1}}(t) = & (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + t}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - t}{1 + \sum_{k \in \mathbb{S}} v_k} \\ & - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{t}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - t}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P(|\mathbb{F}|)), \end{aligned}$$

which is strictly convex in t on $[0, v_{j'_2}]$. Hence, on $[0, v_{j'_2}]$, $f_{\text{Proposition 1}}(t)$ is strictly decreasing in t , strictly increasing in t , or first strictly decreasing in t and then strictly increasing in t . Note that $f_{\text{Proposition 1}}(v_{i'}) = \Pi_{DT}(\mathbb{D}, \mathbb{F})$, $f_{\text{Proposition 1}}(v_{j'_1}) = \Pi_{DT}(\mathbb{D} \setminus \{i'\} \cup \{j'_1\}, \mathbb{F} \cup \{i'\} \setminus \{j'_1\})$ and $f_{\text{Proposition 1}}(v_{j'_2}) = \Pi_{DT}(\mathbb{D} \setminus \{i'\} \cup \{j'_2\}, \mathbb{F} \cup \{i'\} \setminus \{j'_2\})$.

$\{j'_2\}, \mathbb{F} \cup \{i'\} \setminus \{j'_2\}$). We thus have either $\Pi_{DT}(\mathbb{D} \setminus \{i'\} \cup \{j'_1\}, \mathbb{F} \cup \{i'\} \setminus \{j'_1\}) > \Pi_{DT}(\mathbb{D}, \mathbb{F})$ or $\Pi_{DT}(\mathbb{D} \setminus \{i'\} \cup \{j'_2\}, \mathbb{F} \cup \{i'\} \setminus \{j'_2\}) > \Pi_{DT}(\mathbb{D}, \mathbb{F})$, or both. Since $(\mathbb{D} \setminus \{i'\} \cup \{j'_1\}) \cup (\mathbb{F} \cup \{i'\} \setminus \{j'_1\}) = \mathbb{S}$ and $(\mathbb{D} \setminus \{i'\} \cup \{j'_2\}) \cup (\mathbb{F} \cup \{i'\} \setminus \{j'_2\}) = \mathbb{S}$, (\mathbb{D}, \mathbb{F}) cannot be optimal given fixed \mathbb{S} . As a result, any optimal assignment $(\mathbb{D}^*, \mathbb{F}^*)$ must satisfy that a variant in \mathbb{D}^* is either no more popular than all variants in \mathbb{F}^* or no less popular than all variants in \mathbb{F}^* . \square

Proof of Proposition 2 Consider any given assortment \mathbb{S} . Let $\sigma_1 < \sigma_2$, and $(\mathbb{D}_i^*, \mathbb{F}_i^*)$ be any optimal assignment under the condition $\sigma = \sigma_i$ ($i = 1, 2$). By Proposition 1, $(\mathbb{D}_i^*, \mathbb{F}_i^*)$ ($i = 1, 2$) must satisfy the clustering structure. If $\mathbb{F}_1^* = \emptyset$, we must have $\max\{v_i : i \in \mathbb{F}_2^*\} \geq 0 = \max\{v_i : i \in \mathbb{F}_1^*\}$. Now assume $\mathbb{F}_1^* \neq \emptyset$. Consider any assignment (\mathbb{D}, \mathbb{F}) that satisfies the clustering structure, $|\mathbb{F}| \leq |\mathbb{F}_1^*|$ and $\max\{v_i : i \in \mathbb{F}\} < \max\{v_i : i \in \mathbb{F}_1^*\}$. We must have $\sum_{i \in \mathbb{F}_1^*} v_i > \sum_{i \in \mathbb{F}} v_i$ and $\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i > \sum_{i \in \mathbb{F} \setminus \mathbb{F}_1^*} v_i$. In addition, we have $\mathbb{D}_1^* \setminus \mathbb{D} = \mathbb{F} \setminus \mathbb{F}_1^*$ and $\mathbb{D} \setminus \mathbb{D}_1^* = \mathbb{F}_1^* \setminus \mathbb{F}$ because $\mathbb{D} \cup \mathbb{F} = \mathbb{D}_1^* \cup \mathbb{F}_1^* = \mathbb{S}$.

If condition (A) holds, then either $c_D \geq \frac{p}{2}$, or $c_D < \frac{p}{2}$ and $c_F(2) \geq p - c_D > \frac{p}{2}$. Note that, the function

$$f_{\text{Proposition 2.1}}(c) = \frac{p\phi(\Phi^{-1}(1 - \frac{c}{p}))}{\Lambda}$$

increases in c when $c < \frac{p}{2}$ and decreases in c when $c \geq \frac{p}{2}$. In addition, $f_{\text{Proposition 2.1}}(\cdot)$ is symmetric with respect to $c = \frac{p}{2}$, i.e., $f_{\text{Proposition 2.1}}(c) = f_{\text{Proposition 2.1}}(p - c)$. If $c_D \geq \frac{p}{2}$, we have $m_T(2) = f_{\text{Proposition 2.1}}(c_T(2)) < f_{\text{Proposition 2.1}}(c_D) = m_D$ and $m_T(2) > m_T(3) > \dots > m_T(|\mathbb{S}|)$; if $c_F(2) \geq p - c_D > \frac{p}{2}$, we also have $m_T(2) = f_{\text{Proposition 2.1}}(c_T(2)) < f_{\text{Proposition 2.1}}(p - c_D) = f_{\text{Proposition 2.1}}(c_D) = m_D$ and $m_T(2) > m_T(3) > \dots > m_T(|\mathbb{S}|)$. Let \mathbb{G} denote a set of the $|\mathbb{F}|$ most popular variants in \mathbb{F}_μ^* , then $|\mathbb{G}| = |\mathbb{F}|$ and the k th popular variant in \mathbb{G} is more popular than the k th popular variant in \mathbb{F} for $k \in \{1, \dots, |\mathbb{F}|\}$. Also, consider a fact that the function

$$f_{\text{Proposition 2.2}}(x_1, \dots, x_n) = \sum_{i=1}^n \sqrt{x_i} - \sqrt{\sum_{i=1}^n x_i},$$

where $x_i > 0$, increases in all x_i s. With all above facts we have

$$\begin{aligned}
& \mathcal{M}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{M}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) \\
&= \frac{\sigma}{1 + \sum_{k \in \mathbb{S}} v_k} \left[m_T(|\mathbb{F}_1^*|) \sqrt{\sum_{i \in \mathbb{F}_1^*} v_i} - m_T(|\mathbb{F}|) \sqrt{\sum_{i \in \mathbb{F}} v_i} + m_D \sum_{i \in \mathbb{D}_1^*} \sqrt{v_i} - m_D \sum_{i \in \mathbb{D}} \sqrt{v_i} \right] \\
&< \frac{\sigma}{1 + \sum_{k \in \mathbb{S}} v_k} \left[m_T(|\mathbb{F}|) \left(\sqrt{\sum_{i \in \mathbb{F}_1^*} v_i} - \sqrt{\sum_{i \in \mathbb{F}} v_i} \right) + m_D \sum_{i \in \mathbb{D}_1^*} \sqrt{v_i} - m_D \sum_{i \in \mathbb{D}} \sqrt{v_i} \right] \\
&< \frac{\sigma m_D}{1 + \sum_{k \in \mathbb{S}} v_k} \left[\sqrt{\sum_{i \in \mathbb{F}_1^*} v_i} + \sum_{i \in \mathbb{D}_1^*} \sqrt{v_i} - \sqrt{\sum_{i \in \mathbb{F}} v_i} - \sum_{i \in \mathbb{D}} \sqrt{v_i} \right] \\
&< \frac{\sigma m_D}{1 + \sum_{k \in \mathbb{S}} v_k} \left[\sqrt{\sum_{i \in \mathbb{G}} v_i} + \sum_{i \in \mathbb{S} \setminus \mathbb{G}} \sqrt{v_i} - \sqrt{\sum_{i \in \mathbb{F}} v_i} - \sum_{i \in \mathbb{D}} \sqrt{v_i} \right] \text{ (by Jensen's inequality)} \\
&= \frac{\sigma m_D}{1 + \sum_{k \in \mathbb{S}} v_k} \left[\left(\sqrt{\sum_{i \in \mathbb{G}} v_i} - \sum_{i \in \mathbb{G}} \sqrt{v_i} \right) - \left(\sqrt{\sum_{i \in \mathbb{F}} v_i} - \sum_{i \in \mathbb{F}} \sqrt{v_i} \right) \right] \leq 0 \text{ (by the property of } f_{\text{Proposition 2.2}} \text{)}.
\end{aligned}$$

If condition (B) holds, note that

$$\begin{aligned}
& \mathcal{P}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{P}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) \\
&= (c_D - c_T(|\mathbb{F}_1^*|)) \frac{\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} - (c_D - c_T(|\mathbb{F}|)) \frac{\sum_{i \in \mathbb{F} \setminus \mathbb{F}_1^*} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} + (c_T(|\mathbb{F}|) - c_T(|\mathbb{F}_1^*|)) \frac{\sum_{i \in \mathbb{F} \cap \mathbb{F}_1^*} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} \\
&< (c_D - c_T(|\mathbb{F}_1^*|)) \frac{\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} - (c_D - c_T(|\mathbb{F}|)) \frac{\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} + (c_T(|\mathbb{F}|) - c_T(|\mathbb{F}_1^*|)) \frac{\sum_{i \in \mathbb{F} \cap \mathbb{F}_1^*} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} \\
&= (c_T(|\mathbb{F}|) - c_T(|\mathbb{F}_1^*|)) \frac{\sum_{i \in \mathbb{F}_1^*} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} \leq 0.
\end{aligned}$$

Also,

$$\begin{aligned}
& \mathcal{K}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{K}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) \\
&= \frac{1}{\Lambda} (K_D \cdot (|\mathbb{D}_1^*| - |\mathbb{D}|) + K_T(|\mathbb{F}_1^*|) - K_T(|\mathbb{F}|)) \\
&= \frac{1}{\Lambda} (K_D \cdot (|\mathbb{F}| - |\mathbb{F}_1^*|) + K_T(|\mathbb{F}_1^*|) - K_T(|\mathbb{F}|)) \\
&\geq \frac{1}{\Lambda} (-K_D \cdot (|\mathbb{F}_1^*| - |\mathbb{F}|) + K_D \cdot (|\mathbb{F}_1^*| - |\mathbb{F}|)) = 0.
\end{aligned}$$

Because $(\mathbb{D}_1^*, \mathbb{F}_1^*)$ is optimal under $\sigma = \sigma_1$,

$$\begin{aligned}
& \Pi_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \Pi_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) \\
&= [\mathcal{P}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{P}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1)] \\
&\quad - [\mathcal{M}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{M}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1)] \\
&\quad - [\mathcal{K}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{K}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1)] \geq 0.
\end{aligned}$$

Thus we must have

$$\mathcal{M}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{M}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) < 0.$$

Consequently, under either condition (A) or condition (B), we have

$$\mathcal{M}_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \mathcal{M}_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) < 0,$$

which means

$$m_D \frac{\sum_{i \in \mathbb{D}_1^*} \sqrt{v_i}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} + m_T(|\mathbb{F}_1^*|) \frac{\sqrt{\sum_{j \in \mathbb{F}_1^*} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_D \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_T(|\mathbb{F}|) \frac{\sqrt{\sum_{j \in \mathbb{F}} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} < 0.$$

As a result,

$$\begin{aligned} & [\Pi_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_2) - \Pi_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_2)] - [\Pi_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \Pi_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1)] \\ &= -(\sigma_2 - \sigma_1) \left[m_D \frac{\sum_{i \in \mathbb{D}_1^*} \sqrt{v_i}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} + m_T(|\mathbb{F}_1^*|) \frac{\sqrt{\sum_{j \in \mathbb{F}_1^*} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_D \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_T(|\mathbb{F}|) \frac{\sqrt{\sum_{j \in \mathbb{F}} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} \right] > 0 \end{aligned}$$

and thus

$$\Pi_{DT}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_2) - \Pi_{DT}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_2) > 0.$$

It means that, any assignment (\mathbb{D}, \mathbb{F}) that satisfies the clustering structure, $|\mathbb{F}| \leq |\mathbb{F}_1^*|$ and $\max\{v_i : i \in \mathbb{F}\} < \max\{v_i : i \in \mathbb{F}_1^*\}$ must be sub-optimal under $\sigma = \sigma_2$. But $(\mathbb{D}_2^*, \mathbb{F}_2^*)$ is optimal. Hence, either $\mathbb{F}_2^* > \mathbb{F}_1^*$, or $\mathbb{F}_2^* \leq \mathbb{F}_1^*$ and $\max\{v_i : i \in \mathbb{F}\} \geq \max\{v_i : i \in \mathbb{F}_1^*\}$. \square

Proof of Proposition 3 Let

$$\Delta_{DT}(\mathbb{D}, \mathbb{F}) = \Pi_{DT}(\mathbb{D}, \mathbb{F}) - \mathcal{K}_{DT}(\mathbb{D}, \mathbb{F}),$$

i.e., the normalized profit without charging fixed costs.

Consider any assortment \mathbb{S} that violates the ordered structure, then we have $\min\{v_k : k \in \mathbb{S}\} < \max\{v_l : l \in \mathbb{W}\}$. Let k' be one least popular variant in \mathbb{S} and l' be one most popular variant in \mathbb{W} , then $v_{k'} < v_{l'}$. For any assignment of \mathbb{S} , (\mathbb{D}, \mathbb{F}) , we consider the following two cases depending on which technology variant k' is assigned to.

(i) $k' \in \mathbb{D}$. Consider the following three assortment-and-assignment strategies:

(1) Assortment \mathbb{S} and assignment (\mathbb{D}, \mathbb{F}) with

$$\begin{aligned} \Delta_{DT}(\mathbb{D}, \mathbb{F}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} v_i + v_{k'}}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{k'}} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F}} v_j}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{k'}} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} \sqrt{v_i} + \sqrt{v_{k'}}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{k'}}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F}} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{k'}}}, \end{aligned}$$

(2) assortment $\mathbb{S} \setminus \{k'\}$ and assignment $(\mathbb{D} \setminus \{k'\}, \mathbb{F})$ with

$$\begin{aligned} \Delta_{DT}(\mathbb{D} \setminus \{k'\}, \emptyset) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} v_i + 0}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + 0} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F}} v_j}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + 0} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} \sqrt{v_i} + \sqrt{0}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + 0}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F}} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + 0}}, \end{aligned}$$

and (3) assortment $\mathbb{S} \setminus \{k'\} \cup \{l'\}$ and assignment $(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F})$ with

$$\begin{aligned} \Delta_{DT}(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} v_i + v_{l'}}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{l'}} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F}} v_j}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{l'}} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} \sqrt{v_i} + \sqrt{v_{l'}}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{l'}}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F}} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + v_{l'}}}. \end{aligned}$$

The function

$$\begin{aligned} f_{\text{Proposition 3.1}}(t) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{k'\}} v_i + t}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F}} v_j}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{S} \setminus \{k'\}} \sqrt{v_i} + \sqrt{t}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F}} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t}} \\ &= \frac{(p - c_D) \left(\sum_{i \in \mathbb{D} \setminus \{k'\}} v_i + t \right) + (p - c_T(|\mathbb{F}|)) \sum_{j \in \mathbb{F}} v_j}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t} \\ &\quad + \frac{-m_D \sigma \left(\sum_{i \in \mathbb{D} \setminus \{k'\}} \sqrt{v_i} + \sqrt{t} \right) \left(\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t} \right) - m_T(|\mathbb{F}|) \sigma \left(\sqrt{\sum_{j \in \mathbb{F}} v_j} \right) \left(\sqrt{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t} \right)}{1 + \sum_{k \in \mathbb{S} \setminus \{k'\}} v_k + t} \end{aligned}$$

is strictly quasi-convex in t by the result in Mangasarian (1969) (Problem 1 in Chapter 9: $\Phi(\cdot)/\psi(\cdot)$ is quasi-convex on convex set Γ if $\Phi(\cdot)$ is convex on Γ and $\psi(\cdot)$ is positive and linear on Γ) and the fact that a function in t with form $(a + \sqrt{t})(\sqrt{b+t})$ where $a \geq 0$ and $b > 0$ is strictly concave on $[0, \infty)$. Hence it is

strictly decreasing, strictly increasing, or first strictly decreasing and then strictly increasing on $[0, v_{l'}]$. Note that $f_{\text{Proposition 3.1}}(v_{k'}) = \Delta_{DT}(\mathbb{D}, \mathbb{F})$, $f_{\text{Proposition 3.1}}(0) = \Delta_{DT}(\mathbb{D} \setminus \{k'\}, \mathbb{F})$, and $f_{\text{Proposition 3.1}}(v_{l'}) = (\mathbb{D} \setminus \{k'\}) \cup \{l'\}, \mathbb{F}$. Hence, by $0 < v_{k'} < v_{l'}$, we have either $\Delta_{DT}(\mathbb{D} \setminus \{k'\}, \mathbb{F}) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, or $\Delta_{DT}(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F}) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, or both. If $\Delta_{DT}(\mathbb{D} \setminus \{k'\}, \mathbb{F}) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, $\Pi_{DT}(\mathbb{D} \setminus \{k'\}, \mathbb{F}) = \Delta_{DT}(\mathbb{D} \setminus \{k'\}, \mathbb{F}) - \frac{\kappa_D}{\Lambda} \cdot (|\mathbb{D}| - 1) - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} > \Delta_{DT}(\mathbb{D}, \mathbb{F}) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} = \Pi_{DT}(\mathbb{D}, \mathbb{F})$. If $\Delta_{DT}(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F}) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, $\Pi_{DT}(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F}) = \Delta_{DT}(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F}) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} > \Delta_{DT}(\mathbb{D}, \mathbb{F}) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} = \Pi_{DT}(\mathbb{D}, \mathbb{F})$. Hence, either $\Pi_{DT}(\mathbb{D} \setminus \{k'\}, \mathbb{F}) > \Pi_{DT}(\mathbb{D}, \mathbb{F})$, or $\Pi_{DT}(\mathbb{D} \setminus \{k'\} \cup \{l'\}, \mathbb{F}) > \Pi_{DT}(\mathbb{D}, \mathbb{F})$, or both.

(ii) $k' \in \mathbb{F}$. Consider the following three assortment-and-assignment strategies:

(1) Assortment \mathbb{S} and assignment (\mathbb{D}, \mathbb{F}) with

$$\begin{aligned} \Delta_{DT}(\mathbb{D}, \mathbb{F}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D}} v_i}{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}}{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}}}, \end{aligned}$$

(2) assortment \mathbb{D} and assignment (\mathbb{D}, \emptyset) with

$$\begin{aligned} \Delta_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D}} v_i}{1 + \sum_{i \in \mathbb{D}} v_i + 0} + (p - c_T(|\mathbb{F}|)) \frac{0}{1 + \sum_{i \in \mathbb{D}} v_i + 0} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + 0}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{0}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + 0}}, \end{aligned}$$

and (3) assortment $\mathbb{S} \setminus \{k'\} \cup \{l'\}$ and assignment $(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\})$ with

$$\begin{aligned} \Delta_{DT}(\mathbb{D}, \mathbb{F}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D}} v_i}{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}} + (p - c_T(|\mathbb{F}|)) \frac{\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}}{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}}} - m_T(|\mathbb{F}|) \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}}}. \end{aligned}$$

The function

$$\begin{aligned} f_{\text{Proposition 3.2}} &= (p - c_D) \frac{\sum_{i \in \mathbb{D}} v_i}{1 + \sum_{i \in \mathbb{D}} v_i + t} + (p - c_T(|\mathbb{F}|)) \frac{t}{1 + \sum_{i \in \mathbb{D}} v_i + t} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + t}} - m_T \sigma (c_T(|\mathbb{F}|)) \frac{\sqrt{t}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + t}} \end{aligned}$$

is strictly quasi-convex in t by the result in Mangasarian (1969). Thus the function is strictly decreasing, strictly increasing, or first strictly decreasing and then strictly increasing on $[0, \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}]$. Note that $f_{\text{Proposition 3.2}}(\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'}) = \Delta_{DT}(\mathbb{D}, \mathbb{F})$, $f_{\text{Proposition 3.2}}(0) = \Delta_{DT}(\mathbb{D}, \emptyset)$ and $f_{\text{Proposition 3.2}}(\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}) = \Delta_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\})$. Hence, by $\sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{k'} < \sum_{j \in \mathbb{F} \setminus \{k'\}} v_j + v_{l'}$, we have either $\Delta_{DT}(\mathbb{D}, \emptyset) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, or $\Delta_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\}) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, or both. If $\Delta_{DT}(\mathbb{D}, \emptyset) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, $\Pi_{DT}(\mathbb{D}, \emptyset) = \Delta_{DT}(\mathbb{D}, \emptyset) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| > \Delta_{DT}(\mathbb{D}, \mathbb{F}) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} = \Pi_{DT}(\mathbb{D}, \mathbb{F})$. If $\Delta_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\}) > \Delta_{DT}(\mathbb{D}, \mathbb{F})$, $\Pi_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\}) = \Delta_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\}) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} > \Delta_{DT}(\mathbb{D}, \mathbb{F}) - \frac{\kappa_D}{\Lambda} \cdot |\mathbb{D}| - \frac{\kappa_P(|\mathbb{F}|)}{\Lambda} = \Pi_{DT}(\mathbb{D}, \mathbb{F})$. Hence, either $\Pi_{DT}(\mathbb{D}, \emptyset) > \Pi_{DT}(\mathbb{D}, \mathbb{F})$, or $\Pi_{DT}(\mathbb{D}, \mathbb{F} \setminus \{k'\} \cup \{l'\}) > \Pi_{DT}(\mathbb{D}, \mathbb{F})$, or both.

Combining (i) and (ii), we conclude that any technology assignment of \mathbb{S} is dominated by some other assignment of another assortment. Hence, \mathbb{S} must be non-optimal. By contradiction, any optimal assortment must satisfy the ordered structure. \square

B.2. Strategy under 3D Printing

Since $c_P(|\mathbb{F}|)$ and $m_P(|\mathbb{F}|)$ are constant functions, we write them as c_P and m_P throughout Appendix B.

Proof of Proposition 4 (a) Let

$$\Delta_{DP}(\mathbb{D}, \mathbb{F}) = \Pi_{DP}(\mathbb{D}, \mathbb{F}) - \mathcal{K}_{DP}(\mathbb{D}, \mathbb{F}).$$

Suppose a technology assignment $(\mathbb{D}, \mathbb{F}) > 0$ violates the ordered structure, then we must have $\min\{v_i : i \in \mathbb{D}\} < \max\{v_j : j \in \mathbb{F}\}$. Let i' be one least popular variant in \mathbb{D} and j' be one most popular variant in \mathbb{F} , then $v_{i'} < v_{j'}$. Consider the following three assignments:

(1) (\mathbb{D}, \mathbb{F}) with

$$\Delta_{DP}(\mathbb{D}, \mathbb{F}) = (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + v_{i'}}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_P) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{i'}}{1 + \sum_{k \in \mathbb{S}} v_k} - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{v_{i'}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_P \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{i'}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}},$$

(2) $(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\})$ with

$$\begin{aligned} \Delta_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + 0}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_P) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - 0}{1 + \sum_{k \in \mathbb{S}} v_k} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{0}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_P \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - 0}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}}, \end{aligned}$$

and (3) $(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\})$ with

$$\begin{aligned} \Delta_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\}) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + v_{j'}}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_P) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{j'}}{1 + \sum_{k \in \mathbb{S}} v_k} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{v_{j'}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_P \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - v_{j'}}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}}. \end{aligned}$$

The function

$$f_{\text{Proposition 4.1}}(t) = (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} v_i + t}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_P) \frac{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - t}{1 + \sum_{k \in \mathbb{S}} v_k} - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \{i'\}} \sqrt{v_i} + \sqrt{t}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - m_P \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \cup \{i'\}} v_j - t}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}}$$

is strictly convex in $t \in [0, v_{j'}]$. Hence, it is strictly decreasing, strictly increasing, or first strictly decreasing and then strictly increasing on $[0, v_{j'}]$. Note that $f_{\text{Proposition 4.1}}(v_{i'}) = \Delta_{DP}(\mathbb{D}, \mathbb{F})$, $f_{\text{Proposition 4.1}}(0) = \Delta_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\})$, $f_{\text{Proposition 4.1}}(v_{j'}) = \Delta_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\})$. Hence, by $0 < v_{i'} < v_{j'}$, we have either $\Delta_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\}) > \Delta_{DP}(\mathbb{D}, \mathbb{F})$ or $\Delta_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\}) > \Delta_{DP}(\mathbb{D}, \mathbb{F})$, or both. Recall that $K_P(n+1) - K_P(n) \leq K_D$. If $\Delta_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\}) > \Delta_{DP}(\mathbb{D}, \mathbb{F})$, then $\Pi_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\}) = \Delta_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\}) - \frac{K_D}{\Lambda} \cdot (|\mathbb{D}| - 1) - \frac{K_P(|\mathbb{F}|+1)}{\Lambda} > \Delta_{DP}(\mathbb{D}, \mathbb{F}) - \frac{K_D}{\Lambda} \cdot |\mathbb{D}| - \frac{K_P(|\mathbb{F}|)}{\Lambda} = \Pi_{DP}(\mathbb{D}, \mathbb{F})$. If $\Delta_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\}) > \Delta_{DP}(\mathbb{D}, \mathbb{F})$, then $\Pi_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\}) = \Delta_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\}) - \frac{K_D}{\Lambda} \cdot |\mathbb{D}| - \frac{K_P(|\mathbb{F}|)}{\Lambda} > \Delta_{DP}(\mathbb{D}, \mathbb{F}) - \frac{K_D}{\Lambda} \cdot |\mathbb{D}| - \frac{K_P(|\mathbb{F}|)}{\Lambda} = \Pi_{DP}(\mathbb{D}, \mathbb{F})$. Hence, either $\Pi_{DP}(\mathbb{D} \setminus \{i'\}, \mathbb{F} \cup \{i'\}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$ or $\Pi_{DP}(\mathbb{D} \setminus \{i'\} \cup \{j'\}, \mathbb{F} \cup \{i'\} \setminus \{j'\}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$, or both.

In conclusion, a technology assignment that violates the ordered structure must be prevailed by some other assignment. By contradiction, any optimal technology assignment must be ordered.

(b) Consider any strategy (\mathbb{D}, \mathbb{F}) in which there are $i' \in \mathbb{D}$ and $j' \in \mathbb{F}$ satisfying $v_{i'} = v_{j'} = r$. Let $\mathbb{R} = \{v_i \in \mathbb{D} \cup \mathbb{F} : v_i = r\}$. Define

$$\begin{aligned} f_{\text{Proposition 4.2}}(n) &= (p - c_D) \frac{\sum_{i \in \mathbb{D} \setminus \mathbb{R}} v_i + nr}{1 + \sum_{k \in \mathbb{S}} v_k} + (p - c_P) \frac{\sum_{j \in \mathbb{F} \setminus \mathbb{R}} v_j + (|\mathbb{R}| - n)r}{1 + \sum_{k \in \mathbb{S}} v_k} - m_D \sigma \frac{\sum_{i \in \mathbb{D} \setminus \mathbb{R}} \sqrt{v_i} + n\sqrt{r}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} \\ &\quad - m_P \sigma \frac{\sqrt{\sum_{j \in \mathbb{F} \setminus \mathbb{R}} v_j + (|\mathbb{R}| - n)r}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} - \frac{K_D}{\Lambda} (|\mathbb{D} \setminus \mathbb{R}| + n) - \frac{K_P(|\mathbb{F} \setminus \mathbb{R}| + |\mathbb{R}| - n)}{\Lambda}, \end{aligned}$$

which is a strictly convex function for $0 \leq n \leq |\mathbb{R}|$. Hence, it is strictly decreasing, strictly increasing or first strictly decreasing and then strictly increasing on $[0, |\mathbb{R}|]$. Note that $f_{\text{Proposition 4.2}}(0) = \Pi_{DP}(\mathbb{D} \setminus \mathbb{R}, \mathbb{F} \cup \mathbb{R})$, $f_{\text{Proposition 4.2}}(|\mathbb{D} \cap \mathbb{R}|) = \Pi_{DP}(\mathbb{D}, \mathbb{F})$, and $f_{\text{Proposition 4.2}}(|\mathbb{R}|) = \Pi_{DP}(\mathbb{D} \cup \mathbb{R}, \mathbb{F} \setminus \mathbb{R})$. Hence, either $\Pi_{DP}(\mathbb{D} \setminus \mathbb{R}, \mathbb{F} \cup \mathbb{R}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$ or $\Pi_{DP}(\mathbb{D} \cup \mathbb{R}, \mathbb{F} \setminus \mathbb{R}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$, or both.

$\mathbb{R}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$ or $\Pi_{DP}(\mathbb{D} \cup \mathbb{R}, \mathbb{F} \setminus \mathbb{R}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$, or both. (\mathbb{D}, \mathbb{F}) must be non-optimal. By contradiction, any optimal technology assignment must assign equally popular variants to the same technology. \square

Proof of Proposition 5 Consider any given \mathbb{S} . Let $\sigma_1 < \sigma_2$, and $(\mathbb{D}_i^*, \mathbb{F}_i^*)$ be any optimal assignment under the condition $\sigma = \sigma_i$ ($i = 1, 2$). If $\mathbb{F}_1^* = \emptyset$, we must have $|\mathbb{F}_1^*| \leq |\mathbb{F}_2^*|$. Now assume $\mathbb{F}_1^* \neq \emptyset$. Consider any assignment (\mathbb{D}, \mathbb{F}) that satisfies (a) and (b) in Proposition 4 and $|\mathbb{F}| < |\mathbb{F}_1^*|$. We must have $\mathbb{D}_1^* \subset \mathbb{D}$, $\mathbb{F} \subset \mathbb{F}_1^*$. Note that $\mathbb{D} \setminus \mathbb{D}_1^* = \mathbb{F}_1^* \setminus \mathbb{F}$ because $\mathbb{D} \cup \mathbb{F} = \mathbb{D}_1^* \cup \mathbb{F}_1^* = \mathbb{S}$. Thus:

$$\begin{aligned} & \Pi_{DP}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \Pi_{DP}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1) \\ &= (c_D - c_P) \frac{\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} + \sigma_1 \frac{m_D \sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + m_P \sqrt{\sum_{j \in \mathbb{F}} v_j} - m_P \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} \\ & \quad + \frac{1}{\Lambda} (K_D \cdot |\mathbb{F}_1^* \setminus \mathbb{F}| + K_P(|\mathbb{F}|) - K_P(|\mathbb{F}_1^*|)) \geq 0. \end{aligned}$$

If condition (A) holds, then $m_D \geq m_P$ following the same rationale in the proof of Proposition 2. By Jensen's inequality,

$$m_D \sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + m_P \sqrt{\sum_{j \in \mathbb{F}} v_j} - m_P \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j} \geq m_P \left(\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + \sqrt{\sum_{j \in \mathbb{F}} v_j} - \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j} \right) > 0.$$

If condition (B) holds, then

$$K_D \cdot |\mathbb{F}_1^* \setminus \mathbb{F}| + K_P(|\mathbb{F}|) - K_P(|\mathbb{F}_1^*|) = -\kappa \cdot \mathbf{1}_{\{\mathbb{F}=\emptyset\}} \leq 0.$$

We also have

$$(c_D - c_P) \frac{\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} < 0.$$

Consequently,

$$m_D \sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + m_P \sqrt{\sum_{j \in \mathbb{F}} v_j} - m_P \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j} > 0$$

must hold given

$$(c_D - c_P) \frac{\sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} v_i}{1 + \sum_{k \in \mathbb{S}} v_k} + \sigma_1 \frac{m_D \sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + m_P \sqrt{\sum_{j \in \mathbb{F}} v_j} - m_P \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} + \frac{1}{\Lambda} (K_D \cdot |\mathbb{F}_1^* \setminus \mathbb{F}| + K_P(|\mathbb{F}|) - K_P(|\mathbb{F}_1^*|)) \geq 0.$$

In summary,

$$m_D \sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + m_P \sqrt{\sum_{j \in \mathbb{F}} v_j} - m_P \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j} > 0$$

holds under either (A) or (B). By $\sigma_1 < \sigma_2$,

$$\begin{aligned} & [\Pi_{DP}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_2) - \Pi_{DP}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_2)] - [\Pi_{DP}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_1) - \Pi_{DP}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_1)] \\ &= (\sigma_2 - \sigma_1) \frac{m_D \sum_{i \in \mathbb{F}_1^* \setminus \mathbb{F}} \sqrt{v_i} + m_P \sqrt{\sum_{j \in \mathbb{F}} v_j} - m_P \sqrt{\sum_{j \in \mathbb{F}_1^*} v_j}}{\sqrt{1 + \sum_{k \in \mathbb{S}} v_k}} > 0, \end{aligned}$$

and thus $\Pi_{DP}(\mathbb{D}_1^*, \mathbb{F}_1^* | \sigma = \sigma_2) - \Pi_{DP}(\mathbb{D}, \mathbb{F} | \sigma = \sigma_2) > 0$ for any assignment (\mathbb{D}, \mathbb{F}) satisfying (a) and (b) in Proposition 4 and $|\mathbb{F}| < |\mathbb{F}_1^*|$. Because $(\mathbb{D}_2^*, \mathbb{F}_2^*)$ is optimal under $\sigma = \sigma_2$, we must have $|\mathbb{F}_1^*| \leq |\mathbb{F}_2^*|$. \square

Proof of Proposition 6 The proof follows by replacing $c_T(|\mathbb{F}|)$ with c_P in the proof of Proposition 3. \square

Proof of Proposition 7 Since $K_P(\cdot)$ is constant, we denote it as K_P in this proof. We consider a different process of determining the firm's optimal assortment-and-assignment strategy: the firm chooses \mathbb{D} first, and then \mathbb{F} ; the assortment is thus given by $\mathbb{S} = \mathbb{D} \cup \mathbb{F}$. For any given \mathbb{D} , consider the function

$$\begin{aligned} f_{\text{Proposition 7}}(t) &= (p - c_D) \frac{\sum_{i \in \mathbb{D}} v_i}{1 + \sum_{i \in \mathbb{D}} v_i + t} + (p - c_P) \frac{t}{1 + \sum_{i \in \mathbb{D}} v_i + t} \\ &\quad - m_D \sigma \frac{\sum_{i \in \mathbb{D}} \sqrt{v_i}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + t}} - m_P \sigma \frac{\sqrt{t}}{\sqrt{1 + \sum_{i \in \mathbb{D}} v_i + t}} - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P) \\ &= \frac{(p - c_D) (\sum_{i \in \mathbb{D}} v_i) + (p - c_P) t - m_D \sigma (\sum_{i \in \mathbb{D}} \sqrt{v_i}) \sqrt{1 + \sum_{i \in \mathbb{D}} v_i + t} - m_P \sigma \sqrt{t} \sqrt{1 + \sum_{i \in \mathbb{D}} v_i + t}}{1 + \sum_{i \in \mathbb{D}} v_i + t} \\ &\quad - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P), \end{aligned}$$

which is strictly quasi-convex by Mangasarian (1969). Hence, it is strictly decreasing, strictly increasing or first strictly decreasing and then strictly increasing on $[0, \infty)$. Thus, $\forall \mathbb{F} \subset \mathbb{U} \setminus \mathbb{D}$ satisfying $\mathbb{F} \neq \emptyset$, we have either $f_{\text{Proposition 7}}(0) > f_{\text{Proposition 7}}(\sum_{j \in \mathbb{F}} v_j)$ or $f_{\text{Proposition 7}}(\mathbb{U} \setminus \mathbb{D}) > f_{\text{Proposition 7}}(\sum_{j \in \mathbb{F}} v_j)$, or both. That is, either $\Pi_{DP}(\mathbb{D}, \emptyset) > \Pi_{DP}(\mathbb{D}, \emptyset) - K_P > \Pi_{DP}(\mathbb{D}, \mathbb{F})$ or $\Pi_{DP}(\mathbb{D}, \mathbb{U} \setminus \mathbb{D}) > \Pi_{DP}(\mathbb{D}, \mathbb{F})$, or both.

Note that the above argument holds for any given \mathbb{D} . We thus conclude that, in any optimal assortment-and-assignment strategy, it must hold that $\mathbb{S} = \mathbb{U}$ or $\mathbb{S} \subset \mathbb{U}$ and $\mathbb{F} = \emptyset$. \square

B.3. Impact of Flexible Technologies on Product Variety

Proof of Proposition 8 Consider the following constructing process, from (A) to (I):

(A) Let

$$\Delta_{DT}(\mathbb{D}, \mathbb{F}) = \Pi_{DT}(\mathbb{D}, \mathbb{F}) - \mathcal{K}_{DT}(\mathbb{D}, \mathbb{F}).$$

We claim that $\Delta_{DT}(\{1, 2, 3\}, \emptyset) > \Delta_{DT}(\{1, 2\}, \emptyset)$ implies that $\Delta_{DT}(\{1, 2\}, \emptyset) > \Delta_{DT}(\{1\}, \emptyset) > \Delta_{DT}(\emptyset, \emptyset)$.

Because of the strict quasi-convexity (by Mangasarian, 1969) of function

$$f_{\text{Proposition 8.1}}(x) = (p - c_D) \frac{v_1 + v_2 + x}{1 + v_1 + v_2 + x} - m_D \sigma \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{x}}{\sqrt{1 + v_1 + v_2 + x}} \right),$$

$f_{\text{Proposition 8.1}}(v_3) = \Delta_{DT}(\{1, 2, 3\}, \emptyset) > \Delta_{DT}(\{1, 2\}, \emptyset) = f_{\text{Proposition 8.1}}(0)$ implies that $f_{\text{Proposition 7.1}}(v_2) > f_{\text{Proposition 8.1}}(0)$ since $v_2 \geq v_3$. However, the function

$$f_{\text{Proposition 8.2}}(n) = (p - c_D) \frac{nv_1}{1 + nv_1} - m_D \sigma \frac{n\sqrt{v_1}}{\sqrt{1 + nv_1}}$$

is strictly quasi-concave (by Mangasarian, 1969). Hence $f_{\text{Proposition 8.2}}(3) = f_{\text{Proposition 8.1}}(v_2) > f_{\text{Proposition 8.1}}(0) = f_{\text{Proposition 8.2}}(2)$ implies that $\Delta_{DT}(\{1, 2\}, \emptyset) = f_{\text{Proposition 8.2}}(2) > f_{\text{Proposition 8.2}}(1) = \Delta_{DT}(\{1\}, \emptyset) > f_{\text{Proposition 8.2}}(0) = \Delta_{DT}(\emptyset, \emptyset)$.

(B) $\Delta_{DT}(\{1, 2, 3\}, \emptyset) > \Delta_{DT}(\{1, 2\}, \emptyset)$ iff

$$\sigma < \frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1} + \sqrt{v_2}}{\sqrt{1 + v_1 + v_2}} \right).$$

By simple algebra,

$$\begin{aligned} & \frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1} + \sqrt{v_2}}{\sqrt{1 + v_1 + v_2}} \right) \\ & < \frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1} + \sqrt{v_2}}{\sqrt{1 + v_1 + v_2}} \right). \end{aligned}$$

Thus, we can take $\bar{\sigma}$ satisfying

$$\begin{aligned} & \frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1} + \sqrt{v_2}}{\sqrt{1 + v_1 + v_2}} \right) < \bar{\sigma} \\ & < \frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1} + \sqrt{v_2}}{\sqrt{1 + v_1 + v_2}} \right) \end{aligned}$$

and let $\epsilon_1 = \Delta_{DT}(\{1, 2, 3\}, \emptyset | \sigma = \bar{\sigma}) - \Delta_{DT}(\{1, 2\}, \emptyset | \sigma = \bar{\sigma})$. If $\sigma < \bar{\sigma}$ then $\Delta_{DT}(\{1, 2, 3\}, \emptyset) - \Delta_{DT}(\{1, 2\}, \emptyset) > \epsilon_1$. Also, let $\epsilon_2 = \Delta_{DT}(\{1, 2\}, \emptyset | \sigma = \bar{\sigma}) - \Delta_{DT}(\{1\}, \emptyset | \sigma = \bar{\sigma}) > 0$ and $\epsilon_3 = \Delta_{DT}(\{1\}, \emptyset | \sigma = \bar{\sigma}) - \Delta_{DT}(\emptyset, \emptyset | \sigma = \bar{\sigma}) > 0$. If $\sigma < \bar{\sigma}$, then $\Delta_{DT}(\{1, 2\}, \emptyset) - \Delta_{DT}(\{1\}, \emptyset) > \epsilon_2$ and $\Delta_{DT}(\{1\}, \emptyset) - \Delta_{DT}(\emptyset, \emptyset) > \epsilon_3$.

(C) Now, take $\underline{\sigma}$ satisfying

$$\frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1} + \sqrt{v_2}}{\sqrt{1 + v_1 + v_2}} \right) < \underline{\sigma} < \bar{\sigma}.$$

Let $2\epsilon_4 = \Delta_{DT}(\{3\}, \{1, 2\} | c_T(2) \rightarrow c_D, \sigma = \underline{\sigma}) - \Delta_{DT}(\{1\}, \{2, 3\} | c_T(2) \rightarrow c_D, \sigma = \underline{\sigma}) > 0$. By continuity, $\exists c_{21} > c_D$ such that when $c_D < c_T(2) < c_{21}$, $\Delta_{DT}(\{3\}, \{1, 2\} | \sigma = \underline{\sigma}) - \Delta_{DT}(\{1\}, \{2, 3\} | \sigma = \underline{\sigma}) > \epsilon_4$,

$$m_T(2) \frac{\sqrt{v_2 + v_3} - \sqrt{v_1 + v_2}}{\sqrt{1 + v_1 + v_2 + v_3}} + m_D \frac{\sqrt{v_1} - \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} > 0,$$

and thus $\Delta_{DT}(\{3\}, \{1, 2\}) - \Delta_{DT}(\{1\}, \{2, 3\})$ increases in σ . As a result, when $c_D < c_T(2) < c_{21}$ and $\sigma > \underline{\sigma}$, $\Delta_{DT}(\{3\}, \{1, 2\}) - \Delta_{DT}(\{1\}, \{2, 3\}) > \epsilon_4$.

(D) Similarly, we are able to find $c_{22} > c_D$ and $\epsilon_5 > 0$, such that when $c_D < c_T(2) < c_{22}$ and $\sigma > \underline{\sigma}$, $\Delta_{DT}(\{1\}, \{2, 3\}) - \Delta_{DT}(\{1, 2, 3\}, \emptyset) > \epsilon_5$.

(E) Because

$$\frac{p - c_D}{m_D} \cdot \frac{v_3}{(1 + v_1 + v_2 + v_3)(1 + v_1 + v_2)} / \left(\frac{\sqrt{v_1 + v_2} + \sqrt{v_3}}{\sqrt{1 + v_1 + v_2 + v_3}} - \frac{\sqrt{v_1 + v_2}}{\sqrt{1 + v_1 + v_2}} \right) < \underline{\sigma},$$

$\Delta(\emptyset, \{1, 2\} | \sigma = \underline{\sigma}, c_T(2) \rightarrow c_D) - \Delta(\{3\}, \{1, 2\} | \sigma = \underline{\sigma}, c_T(2) \rightarrow c_D) > 0$. Let $2\epsilon_6 = \Delta(\emptyset, \{1, 2\} | \sigma = \underline{\sigma}, c_T(2) \rightarrow c_D) - \Delta(\{3\}, \{1, 2\} | \sigma = \underline{\sigma}, c_T(2) \rightarrow c_D)$. Similar to C), there is $c_{23} > c_D$, such that, when $c_D < c_T(2) < c_{23}$ and $\sigma > \underline{\sigma}$, $\Delta_{DT}(\emptyset, \{1, 2\}) - \Delta_{DT}(\{3\}, \{1, 2\}) > \epsilon_6$.

(F) Let $c_2 = \min\{c_{21}, c_{22}, c_{23}\}$.

(G) $\Delta_{DT}(\emptyset, \{1, 2, 3\} | \sigma = \bar{\sigma}, c_T(2) = c_2, c_T(3) \rightarrow p) = 0$. Let $2\epsilon_7 = \Delta_{DT}(\emptyset, \{1, 2\} | \sigma = \bar{\sigma}, c_T(2) = c_2, c_T(3) \rightarrow p) - \Delta_{DT}(\emptyset, \{1, 2, 3\} | \sigma = \bar{\sigma}, c_T(2) = c_2, c_T(3) \rightarrow p)$. $\exists c_3 < p$, such that when $c_T(3) > c_3$, $\Delta_{DT}(\emptyset, \{1, 2\} | \sigma = \bar{\sigma}, c_T(2) = c_2) - \Delta_{DT}(\emptyset, \{1, 2, 3\} | \sigma = \bar{\sigma}, c_T(2) = c_2) > \epsilon_7$,

$$m_T(2) \frac{\sqrt{v_1 + v_2}}{\sqrt{1 + v_1 + v_2 + v_3}} - m_T(3) \frac{\sqrt{v_1 + v_2 + v_3}}{1 + v_1 + v_2 + v_3} > 0,$$

and thus $\Delta_{DT}(\emptyset, \{1, 2\} | c_T(2) = c_2) - \Delta_{DT}(\emptyset, \{1, 2, 3\} | c_T(2) = c_2)$ decreases in σ . Note that $\Delta_{DT}(\emptyset, \{1, 2\})$ decreases in $c_T(2)$. Consequently, when $\sigma < \bar{\sigma}$, $c_T(2) < c_2$ and $c_T(3) > c_3$, $\Delta_{DT}(\emptyset, \{1, 2\}) - \Delta_{DT}(\emptyset, \{1, 2, 3\}) > \epsilon_7$.

(H) Let $K = \frac{1}{2} \min\{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6, \epsilon_7\}$.

(I) It is straightforward to show that the optimal assortment decision in Case \mathbb{D} must be ordered. Hence in Case \mathbb{D} , we only need to consider assortments \emptyset , $\{1\}$, $\{1, 2\}$, and $\{1, 2, 3\}$ to find the optimal one. Also, by Proposition 1 and Proposition 3, we only need to consider assignments (\emptyset, \emptyset) , $(\{1\}, \emptyset)$, $(\{1, 2\}, \emptyset)$, $(\{1, 2, 3\}, \emptyset)$, $(\emptyset, \{1, 2\})$, $(\emptyset, \{1, 2, 3\})$, $(\{3\}, \{1, 2\})$ and $(\{1\}, \{2, 3\})$ to find the optimal assortment-and-assignment strategy in Case DT .

Note that $\Pi_{DT}(\emptyset, \emptyset) = \Omega_D(\emptyset)$, $\Pi_{DT}(\{1\}, \emptyset) = \Omega_D(\{1\})$, $\Pi_{DT}(\{1, 2\}, \emptyset) = \Omega_D(\{1, 2\})$, and $\Pi_{DT}(\{1, 2, 3\}, \emptyset) = \Omega_D(\{1, 2, 3\})$. When $\underline{\sigma} < \sigma < \bar{\sigma}$, $c_T(2) < c_2$, $c_T(3) > c_3$, and $K_D, K_T(3) < K$ (which implies $K_T(2) < K$ since $K_T(2) \leq K_T(3)$), the following hold:

(i) $\Omega_D(\{1, 2, 3\}) > \Omega_D(\{1, 2\}) > \Omega_D(\{1\}) > \Omega_D(\emptyset)$;

(ii) $\Pi_{DT}(\emptyset, \{1, 2\}) > \Pi_{DT}(\{3\}, \{1, 2\}) > \Pi_{DT}(\{1\}, \{2, 3\}) > \Pi_{DT}(\{1, 2, 3\}, \emptyset) > \Pi_{DT}(\{1, 2\}, \emptyset) > \Pi_{DT}(\{1\}, \emptyset) > \Pi_{DT}(\emptyset, \emptyset)$;

(iii) $\Pi_{DT}(\emptyset, \{1, 2\}) > \Pi_{DT}(\emptyset, \{1, 2, 3\})$.

Hence, the optimal assortment in Case D is $\{1, 2, 3\}$ and the optimal assortment in Case DP is $\{1, 2\}$ when $\underline{\sigma} < \sigma < \bar{\sigma}$, $c_T(2) < c_2$, $c_T(3) > c_3$, and $K_D, K_T(3) < K$. \square

By Proposition 4(a) and Proposition 6, we only need to consider the ordered assortment-and-assignment strategies to search for the optimal one in Case DP . Define $\mathbb{A}^S = \{\emptyset, \{1\}, \{1, 2\}, \dots, \{1, \dots, |\mathbb{U}|\}\}$, i.e., the “popular assortment set” in Kök et al. (2009). Next we only consider assortments in the “popular assortment set,” which can represent other ordered assortments not in the “popular assortment set.” For example, $\{1, 3\}$ can be represented by $\{1, 2\}$ when $v_2 = v_3$. In addition, given $\mathbb{S} \in \mathbb{A}^S$, any optimal technology assignment must be in $\mathbb{A}^{DF} = \{(\emptyset, \{1, \dots, |\mathbb{S}|\}), (\{1\}, \{2, \dots, |\mathbb{S}|\}), \dots, (\{1, \dots, |\mathbb{S}|\}, \emptyset)\}$ by both Proposition 4(a) and Proposition 4(b). Consequently, when $\mathbb{S} \in \mathbb{A}^S$ and $(\mathbb{D}, \mathbb{F}) \in \mathbb{A}^{DF}$, the firm’s first-stage assortment decision only depends on $|\mathbb{S}|$ and second-stage technology assignment decision only depends on $|\mathbb{D}|$ and $|\mathbb{F}|$. We can thus formulate an algebraic form of Π_{DP} as

$$\begin{aligned} \Pi_{DP}^A(|\mathbb{D}|, |\mathbb{F}|) = & (p - c_D) \frac{\sum_{i=1}^{|\mathbb{D}|} v_i}{1 + \sum_{i=1}^{|\mathbb{D}|+|\mathbb{F}|} v_i} + (p - c_P) \frac{\sum_{i=|\mathbb{D}|+1}^{|\mathbb{D}|+|\mathbb{F}|} v_i}{1 + \sum_{i=1}^{|\mathbb{D}|+|\mathbb{F}|} v_i} \\ & - m_D \sigma \frac{\sum_{i=1}^{|\mathbb{D}|} \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{|\mathbb{D}|+|\mathbb{F}|} v_i}} - m_P \sigma \frac{\sqrt{\sum_{i=|\mathbb{D}|+1}^{|\mathbb{D}|+|\mathbb{F}|} v_i}}{\sqrt{1 + \sum_{i=1}^{|\mathbb{D}|+|\mathbb{F}|} v_i}} - \frac{1}{\Lambda} (K_D \cdot |\mathbb{D}| + K_P(|\mathbb{F}|)), \end{aligned}$$

in which $\sum_{i=n}^{n-1} v_i := 0$ for any $n \in \mathbb{N}^+$. Also, an algebraic form of Ω_{DP} is

$$\Omega_{DP}^A(|\mathbb{S}|) = \max_{|\mathbb{D}|+|\mathbb{F}|=|\mathbb{S}|, |\mathbb{D}|, |\mathbb{F}| \in \mathbb{N}} \Pi_{DP}^A(|\mathbb{D}|, |\mathbb{F}|).$$

Similarly, in Case D , we can also define

$$\Omega_D^A(|\mathbb{S}|) = (p - c_D) \frac{\sum_{i=1}^{|\mathbb{S}|} v_i}{1 + \sum_{i=1}^{|\mathbb{S}|} v_i} - m_D \sigma \frac{\sum_{i=1}^{|\mathbb{S}|} \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{|\mathbb{S}|} v_i}} - \frac{1}{\Lambda} K_D |\mathbb{S}|.$$

To compare the product varieties in Case D and Case DP , we only need to compare the largest maximizers of $\Omega_D^A(|\mathbb{S}|)$ and $\Omega_{DP}^A(|\mathbb{S}|)$.

In order to prove Proposition 9, we need two lemmas.

LEMMA 1. Let $f(x)$ and $g(x)$ be functions defined on $\{0, \dots, n\}$. If $f(x) - g(x) > f(x-1) - g(x-1)$ for $\forall x \in \{1, \dots, n\}$, then the largest maximizer of $f(x)$ is no smaller than the largest maximizer of $g(x)$.

Proof of Lemma 1 Let x_f^* be the largest maximizer of $f(x)$, we thus have $\forall x > x_f^*$, $f(x) - f(x_f^*) < 0$. Consequently, $\forall x > x_f^*$,

$$g(x) - g(x_f^*) = \sum_{i=x_f^*+1}^x [g(i) - g(i-1)] \leq \sum_{i=x_f^*+1}^x [f(i) - f(i-1)] = f(x) - f(x_f^*) < 0.$$

That is, the largest maximizer of $g(x)$ is less than or equal to x_f^* . \square

LEMMA 2. If positive numbers a, b, c, k_1, k_2, k_3 satisfy: (i) $a > b$ and (ii) $\frac{k_1\sqrt{c-k_2\sqrt{b}}}{\sqrt{a}} - k_3\frac{b}{a} > 0$, then $\forall 0 < v \leq \frac{b^2}{c}$, $\frac{k_1\sqrt{c-k_2\sqrt{b}}}{\sqrt{a}} - k_3\frac{b}{a} < \frac{k_1(\sqrt{c+\sqrt{v}})-k_2\sqrt{b+v}}{\sqrt{a+v}} - k_3\frac{b+v}{a+v}$.

Proof of Lemma 2 Without loss of generality, let $k_1 = 1$. Otherwise we can simply replace k_2 and k_3 with k_2/k_1 and k_3/k_1 respectively.

By $0 < v \leq \frac{b^2}{c}$, we have $\sqrt{c} < \frac{b}{\sqrt{v}}$. By $\frac{\sqrt{c-k_2\sqrt{b}}}{\sqrt{a}} - k_3\frac{b}{a} > 0$, we have $k_2 < \sqrt{\frac{c}{b}} - k_3\sqrt{\frac{b}{a}} < \sqrt{\frac{b}{v}} - k_3\sqrt{\frac{b}{a}}$, and $k_3 < \frac{\sqrt{ac}}{b} - k_2\sqrt{\frac{a}{b}} < \sqrt{\frac{a}{v}}$. Also, note that $\frac{b+v}{a+v} > \frac{b}{a}$ since $a > b > 0$ and $v > 0$. Hence

$$\begin{aligned} & \sqrt{c} \left(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{a+v}} \right) - \sqrt{\frac{v}{a+v}} + k_2 \left(\sqrt{\frac{b+v}{a+v}} - \sqrt{\frac{b}{a}} \right) + k_3 \left(\frac{b+v}{a+v} - \frac{b}{a} \right) \\ & < \frac{b}{\sqrt{v}} \left(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{a+v}} \right) - \sqrt{\frac{v}{a+v}} + \left(\sqrt{\frac{b}{v}} - k_3\sqrt{\frac{b}{a}} \right) \left(\sqrt{\frac{b+v}{a+v}} - \sqrt{\frac{b}{a}} \right) + k_3 \left(\frac{b+v}{a+v} - \frac{b}{a} \right) \\ & = \frac{b}{\sqrt{av}} - \frac{b}{\sqrt{(a+v)v}} - \sqrt{\frac{v}{a+v}} + \sqrt{\frac{b}{v}} \left(\sqrt{\frac{b+v}{a+v}} - \sqrt{\frac{b}{a}} \right) + k_3 \left(\frac{b+v}{a+v} - \sqrt{\frac{b+v}{a+v}} \sqrt{\frac{b}{a}} \right) \\ & < \frac{b}{\sqrt{av}} - \frac{b}{\sqrt{(a+v)v}} - \sqrt{\frac{v}{a+v}} + \sqrt{\frac{b}{v}} \left(\sqrt{\frac{b+v}{a+v}} - \sqrt{\frac{b}{a}} \right) + \sqrt{\frac{a}{v}} \left(\frac{b+v}{a+v} - \sqrt{\frac{b+v}{a+v}} \sqrt{\frac{b}{a}} \right) \\ & = -\frac{b}{\sqrt{(a+v)v}} - \sqrt{\frac{v}{a+v}} + \sqrt{\frac{a}{v}} \frac{b+v}{a+v} \\ & = \frac{b+v}{\sqrt{(a+v)v}} \left(\sqrt{\frac{a}{a+v}} - 1 \right) < 0, \end{aligned}$$

which means that

$$\frac{\sqrt{c-k_2\sqrt{b}}}{\sqrt{a}} - k_3\frac{b}{a} < \frac{(\sqrt{c+\sqrt{v}})-k_2\sqrt{b+v}}{\sqrt{a+v}} - k_3\frac{b+v}{a+v}.$$

The proof is thus concluded. \square

Now we can proceed to prove Proposition 9.

Proof of Proposition 9 Define $\eta_{DP} = \min\{|\mathbb{S}| \in \{0, 1, \dots, |\mathbb{U}|\} : \max\{\arg \max_{|\mathbb{D}| \in \{0, 1, \dots, |\mathbb{S}|\}} \Pi_{DP}^A(|\mathbb{D}|, |\mathbb{S}| - |\mathbb{D}|)\} < |\mathbb{S}|\}$, i.e., the smallest assortment size with which any optimal technology assignment is not dedicated-only. Clearly, $\eta_{DP} \geq 2$. Also, we have $\Omega_{DP}^A(\eta_{DP}) > \Omega_D^A(\eta_{DP})$ as $\Omega_D^A(\eta_{DP}) = \Pi_{DP}^A(|\mathbb{S}|, 0)$. We thus have

$\Omega_{DP}^A(\eta_{DP}) - \Omega_D^A(\eta_{DP}) > 0 = \Omega_{DP}^A(\eta_{DP} - 1) - \Omega_D^A(\eta_{DP} - 1) = 0$. In the following, by induction, we prove that the following two conditions hold for $\forall n \in \{\eta_{DP}, \dots, |\mathbb{U}|\}$:

- (i) $\Omega_{DP}^A(n) - \Omega_D^A(n) > \Omega_{DP}^A(n-1) - \Omega_D^A(n-1)$;
- (ii) $\Omega_{DP}^A(n) > \Omega_D^A(n)$.

If $\Omega_{DP}^A(n) - \Omega_D^A(n) > \Omega_{DP}^A(n-1) - \Omega_D^A(n-1)$ and $\Omega_{DP}^A(n) > \Omega_D^A(n)$ hold for $\eta_{DP} \leq n \leq |\mathbb{U}| - 1$, consider the $n+1$ case as follows. Let $l_n = \min\{\arg \max_{l \in \{0, \dots, n\}} \Pi_{DP}^A(l, n-l)\}$. $l_n < n$ because $\Omega_{DP}^A(n) > \Omega_D^A(n) = \Pi_{DP}^A(n, 0)$. We thus have

$$\begin{aligned} \Omega_{DP}^A(n+1) - \Omega_D^A(n+1) &\geq \Pi_{DP}^A(l_n, n+1-l_n) - \Omega_D^A(n+1) \\ &= \left[(p-c_D) \frac{\sum_{i=1}^{l_n} v_i}{1 + \sum_{i=1}^{n+1} v_i} + (p-c_P) \frac{\sum_{i=l_n+1}^{n+1} v_i}{1 + \sum_{i=1}^{n+1} v_i} - m_D \sigma \frac{\sum_{i=1}^{l_n} \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - m_P \sigma \frac{\sqrt{\sum_{i=l_n+1}^{n+1} v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} \right. \\ &\quad \left. - K_d \cdot (n+1) - \kappa \right] - \left[(p-c_D) \frac{\sum_{i=1}^{n+1} v_i}{1 + \sum_{i=1}^{n+1} v_i} - m_D \sigma \frac{\sum_{i=1}^{n+1} \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - K_d \cdot (n+1) \right] \\ &= \frac{m_D \sigma \sum_{i=l_n+1}^{n+1} \sqrt{v_i} - m_P \sigma \sqrt{\sum_{i=l_n+1}^{n+1} v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - (c_P - c_D) \frac{\sum_{i=l_n+1}^{n+1} v_i}{1 + \sum_{i=1}^{n+1} v_i} - \kappa. \end{aligned}$$

We also have

$$\Omega_{DP}^A(n) - \Omega_D^A(n) = \Pi_{DP}^A(l_n, n-l_n) - \Omega_D^A(n) = \frac{m_D \sigma \sum_{i=l_n+1}^n \sqrt{v_i} - m_P \sigma \sqrt{\sum_{i=l_n+1}^n v_i}}{\sqrt{1 + \sum_{i=1}^n v_i}} - (c_P - c_D) \frac{\sum_{i=l_n+1}^n v_i}{1 + \sum_{i=1}^n v_i} - \kappa > 0.$$

Let $a = 1 + \sum_{i=1}^n v_i > 0$, $b = \sum_{i=l_n+1}^n v_i > 0$, $c = (\sum_{i=l_n+1}^n \sqrt{v_i})^2 > 0$, $k_1 = m_D \sigma > 0$, $k_2 = m_P \sigma > 0$, $k_3 = c_P - c_D > 0$ and $v = v_{n+1} > 0$. Then $a > b$, and $(n-l_n)b \geq c$ by Cauchy's Inequality. Consequently, $v \leq \frac{b}{n-l_n} \leq \frac{b^2}{c}$. Note that $\Pi_{DP}^A(l_n, n-l_n) - \Omega_D^A(n) = \frac{k_1 \sqrt{c} - k_2 \sqrt{b}}{\sqrt{a}} - k_3 \frac{b}{a} - \kappa > 0$ and thus $\Pi_{DP}^A(l_n, n-l_n) - \Omega_D^A(n) = \frac{k_1 \sqrt{c} - k_2 \sqrt{b}}{\sqrt{a}} - k_3 \frac{b}{a} > 0$. Also, $\Pi_{DP}^A(l_n, n+1-l_n) - \Omega_D^A(n+1) = \frac{k_1(\sqrt{c} + \sqrt{v}) - k_2 \sqrt{b+v}}{\sqrt{a+v}} - k_3 \frac{b+v}{a+v} - \kappa$. By Lemma 2, we have

$$\Omega_{DP}^A(n+1) - \Omega_D^A(n+1) \geq \Pi_{DP}^A(l_n, n+1-l_n) - \Omega_D^A(n+1) > \Pi_{DP}^A(l_n, n-l_n) - \Omega_D^A(n) = \Omega_{DP}^A(n) - \Omega_D^A(n).$$

Also, by $\Omega_{DP}^A(n) - \Omega_D^A(n) > 0$, we have $\Omega_{DP}^A(n+1) - \Omega_D^A(n+1) > 0$.

By induction, $\forall n \in \{\eta_{DP}, \dots, |\mathbb{U}|\}$, $\Omega_{DP}^A(n) - \Omega_D^A(n) > \Omega_{DP}^A(n-1) - \Omega_D^A(n-1)$.

In addition, by the definition of η_{DP} , $\forall n \in \{0, \dots, \eta_{DP} - 1\}$, $\Omega_{DP}^A(n) = \Omega_D^A(n)$. Hence, $\Omega_{DP}^A(|\mathbb{S}|) - \Omega_D^A(|\mathbb{S}|)$ weakly increases in $|\mathbb{S}|$, which means that $\Omega_{DP}(\mathbb{S}) - \Omega_D(\mathbb{S})$ weakly increases in $|\mathbb{S}|$ for all ordered assortment decisions.

(ii) This follows immediately from (i) by Lemma 1. \square

Now we present the details of the numerical study on how the firm's profit changes if it chooses an ordered assortment larger than the optimal one. In both Case *DT* and Case *DP*, we let $\mathbb{U} = \{1, \dots, 50\}$ with $(v_1, \dots, v_{50}) = (0.5, 0.3, 0.3, 0.15, 0.15, 0.15, 0.1, \dots, 0.1)$. Each parameter takes three numerical values: low, medium, and high: $p \in \{1.6, 1.8, 2\}$, $c_D \in \{0.5, 0.8, 1\}$, $\sigma \in \{0.1, 0.2, 0.3\}$, $\Lambda \in \{1, 1.5, 2\}$, $K_D \in \{0.003, 0.004, 0.005\}$, $c_T(n) \in \{c_D + 0.03n, c_D + 0.05n, c_D + 0.08n\}$, $K_T(n) \in \{0.001n, 0.002n, 0.003n\}$, $c_P \in \{c_D + 0.2, c_D + 0.3, c_D + 0.4\}$, and $K_P(n) \in \{0.001n, 0.002n, 0.003n\}$. For each combination of parameters, we examine the profits under $|\mathbb{S}^*| + 1$ (\mathbb{S}^* is the optimal assortment), $|\mathbb{S}^*| + 2, \dots, 50$, and calculate the average of all these profits. Finally, we calculate the deviation percentage of this average profit from the optimal profit. For either Case *DT* or Case *DP*, we have 2187 parameter combinations. The average deviation for Case *DT* is 30.97%, whereas the average deviation for Case *DP* is 5.44%.

To prove Proposition 10, we need one more lemma.

LEMMA 3. Consider real numbers $a > b > c \geq 0$, $d > 0$, $e \geq 0$, $k_1 > 0$, $k_2 > 0$ and $k_3 > 0$. If $k_2 \frac{\sqrt{d}}{\sqrt{a}} - k_3 \frac{\sqrt{b-c}}{\sqrt{a}} - k_1 \frac{b-c}{a} + e \geq 0$, then $k_2 \frac{\sqrt{d}}{\sqrt{a+v}} - k_3 \frac{\sqrt{b+v-c+v}}{\sqrt{a+v}} - k_1 \frac{b-c}{a+v} + e > 0$ for $\forall v > 0$.

Proof of Lemma 3 Let $f_{Lemma\ 3}(v) = k_2 \frac{\sqrt{d}}{\sqrt{a+v}} - k_3 \frac{\sqrt{b+v-c+v}}{\sqrt{a+v}} - k_1 \frac{b-c}{a+v} + e$. By the condition provided, $f_{Lemma\ 3}(0) \geq 0$. Note that

$$f_{Lemma\ 3}(v) = \frac{k_2 \sqrt{d(a+v)} + k_3 (\sqrt{a+v} \sqrt{c+v} - \sqrt{a+v} \sqrt{b+v}) - k_1 (b-c) + e(a+v)}{a+v},$$

where $\sqrt{d(a+v)}$ is concave in v and $a+v$ is increasingly linear. We also have

$$\begin{aligned} & d^2 (\sqrt{a+v} \sqrt{c+v} - \sqrt{a+v} \sqrt{b+v}) / dv^2 \\ &= -\frac{1}{4} (((a+v)^{\frac{1}{4}} (c+v)^{-\frac{3}{4}} - (a+v)^{-\frac{3}{4}} (c+v)^{\frac{1}{4}})^2 - ((a+v)^{\frac{1}{4}} (b+v)^{-\frac{3}{4}} - (a+v)^{-\frac{3}{4}} (b+v)^{\frac{1}{4}})^2) < 0, \end{aligned}$$

because $(a+v)^{\frac{1}{4}} (c+v)^{-\frac{3}{4}} - (a+v)^{-\frac{3}{4}} (c+v)^{\frac{1}{4}} > (a+v)^{\frac{1}{4}} (b+v)^{-\frac{3}{4}} - (a+v)^{-\frac{3}{4}} (b+v)^{\frac{1}{4}} > 0$ when $a > b > c \geq 0$ and $v > 0$. Hence $\sqrt{a+v} \sqrt{c+v} - \sqrt{a+v} \sqrt{b+v}$ is concave in v . Also note that $k_2 > 0$ and $k_3 > 0$. By the result in Mangasarian (1969), $f_{Lemma\ 3}(v)$ is strictly quasi-concave.

Consider the fact that $f_{Lemma\ 3}(0) \geq 0$, $f_{Lemma\ 3}(+\infty) = e \geq 0$. Also, $f_{Lemma\ 3}(v)$ must be strictly increasing, strictly decreasing, or first strictly increasing then strictly decreasing in v on $(0, \infty)$. Hence, $\forall v \in (0, \infty)$, $f_{Lemma\ 3}(v) > 0$, i.e., $k_2 \frac{\sqrt{d}}{\sqrt{a+v}} - k_3 \frac{\sqrt{b+v-c+v}}{\sqrt{a+v}} - k_1 \frac{b-c}{a+v} + e > 0$. \square

Proof of Proposition 10 $\forall n \in \{\mu_{DP} + 1, \dots, |\mathbb{U}| - 1\}$, let $l_n = \min\{\arg \max_{m \in \{0, \dots, n\}} \Pi_{DP}(m, n - m)\}$. By definition,

$$\begin{aligned} & (p - c_D) \frac{\sum_{i=1}^{l_n} v_i}{1 + \sum_{i=1}^{l_n} v_i} + (p - c_P) \frac{\sum_{i=l_n+1}^n v_i}{1 + \sum_{i=1}^n v_i} - m_D \sigma \frac{\sum_{i=1}^{l_n} \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^n v_i}} - m_P \sigma \frac{\sqrt{\sum_{i=l_n+1}^n v_i}}{1 + \sum_{i=1}^n v_i} - K_D \cdot l_n - K_P(n - l_n) \\ & \geq (p - c_D) \frac{\sum_{i=1}^l v_i}{1 + \sum_{i=1}^n v_i} + (p - c_P) \frac{\sum_{i=l+1}^n v_i}{1 + \sum_{i=1}^n v_i} - m_D \sigma \frac{\sum_{i=1}^l \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^n v_i}} - m_P \sigma \frac{\sqrt{\sum_{i=l+1}^n v_i}}{1 + \sum_{i=1}^n v_i} - K_D \cdot l - K_P(n - l) \end{aligned}$$

for $\forall l \in \{l_n + 1, \dots, n - 1\}$. By algebraic transformation:

$$m_D \sigma \frac{\sum_{i=l_n+1}^l \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^n v_i}} - m_P \sigma \frac{\sqrt{\sum_{l_n+1}^n v_i} - \sqrt{\sum_{l+1}^n v_i}}{\sqrt{1 + \sum_{i=1}^n v_i}} - (c_P - c_D) \frac{\sum_{i=l_n+1}^l v_i}{1 + \sum_{i=1}^n v_i} + K_D \cdot (l - l_n) - K_P(n - l_n) + K_P(n - l) \geq 0.$$

Now, let $a = 1 + \sum_{i=1}^n v_i$, $b = \sum_{i=1}^n v_i$, $c = \sum_{i=l+1}^n v_i$, $d = (\sum_{i=l_n+1}^l \sqrt{v_i})^2$, $e = K_D \cdot (l - l_n) - K_P(n - l_n) + K_P(n - l)$, $k_1 = c_P - c_D$, $k_2 = m_D \sigma$, and $k_3 = m_P \sigma$. We have $a > b > c \geq 0$, $d > 0$, $e \geq 0$, $k_i > 0$ for $i = 1, 2, 3$ and $k_2 \frac{\sqrt{d}}{\sqrt{a}} - k_3 \frac{\sqrt{b-c}}{\sqrt{a}} - k_1 \frac{b-c}{a} + e \geq 0$. By Lemma 3, we further have $k_2 \frac{\sqrt{d}}{\sqrt{a+v}} - k_3 \frac{\sqrt{b+v-c+v}}{\sqrt{a+v}} - k_1 \frac{b-c}{a+v} + e > 0$ for $\forall v > 0$, which indicates

$$\begin{aligned} & m_D \sigma \frac{\sum_{i=l_n+1}^l \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^n v_i + v_{n+1}}} - m_P \sigma \frac{\sqrt{\sum_{l_n+1}^n v_i + v_{n+1}} - \sqrt{\sum_{l+1}^n v_i + v_{n+1}}}{\sqrt{1 + \sum_{i=1}^n v_i + v_{n+1}}} \\ & - (c_P - c_D) \frac{\sum_{i=l_n+1}^l v_i}{1 + \sum_{i=1}^n v_i + v_{n+1}} + K_D \cdot (l - l_n) - K_P(n - l_n) + K_P(n - l) \\ & = m_D \sigma \frac{\sum_{i=l_n+1}^l \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - m_P \sigma \frac{\sqrt{\sum_{l_n+1}^{n+1} v_i} - \sqrt{\sum_{l+1}^{n+1} v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - (c_P - c_D) \frac{\sum_{i=l_n+1}^l v_i}{1 + \sum_{i=1}^{n+1} v_i} + K_D \cdot (l - l_n) - K_P(n - l_n) + K_P(n - l) > 0. \end{aligned}$$

Thus, by algebraic transformation,

$$\begin{aligned} & (p - c_D) \frac{\sum_{i=1}^{l_n} v_i}{1 + \sum_{i=1}^{n+1} v_i} + (p - c_P) \frac{\sum_{i=l_n+1}^{n+1} v_i}{1 + \sum_{i=1}^{n+1} v_i} - m_D \sigma \frac{\sum_{i=1}^{l_n} \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - m_P \sigma \frac{\sqrt{\sum_{i=l_n+1}^{n+1} v_i}}{1 + \sum_{i=1}^{n+1} v_i} - K_D \cdot l_n - K_P(n + 1 - l_n) \\ & > (p - c_D) \frac{\sum_{i=1}^l v_i}{1 + \sum_{i=1}^{n+1} v_i} + (p - c_P) \frac{\sum_{i=l+1}^{n+1} v_i}{1 + \sum_{i=1}^{n+1} v_i} - m_D \sigma \frac{\sum_{i=1}^l \sqrt{v_i}}{\sqrt{1 + \sum_{i=1}^{n+1} v_i}} - m_P \sigma \frac{\sqrt{\sum_{i=l+1}^{n+1} v_i}}{1 + \sum_{i=1}^{n+1} v_i} - K_D \cdot l - K_P(n + 1 - l), \end{aligned}$$

because $K_P(n - l_n) - K_P(n - l) \geq K_P(n + 1 - l_n) - K_P(n + 1 - l)$ by the concavity of $K_P(\cdot)$. That is, $\Pi_{DP}(l_n, n + 1 - l_n) > \Pi_{DP}(l, n + 1 - l_n), \forall l \in \{l_n + 1, \dots, n - 1\}$. Then any assignment with $|\mathbb{D}| \in \{l_n + 1, \dots, n - 1\}$ is non-optimal for assortment size $n + 1$. The dedicated-only assignment, which has $|\mathbb{D}| = n + 1$, is also non-optimal by the definition of μ_{DP} . If $K_P(1) \geq K_D$, the assignment with $|\mathbb{D}| = n$ is dominated by the assignment with $|\mathbb{D}| = n + 1$ by simple algebra. If $K_P(1) < K_D$, $K_D \cdot (n - l_n) - K_P(n - l_n) + K_P(0) > 0$ and the above process for $l \in \{l_n + 1, \dots, n - 1\}$ can be carried over to $l = n$, i.e., the assignment with $|\mathbb{D}| = n$ is also non-optimal. In summary, any optimal assignment must assign no more than l_n variants to the dedicated technology for the assortment size $n + 1$.

In conclusion, if $\mu_{DP} < |\mathbb{S}_1| < |\mathbb{S}_2|$ and $(\mathbb{D}_i^*, \mathbb{F}_i^*)$ is any optimal assignment given assortment \mathbb{S}_i ($i = 1, 2$), then $|\mathbb{D}_1^*| \geq |\mathbb{D}_2^*|$. \square

B.4. Extensions

Proof of Proposition 11 (a) Similar to the proof of Proposition 1, we can get: $\forall i \in \mathbb{R}_{m_1}^*$, either $v_i \geq \max\{v_j : j \in \mathbb{R}_{m_2}^*\}$ or $v_i \leq \min\{v_j : j \in \mathbb{R}_{m_2}^*\}$; also, $\forall j \in \mathbb{R}_{m_2}^*$, either $v_j \geq \max\{v_i : i \in \mathbb{R}_{m_1}^*\}$ or $v_j \leq \min\{v_i : i \in \mathbb{R}_{m_1}^*\}$. Combine these two facts together, we have either $\min\{v_j : j \in \mathbb{R}_{m_2}^*\} \geq \max\{v_i : i \in \mathbb{R}_{m_1}^*\}$ or $\max\{v_j : j \in \mathbb{R}_{m_2}^*\} \leq \min\{v_i : i \in \mathbb{R}_{m_1}^*\}$, i.e., either all variants in $\mathbb{R}_{m_1}^*$ are (weakly) more popular than all variants in $\mathbb{R}_{m_2}^*$ or all variants in $\mathbb{R}_{m_1}^*$ are (weakly) less popular than all variants in $\mathbb{R}_{m_2}^*$.

(b) The proof is similar to that of Proposition 3 and thus omitted. \square