Strategic Commitment to a Production Schedule with Uncertain Supply and Demand: Renewable Energy in Day-Ahead Electricity Markets

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We consider a day-ahead electricity market that consists of multiple competing renewable firms (e.g., wind generators) and conventional firms (e.g., coal-fired power plants) in a discrete-time setting. The market is run in every period, and all firms submit their price-contingent production schedules in every day-ahead market. Following the clearance of a day-ahead market, in the next period, each (respectively, renewable) firm chooses its associated production quantity (respectively, after observing its available supply). If a firm produces less than its cleared day-ahead commitment, the firm pays an undersupply penalty in proportion to its underproduction. Using differential equations theory, we explicitly characterize equilibrium strategies of firms. The purpose of an undersupply penalty is to improve reliability by motivating each firm to commit to quantities it can produce in the following day. We prove that in equilibrium, imposing or increasing a market-based undersupply penalty rate in a period can result in a strictly larger renewable energy commitment at all prices in the associated day-ahead market, leading to lower equilibrium reliability in all periods with probability 1. We also show in an extension that firms with diversified technologies result in lower equilibrium reliability than single-technology firms in all periods with probability 1.

Key words: renewable energy, production schedule, supply uncertainty, day-ahead electricity market, demand uncertainty, supply function equilibrium, reliability, production quantity, penalty, subsidy

1. Introduction

The penetration of variable renewable energy (such as wind and solar) in the electricity generation mix has significantly increased in the world (REN21 2015). In the E.U., wind and solar power accounted for 43.7% and 29.7% of the new power capacity in 2014, respectively (EWEA 2015). Many countries have implemented policies that set targets for renewable energy generation, which boosted the capacity and production of wind and solar energy. For example, Denmark targets to meet all of its electricity demand with renewable power by 2050, and it produced almost 33% of its electricity with wind energy in 2013 (Vittrup 2014, REN21 2015). In fact, during 2015, there have been days in which Denmark met 100% of its electricity demand with wind power (Guardian 2015). Similarly, in the U.S., with the renewable portfolio standard and other renewable energy policies, variable renewable energy capacity and generation have substantially increased. In 2014, the U.S. ranked first for wind energy generation and second for total wind capacity in the world (REN21 2015).

With increasing penetration of wind and solar in the generation mix, the participation of variable renewable power producers in U.S. day-ahead electricity markets has significantly increased, mainly
because of various rules implemented in regional electricity markets. For example, recently, Midcontinent Independent System Operator, Inc. (MISO) has created a new category of wind energy resource called “Dispatchable Intermittent Resource (DIR)” (MISO 2014a). DIRs are treated as any conventional firm in MISO’s day-ahead electricity markets (MISO 2013), which means that wind producers’ day-ahead committed production schedule must be taken into account in MISO’s day-ahead market clearance (MISO 2016b). This initiative enabled scheduling large amount of wind power in MISO’s day-ahead markets: After the addition of DIRs, in 2014, around 33 million MWh of wind power was cleared in MISO’s day-ahead markets (MISO 2015b). Other regional electricity markets (such as PJM Interconnection (PJM) and ISO New England (ISO-NE)) also implemented rules that allowed for more wind energy participation and scheduling in day-ahead ahead electricity markets (UWIG 2011). The caveat is that in all of these markets, wind producers are subject to penalties if there is a mismatch between their actual production quantity and day-ahead production commitment (UWIG 2011).

The fundamental challenge with variable renewable energy is supply uncertainty. Despite increasing production commitments by variable renewable energy producers in day-ahead electricity markets and the high penetration of wind and solar in the electricity generation mix, the optimal day-ahead production schedule of a strategic firm that faces supply uncertainty and penalties has not yet been studied. Our paper analyzes this issue by considering a day-ahead market with competing conventional firms (such as coal-fired or nuclear power plants) and variable renewable firms that can commit to their production schedules in day-ahead markets. In this context, our paper studies four main research questions. First, (i) what are the optimal day-ahead production commitment and actual production quantity of each firm in equilibrium? Our second research question is related to “reliability” that is a key performance metric for an independent system operator (MISO 2014a). In a day-ahead electricity market, reliability can be measured by the probability of the actual aggregate production in the operating day to be larger than the total day-ahead production commitment. (The subsequent paragraph will explain how a typical day-ahead market operates; the formal definition of the reliability will be introduced in Section 2.) To improve the reliability in the presence of supply uncertainty, independent system operators impose penalties on variable renewable energy imbalances, as mentioned above. However, these penalty rules have been subject to many changes in the U.S. (UWIG 2009, 2011). In light of this, our second research question is (ii) what are the implications of penalty rules for reliability, renewable firms’ committed production schedules and actual production strategies in equilibrium? Third, in day-ahead electricity markets, there can be firms that generate electricity from both variable renewable resources and conventional resources. Based on this, (iii) what are the implications of firms that can generate electricity from different technologies for day-ahead commitments, actual production quantity and the reliability
in equilibrium? Finally, (iv) how does subsidizing renewable firms for their production affect their committed production schedules and actual production strategies in equilibrium?

In the U.S., day-ahead markets consist of multiple competing firms (U.S. EIA 2014b), and these markets typically operate as follows. A day-ahead market is run one day before every operating day (i.e., the actual production day). Each firm submits its production schedule to the independent system operator without observing the day-ahead demand for electricity or other firms’ submitted production schedules. A submitted production schedule represents how much the submitting firm commits to produce in the following day for any potential day-ahead market clearing price. After firms commit to their production schedules and the day-ahead demand is realized, later in the same day, the independent system operator determines a market clearing price that matches day-ahead demand and day-ahead commitments. In the following day, each firm must produce its committed production quantity at the aforementioned day-ahead market clearing price. A firm is subject to penalties if there is a mismatch between its cleared commitment in the day-ahead market and its production in the operating day, which is the day following the day-ahead market clearance.

Variable renewable energy firms (such as DIRs in the MISO system), in short renewable firms, have the flexibility to commit to a different production schedule in every day-ahead electricity market. However, nuclear or coal-fired power generators do not have such flexibility because they are used as base-load generators that must run continuously and at a constant rate to achieve operational efficiency (U.S. EIA 2016). Therefore, these conventional firms do not change their production quantities or commitments for a long time. Based on this, hereafter, firms with this type of electricity generation will be called inflexible firms. In fact, inflexible firms and renewable firms together meet a major portion of electricity demand in the U.S. markets. For instance, in 2014, nearly 80% of the electricity demand was met by inflexible firms and wind power producers in MISO (MISO 2015b). Apart from the flexibility aspect, inflexible firms and renewable firms differ in the following two main aspects. First, inflexible firms have positive cost of production, whereas renewable firms incur no (or negligibly small) cost for production. Second, the renewable energy potential in the following day is uncertain (i.e., a firm does not exactly know how much wind is going to blow in the following day); on the other hand, an inflexible firm is not exposed to such uncertainty in its available supply.

Considering these practical issues, to answer our research questions, we study a discrete time setting in which a day-ahead market is run in every period for a finite period of time. The market consists of multiple competing inflexible firms and renewable firms. In every period, each renewable firm’s strategy is to choose (a) a committed production schedule, which is a function that maps any possible day-ahead market clearing price to a committed production quantity (that must be delivered in the following period) and (b) an actual production quantity (that is related to the cleared
day-ahead commitment in the previous period); the strategy of each inflexible firm consists of (c) a constant production commitment and (d) actual production quantity. In each day-ahead market, all firms simultaneously submit their production commitments before the realization of the day-ahead demand, which is time-dependent and subject to a random shock in every period. At the time of the day-ahead commitment, a renewable firm cannot observe its available supply in the following period; hence, it is a random variable for the firm. The length of the finite horizon represents the time frame during which inflexible firms cannot change their commitments; thus, an inflexible firm chooses its commitment in the initial day-ahead market, and that commitment remains the same throughout the finite horizon. In contrast, renewable firms can change their day-ahead committed production schedules in every period. Firms with the same generation technology (i.e., inflexible or renewable) are identical and consider same strategies. If a firm produces less (respectively, more) than its cleared production commitment determined in the previous period’s day-ahead market, the firm has to pay an undersupply penalty rate for each unit of its underproduction (respectively, an oversupply penalty rate for each unit of its overproduction). Our paper also analyzes various variants of this base model (see Sections 4 and 5, and Sections EC.1.1 through EC.1.4 in the Electronic Companion).

1.1. Overview of Main Results
Using the setting explained above, Proposition 2 characterizes firms’ equilibrium day-ahead committed production schedules and actual production strategies in every period. These issues have not been studied in the literature (see Section 1.2 for a detailed discussion). By Proposition 2, a renewable firm’s day-ahead committed production schedule in every period is characterized by an ordinary differential equation (ODE) subject to a monotonicity constraint and an initial condition.

Our paper establishes three unexpected results related to the implementation of the undersupply penalty rates. First, one might expect that if there is an increase in the undersupply penalty rate in a period, each renewable firm would optimally decrease its associated day-ahead commitment in equilibrium to reduce its underproduction (and hence undersupply penalty). In contrast, Proposition 4 shows that increasing or imposing a market-based undersupply penalty rate can lead to a strictly larger day-ahead commitment by each renewable firm for any price in equilibrium. Second, intuition suggests that imposing or increasing a penalty rate on underproduction would improve the reliability by motivating firms to submit day-ahead commitments that they can deliver in the following period. However, Propositions 5 and 6 show that increasing or imposing a market-based undersupply penalty rate in a period can lead to strictly lower reliability with probability 1 (i.e., for any realization of random day-ahead demand shock) in the associated day-ahead market. Third, Proposition 6 proves that increasing or imposing a market-based undersupply penalty rate in one period can indeed result in strictly lower equilibrium reliability in all periods with probability 1. The intuition behind
Propositions 4 and 5 relies on renewable firms’ manipulation power on the undersupply penalty rate. If the penalty rate is linked to the day-ahead market price or to the price in a market that is related to the day-ahead electricity market (e.g., as in MISO (MISO 2016b) and PJM (PJM 2015)), renewable firms can manipulate the undersupply penalty rate through their day-ahead commitments. When said manipulation power is large, as a response to an increase in the penalty rate, each renewable firm can profitably inflate its production commitment to mitigate the increase in the undersupply penalty rate by reducing the day-ahead market clearing price. Proposition 6 is due to the aforementioned commitment inflation by renewable firms and inflexible firms’ strategic reaction to that inflation.

Sections 4 and 5, and Sections EC.1.1 through EC.1.4 in the Electronic Companion show that the explained unexpected results hold in various variants of the base model, and hence they are robust. Section EC.1.4 analyzes a variant of the model where renewable firms gain revenue for their overproduction instead of paying an oversupply penalty. In this setting, Proposition EC.4 shows that all three unexpected results hold for a more general set of conditions.

Section 4 analyzes a variant of the base model where each firm generates electricity from both inflexible and renewable resources. Proposition 9 shows that in a day-ahead market, firms with diversified technologies result in lower equilibrium reliability than single-technology firms in all periods with probability 1.

Section 5 extends the base model to consider production subsidies for renewable firms. Proposition 11 demonstrates an interesting theoretical property of a renewable firm’s equilibrium committed production schedule with subsidies. Specifically, it shows that with a large initial subsidy rate, there exists a critical quantity that divides up the quantity space into two intervals, and in each of these two intervals, each renewable firm optimally commits to a production schedule that is a solution of a different ODE.

The numerical example in Section 6 demonstrates that the penalty rule has a large impact on firms’ day-ahead commitments and reliability.

1.2. Literature Review
Our paper belongs to the growing literature on renewable energy/resources operations. Aflaki and Netessine (2012) show that the intermittency of renewable energy significantly impacts the effectiveness of the environmental policies, and find that increasing the emissions cost can decrease the share of renewable capacity investments in an energy investment portfolio. Kök et al. (2014) show that for a utility firm, flat pricing results in a higher solar energy investment than peak pricing, and the impact of pricing scheme on wind energy investment depends on the output pattern. Murali et al. (2015) characterize the optimal groundwater allocation and control policies when the water can be transferred. Hu et al. (2015) establish that the granularity of the demand and renewable output data plays a crucial role in renewable capacity investments. Zhou et al. (2011)
study the performance of various heuristic policies and show that the energy storage adds significant value to renewable generators. Wu and Kapuscinski (2013) show that reducing renewable energy output can have a higher economic value with energy storage. Lobel and Perakis (2011) study welfare-maximizing subsidy design for customers’ solar technology adoption, and conclude that current subsidy rate in Germany is very low. Alizamir et al. (2016) analyze the socially-optimal design of feed-in-tariffs for renewable technology, and show that a constant profitability policy is mostly suboptimal. To the best of our knowledge, our paper is the first that analyzes strategic renewable energy producers’ optimal production schedule commitments in a day-ahead market with various penalty rules and subsidies in effect.

Our paper is related to the supply function equilibrium (SFE) literature. A comprehensive literature review on SFE models can be found in Holmberg and Newbery (2010); here we only include papers that are most relevant to ours. Klemperer and Meyer (1989) introduce and analyze a supply function competition model with demand uncertainty and convex production cost, and show the existence of a symmetric SFE. Green and Newbery (1992) and Green (1996) are among the first that apply SFE models in an electricity market. Green and Newbery (1992) analyze a linear supply function competition model; Green (1996) calibrates a supply competition model for British spot electricity market. It is well-established that the explicit analysis of an asymmetric SFE in a general setting is usually infeasible (see Johari and Tsitsiklis (2011) for a discussion). Thus, most of the SFE literature focuses on the analysis of a symmetric SFE. Rudkevich et al. (1998) analyze a symmetric SFE when the demand is inelastic and the cost of production is a piecewise constant and convex function. Anderson and Philpott (2002) solve a symmetric SFE with an inelastic demand and convex production cost. Holmberg (2008) identify the conditions under which there exists a unique symmetric SFE when the demand is inelastic. Focusing primarily on deterministic demand and the linear supply functions, Vives (2011) studies a symmetric SFE model with private information about production cost, and establishes that the supply function can be decreasing in price. Unlike these papers, our paper introduces and analyzes a supply function competition model with both supply and demand uncertainty under various penalty/credit rules, which has not been studied in the SFE literature. Considering the supply uncertainty together with the penalty rules in day-ahead markets give rise to results (summarized in Section 1.1), which have not been identified in the literature. Furthermore, different from the literature that analyzes linear supply functions, in our paper, each renewable firm’s committed production schedule is allowed to be any function in equilibrium.

Our paper complements the recent “priority dispatch” literature that studies settings where renewable energy is prioritized in the electricity dispatch. Some of the leading papers in that literature are Buygi et al. (2012) and Al-Gwaiz et al. (2014). The work by Al-Gwaiz et al. (2014) is relevant to our paper, but its focus is very different than ours. Al-Gwaiz et al. (2014) analyze an electricity
market where there are multiple competing conventional firms, and renewable energy is prioritized. They show that the implementation of economic curtailment policy for renewable energy results in intensified market competition. In their setting, there is no decision to be made by renewable firms in the market, meaning that renewable firms are not allowed to submit their production schedules in the market and cannot choose their production quantities; a fixed price is paid for unit renewable energy output. In contrast, our paper analyzes optimal decisions of renewable firms in equilibrium. Specifically, motivated by the recent developments in the U.S. electricity markets, our paper studies how each renewable firm should dynamically commit to its production schedule and optimally set its production strategy when there are multiple competing renewable firms and inflexible firms.

2. Model

Consider a day-ahead electricity market consisting of finitely many competing firms in a discrete-time setting. There are two types of firms in the market: \( N_r \geq 2 \) renewable firms indexed by \( n = 1, 2, \ldots, N_r \), and \( N_i \geq 2 \) inflexible (conventional) firms (such as coal-fired power plants) indexed by \( k = N_r + 1, \ldots, N_r + N_i \). There are three important differences between a renewable firm and an inflexible firm. First, different from an inflexible firm, the available supply of a renewable firm is uncertain in the future. Second, compared to an inflexible firm, a renewable firm incurs no (or negligibly small) production cost. Third, a renewable firm can change its committed production schedule in every day-ahead market. However, an inflexible firm does not change its production schedule that frequently. In fact, an inflexible firm continuously produces electricity at a constant rate for a long time (U.S. EIA 2016) because changing the production quantity leads to excessive operational inefficiencies for the firm. All of these elements will be included in our formulation.

There are \( \tau \) commitment periods indexed by \( t = 1, \ldots, \tau \). In our formulation, \( \tau \) represents the number of periods during which each inflexible firm commits to the same production quantity. A day-ahead market is run in each commitment period.

Day-ahead electricity demand in period \( t \) is \( D_t(p, \epsilon_t) \), which is a function of random shock \( \epsilon_t \) and price \( p \in \mathbb{R} \); \( D_t(p, \epsilon_t) \) strictly increases in \( \epsilon_t \), and strictly decreases and concave in \( p \). The day-ahead demand shock \( \epsilon_t \) is a random variable with a cumulative distribution function \( \Phi_t(\cdot) \) and a density function \( \phi_t(\cdot) > 0 \) on support \( [\hat{\epsilon}_t(0, p), \infty) \). Here, \( \hat{\epsilon}_t(y, p) \) is the realization of \( \epsilon_t \) that results in demand \( y \) at price \( p \) in the period-\( t \) day-ahead market and \( p_t \) is the minimum possible market clearing price over \( \tau \) periods. (Because \( D_t(p, \epsilon_t) \) strictly increases in \( \epsilon_t \) for any \( p \), we can take the inverse of \( D_t(p, \epsilon_t) \) with respect to \( \epsilon_t \) to obtain the function \( \hat{\epsilon}_t : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R} \).

For \( t = 1, \ldots, \tau \), the period-\( t \) day-ahead market is run as follows. Before the uncertainty in the period-\( t \) day-ahead demand is resolved, all firms simultaneously submit their production schedules which represent the amount of electricity they commit to generate in the following period as a
function of price. Specifically, each renewable firm \( n \leq N_r \) chooses its twice continuously differentiable production schedule, i.e., supply function
\[
S_{n,t} : \mathbb{R} \rightarrow \mathbb{R}_+.
\]
The production schedule \( S_{n,t}() \) gives the quantity renewable firm \( n \) commits to produce in period \( t+1 \) as a function of period-\( t \) day-ahead market clearing price. If \( t = 1 \), each inflexible firm \( k \) chooses its day-ahead production commitment \( \tilde{S}_k \leq K \) for \( p \geq p_t \), where \( K \) is the capacity of firm \( k \); for \( t > 1 \), inflexible firm \( k \)’s day-ahead production commitment remains the same and is equal to \( \tilde{S}_k \). After the day-ahead demand uncertainty is resolved, that is, after \( \epsilon_t \) is realized, in the same period, the independent system operator (ISO) determines the day-ahead market clearing price \( p_t^* \) that matches the day-ahead demand and total production commitment in period \( t \):
\[
D_t(p_t^*, \epsilon_t) = \sum_{n=1}^{N_r} S_{n,t}(p_t^*) + \sum_{k=N_r+1}^{N_r+N_i} \tilde{S}_k.
\] (1)

This market clearing procedure is well-established in the literature; see for instance Anderson and Philpott (2002), Rudkevich (1999), Bolle (1992) and Klemperer and Meyer (1989) among others. With (1) the ISO meets the period-\( t \) day-ahead demand at a minimum cost because day-ahead demand decreases in price and we will establish later in Section 3 that in equilibrium, total production commitment increases in price. Outcomes of the period-\( t \) day-ahead market determine how much electricity each firm must produce in period \( t+1 \). Here, \( S_{n,t}(p_t^*) \) represents renewable firm \( n \)'s cleared production commitment in period \( t \), which is the amount of electricity firm \( n \) commits to deliver in period \( t+1 \).

In period \( t \), there is uncertainty about the renewable firms’ available supply in period \( t+1 \) (e.g., because firms do not exactly know how much wind is going to blow in the following period). Therefore, at the time of the commitment in period \( t \), the available supply of renewable firm \( n \) in period \( t+1 \) is a random variable \( Q_{n,t+1} \), independent of \( \epsilon_t \). The random variable \( Q_{n,t+1} \) has a cumulative distribution function \( F_{t+1}(\cdot) \) and a probability density function \( f_{t+1}(\cdot) \) for all \( n \leq N_r \).

At the beginning of period \( t+1 \), the available supply of each renewable firm is realized. Observing \( Q_{n,t+1} \), each renewable firm \( n \) chooses its production quantity \( q_{n,t+1} \) subject to \( q_{n,t+1} \leq Q_{n,t+1} \). Because renewable firm \( n \) cannot produce more than \( Q_{n,t+1} \) in period \( t+1 \), \( Q_{n,t+1} \) can be interpreted as the period-(\( t+1 \)) capacity constraint that is unobservable to firm \( n \) in period \( t \). Simultaneously with all other firms, each inflexible firm \( k \) chooses its production quantity \( q_{k,t+1} \leq K \) in period \( t+1 \). Based on the choice of production quantity in period \( t+1 \), a firm can incur two types of costs: undersupply or oversupply penalty, and the cost of production, all of which will be detailed below.

\footnote{Later, we will establish that the explained commitment strategy of inflexible firms ensures that we focus on a practically relevant equilibrium in which each inflexible firm produces at a constant rate in all periods (U.S. EIA 2016).}
If a firm’s production quantity in period $t+1$ is less than its cleared production commitment in period $t$, the firm pays an undersupply penalty rate $\eta_{u,t+1}(p^*_{t}) = \beta_{u,t+1}p^*_{t} + b_{u,t+1}$ per its unit underproduction, where $b_{u,t+1} > 0$ and $\beta_{u,t+1} \in [0,1]$ for all $t$. For example, if $q_{n,t+1} < S_{n,t}(p^*_{t})$ for renewable firm $n$, then firm $n$ must pay a total undersupply penalty of $\eta_{u,t+1}(p^*_{t})(S_{n,t}(p^*_{t}) - q_{n,t+1})$ in period $t+1$. If a firm’s production quantity in period $t+1$ is more than its cleared production commitment in period $t$, the firm pays an oversupply penalty rate $\eta_{o,t+1}(p^*_{t})$ per its unit overproduction. Here, $\eta_{o,t+1}(p) = \beta_{o,t+1}p + b_{o,t+1}$ for $p \geq -b_{o,t+1}/\beta_{o,t+1}$, otherwise $\eta_{o,t+1}(p) = b_{t+1} > 0$ for $t \leq \tau$, where $b_{o,t+1} > 0$ and $\beta_{o,t+1} \in [0,1]$. (The condition that $\eta_{o,t+1}(p) = b_{t+1} > 0$ for $p < -b_{o,t+1}/\beta_{o,t+1}$ is necessary to eliminate unrealistic equilibrium strategies, such as committing to arbitrarily small production quantities at extremely small negative prices in period $t$ and producing the available supply in period $t+1$.) In markets where penalty rates are linked to the realized price in an electricity market that is related to the day-ahead market (e.g., as in MISO, PJM and ISO-NE (MISO 2016b, PJM 2015, UWIG 2011)), the parameters $\beta_{u,t+1}$ and $\beta_{o,t+1}$ represent the ultimate dependence of the undersupply and oversupply penalty rates in period $t+1$ on the day-ahead price $p^*_{t}$, respectively. The parameters $\beta_{u,t+1}$ and $\beta_{o,t+1}$ can also be viewed as a measure of dependence between the day-ahead price $p^*_{t}$ and the realized price in the market based on which the undersupply penalty and oversupply penalty rates are calculated, respectively.

The cost of production for each renewable firm is zero, and a renewable firm incurs no operating cost in reducing its output. For instance, wind generators can reduce their output by simply pitching the blades, which incurs no (or negligibly small) operating cost to wind generators (Wu and Kapuscinski 2013). In contrast, production is costly for inflexible firms. Each inflexible firm’s production cost function in period $t+1$ is $C_{t+1}(\cdot)$ which is twice continuously differentiable, convex and strictly increasing, and it satisfies the following standard properties: $C_{t+1}(0) = 0$ and $C'_{t+1}(0) = 0$ for $t \leq \tau$.

In practice, an inflexible firm’s average marginal cost of production is small relative to the average penalty rate (see for instance U.S. EIA (2013) and PJM (2015)) and inflexible firms are not generally exposed to supply uncertainty. Therefore, it is never profitable for an inflexible firm to produce less than its commitment to save from the production cost. Also, any positive oversupply penalty deters an inflexible firm from overproducing. As a result, each inflexible firm $k$’s optimal production quantity is $q^*_{k,t+1} = \bar{S}_k$ for $t \leq \tau$, which is in line with the constant and continuous production schedules of inflexible firms observed in practice (U.S. EIA 2016). On the other hand, due to supply uncertainty, renewable firms might not be able to satisfy their production commitments, making their profits prone to the undersupply penalty rate. The objective of each type of firm will be formally defined in Section 2.1. The following table summarizes the explained sequence of events related to firms’ production in period $t+1$. 
Sequence of events related to the production in period $t+1$
(in the order of occurrence)

Day-ahead commitment and clearance in period $t$:

1. Each renewable firm $n$ commits to a production schedule $S_{n,t}(\cdot)$ for $n=1,\ldots,N_r$, and each inflexible firm $k$ commits to a production quantity $\hat{S}_k$ for $k=N_r+1,\ldots,N_r+N_i$.
2. Period-$t$ day-ahead demand shock $\epsilon_t$ is realized.
3. Day-ahead market clearing price $p_t^*$ is determined.

Production in period $t+1$ (based on the cleared day-ahead commitment in period $t$):

4. Available supply $Q_{n,t+1}$ is realized for $n=1,\ldots,N_r$.
5. Each firm $j$ chooses its production quantity $q_{j,t+1}$ for $j=1,\ldots,N_r+N_i$.
6. Total penalty for each firm $j$ is determined based on its cleared commitment in period $t$ and $q_{j,t+1}$ for $j=1,\ldots,N_r+N_i$.

2.1. Firms’ Objectives and the Equilibrium Definition

As explained above, an inflexible firm $k$’s strategy is to commit to a production quantity $\hat{S}_k$ at $t=1$ and produce that quantity for all $t>1$; a renewable firm $n$’s strategy is to choose (a) a committed production schedule $S_{n,t}(\cdot)$ in each day-ahead market $t$ and (b) a production quantity $q_{n,t+1}$ for $t\leq \tau$. To identify equilibrium commitments and production quantities, we shall analyze the problem backwards.

In period $t+1$, after observing $Q_{n,t+1}$, each renewable firm $n$ chooses its production quantity $q_{n,t+1}$ to minimize the total realized net penalty associated with its cleared commitment $S_{n,t}(p_t^*)$. Hence, renewable firm $n$’s optimal production quantity in period $t+1$ is

$$q_{n,t+1}^* = \arg\min_{0\leq q_{n,t+1} \leq Q_{n,t+1}} \left[ \eta_{n,t+1}(p_t^*) (S_{n,t}(p_t^*) - q_{n,t+1})^+ + \eta_{n,t+1}(p_t^*) (q_{n,t+1} - S_{n,t}(p_t^*))^+ \right]. \quad (2)$$

In period $t$, given the optimal production quantity $q_{n,t+1}^*$ and other firms’ period-$t$ commitment profiles $S_{-n,t} = (S_{1,t},\ldots,S_{n-1,t},S_{n+1,t},\ldots,S_{N_r,t})$ and $\bar{S} = (\hat{S}_{N_r+1},\ldots,\hat{S}_{N_r+N_i})$, each renewable firm $n$ chooses its production schedule $S_{n,t}(\cdot)$ that achieves its commitment-related ex-post maximum expected profit with respect to period-$t$ day-ahead demand. Given other firms’ period-$t$ commitments, the aforementioned production schedule ex-ante achieves the commitment-related maximum expected profit that renewable firm $n$ would have achieved if the random shock $\epsilon_t$ was known to the firm $n$ before its period-$t$ commitment. In period $t$, given $S_{-n,t}$ and $\bar{S}$, renewable firm $n$ achieves its commitment-related ex-post maximum expected profit with respect to period-$t$ day-ahead demand by choosing $S_{n,t}(\cdot)$ if

$$\Pi_n(p_t^*(S_{n,t},S_{-n,t},\bar{S},\epsilon_t);\epsilon_t, S_{-n,t}, \bar{S}) \geq \Pi_n(p_t^*(S, S_{-n,t}, \bar{S}, \epsilon_t); \epsilon_t, S_{-n,t}, \bar{S}), \quad \text{for } S: \mathbb{R} \to \mathbb{R}_+ \quad (3)$$
and any realization of random shock $\epsilon_t$. Here, $\Pi_n(p^*_t(S_{n,t}, S_{-n,t}, \tilde{S}, \epsilon_t); \epsilon_t, S_{-n,t}, \tilde{S})$ is the renewable firm $n$’s commitment-related expected profit in period $t$ at the market clearing price $p^*_t(S_{n,t}, S_{-n,t}, \tilde{S}, \epsilon_t)$, when firm $n$’s period-$t$ committed production schedule is $S_{n,t}$, other firms’ period-$t$ commitments are $S_{-n,t}$ and $\tilde{S}$, and the period-$t$ day-ahead shock is $\epsilon_t$. For example, given $S_{-n,t}$ and $\tilde{S}$, renewable firm $n$’s commitment-related period-$t$ expected profit at price $p$ and random shock $\epsilon_t$ is $\Pi_n(p; \epsilon_t, S_{-n,t}, \tilde{S})$, which is equal to the revenue from period-$t$ day-ahead commitment minus total expected penalty associated with that commitment. Specifically,

$$\Pi_n(p; \epsilon_t, S_{-n,t}, \tilde{S}) = \mathbb{E}_{Q_{n,t+1}} \left[ pR_{n,t}(p; \epsilon_t) - \eta_{n,t+1}(p) \left( R_{n,t}(p; \epsilon_t) - q^*_{n,t+1} \right) + \eta_{o,t+1}(p) \left( q^*_{n,t+1} - R_{n,t}(p; \epsilon_t) \right) \right],$$

(4)

where $q^*_{n,t+1}$ is a function of $Q_{n,t+1}$ by (2), and $R_{n,t}(\cdot; \epsilon_t)$ is the firm $n$’s period-$t$ residual demand curve at random shock $\epsilon_t$:

$$R_{n,t}(p; \epsilon_t) = D_t(p, \epsilon_t) - \sum_{j \neq n} S_{j,t}(p) - \sum_{k=K_{n,t+1}}^{N_r+N_{i,t}} \bar{S}_k.$$

(5)

A collection of renewable production schedules that satisfies (3) constitutes an ex-post period-$t$ equilibrium with respect to day-ahead demand given $\tilde{S}$. The formal definition of such an equilibrium is as follows.

**Definition 1. (Period-$t$ Supply Function Equilibrium)** For any given commitment profile of inflexible firms $\bar{S}$, renewable firms’ commitment profile $S_t \equiv (S_{1,t}, \ldots, S_{N_r,t})$ is called “period-$t$ supply function equilibrium” or in short “period-$t$ equilibrium” if $S_{n,t}(\cdot)$ satisfies (3) for $n = 1, \ldots, N_r$.

It is perhaps worth noting that the production schedule $S_{n,t}(\cdot)$ that satisfies (3) also achieves the commitment-related ex-ante maximum expected profit $\mathbb{E}_{\epsilon_t}[\Pi_n(p^*_t(\cdot, S_{-n,t}, \tilde{S}, \epsilon_t); \epsilon_t, S_{-n,t}, \tilde{S})]$ given $\tilde{S}$ and $S_{-n,t}$ because (3) holds for every realization of $\epsilon_t$. Thus, given $\tilde{S}$ and $S_{-n,t}$, $S_{n,t}(\cdot)$ that satisfies (3) achieves not only the ex-post maximum expected profit but also ex-ante maximum expected profit with respect to period-$t$ day-ahead demand.

In period 1, given other firms’ all commitment profiles $\bar{S}_{-k} \equiv (S_{N_r+1,t}, \ldots, S_{k-1,t}, S_{k+1,t}, \ldots, S_{N_r+N_{i,t}})$ and $(S_{1,t}, \ldots, S_{r,t})$, each inflexible firm $k$ chooses its production commitment $\tilde{S}_k \in [0, K]$ to maximize its total expected profit, which is equal to the total revenue from day-ahead commitments minus the total production cost:

$$\max_{\tilde{S}_k \in [0, K]} \Pi_k(\tilde{S}_k; \bar{S}_{-k}, S_{1,t}, \ldots, S_{r,t}) \equiv \mathbb{E}_{\epsilon_1, \ldots, \epsilon_r} \left[ \sum_{t=1}^{T} p^*_t(\tilde{S}_k; \bar{S}_{-k}, S_{t}, \epsilon_t) \tilde{S}_k - C_{t+1}(\tilde{S}_k) \right].$$

(6)

Note that firm $k$’s production cost is $C_{t+1}(\tilde{S}_k)$ in period $t+1$ because $q^*_{k,t+1} = \tilde{S}_k$ (due to the reasons explained before). We focus on a parameter region in which the unconstrained optimizer of (6) is a real number, that is, for $t \leq T$,

$$\partial \left( C_{t+1}(x)(1 - \Phi_x(\epsilon_t(N,x,0))) \right) / \partial x \geq a_t > 0, \quad x \geq \bar{x}_t,$$

(7)
for some constants \(a_t\) and \(\tilde{x}_t\), and (6) has a unique stationary point. The condition (7) means that the rate of increase in the production cost function \(C_{t+1}(\cdot)\) ultimately dominates the rate of decrease in \((1 - \Phi_t(\xi_t(Nx, 0)))\), which is a tail probability of the day-ahead demand shock in period \(t\).

Letting \(\vec{q} \doteq (q_{j,t+1}; \ t \leq \tau, \ j \leq N_r + N_i)\) be the production profile of all firms in all periods, an equilibrium in this multi-period setting is defined as follows.

**Definition 2. (Equilibrium)** A strategy profile \((S_1, \ldots, S_r, \vec{S}, \vec{q})\) is an equilibrium in the explained multi-period setting if

\[
\Pi_k(S_k; \vec{S}_{-k}, S_1, \ldots, S_r) \geq \Pi_k(s; \vec{S}_{-k}, S_1, \ldots, S_r) \quad \text{for} \quad s \in \mathbb{R}_+ \quad \text{and} \quad k = N_r + 1, \ldots, N_r + N_i, \quad (8)
\]

\(S_{n,t}(\cdot)\) satisfies (3) for \(n \leq N_r\) and \(t \leq \tau\), (9) subject to the fact that \(\vec{q}\) is the optimal production profile \((q_{j,t+1}^*; \ t \leq \tau, \ j \leq N_r + N_i)\) under the commitment strategy profile \((S_1, \ldots, S_r, \vec{S})\).

\[2.2. \text{A performance metric for an ISO}\]

An important performance metric for an ISO is the supply security for cleared day-ahead commitments. “Period-\(t\) reliability” is a measure of supply security for period-\(t\) day-ahead commitments, and it is defined as the probability that the aggregate cleared commitment in period \(t\) to be smaller than total production quantity of all firms in period \(t+1\). Its formal definition is as follows.

**Definition 3. (Period-\(t\) Reliability)** Under an equilibrium strategy profile \((S_1, \ldots, S_r, \vec{S}, \vec{q})\), the period-\(t\) reliability is defined as

\[
\rho_t(\epsilon_t) \doteq \mathbb{P}_{Q_{1,t+1}, \ldots, Q_{N_r,t+1}} \left( \sum_{n=1}^{N_r} S_{n,t}(p_{t}^*) + \sum_{k=N_r+1}^{N_r+N_i} \tilde{S}_k \leq \sum_{j=1}^{N_r+N_i} q_{j,t+1}^* \bigg| \epsilon_t \right), \ t \leq \tau. \quad (10)
\]

In our analysis, we focus attention on an equilibrium in which same-type firms consider same commitment strategies:

\[
\vec{S} \doteq \tilde{S}_{N_r+1} = \tilde{S}_{N_r+2} = \ldots = \tilde{S}_{N_r+N_i} \quad \text{and} \quad \vec{S}(p) \doteq S_{1,t}(p) = \ldots = S_{N_r,t}(p) \quad \text{for} \quad t \leq \tau, \ p \in \mathbb{R}. \quad (11)
\]

For ease of exposition, hereafter, we consider a linear demand function \(D_t(p, \epsilon_t) = v_t - \alpha_t p + \epsilon_t\) where \(v_t > 0\) and \(\alpha_t > 0\) are constants. All problem parameters introduced in this section are common knowledge to all firms.

\[3. \text{Analysis}\]

To characterize equilibrium strategies, we solve backwards. First, Lemma 1 identifies the optimal production quantity of renewable firm \(n\) in period \(t+1\), given other firms’ committed production schedules in a period-\(t\) day-ahead market. The proof of all formal results in our paper can be found in Appendices A through P of the Electronic Companion. Recall the residual demand notation \(\mathcal{R}_{n,t}(p; \epsilon_t)\) in (5).
Lemma 1. Consider any renewable firm \( n \). Given that committed production schedules of other firms are \( \overline{S} \) and \( S_{-n,t} \) in a period-\( t \) day-ahead market, the renewable firm \( n \)’s optimal production quantity in period \( t+1 \) is

\[
q^*_{n,t+1}(p; \epsilon_t, Q_{n,t+1}) = \min\{Q_{n,t+1}, R_{n,t}(p; \epsilon_t)\}
\]

for any realization of the renewable firm \( n \)’s available supply \( Q_{n,t+1} \) and period-\( t \) market clearing price \( p > 0 \). Furthermore, given \( \epsilon_t, p > 0, S_{-n,t} \) and \( \overline{S} \), firm \( n \)’s commitment-related expected profit (4) in period \( t \) is equivalent to

\[
\Pi_n(p; \epsilon_t, S_{-n,t}, \overline{S}) = \mathbb{E}_{Q_{n,t+1}}\left[pR_{n,t}(p; \epsilon_t) - \eta_{u,t+1}(p) (R_{n,t}(p; \epsilon_t) - Q_{n,t+1})^+\right].
\]

In a period-\( t \) day-ahead market, a renewable firm’s residual demand at market clearing price is equal to the firm’s cleared production commitment by (1). Based on this, (13) implies that renewable firm \( n \) optimally produces at most its period-\( t \) cleared commitment in period \( t+1 \). The reason is that each firm must pay a penalty if the firm’s production quantity in period \( t+1 \) exceeds its period-\( t \) cleared commitment. This positive overproduction penalty deters a renewable firm from producing more than its cleared commitment. The dependence of the optimal production quantity (13) on the cleared commitment underscores the importance of day-ahead commitment strategies for renewable firms.

Using Lemma 1, we now present an intuitive approach to derive a period-\( t \) equilibrium defined in Definition 1. (The formal statement and analysis of a period-\( t \) equilibrium are provided in Proposition 1 and Appendix B of the Electronic Companion, respectively.) Recall from (4) that \( \Pi_n(\cdot; \epsilon_t, S_{-n,t}, \overline{S}) \) represents renewable firm \( n \)’s (commitment-related) expected profit in period \( t \), given \( \epsilon_t \) and other firms’ commitments \( S_{-n,t} \) and \( \overline{S} \). To maximize (4) for any \( \epsilon_t \) given \( S_{-n,t} \) and \( \overline{S} \), firm \( n \) must commit to a production schedule inducing a market clearing price that maximizes \( \Pi_n(\cdot; \epsilon_t, S_{-n,t}, \overline{S}) \) for each \( \epsilon_t \). With such a commitment, firm \( n \) achieves the maximum expected profit it would achieve in period \( t \) if it observed the random shock \( \epsilon_t \) before its commitment in period \( t \). Note that a market clearing price \( p < 0 \) is never optimal for firm \( n \) as \( \Pi_n(p; \epsilon_t, S_{-n,t}, \overline{S}) < 0 \) for \( p < 0 \). By Lemma 1, \( \Pi_n(p; \epsilon_t, S_{-n,t}, \overline{S}) \) is equivalent to (14) for \( p > 0 \). Then, for a given \( \epsilon_t \) and other firms’ commitments \( S_{-n,t} \) and \( \overline{S} \), the price \( p \) that maximizes (14) satisfies the following first order condition:

\[
R_{n,t}(p; \epsilon_t) + R'_{n,t}(p; \epsilon_t) \left[p - \eta_{u,t+1}(p) F_{t+1}(R_{n,t}(p; \epsilon_t))\right] - \beta_{u,t+1} \int_{0}^{R_{n,t}(p; \epsilon_t)} (R_{n,t}(p; \epsilon_t) - x) dF_{t+1}(x) = 0.
\]

(15)

Recall the notation \( \epsilon_t(\cdot, \cdot) \) from Section 2, and denote by \( A_t(p) \) the aggregate production commitment by all firms at \( p \) in period \( t \). Then, from (1), (5) and (15), we have

\[
S_{n,t}(p) + \left( \frac{\partial D_t(p, \epsilon_t(A_t(p), p))}{\partial p} - \sum_{j \neq n} S_{j,t}(p) \right) [p - \eta_{u,t+1}(p) F_{t+1}(S_{n,t}(p))].
\]
which characterizes the best-response committed production schedule of firm \( n \) in period \( t \). Observe from (12) and the fact that \( \partial D_t(p, \bar{\ell}_t, A_t(p), p)) / \partial p = -\alpha_t \), (16) reduces to the following ordinary differential equation:

\[
S_t'(p) = \frac{1}{N_t - 1} \left[ S_t(p) - \beta_{u,t+1} \int_0^{S_t(p)} (S_t(p) - x) dF_{t+1}(x) \right] - \alpha_t.
\]

(17)

The following proposition establishes the existence of a period-\( t \) equilibrium and formally characterizes renewable firms’ committed production schedules in such an equilibrium.

**Proposition 1.** *(Existence and Characterization of a Period-\( t \) Equilibrium)*

(i) For any given commitment profile of inflexible firms \( \mathbb{S} \) and \( t = 1, \ldots, \tau \), a function \( S_t(\cdot) \) is each renewable firm’s committed production schedule in a period-\( t \) supply function equilibrium that satisfies (12) if and only if \( S_t(\cdot) \) satisfies the ordinary differential equation (17) subject to a monotonicity constraint and an initial condition, respectively:

\[
0 < S_t'(p) < \infty, \quad p \geq p_{\ell,t}, \quad \text{and} \quad S_t(p_{\ell,t}) = 0, \quad \text{where} \quad p_{\ell,t} \geq 0.
\]

(18)

(ii) For \( t = 1, \ldots, \tau \), there exists a function \( S_t(\cdot) \) that satisfies (17) subject to (18). Therefore, there exists a period-\( t \) supply function equilibrium that satisfies (12) for any given \( \mathbb{S} \).

Let us develop an understanding about period-\( t \) equilibrium conditions (17) and (18). Suppose that \( p \) is the period-\( t \) market clearing price. Then, the term \( p - (\beta_{u,t+1} p + b_{u,t+1}) F_{t+1}(S_t(p)) \) in (17), which is equal to \( p - \eta_{u,t+1}(p) F_{t+1}(S_t(p)) \), can be interpreted as each renewable firm’s expected marginal profit for an additional unit of commitment in period \( t \). This is because if a renewable firm’s commitment increases by one unit in period \( t \), it gains \( p \) and incurs an expected marginal cost \( \eta_{u,t+1}(p) F_{t+1}(S_t(p)) \). The reason for the latter cost is as follows. There are two possible scenarios about a renewable firm’s available supply in period \( t+1 \): It is either smaller than the firm’s period-\( t \) cleared commitment \( S_t(p) \) or strictly larger than that. The probability of the former scenario is \( F_{t+1}(S_t(p)) \), which equals the underproduction probability in period \( t+1 \). Given that the firm underproduces in period \( t+1 \), an additional unit of commitment at price \( p \) costs \( \eta_{u,t+1}(p) \) to the firm. The probability of the latter scenario is \( (1 - F_{t+1}(S_t(p))) \). In this case, the firm produces its cleared production commitment by Lemma 1, and thus the cost of an additional unit of commitment is equal to the firm’s marginal cost of production, which is zero. Taking the expectation of the cost of an additional unit of commitment with respect to the random available supply in period \( t+1 \), we obtain expected marginal cost \( \eta_{u,t+1}(p) F_{t+1}(S_t(p)) \) in period \( t \).

The term \( S_t(p) - \beta_{u,t+1} \int_0^{S_t(p)} (S_t(p) - x) dF_{t+1}(x) \) in (17) can be interpreted as each renewable firm’s expected marginal profit as a result of a unit increase in price. This is because for a given
period-t commitment, if price $p$ increases by one unit, a renewable firm’s revenue increases by $S_t(p)$ whereas its expected undersupply penalty increases by $\beta_{o,t+1} \int_0^{S_t(p)} (S_t(p) - x) dF_{t+1}(x)$.

The monotonicity constraint in (18) suggests that each renewable firm commits to a larger quantity at a higher price for any $t$. The conditions in (18) also show that each renewable firm commits to a positive production quantity only for $p > 0$.

Using Proposition 1, Proposition 2 explicitly identifies equilibrium strategies of all firms in all periods, including each inflexible firm’s equilibrium production commitment at $t = 1$. To state Proposition 2, we shall introduce the following notation:

$$\bar{z}_t(\bar{s}) = N_i \bar{s} - v_t.$$  \hfill (19)

For $\bar{s} \leq K$, the term $\bar{z}_t(\bar{s})$ represents period-t day-ahead demand shock that results in $p_t^* = 0$ when each inflexible firm’s production commitment is $\bar{s}$.

**Proposition 2.** (i) There exists an equilibrium $(S_1, \ldots, S_r, \bar{S}, q)$ that satisfies (11) and (12). In such an equilibrium, firms’ strategies are as follows.

(ii) For $t = 1, \ldots, \tau$, each renewable firm $n$ commits to a production schedule $S_t(\cdot)$ that satisfies (17) and (18) in period $t$, and firm $n$ produces $q_{n,t+1}^*(p_t^*, S_t, Q_{n,t+1}) = \min\{Q_{n,t+1}, S_t(p_t^*)\}$ in period $t+1$.

(iii) At $t = 1$, each inflexible firm commits to a production quantity $\bar{S} = \min\{\bar{s}^*, K\}$ such that $\bar{s}^*$ satisfies

$$\Psi(\bar{s}) \equiv \sum_{t=1}^{\tau} \left[ \int_{\bar{z}_t(\bar{s})}^{\infty} \left( p_t^* (\bar{s}; z) - \frac{\bar{s}}{\alpha_t + N_i S_t(p_t^* (\bar{s}; z))} - C_{t+1}'(\bar{s}) \right) d\Phi_t(z) + C_{t+1}(\bar{s}) \phi_t(\bar{z}_t(\bar{s})) \right] = 0, \quad (20)$$

where $S_t(\cdot)$ is as identified in part (ii), and $p_t^* (\bar{s}; z)$ is the price that satisfies $v_t - \alpha_t p_t^* (\bar{s}; z) + z = N_i S_t(p_t^* (\bar{s}; z)) + N_i \bar{s}$.

**Proposition 3.** There exists a unique equilibrium $(S_1, \ldots, S_r, \bar{S}, q)$ that satisfies (11) and (12).

By (20), an inflexible firm’s committed production quantity $\bar{S}$ in equilibrium is dependent on renewable firms’ committed production schedules in all periods. The reason is that, by (1), renewable firms’ committed production schedules impact the market clearing price in each period, thereby affecting an inflexible firm’s total expected profit (6).

Proposition 2 verifies that the interpretations related to $S_t(\cdot)$ explained below Proposition 1 are in fact valid in an equilibrium where inflexible firms also strategically commit to their production quantities. Hereafter, the term “committed production schedule” in period $t$ is used to refer to the function $S_t(\cdot)$ that satisfies the equilibrium conditions (17) and (18).

Because each renewable firm chooses not to overproduce in the equilibrium by Proposition 2-(ii), the period-t committed production schedule is not dependent on the oversupply penalty parameters $\beta_{o,t+1}$ and $b_{o,t+1}$ for any $t$. On the other hand, the undersupply penalty rate significantly influence renewable firms’ equilibrium commitments by (17) because supply uncertainty brings forth the
possibility of underproduction for a renewable firm under the optimal production strategy identified in Proposition 2-(ii).

The penalty rules for renewable energy producers have been subject to various changes in the U.S. electricity markets (UWIG 2009, 2011). Yet, the implications of penalties for renewable firms’ day-ahead committed production schedules and the reliability have not been investigated. Below, Propositions 4 through 7 will show that seemingly intuitive undersupply penalty rules can have unintended consequences.

In our setting, for any \( t \), a higher \( \beta_{u,t+1} \) implies a strictly higher undersupply penalty rate \( \eta_{u,t+1}(p) = \beta_{u,t+1}p + b_{u,t+1} \) for \( p > 0 \). Therefore, one might expect that increasing \( \beta_{u,t+1} \) or imposing a positive market-based penalty rate \( \beta_{u,t+1}p \) in addition to the fixed rate \( b_{u,t+1} \) would motivate renewable firms to lower their production commitments in period \( t \) to avoid high underproduction penalty. Contrary to this intuition, Proposition 4 shows that if there is an increase in \( \beta_{u,t+1} \) or if the market-based undersupply penalty rate \( \beta_{u,t+1}p \) is imposed in addition to the fixed rate \( b_{u,t+1} \), each renewable firm can commit to a strictly larger production quantity at all prices in equilibrium.

**Proposition 4.** *(Uniform Inflation of Committed Production Schedules)*

Consider the equilibrium characterized in Proposition 2 and any \( t = 1, \ldots, \tau \).

(i) Each renewable firm’s period-\( t \) committed production schedule is uniformly and strictly larger with a higher \( \beta_{u,t+1} \), i.e., \( \partial S_t(p; \beta_{u,t+1}) / \partial \beta_{u,t+1} > 0 \) for \( p > 0 \), if and only if the number of firms \( N_r \) and the undersupply penalty parameter \( b_{u,t+1} \) satisfy

\[
N_r \leq \Gamma_{t+1} \quad \text{and} \quad b_{u,t+1} < \Delta_{t+1},
\]

where \( \Gamma_{t+1} \) and \( \Delta_{t+1} \) are positive constants.

(ii) If (21), imposing a market-based penalty rate \( \beta_{u,t+1}p > 0 \) in addition to the fixed rate \( b_{u,t+1} \) for underproduction results in a uniformly and strictly larger committed production schedule \( S_t(\cdot) \) for each renewable firm in period \( t \).

**Remark 1.** This proposition and subsequent propositions in this section do not require the uniqueness of the equilibrium as results hold for any equilibrium that satisfies (11) and (12). Thus, to facilitate the extensions of Propositions 4 through 7 to other settings, proofs of Propositions 4 through 7 (in Appendices E through H of the Electronic Companion) are presented for any equilibrium that satisfies (11) and (12).

Let us explain the intuition behind the conditions in (21). If there is an increase in \( \beta_{u,t+1} \), a renewable firm can mitigate the increase in the undersupply penalty rate \( \eta_{u,t+1}(\cdot) \) by uniformly inflating its committed production schedule in period \( t \), and thereby reducing the day-ahead market clearing price \( p_t^* \) for any commitment profile of inflexible firms. However, a renewable firm benefits from such a commitment inflation in equilibrium if and only if (a) the market clearing price is
highly sensitive to the changes in the committed production schedules so that a slight increase in the committed production schedule can achieve a significant reduction in the realized equilibrium undersupply penalty rate with the new $\beta_{u,t+1}$, and (b) the expected cost of commitment is low. Condition (b) is equivalent to having a sufficiently small $b_{u,t+1}$. Condition (a) is equivalent to having a small number of renewable firms in the market because if the market is extremely competitive (i.e., $N_r \to \infty$), a renewable firm has very limited or negligibly small market power to influence the market clearing price with its commitment.

Figure 1 pictures a numerical example where in equilibrium, a uniformly larger $\eta_{u,t+1}(\cdot)$ (due to a higher $\beta_{u,t+1}$) results in a uniformly and strictly larger committed production schedule for each renewable firm in period $t$, as suggested by Proposition 4. For any given $S$, although such commitment inflations result in a smaller market clearing price (due to uniformly and strictly larger aggregate committed production schedule by all renewable firms in period $t$), the realized undersupply penalty rate $\eta_{u,t+1}(p^*_t)$ with the new $\beta_{u,t+1}$ can be strictly larger than the one with the original $\beta_{u,t+1}$. This is because, in certain cases, the resulting decrease in the market clearing price might not be large enough to suppress the increase in $\beta_{u,t+1}$. In fact, Proposition EC.9 (in Appendix P) shows that, if (21), for any commitment profile of inflexible firms, an increase in $\beta_{u,t+1}$ results in a strictly larger realized undersupply penalty rate $\eta_{u,t+1}(p^*_t)$ in equilibrium when $\beta_{u,t+1}$ is small and $\epsilon_t$ is moderate.

Let us now study the implications of $\beta_{u,t+1}$ on period-$t$ reliability (10) in equilibrium, which is an important performance metric for an ISO (MISO 2014a). The rationale behind the existence of an
undersupply penalty is to improve reliability by motivating firms to commit to production quantities they can deliver in the following day. The following two propositions show that an undersupply penalty rate can defeat this purpose.

**Proposition 5.** (Period-t Reliability Degradation with a Larger \( \eta_{u,t+1}(\cdot) \))

Consider any \( t = 1, \ldots, \tau \) and \( \bar{S} \). (i) An increase in \( \beta_{u,t+1} \) results in strictly lower period-t reliability (10) for every realization of \( \epsilon_t \) (i.e., with probability 1) in equilibrium if and only if (21). (ii) If (21), imposing a positive market-based penalty rate \( \beta_{u,t+1} p \) in addition to the fixed rate \( b_{u,t+1} \) per unit underproduction results in strictly lower period-t reliability (10) for every realization of \( \epsilon_t \) (i.e., with probability 1) in equilibrium.

Proposition 6 proves a stronger result on reliability with some additional conditions.

**Proposition 6.** (Reliability Degradation in All Periods with a Larger \( \eta_{u,t+1}(\cdot) \))

Suppose that (21) holds for some \( t = 1, \ldots, \tau \). (i) In equilibrium, an increase in \( \beta_{u,t+1} \) results in lower reliability in all periods for every realization of \( \epsilon_j \), \( j = 1, \ldots, \tau \) (i.e., with probability 1), if either (1) \( K < K_L \) where \( K_L > 0 \) is a constant, or (2) the following set of conditions hold for a particular \( \gamma > 0 \) and some positive constants \( \tilde{s} \) and \( K_H \) such that \( \tilde{s} \leq K \) and \( K_L < K_H \):

\[
\left( \mathbb{E}_{\epsilon_t} \left[ \frac{\partial S_t(p_t(\tilde{s}))}{\partial \beta_{u,t+1}} \left( \epsilon_t \geq z_t(\tilde{s}) \right) \right] \right)^2 > \gamma, \quad K > K_H \quad \text{and} \quad S_t''(\cdot) > -\alpha_t^2/(N, K). \tag{22}
\]

(ii) In equilibrium, if either the conditions in (22) hold for \( \beta_{u,t+1} \in [0, 1) \) or \( K < K_L \), imposing a positive market-based penalty rate \( \beta_{u,t+1} p \) (in addition to the fixed penalty rate \( b_{u,t+1} \)) per unit underproduction results in lower reliability (10) in all periods with probability 1.

Let us explain the conditions in (22). Proposition 4 proves that, if (21), each renewable firm uniformly inflates its committed production schedule in period \( t \) as a response to an increase in \( \beta_{u,t+1} \). Based on this, the first condition in (22) introduces a measure for the magnitude of such inflations, and represents a scenario in which the aforementioned measure of inflation is sufficiently large. The second condition in (22) implies that each inflexible firm’s capacity is non-binding in equilibrium for all \( \beta_{u,t+1} \in [0, 1) \) so that the commitments of inflexible firms are sensitive to changes in \( \beta_{u,t+1} \). The third condition in (22) ensures that, when \( \beta_{u,t+1} \) increases, the period-t market clearing price does not become too insensitive to the production commitment of inflexible firms. To see this, suppose \( \beta_{u,t+1} \) is increased. For a given \( \bar{S} \), this would reduce period-t market clearing price due to the renewable firms’ commitment inflation (explained in Proposition 4).

Note that the expression on the left hand side of the first condition in (22) is a 0.5-norm \( \| \cdot \|_{0.5} \) of the production commitment inflation.
the inverse of \( S_t(p) \) became very flat for smaller \( p \), then period-\( t \) market clearing price would have been virtually unresponsive to the inflexible firms’ production commitments.) As a result, if the magnitude of renewable commitment inflation is large (which corresponds to the first condition in (22)), and if the market clearing price is sufficiently sensitive to the inflexible firms’ production commitment (which is guaranteed by the third condition in (22)), each inflexible firm commits strictly less to increase the equilibrium market clearing price for any \( \epsilon_t \). Strictly less commitment \( \bar{S} \) by each inflexible firm results in a uniformly and strictly larger residual demand curve for renewable firms in all periods. With this, a strictly larger commitment is cleared for each renewable firm in every period, resulting in strictly lower system reliability in all periods.

Propositions 4 through 6 hold when the competition among renewable firms is not too intense. In a more general setting, Proposition 7 below shows that each renewable firm can inflate its equilibrium period-\( t \) commitment due to an increase in the undersupply penalty rate \( \eta_{u,t+1}(\cdot) \) even if there is intense competition among renewable firms. However, such an inflation occurs only in a certain price range in equilibrium.

**Proposition 7.** *(Partial Commitment Inflation with a Larger \( \eta_{u,t+1}(\cdot) \))* For any \( t = 1, \ldots, \tau \), consider the following two alternative schemes for the undersupply penalty rate in period \( t+1 \): \( \eta_1(p) = \beta_1 p + b_1 \) and \( \eta_2(p) = \beta_2 p + b_2 \) such that \( \beta_1 > \beta_2 \geq 0 \) and \( 0 < b_1 < b_2 \). Then, in equilibrium, there exists a price interval in which each renewable firm commits to a strictly larger production quantity in period \( t \) under the penalty scheme with a larger \( \eta_{u,t+1}(\cdot) \).

**Corollary 1.** Under the setting stated in Proposition 7, there exists a nonempty period-\( t \) day-ahead demand shock interval in which the scheme with a larger \( \eta_{u,t+1}(\cdot) \) results in a strictly lower period-\( t \) reliability (10) for any \( \bar{S} \) in equilibrium.

The proof of Proposition 7 shows that the scenario in which \( S_t(p; \eta_1) > S_t(p; \eta_2) \) despite \( \eta_1(p) > \eta_2(p) \) can occur when the price is not too small. The numerical example in Section 6 demonstrates that when parameters are set to realistic values, such type of a commitment inflation can be observed for a wide range of prices (i.e., for \( p > \$4.7 \) per MWh). From a reliability standpoint, this means that even when there is intense competition among renewable firms, period-\( t \) reliability can decrease for a very wide range of day-ahead demand shocks due to an increase in the undersupply penalty rate. The numerical example (in Section 6) verifies this, and shows that in fact, the aforementioned type of commitment inflations can strictly decrease the expected reliability in the associated day-ahead market.

Propositions 4 through 7 offer important insights for settings in which firms choose their optimal production schedules in the day-ahead market and pay their undersupply penalty based on the day-ahead market clearing itself or a closely related electricity market such as a balancing market. In the
U.S., example of such settings include regional markets operated by MISO, PJM and ISO-NE (MISO 2016b, PJM 2015, UWIG 2011). As explained in Section 2, when the undersupply penalty depends on the price realized in a closely related market, which in turn depends on the day-ahead price, the coefficient $\beta_{u,t+1}$ represents the ultimate dependence of the undersupply penalty rate on the day-ahead market clearing price. The magnitude of $\beta_{u,t+1}$ can be viewed as a measure of dependence between the day-ahead price and the realized price in the market based on which the undersupply penalty is calculated. In light of this, a higher $\beta_{u,t+1}$ corresponds to a higher dependence between these two market prices.

In this context, Propositions 4 and 7 have important implications for firms. If the undersupply penalty rate is linked to the realized price of another electricity market whose outcomes are strongly dependent on the outcomes of the day-ahead market, firms can manipulate the undersupply penalty rate by changing their production schedules in the day-ahead market. As a result, with a higher dependence, despite the undersupply penalty rate is higher at all $p > 0$, firms can have the financial incentive to uniformly or partially inflate their committed production schedules, thereby mitigating the increase in the undersupply penalty rate, as suggested by Propositions 4 and 7.

Propositions 5 and 6 have key implications for reliability. Accordingly, linking the undersupply penalty rate to the realized price of a market that is more closely related to the day-ahead electricity market can strictly decrease reliability (10) for any realization of the day-ahead demand shock. It is straightforward to show that the magnitude of such type of reliability degradation in period $t$ increases with the realization of random shock $\epsilon_t$. Thus, for an independent system operator, one way to mitigate said reliability degradations is to implement more accurate forecasting techniques to reduce $\epsilon_t$. Another way to mitigate this type of reliability degradations is to link the undersupply penalty rate to a market that is less related to the day-ahead electricity market. In fact, Proposition 6 suggests that for markets in which the aforementioned type of reliability degradation is possible, letting $\beta_{u,t+1} = 0$ maximizes the reliability (10) for any realization of the day-ahead demand shock in all periods. Thus, for such type of markets, using a fixed undersupply penalty rate $b_{u,t+1}$ instead of a market-based undersupply rate achieves the maximum reliability in all periods with probability 1.

Proposition EC.10 (in Appendix P) analyzes the impact of $b_{u,t+1}$ on renewable firms’ equilibrium commitments and the discussion following it explains the reliability implications. In contrast to

---

The positive $\beta_{u,t+1}$ suggests a positive dependence. The intuition behind the positive dependence can be explained as follows. Holding all other problem parameters the same, if the random shock $\epsilon$ increases (respectively, decreases), a higher (respectively, lower) day-ahead market clearing price is realized. This implies a larger (respectively, smaller) total production commitment in equilibrium by the monotonicity constraint in (18). As a result, the total available supply net from the production commitment is smaller (respectively, larger), implying a higher (respectively, a lower) net demand in the rest of the system and hence a higher (respectively, a lower) market-clearing price in the closely related electricity market.
Proposition 4. Proposition EC.10 shows that if there is an increase in \( b_{u,t+1} \), each renewable firm’s equilibrium production commitments in period \( t \) decrease in a non-empty price interval. Thus, in equilibrium, renewable firms do not uniformly inflate their committed production schedules as a response to an increase in \( b_{u,t+1} \). This suggests that a larger undersupply penalty rate \( \eta_{u,t+1}(\cdot) \) can provide financial incentives to renewable firms for uniformly inflating their equilibrium production commitments only when the undersupply penalty rate is market-based (as in MISO, PJM and ISO-NE (MISO 2016b, PJM 2015, UWIG 2011)); a fixed undersupply penalty rate \( b_{u,t+1} \) does not offer such incentives.

4. Firms with both Renewable and Nonrenewable Technologies

Section 3 analyzed a day-ahead market with \( N_r \) renewable and \( N_i \) inflexible firms. To understand the implications of firms with multiple diversified technologies, this section considers a setting where there are \( N \) firms, each of which owns one renewable generator and one inflexible generator. All other modelling elements are the same as in Section 2, with the exception that the period-\( t \) market clearing equation (1) must be modified as

\[
D_t(p_t^*, \epsilon_t) = \sum_{j=1}^{N} S_{j,t}(p_t^*) + \sum_{j=1}^{N} \bar{S}_j.
\]

In this setting, firm \( j \)'s optimal inflexible energy production is \( \bar{S}_j \). Because firm \( j \)'s (production-related) total realized net penalty in period \( t+1 \) is (2), the optimal renewable energy production of firm \( j \) is as identified in Lemma 1. (Proof of Lemma 1 remains the same.) Therefore, firm \( j \)'s period-\( t \) expected profit from renewable energy commitment is (4), implying that firm \( j \)'s committed renewable energy production schedule \( S_{j,t}(\cdot) \) is as identified in Proposition 1 in a period-\( t \) supply function equilibrium that satisfies (12). (The proof of Proposition 1 remains the same.) Firm \( j \)'s inflexible energy commitment at \( t = 1 \) impacts the market-clearing price in every day-ahead market; hence, it affects firm \( j \)'s expected profit from both renewable and inflexible energy in every period. Thus, at \( t = 1 \), each firm \( j \) chooses an inflexible energy commitment \( \bar{S}_j \in [0, K] \) to maximize its total expected profit from its inflexible and renewable energy commitments, respectively:

\[
\mathbb{E}_x \left[ \sum_{t=1}^{\tau} p_t^* \bar{S}_j - C_{t+1}(\bar{S}_j) \right] + \mathbb{E}_{\tilde{\epsilon}} \left[ \sum_{t=1}^{\tau} p_t^* S_{j,t}(p_t^*) - \epsilon_t \bar{S}_j - \eta_{u,t+1}(p_t^*) \mathbb{E}_{Q_{j,t+1}} \left[ (S_{j,t}(p_t^*) - Q_{j,t+1})^+ \right] \right]. \tag{23}
\]

Here, \( \tilde{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_\tau) \) is a vector of day-ahead random shocks, and the period-\( t \) market clearing price \( p_t^* \) is a function of \( \epsilon_t, \bar{S}_j \) and other firms’ period-\( t \) commitment profiles \( \bar{S}_{-j} \) and \( S_t \).

Proposition 8 below identifies firms’ equilibrium strategies. To state Proposition 8, we shall define

\[
\delta_1(p) = \beta_{u,t+1} \int_0^{S_t(p)} (S_t(p) - x) dF_{t+1}(x) \quad \text{and} \quad \delta_2(p) = p - \eta_{u,t+1}(p) F_{t+1}(S_t(p)) \]

where \( S_t(\cdot) \) is a function that satisfies (17) and (18).

\textbf{PROPOSITION 8.} (i) There exists an equilibrium \( (S_1, \ldots, S_{\tau}, \bar{S}, \bar{q}) \) that satisfies (11) and (12). In such an equilibrium, firms’ strategies are as follows.

(ii) For \( t = 1, \ldots, \tau \), each firm \( j \) commits to a renewable production schedule \( S_t(\cdot) \) that satisfies (17)
and (18) in the period-\(t\) day-ahead market, and firm \(j\)'s optimal renewable energy production in period \(t+1\) is \(\min\{S_t(p^*_t), Q_{j,t+1}\}\).

(iii) At \(t=1\), each firm's inflexible energy commitment is \(\bar{S} = \min\{K, \bar{s}^*\}\) such that \(\bar{s}^*\) is the solution of

\[
\Psi_M(\bar{s}) = \sum_{t=1}^\tau \int_{\tilde{z}_t(\bar{s})}^{\infty} \left( \frac{S_t(p^*_t(\bar{s}; z)) - \delta_1(p^*_t(\bar{s}; z)) + S_t^1(p^*_t(\bar{s}; z))}{\alpha_t + NS_t^1(p^*_t(\bar{s}; z))} \right) d\Phi_t(z) \\
- \sum_{t=1}^\tau \int_{\tilde{z}_t(\bar{s})}^{\infty} \left( p^*_t(\bar{s}; z) - \frac{\bar{s}}{\alpha_t + NS_t^1(p^*_t(\bar{s}; z))} - C_{t+1}'(\bar{s}) \right) d\Phi_t(z) + C_{t+1}(\bar{s}) = 0,
\]

where \(\tilde{z}_t(\bar{s})\) is as defined in (19), \(S_t(\cdot)\) is as in Proposition 8-(ii) and \(p^*_t(\bar{s}; z)\) satisfies \(v_t - \alpha_t p^*_t(\bar{s}; z) + z = N(\bar{s} + S_t(p^*_t(\bar{s}; z)))\).

To compare the equilibrium characterized in Proposition 8 with the one identified in Proposition 2, we let \(N_r = N_i = N\). Proposition 8 shows that, compared to Proposition 2, each firm’s inflexible energy commitment is the only element that differs in equilibrium. Regarding other results in Section 3, the statements and the proofs of Propositions 3, 4, 5 and 7 remain unchanged in this setting. Proposition 6 holds with a larger lower bound on \(S''_t(\cdot)\) in (22). As a result, the main results and insights in Section 3 hold in this setting.

Finally, the following proposition studies the implications of these multi-technology firms for inflexible energy commitments and reliability in equilibrium.

**Proposition 9.** (i) A multi-technology firm’s equilibrium inflexible energy commitment (identified in Proposition 8-(iii)) is smaller than an inflexible-only firm’s equilibrium commitment (identified in Proposition 2-(iii)).

(ii) For any \(t = 1, \ldots, \tau\), a day-ahead electricity market with multi-technology firms results in lower period-\(t\) reliability (10) for any realization of period-\(t\) day-ahead demand shock \(\epsilon_t\) (i.e., with probability 1) in equilibrium, compared to a day-ahead electricity market explained in Section 2.

If firms have multiple diversified technologies, each restricts its inflexible energy commitment to increase the market clearing price in all periods with probability 1, thereby gaining a larger expected profit in equilibrium. As a result, when firms have multiple diversified technologies, a larger renewable energy commitment is cleared to satisfy a particular day-ahead demand in each period.

This immediately implies lower equilibrium reliability in all periods with probability 1.

### 5. Subsidy

In the U.S., various renewable firms receive subsidies for their production. For example, U.S. wind energy producers receive a “production tax credit” for their production (DSIRE 2015). Motivated by this fact, this section extends our base model (in Section 2) to consider production-based subsidies for renewable firms.
Each renewable firm \( n \) receives a total subsidy of \( T_t(q_{n,t}) \) by producing \( q_{n,t} \) in period \( t \) for any \( t \). Here, \( T_t(\cdot) \) is a twice continuously differentiable and concave function that satisfies \( T_t(0) = 0 \) and \( T_t'(\cdot) > 0 \). All other modelling elements are the same as the ones explained in Section 2, except the following: In this setting, the optimal production quantity of renewable firm \( n \) in period \( t+1 \) is the one that minimizes the firm’s realized net penalty in that period:

\[
q_{n,t+1}^* \doteq \arg\min_{0 \leq q_{n,t+1} \leq Q_{n,t+1}} \eta_{u,t+1}(p_t^*) \left( S_{n,t}(p_t^*) - q_{n,t+1} \right)^+ + \eta_{o,t+1}(p_t^*) \left( q_{n,t+1} - S_{n,t}(p_t^*) \right)^+ - T_{t+1}(q_{n,t+1}).
\]

Thus, in period \( t \), renewable firm \( n \) chooses a production schedule \( S_{n,t}(\cdot) \) to maximize its commitment-related expected profit (26) for any random shock \( \epsilon_t \), given other firms’ commitments \( S_{-n,t} \) and \( \bar{S} \):

\[
\mathbb{E}_{Q_{n,t+1}} \left[ pR_{n,t}(p; \epsilon_t) + T_{t+1}(q_{n,t+1}^*) - \eta_{u,t+1}(p) \left( R_{n,t}(p; \epsilon_t) - q_{n,t+1}^* \right)^+ - \eta_{o,t+1}(p) \left( q_{n,t+1}^* - R_{n,t}(p; \epsilon_t) \right)^+ \right],
\]

where \( q_{n,t+1}^* \) is a function of \( Q_{n,t+1} \) by (25). Accounting for these differences, our aim is to characterize firms’ committed production schedules and optimal production strategies in an equilibrium that satisfies (11) and (12). The analysis in this section requires considering two possible cases about the initial subsidy rate \( T_t^r(0) \) for all \( t \): \( b_{o,t} < T_t^r(0) \) and \( b_{o,t} \geq T_t^r(0) \). To state our results for \( b_{o,t} < T_t^r(0) \), we define the critical supply level \( \hat{s}_t \) as the solution of

\[
T_t^r(\hat{s}_t) = b_{o,t},
\]

For \( b_{o,t} < T_t^r(0) \), the attention is restricted to \( T_t^r(\cdot) \) that ensures the uniqueness of \( \hat{s}_t \). Below, Proposition 10 establishes the existence of a solution function \( S_t(\cdot) \) to a particular set of conditions. Later, Proposition 11 will verify that \( S_t(\cdot) \) characterizes each renewable firm’s committed production schedule in equilibrium.

**PROPOSITION 10.** Consider any \( t = 1, \ldots, \tau \). \( i) \) If \( b_{o,t+1} < T_{t+1}^r(0) \), there exists a function \( S_t(\cdot) \) that satisfies the following two-piece ordinary differential equation

\[
S_t'(p) = \begin{cases} 
\left[ \frac{S_t(p) - \beta_{u,t+1} \int_0^{S_t(p)} (S_t(p) - x) dF_{t+1}(x)}{p - \eta_{u,t+1}(p)F_{t+1}(S_t(p)) + b_{o,t+1} (1 - F_{t+1}(S_t(p)))} \right] \lambda_r - \alpha_t & \text{if } S_t(p) < \hat{s}_{t+1}, \\
\left[ \frac{S_t(p) - \beta_{u,t+1} \int_0^{S_t(p)} (S_t(p) - x) dF_{t+1}(x)}{p - \eta_{u,t+1}(p)F_{t+1}(S_t(p)) + T_{t+1}(S_t(p))(1 - F_{t+1}(S_t(p)))} \right] \lambda_r - \alpha_t & \text{if } S_t(p) \geq \hat{s}_{t+1},
\end{cases}
\]

subject to a monotonicity constraint and an initial condition, respectively:

\[
0 < S_t'(p) < \infty \text{ for } p \in (p_{\ell,t}, \infty) \quad \text{and} \quad S_t(p_{\ell,t}) = 0,
\]

where \( p_{\ell,t} \doteq -b_{o,t+1} \) and \( \lambda_r \doteq 1/(N_r - 1) \).

\( ii) \) There exists a function \( S_t(\cdot) \) that satisfies (28b) subject to the following monotonicity constraint and initial condition, respectively: \( 0 < S_t'(p) < \infty \) for \( p > p_{\ell,t} \) and \( S_t(p_{\ell,t}) = 0 \) where \( p_{\ell,t} \doteq -T_{t+1}^r(0) \).
Recall from Proposition 2 that, in our base model, each renewable firm’s committed production schedule in a period is characterized by the solution of a single ODE subject to certain conditions. Proposition 11 and the discussion at the end of Appendix K (in the Electronic Companion) establish a new structure: In stark contrast to Proposition 2, Proposition 11 proves that if renewable firms receive subsidies, the characterization of a renewable firm’s committed production schedule in a period can require more than one ODE in equilibrium. For ease of exposition, we focus on a fixed oversupply penalty rate in Proposition 11, and explain at the end of Appendix K that the multiple-piece structure of an equilibrium committed production schedule is preserved with a market-based oversupply penalty rate.

**Proposition 11.** (i) There exists an equilibrium $\langle S_1, \ldots, S_T, \bar{S}, \bar{q} \rangle$ that satisfies (11) and (12). In such an equilibrium, firms’ strategies are as follows.

(ii) Consider any $t = 1, \ldots, \tau$. If $b_{o,t+1} < T'_{t+1}(0)$, then each renewable firm commits to a production schedule $S_t(\cdot)$ that satisfies (28a) through (29) in period-\(t\) day-ahead market, and the optimal production quantity of renewable firm $n$ in period $t+1$ is

$$
q_{n,t+1}^* = \begin{cases} 
Q_{n,t+1} & \text{if } Q_{n,t+1} \leq S_t(p_t^*), \\
S_t(p_t^*) & \text{if } Q_{n,t+1} > S_t(p_t^*) \text{ and } b_{o,t+1} > T'_{t+1}(S_t(p_t^*)), \\
\hat{s}_{t+1} & \text{if } Q_{n,t+1} > S_t(p_t^*) \text{ and } T_{t+1}(Q_{n,t+1}) < b_{o,t+1} \leq T'_{t+1}(S_t(p_t^*)), \\
Q_{n,t+1} & \text{if } Q_{n,t+1} > S_t(p_t^*) \text{ and } b_{o,t+1} \leq T'_{t+1}(Q_{n,t+1}).
\end{cases}
$$

(iii) Consider any $t = 1, \ldots, \tau$. If $b_{o,t+1} \geq T'_{t+1}(0)$, then each renewable firm commits to a production schedule $S_t(\cdot)$ that satisfies said conditions in Proposition 10-(ii), and the optimal production quantity of renewable firm $n$ in period $t+1$ is $q_{n,t+1}^* = \min\{Q_{n,t+1}, S_t(p_t^*)\}$.

(iv) At $t = 1$, each inflexible firm commits to a production quantity $\bar{S} = \min\{\bar{s}^*, K\}$ such that $\bar{s}^*$ satisfies

$$
0 = \sum_{t=1}^{\tau} \left[ \int_{\bar{z}_{t}(\bar{s})}^{\infty} \left( p_t^*(\bar{s};z) - \frac{\bar{s}}{\alpha_t + N_t S_t'(p_t^*(\bar{s};z))} - C_{t+1}'(\bar{s}) \right) d\Phi_t(z) \right] + \left[ C_{t+1}(\bar{s}) - pt\bar{s} \right] \phi_t(\bar{z}_t(\bar{s})),
$$

where $\bar{z}_t(\bar{s}) = N_t\bar{s} + \alpha_t p_t - v_t$, $\bar{z}_t(\bar{s}) = N_t\bar{s} + \alpha_t p_{t-1} - v_t$, $p_t^*(\bar{s};z)$ is the solution of $v_t - \alpha_t p_t'(\bar{s};z) + z = N_t S_t(p_t'(\bar{s};z)) + N_t\bar{s}$, and $S_t(\cdot)$ is as identified in part (ii) if $b_{o,t+1} < T'_{t+1}(0)$; otherwise, $S_t(\cdot)$ is as in part (iii).

**Remark 2.** Proposition 11 demonstrates that with subsidies, each renewable firm commits to a positive production quantity over a range of negative prices.\(^4\)

\(^4\) A negative day-ahead market clearing price means that each firm with a positive production commitment at that price pays for its commitment. Negative electricity prices are observed in practice, especially when the market demand is
By (27), if the initial subsidy rate is large in period \( t + 1 \) (i.e., \( T_{t+1}'(0) > b_{o,t+1} \)), the quantity space is divided into two intervals - the subsidy-prevalent interval \( [0, \hat{s}_{t+1}] \) and the penalty-prevalent interval \( [\hat{s}_{t+1}, \infty) \) - as the subsidy rate is larger than the oversupply penalty rate if and only if the quantity is in \( [0, \hat{s}_{t+1}] \). Proposition 11-(ii) shows that \( S_t(\cdot) \) satisfies a different ODE in each of these intervals. The difference between (28a) and (28b) is that the parameter \( b_{o,t+1} \) in (28a) is replaced with \( T_{t+1}'(S_t(p)) \) in (28b). This is because when \( Q_{n,t+1} \geq S_t(p) \), depending on which interval renewable firm \( n \)'s commitment is in, the firm gains a different benefit per unit period-\( t \) commitment in addition to price \( p \). To see this, suppose that \( S_t(p) \in [0, \hat{s}_{t+1}] \). In this subsidy-prevalent interval, a renewable firm optimally overproduces to benefit from the large subsidy rate. Therefore, by committing to an additional unit in period \( t \), the firm decreases its overproduction by one unit in period \( t + 1 \), thereby saving the associated oversupply penalty rate \( b_{o,t+1} \). On the other hand, if the firm’s commitment \( S_t(p) \) is in the penalty-prevalent interval \( [\hat{s}_{t+1}, \infty) \), the firm’s optimal production quantity is \( S_t(p) \) in period \( t + 1 \). As a result, by increasing \( S_t(p) \) by one unit, the firm also increases its production in period \( t + 1 \) by one unit and gains an additional benefit of \( T_{t+1}'(S_t(p)) \).

(footnote \#4 is continuing) low and there are production subsidies (Malik and Weber 2016, U.S. EIA 2012). U.S. EIA (2012) explains that subsidizing renewable energy is a reason for negative prices observed in practice. With production subsidies, renewable firms can profitably produce even at negative prices due to said additional revenue stream. As a result, they can profitably offer positive production quantities at negative prices in day-ahead markets, leading to a negative market clearing price when the day-ahead demand is low (that is, when either \( \nu_t \) or \( \epsilon_t \) is low). This is consistent with the finding in Proposition 11.
Proposition 11 explicitly characterizes each renewable firm’s optimal production strategy. Accordingly, a renewable firm’s production quantity in period $t+1$ is highly dependent on the firm’s committed production schedule and the day-ahead market clearing price in period $t$, as well as the realized available supply of the firm in period $t+1$. If the initial subsidy rate is sufficiently small (i.e., $T_{t+1}(0) \leq b_{o,t+1}$), the oversupply penalty rate always dominates the subsidy rate in period $t+1$. This deters a renewable firm from overproduction in period $t+1$, as shown in Proposition 11-(iii).

If the initial subsidy rate is not too small, overproduction becomes a viable option for a renewable firm. In this case, Proposition 11-(ii) shows that the space for the day-ahead market clearing price and the available supply is divided into four regions and in each of these regions it is optimal to implement a particular production strategy. Figure 2-(b) pictures these four regions (i.e., regions I through IV) for a numerical example. (Regions I, II, III and IV are characterized by conditions in (30d), (30c), (30b) and (30a), respectively.) Note that overproduction is a feasible strategy in regions I through III where the firm’s available supply is larger than its commitment. In these regions, the firm faces a tradeoff between paying an oversupply penalty versus receiving a subsidy in evaluating its overproduction option. In region I, the firm optimally produces all of its available supply because the firm’s available supply is in the subsidy-prevalent interval $[0, \hat{s}_{t+1}]$ and underproduction is never optimal due to the undersupply penalty. In contrast, in regions II and III, the firm’s available supply is in the (oversupply) penalty-prevalent interval $[\hat{s}_{t+1}, \infty)$. In region II, the firm optimally produces $\hat{s}_{t+1}$ because it is suboptimal to underproduce and $\hat{s}_{t+1}$ is the largest quantity at which the subsidy rate (weakly) dominates the oversupply penalty rate. In region III, the firm optimally produces its day-ahead commitment because underproduction is never optimal and the oversupply penalty rate dominates the subsidy rate at any production quantity in that region. In region IV, the firm’s available supply is smaller than the firm’s commitment. Thus, to minimize the undersupply penalty, the firm optimally produces the maximum feasible quantity, which is its available supply.

Regarding other results in Section 3, proofs of Propositions 4 through 6 extend to this setting in a straightforward fashion, meaning that our insights in Section 3 hold with subsidies. Specifically, the uniform inflation of $S_t(\cdot)$ with a larger $\eta_{u,t+1}(\cdot)$ (in Proposition 4), the period-$t$ reliability degradation with probability 1 in equilibrium due to a larger $\eta_{u,t+1}(\cdot)$ (in Proposition 5) occur if (21) and the fixed oversupply penalty rate is not too large. Furthermore, under aforementioned conditions, Proposition 6 holds as stated.

The discussion at the end of Appendix K (of the Electronic Companion) explains that with a market-based oversupply penalty rate, the aforementioned insights related to the commitment and production strategies of a renewable firm remain the same, except with a market-based oversupply penalty rate, the critical supply $\hat{s}_{t+1}$ is a decreasing function of price rather than a constant. Therefore, in this case, the lower boundary of region II is a decreasing curve rather than a flat line.
In this section, we provided an analysis of general concave subsidy functions. It is perhaps worth noting that the analysis of a linear subsidy is a special case of said analysis.

6. Numerical Example

This section presents a numerical example based on MISO’s day-ahead electricity market. The purpose of the numerical example is to demonstrate the following two insights: (i) even a slight change in the undersupply penalty rate can have a large impact on firms’ equilibrium commitments (and hence reliability), and (ii) unexpected consequences of a larger undersupply penalty rate for firms’ commitments and reliability can be observed at plausible prices when the parameters are set to realistic values. Below, we first explain how these realistic values are obtained for parameters of interest. Then, using these parameters, we solve for the equilibrium characterized in Proposition 2 under two alternative penalty schemes.

In MISO’s day-ahead electricity market, a day is divided into 24 time blocks: 12:00am - 1:00am, 1:00am - 2:00am,. . . , 11:00pm - 12:00am. For any given time block, a firm can commit to a production schedule in the day-ahead market to deliver electricity in the given time block of the following day. The day-ahead market is cleared for each time block, resulting in 24 pairs of MISO system day-ahead electricity prices and total cleared production commitments.

In MISO system, there are \( N_i = 45 \) major inflexible firms (U.S. EIA 2014b). Using the U.S. fuel price data set (U.S. EIA 2014a) and the U.S. power plant operations database (U.S. EIA 2014b), we calculate monthly total cost and monthly total electricity generation of each inflexible firm. Assuming that the variable cost of production for each inflexible firm in month \( m = 1, \ldots, 12 \) is \( c_m q^2 \) at output \( q \), where \( c_m \) is the variable cost coefficient in month \( m \), we estimate \( c_m \) by running an ordinary least squares regression between the monthly total fuel cost and the monthly total electricity generation data for these 45 major inflexible firms.

MISO’s rule on “dispatchable intermittent resources (DIR),” which requires all major wind producers to register as a displaceable intermittent resource and actively participate in the electricity market, was not in full effect at the beginning of 2013 (MISO 2014a). Thus, to estimate the price sensitivity parameter \( \alpha \) for each time block in every month of 2013, we assume that the day-ahead electricity prices in 2013 are driven by \( N_i \) inflexible firms (each of which commit to a particular production quantity that remains the same for a certain time) and \( N_c = 25 \) other conventional firms that can commit to a different production schedule in every day-ahead electricity market (U.S. EIA 2014b). (It is perhaps worth noting that in 2013, the generation by the latter type of firms was only around 10% of the generation by inflexible firms in MISO system (MISO 2015a).) Firms’ equilibrium commitment strategies in this market are characterized by (EC.113) and (EC.114). Based on these, we find the estimate \( \hat{\alpha}_{m,\ell} \) of the true sensitivity parameter \( \alpha_{m,\ell} \) for each month \( m = 1, \ldots, 12 \)
and time block $\ell = 1, \ldots, 24$ by using a method that minimizes residual sum of squares for every time block of each month. Details of this method can be found in Appendix O of the Electronic Companion. For instance, as a result of the aforementioned procedure, we have $\hat{\alpha}_{5,10} = 0.0043$ for the 9:00am - 10:00am time block in May.

The cumulative probability distribution $F_{m,\ell}(\cdot)$ of a wind producer for each time block $\ell = 1, 2, \ldots, 24$ in month $m = 1, 2, \ldots, 12$ is identified as follows. MISO publishes the hourly wind generation data on a daily basis (MISO 2015c). The wind generation is usually smaller than the wind power potential due to curtailment. The MISO data do not include the curtailment percentages. Hence, we estimate the monthly curtailment percentages in 2013 using the curtailment data in (Ruud 2013) and (Wiser and Bolinger 2014). Using the estimated curtailment percentages, actual hourly wind generation data throughout 2013 (MISO 2015c) and the number of wind power producers in MISO system (U.S. EIA 2014b), we fit a Lomax distribution to estimate the wind power potential for each time block in each month, and estimate the scale and shape parameters via maximum likelihood estimation.

The penalty rules and rates vary from market to market. Using the explained parameter estimates for May, we now consider the market explained in Section 3 with $N_r = 25$ renewable firms and $N_i = 45$ inflexible firms, and solve for the equilibrium identified in Proposition 2 under the following two alternative rules for the undersupply penalty rate. Under the first rule (i.e., rule A), the undersupply penalty rate at day-ahead price $p$ is $\eta_A(p) = (p + 0.047)/\text{MWh}$, which is the day-ahead price $p$ plus the average fixed penalty rate in May (MISO 2016a). With the rule A, a firm receives the day-ahead payment only based on its actual delivery. Under the second rule (i.e., rule B), the undersupply penalty rate is equal to the real-time price, which is estimated as $\eta_B(p) = $(0.962$p + 0.225$)/\text{MWh} for the time block 9:00am - 10:00am in May. The average MISO day-ahead market clearing price in 2013 was $31.94/\text{MWh}$ (MISO 2014b). Thus, in equilibrium characterization, we restrict attention to a plausible range of day-ahead prices where $p$ is in between $0/\text{MWh}$ and $150/\text{MWh}$ under each of the undersupply penalty rules A and B.

Note that penalty coefficients are very close to each other under rules A and B in the time block 9:00am - 10:00am of May. Specifically, the fixed penalty terms under two rules differ only by $0.18 per MWh (which is around 0.5% of the average market clearing price in 2013), and the change in the weight assigned to day-ahead price is less than 4%. Despite this, renewable firms’ equilibrium committed production schedules are considerably different under the rules A and B. Starting from very small prices (i.e., $p \geq 4$ per MWh), each renewable firm’s equilibrium commitment under the rule A is more than 15% larger, relative to the one under the rule B.

In fact, average penalty coefficients in May also do not differ too much with respect to the penalty rule. However, because renewable firms’ commitments are very sensitive to the penalty rule, inflexible
firms’ equilibrium commitments are also (indirectly) very sensitive to the penalty rule. Assuming τ is one month, an inflexible firm’s equilibrium production commitment is 900 MW under the rule A whereas the corresponding figure under the rule B is 920 MW, which is around a 22% increase from the former commitment. The increase in the electricity production by all inflexible firms due to switching from the rule A to B is sufficient to meet more than 991,000 residential customers’ 2014 electricity demand in Michigan (U.S. EIA 2015). This and the large difference in renewable firms’ equilibrium commitments under rules A and B demonstrate that the penalty rule can have a drastic effect on equilibrium production commitments in day-ahead markets.

The unintended consequences of a larger undersupply penalty scheme for reliability and renewable firms’ commitments are prevalent in the time block 9:00am - 10:00am of May. Note that \( \eta_A(p) > \eta_B(p) \) for \( p > \$4.7 \) per MWh. However, based on our observation above, the equilibrium commitment of each renewable firm under \( \eta_A \) is strictly larger than the one under \( \eta_B \) for \( p > \$4.7 \) per MWh in said time block. These imply that for a wide range of prices, the production commitment of each renewable firm is strictly larger under the rule that results in a higher undersupply penalty rate, as suggested by Proposition 7. Furthermore, despite that \( \eta_A(\cdot) \) is strictly larger than \( \eta_B(\cdot) \) for \( p > \$4.7 \) per MWh, the penalty rule A results in strictly lower equilibrium reliability than the penalty rule B for any nonnegative random shock \( \epsilon \) in said time block. (Such random shocks imply an equilibrium market clearing price larger than \$4.7 per MWh under both penalty rules in the time block 9:00am - 10:00am of May.) Because the rule A results in strictly lower reliability in such a wide range of \( \epsilon \), it also results in strictly lower expected reliability than the rule B in said time block.

7. Further Extensions

In the electronic companion, Sections EC.1.1 through EC.1.4 analyze various variants of the model in Section 2, and show that the main results and insights developed in Section 3 are robust.

Recall from Section 2 that renewable firm \( n \)’s available supply \( Q_{n,t+1} \) in period \( t+1 \) can be seen as period-(\( t+1 \)) capacity constraint that is unobservable to firm \( n \) in period \( t \). The formulation in Section 2 allows for a cumulative probability distribution \( F_{t+1}(\cdot) \) such that \( Q_{n,t+1} \) is smaller than a finite number with almost probability 1. (Recall that \( Q_{n,t+1} \) has the same distribution function \( F_{t+1}(\cdot) \) for all \( n = 1, \ldots, N \); that property is necessary for tractability.) Section EC.1.1 analyzes a setting where the available supply \( Q_{n,t+1} \) is smaller than a finite constant with probability 1. Proposition EC.1 in Section EC.1.1 characterizes the equilibrium in this setting, and the last paragraph of Section EC.1.1 explains that the main results and insights in Section 3 extend to this setting. (The proof of Proposition 4 in this setting is included at the end of Appendix L in the Electronic Companion.)

Section EC.1.2 studies a model where the penalty rates in period \( t \) are also affected by the realization of \( \sum_{n=1}^{N} Q_{n,t} \) for all \( t \). Proposition EC.2 in Section EC.1.2 identifies the equilibrium in this
extended setting. The last paragraph of Section EC.1.2 explains that the main results and insights in Section 3 hold in this setting, as well. (Appendix M in the Electronic Companion includes the proof of Proposition 4 in this extended model.)

Section EC.1.3 explains that all results and proofs hold as stated if some of the renewable firms sell their supply through long term contracts. Finally, Section EC.1.4 allows renewable firms to receive revenue for their overproduction, and establishes that the main results and insights in Section 3 hold for a more general set of conditions in this extension (see Propositions EC.3 through EC.5 for formal statements in Section EC.1.4 and Appendix N in the Electronic Companion for the proofs of these and other supplementary results).

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