Latent Variable Modeling in Congruence Research: Current Problems and Future Directions
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Latent Variable Modeling in Congruence Research

Current Problems and Future Directions

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During the past decade, the use of polynomial regression has become increasingly prevalent in congruence research. One drawback of polynomial regression is that it relies on the assumption that variables are measured without error. This assumption is relaxed by structural equation modeling with latent variables. One application of structural equation modeling to congruence research is the latent congruence model (LCM). Although the LCM takes measurement error into account and allows tests of measurement equivalence, it is framed around the mean and algebraic difference of the components of congruence (e.g., the person and organization), which creates various interpretational problems. This article discusses problems with the LCM and shows how these problems are resolved by a linear structural equation model that uses the components of congruence as predictors and outcomes. Extensions of the linear model to quadratic equations used in polynomial regression analysis are discussed.

Keywords: congruence; difference scores; polynomial regression; latent variables; structural equation modeling

The concept of congruence maintains a central position in organizational research. Congruence refers to the fit, match, similarity, or agreement between two constructs, such as the personal and organizational values (Chatman, 1989), employee needs and organization rewards (Dawis, 1992), job demands and employee abilities (Edwards, 1996), and organizational strategy and the environment (Venkatraman, 1989). Congruence has been related to various causes and outcomes at the individual, group, and organizational levels (Donaldson, 2001; Edwards, 1991; Kristof-Brown, Zimmerman, & Johnson, 2005; Spokane, Meir, & Catalano, 2000).

The study of congruence has progressed through various methodological stages. Early studies relied on difference scores and profile similarity indices (Edwards, 1991; Kristof, 1996), which are prone to numerous methodological problems (Cronbach, 1958; Edwards, 1994; Johns, 1981). Many of these problems are avoided by polynomial regression (Edwards, 1994, 2002; Edwards & Parry, 1993), which has gained prominence during the past decade. For instance, a recent meta-analysis of person–environment fit research (Kristof-Brown et al., 2005) indicated that polynomial regression was used in about 20% of studies published since the method was introduced. One limitation of polynomial regression is that it rests on the assumption that variables are measured without error (Berry, 1993). This assumption is relaxed by structural equation modeling with latent variables measured without error.
variables (Bollen, 1989; Kline, 2004; Loehlin, 2004), and work that integrates polynomial regression with structural equation modeling is under way (Edwards & Kim, 2002).

One recent application of structural equation modeling to the study of congruence is the latent congruence model (LCM) developed by Cheung (2007). The LCM treats the components of congruence (e.g., the person and environment) as first-order factors with fixed loadings on second-order factors intended to represent the congruence (i.e., algebraic difference) and level (i.e., mean) of the components. The LCM takes measurement error into account by specifying the components of congruence as latent variables with multiple indicators. Cheung (2007) also has demonstrated how the indicators of the components can be tested for measurement equivalence, which is important for interpreting the comparison of the components in terms of congruence.

From a measurement perspective, the LCM is a step forward in congruence research, given that measurement error can bias coefficient estimates in polynomial regression and lead to erroneous conclusions. However, from a substantive perspective, the LCM is a step backward for two reasons. First, the LCM is restricted to linear relationships and, therefore, cannot address curvilinear relationships, which are central to congruence research. For instance, person–organization fit research is based on the premise that outcomes such as job satisfaction, organizational commitment, and performance decrease when the person and organization differ from one another in either direction (Chatman, 1989; Kristof, 1996). This premise implies a curvilinear (i.e., inverted-U) relationship between congruence and outcomes. More generally, any study that uses a squared difference, absolute difference, Euclidean distance, or profile correlation to operationalize congruence implicitly rests on the assumption that the relationship between congruence and other variables is curvilinear (Edwards, 1994). Because the LCM is strictly linear, it cannot test curvilinear relationships or detect whether relationships predicted to be linear are, in fact, curvilinear.

Second, the LCM shifts attention from the components of congruence to the difference and mean of the components. This shift reintroduces problems with difference scores that polynomial regression was designed to solve. For instance, using the difference between two components as a predictor conceals the relationship between each component and the outcome (Edwards, 1994). This problem also occurs when the mean of the components is used as a predictor (Lichtenberg, 1990), as in the LCM. Similar problems occur when the difference and mean of the components are used as outcomes (Edwards, 1995). By obscuring relationships involving the components of congruence, the LCM invites interpretations that are misleading or incorrect. For instance, the difference and mean of two components can relate to an outcome, suggesting support for congruence and level effects, when both relationships represent nothing more than a relationship between one component and the outcome. Furthermore, as shown later, results that satisfy the definitions of level and congruence are antagonistic such that, if evidence for level is found, evidence for congruence is necessarily ruled out and vice versa.

This article identifies problems with the LCM and explains how these problems are avoided by using the components of congruence as predictors and outcomes. This approach clarifies the relationships associated with the components and yields results that can be used to test hypotheses that level and congruence represent. Results from this approach can also be used to compute any information yielded by the LCM, which raises further questions concerning the value of the LCM to congruence research. Although the
LCM takes measurement error into account and can be used to evaluate measurement equivalence, these features also characterize linear structural equation models that use components as predictors and outcomes. The article concludes with future directions for structural equation modeling in congruence research, highlighting recent work on quadratic structural equation models (Edwards & Kim, 2002).

**Defining Level and Congruence**

The distinguishing feature of the LCM is its use of the algebraic difference and mean of the components of congruence as predictors and outcomes of other variables. This feature of the LCM involves the structural equations that relate the latent component variables to other latent variables, not the measurement equations that relate the latent component variables to their indicators. As such, this article focuses on the structural equations that underlie the LCM. These equations are expressed with simple notation that clearly distinguishes level and congruence from the component variables and helps to convey the problems that the LCM creates. Structural equations for the LCM using conventional LISREL notation are provided by Cheung (2007), and LISREL syntax for the alternative linear structural equation model advocated in this article is given in Appendix A.

The mapping of level and congruence onto the component variables can be expressed in equation form. Drawing from Equations 1 and 2 of Cheung (2007), the component variables can be expressed as functions of level and congruence as follows:

\[ Y_1 = L - 0.5C \]
\[ Y_2 = L + 0.5C, \]

where \( Y_1 \) and \( Y_2 \) are component variables, \( L \) is level, and \( C \) is congruence. Solving for \( L \) and \( C \) yields expressions that correspond to Equations 3 and 4 of Cheung (2007):

\[ L = 0.5(Y_1 + Y_2) \]
\[ C = Y_2 - Y_1. \]

Equations 3 and 4 show that \( L \) and \( C \) are the mean and algebraic difference, respectively, of \( Y_1 \) and \( Y_2 \).

Several points concerning the definitions of \( L \) and \( C \) in Equations 3 and 4 should be underscored. First, \( L \) and \( C \) are exact functions of \( Y_1 \) and \( Y_2 \), as indicated by the absence of residuals in Equations 3 and 4. Therefore, any results generated by \( L \) and \( C \) can also be obtained from \( Y_1 \) and \( Y_2 \), as demonstrated later in this article. Second, \( C \) is not defined as the match between \( Y_1 \) and \( Y_2 \), such that \( C \) is maximized when \( Y_1 \) and \( Y_2 \) are equal. Rather, \( C \) increases as \( Y_2 \) increases toward \( Y_1 \) and continues to increase as \( Y_2 \) exceeds \( Y_1 \). Thus, when \( Y_2 \) is less than \( Y_1 \), an increase in \( C \) means that the match between \( Y_1 \) and \( Y_2 \) is increasing, whereas when \( Y_2 \) is greater than \( Y_1 \), an increase in \( C \) means that the match between \( Y_1 \) and \( Y_2 \) is decreasing. Hence, an increase in \( C \) does not necessarily mean that two variables are closer to one another, which is how congruence is usually conceptualized in research on fit, similarity, and agreement (Edwards, 1994). Rather,
higher scores on C could mean that Y₁ and Y₂ are becoming closer together or further apart, depending on the relative magnitudes of Y₁ and Y₂.

A third point is that, when L and C are used as predictors or outcomes, the definitions of L and C imply specific patterns of relationships for Y₁ and Y₂. For instance, because L assigns equal weights of .5 to Y₁ and Y₂, using L as a predictor implies that Y₁ and Y₂ have equal effects on the outcome (Lichtenberg, 1990). Analogously, because C is an algebraic difference, it assigns equal but opposite weights of –1 and +1, respectively, to Y₁ and Y₂. As such, using C as a predictor implies that Y₁ and Y₂ have equal but opposite effects on the outcome (Edwards, 1994). Similarly, using L as an outcome implies that the causes of L have equal effects on Y₁ and Y₂, and using C as an outcome implies that the causes of C have equal but opposite effects on Y₁ and Y₂. The relationships for Y₁ and Y₂ implied by L and C follow from the definitions of L and C, and researchers attempting to interpret results for L and C are likely to infer the patterns described above. For example, if results show that L influences an outcome, it stands to reason that a researcher will interpret this result not as an effect for either Y₁ or Y₂ alone but rather as an effect that involves both Y₁ and Y₂, presumably with equal weights in light of how L is defined in Equation 3. Unfortunately, the coefficient on L provides no information about whether the effects of Y₁ and Y₂ are equal. Similar ambiguities arise when L is used as an outcome and when C is used as a predictor or outcome. These ambiguities undermine the interpretation of results for L and C and constitute what is perhaps the most serious drawback of the LCM. The following discussion traces the sources of these ambiguities and shows how they are avoided by replacing L and C with Y₁ and Y₂, supplemented by analyses that address conceptual issues that might motivate the use of L and C.

**Level and Congruence as Predictors**

To demonstrate the problems that occur when L and C are used as predictors, consider the following equation:

\[ Z = a₀ + a₁L + a₂C + e, \]  

where L and C are defined according to Equations 3 and 4, Z is the outcome, e is a residual, and a₀, a₁, and a₂ are unstandardized coefficients. Throughout this article, coefficients for L and C are labeled aᵢ, and coefficients for Y₁ and Y₂ are labeled bᵢ. Although this notation departs from the usual LISREL convention, it clearly differentiates models involving L and C versus Y₁ and Y₂. Traditional LISREL notation is used in the syntax provided in Appendix A.

A basic question concerning Equation 5 is how to interpret a₁ and a₂. This question might be addressed by considering how L and C are defined in Equations 3 and 4. As noted earlier, because L assigns the same weight of .5 to Y₁ and Y₂, a₁ would seem to represent equal effects of Y₁ and Y₂. Likewise, because C assigns equal but opposite weights of –1 and +1 to Y₁ and Y₂, a₂ appears to capture equal but opposite effects for Y₁ and Y₂. These interpretations of a₁ and a₂ are reinforced by substituting Equations 3 and 4 into Equation 5 and expanding:
\[ Z = a_0 + a_1(Y_1 + Y_2) + a_2(Y_2 - Y_1) + e \\
= a_0 + 0.5a_1Y_1 + 0.5a_1Y_2 + a_2Y_2 - a_2Y_1 + e. \] (6)

In Equation 6, the terms \(0.5a_1Y_1\) and \(0.5a_1Y_2\) result from the expansion of \(L\). Because \(Y_1\) and \(Y_2\) have the same coefficient, it appears that \(a_1\) implies equal effects of \(Y_1\) and \(Y_2\) on \(Z\). Similarly, the terms \(a_2Y_2\) and \(-a_2Y_1\) are produced by the expansion of \(C\), and given that these terms assign coefficients on \(Y_1\) and \(Y_2\) that have the same magnitude and opposite signs, it follows that \(a_2\) implies equal but opposite effects of \(Y_1\) and \(Y_2\) on \(Z\).

Although these interpretations of \(a_1\) and \(a_2\) might seem plausible, they are incorrect. The correct interpretations of \(a_1\) and \(a_2\) are revealed by collecting like terms in Equation 6 to obtain:

\[ Z = a_0 + (0.5a_1 - a_2)Y_1 + (0.5a_1 + a_2)Y_2 + e. \] (7)

Equation 7 shows that \(a_1\) and \(a_2\) are elements of compound coefficients on \(Y_1\) and \(Y_2\). Neither \(a_1\) nor \(a_2\) alone captures the magnitude or sign of the effects of \(Y_1\) or \(Y_2\) and, hence, whether these effects are consistent with the expansions of \(L\) or \(C\) in Equation 6. For instance, if \(Y_1\) and \(Y_2\) have equal effects, as implied by \(L\), then the compound coefficients on \(Y_1\) and \(Y_2\) in Equation 7 would be equal, such that \(0.5a_1 - a_2 = 0\). Subtracting \(0.5a_1\) from both sides of this equality and adding \(a_2\) to both sides yields \(2a_2 = 0\), which simplifies to \(a_2 = 0\). Hence, the extent to which \(Y_1\) and \(Y_2\) have equal effects on \(Z\) is not indicated by \(a_1\), the coefficient on \(L\), but rather by \(a_2\), the coefficient on \(C\). If \(a_2\) equals 0, then the coefficients on \(Y_1\) and \(Y_2\) are equal. If \(a_2\) differs from 0, the coefficients on \(Y_1\) and \(Y_2\) differ from one another by the value \(2a_2\).

Like \(a_1\), \(a_2\) does not itself indicate whether \(Y_1\) and \(Y_2\) have equal but opposite effects on \(Z\), as implied by \(C\). Rather, this condition holds when the compound coefficients on \(Y_1\) and \(Y_2\) in Equation 7 are equal in magnitude and opposite in sign, such that \(0.5a_1 - a_2 = 0\), or \(0.5a_1 - a_2 = -0.5a_1 - a_2\). Adding \(a_2\) to both sides of this equality and solving for \(a_1\) yields \(a_1 = 0\). Thus, whether \(Y_1\) and \(Y_2\) have equal but opposite effects is indicated not by \(a_2\), the coefficient on \(C\), but rather by \(a_1\), the coefficient on \(L\). If \(a_1 = 0\), then the compound coefficients on \(Y_1\) and \(Y_2\) reduce to \(-a_2\) and \(a_2\), respectively, such that \(Y_1\) and \(Y_2\) have equal but opposite effects. If \(a_1\) differs from 0, the effects of \(Y_1\) and \(Y_2\) are unequal in magnitude and can have the same or different signs.

The relative magnitudes of \(a_1\) and \(a_2\) also have important implications for interpreting results from Equation 5. For instance, if \(a_1 = 2a_2\), then the compound coefficient on \(Y_1\) equals 0, and the compound coefficient on \(Y_2\) equals \(a_1\) (or equivalently, \(2a_2\)). In this case, the coefficients on \(L\) and \(C\) in Equation 5 simply reflect an effect of \(Y_2\) on \(Z\). Likewise, if \(a_1 = -2a_2\), then the compound coefficient on \(Y_2\) equals 0, and the compound coefficient on \(Y_1\) equals \(a_1\) (or equivalently, \(-2a_2\)). In this case, the coefficients on \(L\) and \(C\) in Equation 5 are driven solely by the effect of \(Y_1\) on \(Z\). From a conceptual standpoint, it seems pointless to draw conclusions about level and congruence when results merely represent the effect of one component variable (cf. Cronbach, 1958)

Interpretational problems created when \(L\) and \(C\) are used as predictors are avoided by the following equation, which uses \(Y_1\) and \(Y_2\) as predictors:
Equation 8 directly captures the joint effects of $Y_1$ and $Y_2$ on $Z$, thereby avoiding ambiguities associated with $L$ and $C$. Equation 8 can also be used to test hypotheses that represent level and congruence effects. For instance, to assess whether $Y_1$ and $Y_2$ have equal effects, as implied by the expansion of $L$ in Equation 6, the equality $b_1 = b_2$ can be tested. Likewise, to determine whether $Y_1$ and $Y_2$ have equal but opposite effects, as implied by the expansion of $C$ in Equation 6, the expression $b_1 = -b_2$ can be tested. More generally, Equation 8 can be used to obtain any information yielded by $L$ and $C$, given that $L$ and $C$ are completely determined by $Y_1$ and $Y_2$. For instance, the coefficients on $L$ and $C$ can be computed by substituting Equations 1 and 2 into Equation 8 and rearranging terms, which produces:

$$Z = b_0 + b_1(L - .5C) + b_2(L + .5C) + e = b_0 + (b_1 + b_2)L + .5(b_2 - b_1)C + e. \quad (9)$$

Expressing the coefficients on $L$ and $C$ in this manner reveals the relative contributions of $Y_1$ and $Y_2$, as reflected by $b_1$ and $b_2$, thereby avoiding the ambiguities associated with $a_1$ and $a_2$. For instance, the compound coefficient $(b_1 + b_2)$ that precedes $L$ would indicate whether an effect attributed to $L$ was primarily or solely driven by either $Y_1$ or $Y_2$, based on the magnitudes of $b_1$ and $b_2$. Similar information is yielded by the compound coefficient $.5(b_2 - b_1)$ that precedes $C$.

To illustrate the problems created by using $L$ and $C$ as predictors, data were generated in which $Y_1$ and $Y_2$ had various relationships with $Z$, and structural equation models were estimated that used $Y_1$ and $Y_2$ or $L$ and $C$ as predictors, using LISREL 8.54 (Jöreskog & Sörbom, 2003). For simplicity, $Y_1$, $Y_2$, and $Z$ were specified as single indicators of their corresponding latent variables with loadings fixed at unity and measurement error variances fixed at zero. Using single indicators simplifies the measurement model but has no effect on the specification of the structural equation model, which would be the same regardless of whether single or multiple indicators are used. $L$ and $C$ were specified as latent variables with paths to $Y_1$ and $Y_2$ fixed according to Equations 1 and 2 (Cheung, 2007). Scores for $Y_1$ and $Y_2$ were randomly drawn from a bivariate normal distribution in which $Y_1$ and $Y_2$ had zero means, unit variances, and a correlation of .30. Nine population structural equations were specified in which $b_1$ varied in .20 increments from .00 to .80 and then back to .00 and, concurrently, $b_2$ increased in .20 increments from $-.80$ to $.80.$ For each equation, residuals were randomly drawn from a standard normal distribution and weighted such that the $R^2$ for each equation was approximately .30. A sample size of 250 was used for all analyses, similar to that in the empirical example used by Cheung (2007). All data were generated using the BASIC model of SYSTAT 10 (Wilkinson, 2000).

Results from structural equations using $L$ and $C$ versus $Y_1$ and $Y_2$ as predictors are reported in Table 1. Consider the results for $L$. For the first five cases, the coefficients on $L$ were positive and similar in size, suggesting uniformly strong support for a level effect. However, as noted earlier, level implies that the coefficients on $Y_1$ and $Y_2$ are equal, which cannot be determined from the coefficient on $L$. This condition can be assessed using $Y_1$ and $Y_2$ as predictors and determining whether $b_1 = b_2$, as indicated by imposing
this constraint on the model and testing the reduction in fit using the chi-square difference

\[
\Delta \chi^2(1) = 0.38, \quad p > .05.
\]

For Cases 1, 2, 4, and 5, the effect for L was driven primarily or solely by either Y1 or Y2.

Turning to C, coefficients for the last five cases indicate negative effects of similar magnitude for congruence. However, the coefficients on C conceal whether the coefficients on Y1 and Y2 were equal in magnitude and opposite in sign. Again, this condition can be assessed using Y1 and Y2 as predictors and testing the equality \(b_1 = -b_2\). Imposing this constraint on the model significantly reduced fit for all but Case 7,

\[
\Delta \chi^2(1) = 0.05, \quad p > .05.
\]

For Cases 5, 6, 8, and 9, the coefficient on C primarily or solely reflected the influence of either Y1 or Y2.

Combining the results for L and C reveals a consistent pattern. For L, the condition that Y1 and Y2 have equal coefficients is satisfied only when the coefficient on C is not significant. This property holds because testing \(b_1 = b_2\) is equivalent to testing \(b_1 - b_2 = 0\), which in turn implies \(a_2 = 0\) given that \(b_1 - b_2 = -2a_2\). This equality follows from Equations 7 and 8, which show that \(b_1 = .5a_1 - a_2\) and \(b_2 = .5a_1 + a_2\). Hence, \(b_1 - b_2 = .5a_1 - a_2 - (.5a_1 + a_2) = -2a_2\). Likewise, for C, the condition that the coefficients on Y1 and Y2 are equal in magnitude but opposite in sign is satisfied only when the coefficient on L is not significant. This pattern results from the fact that testing \(b_1 = -b_2\) is equivalent to testing \(b_1 + b_2 = 0\), which implies \(a_1 = 0\) given that \(b_1 + b_2 = a_1\), as again indicated by the coefficients on Y1 and Y2 in Equations 7 and 8. Hence, support for level requires lack of support for congruence and vice versa, meaning that level and congruence effects cannot coexist.

It might be tempting to dismiss the conditions for level and congruence that follow from Equations 3 and 4, instead inferring support for level or congruence when either L or C has a significant coefficient. However, doing so would invite the conclusion that Cases 1,
5, and 9 in Table 1 indicate both level and congruence effects when, in each case, the outcome is related to only one component variable. Little is gained by invoking interpretations of level and congruence when only one component variable is related to the outcome. This problem is avoided when L and C are replaced by Y1 and Y2, which clarifies their relationship with the outcome and permits tests of the conditions implied by the definitions of level and congruence in Equations 3 and 4.

**Level and Congruence as Outcomes**

Problems that occur when L and C are used as predictors are similar to those that arise when L and C are treated as outcomes, as in the following equations:

\[
L = a_{10} + a_{11}X + e_L \tag{10}
\]

\[
C = a_{20} + a_{21}X + e_C \tag{11}
\]

In Equations 10 and 11, L and C are defined as before, X is a predictor variable, and \(a_{10}, a_{11}, a_{20},\) and \(a_{21}\) are unstandardized coefficients. Consider the interpretation of \(a_{11}\) and \(a_{21}\) in Equations 10 and 11. Given that L is defined as an equally weighted composite of Y1 and Y2, it is tempting to conclude that \(a_{11}\) represents equal effects of X on Y1 and Y2. Likewise, because C is defined as the algebraic difference between Y1 and Y2, it effectively assigns opposite weights to Y1 and Y2, and \(a_{21}\) would appear to capture equal but opposite effects of X on Y1 and Y2. These interpretations are seemingly reinforced by replacing L and C with expressions that describe their definitions, as given by Equations 3 and 4:

\[
.5(Y_1 + Y_2) = a_{10} + a_{11}X + e_L \tag{12}
\]

\[
Y_2 - Y_1 = a_{20} + a_{21}X + e_C \tag{13}
\]

In Equation 12, \(a_{11}\) captures the relationship between X and the composite \(.5(Y_1 + Y_2)\), which assigns equal weights of \(.5\) to Y1 and Y2. Similarly, in Equation 13, \(a_{21}\) captures the relationship between X and the composite \(Y_2 - Y_1\), which places equal but opposite weights of \(-1\) and \(+1\) on Y1 and Y2, respectively.

Although these interpretations of \(a_{11}\) and \(a_{21}\) might appear reasonable, they are incorrect. The correct interpretations are revealed by solving Equations 12 and 13 for Y1 and Y2. To solve for Y1, Equation 13 is multiplied by \(.5\), subtracted from Equation 12, and simplified to obtain:

\[
Y_1 = a_{10} - .5a_{20} + (a_{11} - .5a_{21})X + e_L - .5e_C. \tag{14}
\]

To solve for Y2, Equation 13 is multiplied by \(.5\), added to Equation 12, and simplified to yield:

\[
Y_2 = a_{10} + .5a_{20} + (a_{11} + .5a_{21})X + e_L + .5e_C. \tag{15}
\]

Equations 14 and 15 show that the effects of X on Y1 and Y2 are represented by the compound coefficients \((a_{11} - .5a_{21})\) and \((a_{11} + .5a_{21})\), respectively. Hence, neither \(a_{11}\) nor \(a_{21}\)
itself captures the effects of $X$ on both $Y_1$ and $Y_2$, which in turn means that neither $a_{11}$ nor $a_{21}$ indicates whether these effects are consistent with the definitions of $L$ and $C$ in Equations 3 and 4. To illustrate, if the effects of $X$ on $Y_1$ and $Y_2$ are equal, as implied by $L$, then $(a_{11} - .5a_{21}) = (a_{11} + .5a_{21})$, which simplifies to $a_{21} = 0$. Hence, whether the effects of $X$ on $Y_1$ and $Y_2$ are equal is captured not by the coefficient on $X$ when $L$ is the outcome, but instead by the coefficient on $X$ when $C$ is the outcome. If $a_{21} = 0$, then the effects of $X$ on $Y_1$ and $Y_2$ indicated by $a_{11}$ are equal, whereas when $a_{21}$ differs from 0, the effects of $X$ on $Y_1$ and $Y_2$ differ by the value $a_{21}$. Conversely, if the effects of $X$ on $Y_1$ and $Y_2$ are equal in magnitude but opposite in sign, as implied by $C$, then $(a_{11} - .5a_{21}) = -(a_{11} + .5a_{21})$, or $a_{11} - .5a_{21} = -a_{11} - .5a_{21}$. Adding $a_{11}$ and $.5a_{21}$ to both sides of this expression yields $2a_{11} = 0$, which simplifies to $a_{11} = 0$. Thus, whether $X$ has equal but opposite effects on $Y_1$ and $Y_2$ is represented by the coefficient on $X$ not when $C$ is the outcome, but when $L$ is the outcome. If $a_{11} = 0$, the coefficients linking $X$ to $Y_1$ and $Y_2$ are equal in magnitude and opposite in sign. If $a_{11}$ differs from 0, the coefficients on $X$ differ in magnitude and can have the same or different signs.

The interpretation of results from either Equations 10 or 11 also depends on the relative magnitudes of both $a_{11}$ and $a_{21}$. For example, if $a_{21} = 2a_{11}$, then the compound coefficient linking $X$ to $Y_1$ equals 0, and the compound coefficient linking $X$ to $Y_2$ equals $a_{21}$ (or equivalently, $2a_{11}$). In this case, coefficients from Equations 10 and 11 that appear to represent effects of $X$ on both $L$ and $C$ simply reflect an effect of $X$ on $Y_2$. Similarly, if $a_{21} = -2a_{11}$, the compound coefficient relating $X$ to $Y_2$ equals 0, and the compound coefficient relating $X$ to $Y_1$ equals $a_{21}$ (or equivalently, $-2a_{11}$). In this case, coefficients from Equations 10 and 11 would suggest that $X$ is related to both $L$ and $C$ when, in fact, $X$ is simply related to $Y_1$. Little is gained by drawing inferences about level and congruence when $X$ is merely related to one component variable.

The foregoing problems are avoided by using $Y_1$ and $Y_2$ as outcomes, as follows:

\[
Y_1 = b_{10} + b_{11}X + e_1 \tag{16}
\]
\[
Y_2 = b_{20} + b_{21}X + e_2. \tag{17}
\]

Together, Equations 16 and 17 give estimates of the joint relationships of $X$ with both $Y_1$ and $Y_2$. Coefficients from these equations can also be used to test hypotheses stated in terms of level and congruence. For example, to determine whether $X$ has equal effects on $Y_1$ and $Y_2$, as implied when $L$ is used as an outcome, the equality $b_{11} = b_{21}$ can be tested. Similarly, to assess whether the effects of $X$ on $Y_1$ and $Y_2$ are equal in magnitude but opposite in sign, as implied when $C$ is used as an outcome, the equality $b_{11} = -b_{21}$ can be tested. Other questions that might motivate the use of Equations 10 and 11 can be answered using estimates from Equations 16 and 17, given that the coefficients from these equations can be used to compute the coefficients that would be obtained from estimating Equations 10 and 11. The required expressions are given by substituting Equations 1 and 2 into Equations 16 and 17, as follows:

\[
L - .5C = b_{10} + b_{11}X + e_1 \tag{18}
\]
\[
L + .5C = b_{20} + b_{21}X + e_2. \tag{19}
\]
Coefficients that would result from using L as an outcome are found by adding Equations 18 and 19 and multiplying both sides by .5, which yields:

\[ L = .5(b_{10} + b_{20}) + .5(b_{11} + b_{21})X + .5(e_1 + e_2). \]  

(20)

Comparing Equations 10 and 20 shows that \( a_{10} = .5(b_{10} + b_{20}) \) and \( a_{11} = .5(b_{11} + b_{21}) \).

Similarly, coefficients that would result from using C as an outcome are given by subtracting Equation 18 from Equation 19 and collecting like terms, which produces:

\[ C = b_{20} - b_{10} + (b_{21} - b_{11})X + e_2 - e_1. \]  

(21)

Comparing Equations 11 and 21 indicates that \( a_{20} = b_{20} - b_{10} \) and \( a_{21} = b_{21} - b_{11} \). Expressing the coefficients linking X to L and C in this manner reveals the extent to which these coefficients are determined by the effects of X on \( Y_1 \) and \( Y_2 \). These expressions also show that \( a_{11} \) is the average of the effects of X on \( Y_1 \) and \( Y_2 \), and \( a_{21} \) is the difference between these effects. Although the average and difference of these effects might be relevant to certain research questions, focusing exclusively on these quantities is ill advised, because doing so is tantamount to using L and C as outcomes and disregarding the relationships of X with \( Y_1 \) and \( Y_2 \).

To demonstrate the problems that occur when L and C are used as outcomes, data were generated in which X had various relationships with \( Y_1 \) and \( Y_2 \), and structural equation models were estimated in which \( Y_1 \) and \( Y_2 \) or L and C were treated as outcomes. As before, X, \( Y_1 \), and \( Y_2 \) were specified as single indicators of their associated latent variables with loadings fixed at unity and measurement error variances fixed at zero, and paths relating L and C to \( Y_1 \) and \( Y_2 \) were fixed according to Equations 1 and 2. Scores for X were randomly drawn from a standard normal distribution, and nine population structural equations were specified for both \( Y_1 \) and \( Y_2 \). The coefficients relating X to \( Y_1 \) varied in .20 increments from .00 to .80 and back down to .00. Simultaneously, the coefficients relating X to \( Y_2 \) increased in .20 increments from –.80 to .80. Residuals for \( Y_1 \) and \( Y_2 \) were randomly drawn from a standard normal distribution and assigned weights to produce \( R^2 \) values of approximately .30 for each structural equation. As before, the sample size was set at 250, data were generated using SYSTAT 10, and models were estimated with LISREL 8.54.

Results from structural equation models using L and C versus \( Y_1 \) and \( Y_2 \) as outcomes are provided in Table 2. For the first five cases, the coefficients relating X to L were positive and comparable in size, suggesting a level effect. However, as shown earlier, the coefficient relating X to L does not itself indicate whether X has equal effects on \( Y_1 \) and \( Y_2 \), such that \( b_{11} = b_{21} \). This equality can be assessed by specifying \( Y_1 \) and \( Y_2 \) as outcomes and testing the reduction in fit produced by the constraint \( b_{11} = b_{21} \). Results indicated that the difference between \( b_{11} \) and \( b_{21} \) was significant for all but Case 3, \( \Delta \chi^2(1) = 0.09, p > .05 \). For Cases 1, 2, 4, and 5, the coefficients on \( Y_1 \) and \( Y_2 \) were significantly different, such that the coefficient linking X to L was driven primarily or solely by the relationship between X and either \( Y_1 \) or \( Y_2 \). Thus, the coefficients on L concealed substantial variability in the coefficients relating X to \( Y_1 \) and \( Y_2 \), and for Cases 1 and 5, an apparent relationship between X and L was actually a bivariate relationship between X and either \( Y_1 \) or \( Y_2 \).
When C is used as the outcome, results for the last five cases produced coefficients on X of similar magnitude, each suggesting a negative relationship with congruence. However, these coefficients do not reveal whether X has equal but opposite relationships with Y1 and Y2. When Y1 and Y2 were used as outcomes, the coefficients on X were consistent with the equality $b_{11} = -b_{21}$ only for Case 7, where imposing this constraint did not significantly reduce the fit of the model, $\Delta \chi^2(1) = 0.84, p < .05$. For Cases 5, 6, 8, and 9, the coefficient linking X and C was driven primarily by the relationship between X and either Y1 or Y2.

Taken together, the results for L and C as outcomes follow a pattern similar to that when L and C are used as predictors. In particular, the condition implied by L in which X has equal effects on Y1 and Y2 is satisfied only when the coefficient linking X to C is not significant. This result is due to the fact that testing $b_{11} = b_{21}$ is equivalent to testing $b_{11} - b_{21} = 0$, which in turn is equivalent to testing $a_{21} = 0$ given that $a_{21} = b_{21} - b_{11}$. Conversely, the condition associated with C in which X has equal but opposite effects on Y1 and Y2 is fulfilled only when the coefficient relating X to L is not significant. This outcome occurs because testing $b_{11} = -b_{21}$ is the same as testing $b_{11} + b_{21} = 0$, which, in turn, is the same as testing $a_{11} = 0$ given that $a_{11} = .5(b_{11} + b_{21})$. Thus, evidence for level requires the lack of evidence for congruence and vice versa, meaning that a predictor cannot have effects on both level and congruence.

### Table 2

<table>
<thead>
<tr>
<th>Case</th>
<th>L and C as Outcomes</th>
<th>Y1 and Y2 as Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>L 0.40** C 0.72**</td>
<td>Y1 0.04 Y2 0.76**</td>
</tr>
<tr>
<td>Case 2</td>
<td>L 0.46** C 0.40**</td>
<td>Y1 0.26** Y2 0.66**</td>
</tr>
<tr>
<td>Case 3</td>
<td>L 0.42** C 0.02</td>
<td>Y1 0.41** Y2 0.43**</td>
</tr>
<tr>
<td>Case 4</td>
<td>L 0.38** C -0.43**</td>
<td>Y1 0.59** Y2 0.16**</td>
</tr>
<tr>
<td>Case 5</td>
<td>L 0.45** C -0.84**</td>
<td>Y1 0.87** Y2 0.03</td>
</tr>
<tr>
<td>Case 6</td>
<td>L 0.17** C -0.90**</td>
<td>Y1 0.62** Y2 -0.28**</td>
</tr>
<tr>
<td>Case 7</td>
<td>L -0.03 C -0.83**</td>
<td>Y1 0.38** Y2 -0.45**</td>
</tr>
<tr>
<td>Case 8</td>
<td>L -0.20** C -0.83**</td>
<td>Y1 0.22** Y2 -0.61**</td>
</tr>
<tr>
<td>Case 9</td>
<td>L -0.48** C -0.85**</td>
<td>Y1 -0.06 Y2 -0.91**</td>
</tr>
</tbody>
</table>

Note: $N = 250$. Table entries are unstandardized regression coefficients relating X to either L and C or Y1 and Y2 as outcomes. Coefficients to the right of L and C are estimates of $a_{11}$ and $a_{21}$, respectively. Coefficients to the right of Y1 and Y2 are estimates of $b_{11}$ and $b_{21}$, respectively. For each case, estimates of $a_{11}$ and $a_{21}$ relate to estimates of $b_{11}$ and $b_{21}$ according to Equations 14, 15, 20, and 21 (within rounding error).

* $p < .05$. ** $p < .01$. 

When C is used as the outcome, results for the last five cases produced coefficients on X of similar magnitude, each suggesting a negative relationship with congruence. However, these coefficients do not reveal whether X has equal but opposite relationships with Y1 and Y2. When Y1 and Y2 were used as outcomes, the coefficients on X were consistent with the equality $b_{11} = -b_{21}$ only for Case 7, where imposing this constraint did not significantly reduce the fit of the model, $\Delta \chi^2(1) = 0.84, p < .05$. For Cases 5, 6, 8, and 9, the coefficient linking X and C was driven primarily by the relationship between X and either Y1 or Y2.
Again, it might be argued that effects on level and congruence should be inferred solely from \( a_{11} \) and \( a_{21} \), respectively, without requiring equal effects for level and opposite effects for congruence. However, this approach would treat Cases 1, 5, and 9 in Table 2 as examples of both level and congruence, even though \( X \) is related to only \( Y_1 \) or \( Y_2 \). Bivariate relationships such as these hardly justify complex interpretation in terms of level and congruence. By using \( Y_1 \) and \( Y_2 \) instead of \( L \) and \( C \) as outcomes, the relationships of \( X \) with \( Y_1 \) and \( Y_2 \) are clarified, and conditions implied by using \( L \) and \( C \) as outcomes can be directly tested.

**Level and Congruence as Predictors and Outcomes**

When \( L \) and \( C \) are used as both predictors and outcomes, their interpretational problems are compounded. To distinguish between \( L \) and \( C \) as predictors versus outcomes, the notation used up to this point is modified. \( L \) and \( C \) as predictors are expressed as follows:

\[
L_X = \frac{1}{2}(X_1 + X_2) \\
C_X = X_2 - X_1.
\]  

(22)  

(23)

The subscript \( X \) designates \( L \) and \( C \) as predictors, and the component predictor variables are \( X_1 \) and \( X_2 \). \( L \) and \( C \) as outcomes are written as follows:

\[
L_Y = \frac{1}{2}(Y_1 + Y_2) \\
C_Y = Y_2 - Y_1.
\]  

(24)  

(25)

Here, the subscript \( Y \) indicates that \( L \) and \( C \) are outcomes. As before, the component outcome variables are \( Y_1 \) and \( Y_2 \). Equations that combine \( L_X \), \( C_X \), \( L_Y \), and \( C_Y \) as predictors and outcomes can be written as follows:

\[
L_Y = a_{10} + a_{11}L_X + a_{12}C_X + e_L \\
C_Y = a_{20} + a_{21}L_X + a_{22}C_X + e_C.
\]  

(26)  

(27)

We now consider the interpretation of the coefficients in Equation 26 and 27. Based on the definitions of \( L_X \) in Equation 22, it would seem that the coefficients on \( L_X \) represent equal effects of \( X_1 \) and \( X_2 \) on both \( L_Y \) and \( C_Y \). Likewise, given the definition of \( C_X \) in Equation 23, the coefficients on \( C_X \) imply equal but opposite effects of \( X_1 \) and \( X_2 \) on \( L_Y \) and \( C_Y \). However, these interpretations are again incorrect, as seen by substituting Equations 22 and 23 into Equations 26 and 27 and simplifying to obtain:

\[
L_Y = a_{10} + (\frac{1}{2}a_{11} - a_{12})X_1 + (\frac{1}{2}a_{11} + a_{12})X_2 + e_L \\
C_Y = a_{20} + (\frac{1}{2}a_{21} - a_{22})X_1 + (\frac{1}{2}a_{21} + a_{22})X_2 + e_C.
\]  

(28)  

(29)

Like Equation 7, Equations 28 and 29 show that the coefficients on \( L_X \) and \( C_X \) are elements of compound coefficients on \( X_1 \) and \( X_2 \). If \( X_1 \) and \( X_2 \) have equal effects on \( L_Y \)
and $C_Y$, as implied by using $L_X$ as a predictor, then the coefficients on $C_X$ (i.e., $a_{12}$ and $a_{22}$) must be zero. Under this condition, the coefficients on $X_1$ and $X_2$ reduce to $0.5a_{11}$ in Equation 28 and $0.5a_{21}$ in Equation 29. Conversely, if $X_1$ and $X_2$ have equal but opposite effects on $L_Y$ and $C_Y$, as implied when $C_X$ is used as a predictor, then the coefficients on $L_X$ (i.e., $a_{11}$ and $a_{21}$) must be zero. In this case, the coefficients on $X_1$ and $X_2$ are $-a_{12}$ and $a_{12}$ in Equation 28 and $-a_{22}$ and $a_{22}$ in Equation 29.

Although Equations 28 and 29 resolve ambiguities created when $L_X$ and $C_X$ are used as predictors, the interpretation of these equations remains problematic due to the use of $L_Y$ and $C_Y$ as outcomes. These problems are addressed by substituting Equations 24 and 25 into Equations 28 and 29 and solving for $Y_1$ and $Y_2$, which yields:

\[
Y_1 = a_{10} - 0.5a_{20} + (0.5a_{11} - a_{12} - 0.25a_{21} + 0.5a_{22})X_1
\]
\[
= (0.5a_{11} + a_{12} - 0.25a_{21} - 0.5a_{22})X_2 + e_L - 0.5e_C
\]
\[
(30)
\]
\[
Y_2 = a_{10} + 0.5a_{20} + (0.5a_{11} - a_{12} + 0.25a_{21} - 0.5a_{22})X_1
\]
\[
+ (0.5a_{11} + a_{12} + 0.25a_{21} + 0.5a_{22})X_2 + e_L + 0.5e_C.
\]
\[
(31)
\]

Equations 30 and 31 express the coefficients from Equations 26 and 27 as elements of compound coefficients linking $X_1$ and $X_2$ to $Y_1$ and $Y_2$. These expressions can be used to identify patterns of coefficients from Equations 26 and 27 implied by using $L_X$ and $C_X$ as predictors and $L_Y$ and $C_Y$ as outcomes. For instance, using $L_X$ implies that the compound coefficients in Equation 30 are equal, or $0.5a_{11} - a_{12} = 0.25a_{21} + 0.5a_{22} = 0.5a_{11} + a_{12} - 0.25a_{21} - 0.5a_{22}$. This equality simplifies to $2a_{12} = a_{22}$. Using $L_X$ also implies that the compound coefficients in Equation 31 are equal, such that $0.5a_{11} - a_{12} + 0.25a_{21} - 0.5a_{22} = 0.5a_{11} + a_{12} + 0.25a_{21} + 0.5a_{22}$, which reduces to $2a_{12} = -a_{22}$. Hence, using $L_X$ as a predictor implies that the equalities $2a_{12} = a_{22}$ and $2a_{12} = -a_{22}$ both hold. Treating these equalities as simultaneous equations and solving for $a_{12}$ and $a_{22}$ yields $a_{12} = 0$ and $a_{22} = 0$, meaning that the coefficients on $C_X$ in Equations 26 and 27 are both zero. Analogously, using $C_X$ implies that the compound coefficients on $X_1$ and $X_2$ are equal in magnitude but opposite in sign. For Equation 30, this condition can be written as $0.5a_{11} - a_{12} - 0.25a_{21} + 0.5a_{22} = 0.5a_{11} - a_{12} + 0.25a_{21} + 0.5a_{22}$, which reduces to $a_{11} = 0.5a_{21}$. For Equation 31, the condition means that $0.5a_{11} - a_{12} + 0.25a_{21} - 0.5a_{22} = 0.5a_{11} - a_{12} - 0.25a_{21} - 0.5a_{22}$, or $a_{11} = -0.5a_{21}$. Combining these equalities and solving for $a_{11}$ and $a_{21}$ gives $a_{11} = 0$ and $a_{21} = 0$, such that the coefficients on $L_X$ in Equations 26 and 27 are both zero.

Turning to $L_Y$ and $C_Y$ as outcomes, using $L_Y$ implies that the coefficients on $X_1$ are equal across Equations 30 and 31. This equality can be written as $0.5a_{11} - a_{12} - 0.25a_{21} + 0.5a_{22} = 0.5a_{11} - a_{12} + 0.25a_{21} + 0.5a_{22}$, which simplifies to $0.5a_{21} = a_{22}$. Using $L_Y$ also implies that the coefficients on $X_2$ are equal, such that $0.5a_{11} + a_{12} - 0.25a_{21} - 0.5a_{22} = 0.5a_{11} + a_{12} + 0.25a_{21} + 0.5a_{22}$, or $0.5a_{21} = -a_{22}$. When combined, the equalities $0.5a_{21} = a_{22}$ and $0.5a_{21} = -a_{22}$ are satisfied when $a_{21} = 0$ and $a_{22} = 0$. Hence, using $L_Y$ as an outcome implies that the coefficients on $L_X$ and $C_X$ in the equation using $C_Y$ as the outcome are both zero. Conversely, using $C_Y$ implies that the coefficients on $X_1$ in Equations 30 and 31 are equal in magnitude but opposite in sign, such that $0.5a_{11} - a_{12} - 0.25a_{21} + 0.5a_{22} = 0.5a_{11} + a_{12} - 0.25a_{21} + 0.5a_{22}$, or $a_{11} = 2a_{12}$. Using $C_Y$ also implies that the coefficients on $X_2$ are equal in magnitude but opposite in sign, which translates into $0.5a_{11} + a_{12} - 0.25a_{21} - 0.5a_{22} = 0.5a_{11} - a_{12} + 0.25a_{21} + 0.5a_{22}$, or $a_{11} = 2a_{12}$.
integrating these conditions reveals patterns of coefficients implied when \( L_X \) and \( C_X \) are combined with \( L_Y \) and \( C_Y \). for example, combining the conditions for \( L_X \) and \( L_Y \) yields \( a_{12} = 0, a_{21} = 0, \) and \( a_{22} = 0, \) such that the only nonzero coefficient is \( a_{11}, \) the coefficient on \( L_X \) predicting \( L_Y \). Similarly, combining the conditions for \( C_X \) and \( L_Y \) gives \( a_{11} = 0, a_{21} = 0, \) and \( a_{22} = 0, \) meaning the only nonzero coefficient is \( a_{12}, \) the coefficient on \( C_X \) predicting \( L_Y \). the conditions for \( L_X \) and \( C_Y \) combine into \( a_{11} = 0, a_{12} = 0, \) and \( a_{22} = 0, \) such that the only nonzero coefficient is \( a_{21}, \) which relates \( L_X \) to \( C_Y \). finally, the conditions for \( C_X \) and \( C_Y \) combine into \( a_{11} = 0, a_{12} = 0, \) and \( a_{21} = 0, \) leaving the only nonzero coefficient \( a_{22}, \) which links \( C_X \) to \( C_Y \). in each case, the coefficient linking the predictor to the outcome is insensitive to whether the conditions implied by the predictor and outcome are fulfilled. rather, these conditions are reflected by the other three coefficients, which differ from zero when the conditions for the predictor and outcome are violated. hence, evidence that supports any single relationship between a predictor and outcome necessarily rules out the other three relationships.

the ambiguities associated with \( L_X, C_X, L_Y, \) and \( C_Y \) are avoided when \( X_1 \) and \( X_2 \) are used as predictors of \( Y_1 \) and \( Y_2 \):

\[
\begin{align*}
Y_1 &= b_{10} + b_{11}X_1 + b_{12}X_2 + e_1 \\
Y_2 &= b_{20} + b_{21}X_1 + b_{22}X_2 + e_2.
\end{align*}
\]

coefficients from equations 32 and 33 can be used to test hypotheses that might motivate the use of \( L_X, C_X, L_Y, \) and \( C_Y \). for example, using \( L_X \) as a predictor implies that \( b_{11} = b_{12} \) and \( b_{21} = b_{22} \), and using \( L_Y \) as an outcome implies that \( b_{11} = b_{21} \) and \( b_{12} = b_{22} \). in combination, these conditions mean that all four coefficients on \( X_1 \) and \( X_2 \) are equal. similarly, using \( C_X \) as a predictor implies that \( b_{11} = -b_{12} \) and \( b_{21} = -b_{22} \), and using \( C_Y \) as a predictor implies that \( b_{11} = -b_{21} \) and \( b_{12} = -b_{22} \). combining these conditions yields \( b_{11} = -b_{12} = -b_{21} = b_{22} \). other equalities can be derived to test coefficient patterns implied by \( L_X \) as a predictor of \( C_Y \) and \( C_X \) as a predictor of \( L_Y \). if desired, the coefficient in equation 32 and 33 can also be used to compute the coefficients that would be produced by equations 26 and 27. the required expressions are obtained by solving equations 22, 23, 24, and 25 for \( X_1, X_2, Y_1, \) and \( Y_2, \) substituting the resulting equalities into equations 32 and 33 and rearranging terms, which yields the following (for details, see appendix b):

\[
\begin{align*}
L_Y &= 0.5(b_{10} + b_{20}) + 0.5(b_{11} + b_{12} + b_{21} + b_{22})L_X \\
&\quad + 0.25(b_{12} - b_{11} + b_{22} - b_{21})C_X + 0.5(e_1 + e_2) \\
C_Y &= b_{20} - b_{10} + (b_{21} - b_{11} + b_{22} - b_{12})L_X \\
&\quad + 0.5(b_{11} - b_{12} - b_{21} + b_{22})C_X + e_2 - e_1.
\end{align*}
\]

equations 34 and 35 show that any information obtained from \( L_X, C_X, L_Y, \) and \( C_Y \) can be derived from \( X_1, X_2, Y_1, \) and \( Y_2 \). however, the value of computing the coefficients on \( L_X, C_X, L_Y, \) and \( C_Y \) is questionable in light of the ambiguities they create.
Problems that result from using LX and CX as predictors and LY and CY as outcomes are illustrated using artificial data in which X1 and X2 had various relationships with Y1 and Y2. Again, X1, X2, Y1, and Y2 were specified as single indicators of their respective latent variables with loadings fixed to unity and measurement error variances fixed to zero.

Paths relating LX, CX, LY, and CY to X1, X2, Y1, and Y2 were fixed according to Equations 22 through 25. X1 and X2 were drawn from a bivariate normal distribution with zero means, unit variances, and a correlation of .30. Nine population equations were generated for both Y1 and Y2 in which b11, b12, b21, and b22 had values ranging from −.80 to .80 in .40 increments. Combinations of these coefficients were chosen to include cases in which the conditions for level or congruence as a predictor or outcome were satisfied as well as cases in which support for level or congruence is partial or absent. Residuals were randomly drawn from a standard normal distribution and weighted to produce $R^2$ values averaging .30 for both Y1 and Y2. For each equation, a sample size of 250 was again used. Data were generated with SYSTAT 10, and models were estimated using LISREL 8.54. Results are reported in Table 3, and as before, selected aspects of these results are discussed to demonstrate the issues at hand.

### Table 3

Results for Level and Congruence Versus Component Variables as Predictors and Outcomes

<table>
<thead>
<tr>
<th>Case</th>
<th>LX and CX as Predictors and LY and CY as Outcomes</th>
<th>X1 and X2 as Predictors and Y1 and Y2 as Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LX</td>
<td>CX</td>
</tr>
<tr>
<td>1</td>
<td>1.12**</td>
<td>0.22**</td>
</tr>
<tr>
<td>2</td>
<td>-1.04**</td>
<td>0.48**</td>
</tr>
<tr>
<td>3</td>
<td>0.86**</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>-0.10</td>
<td>-0.06</td>
</tr>
<tr>
<td>5</td>
<td>0.43**</td>
<td>-0.22**</td>
</tr>
<tr>
<td>6</td>
<td>0.67**</td>
<td>-0.52**</td>
</tr>
<tr>
<td>7</td>
<td>0.01</td>
<td>-0.43**</td>
</tr>
<tr>
<td>8</td>
<td>0.01</td>
<td>0.09</td>
</tr>
<tr>
<td>9</td>
<td>-0.06</td>
<td>-0.39**</td>
</tr>
</tbody>
</table>

Note: $N = 250$. Table entries are unstandardized coefficients. In the columns labeled LX and CX, the coefficients to the right of LY are $a_{11}$ and $a_{12}$, and the coefficients to the right of CY are $a_{21}$ and $a_{22}$, respectively. In the columns labeled X1 and X2, the coefficients to the right of Y1 are $b_{11}$ and $b_{12}$, and the coefficients to the right of Y2 are $b_{21}$ and $b_{22}$, respectively. For each case, estimates of $a_{11}, a_{12}, a_{21},$ and $a_{22}$ relate to estimates of $b_{11}, b_{12}, b_{21},$ and $b_{22}$ according to Equations 30, 31, 34, and 35 (within rounding error).

*p < .05. **p < .01.

Note: $N = 250$. Table entries are unstandardized coefficients. In the columns labeled LX and CX, the coefficients to the right of LY are $a_{11}$ and $a_{12}$, and the coefficients to the right of CY are $a_{21}$ and $a_{22}$, respectively. In the columns labeled X1 and X2, the coefficients to the right of Y1 are $b_{11}$ and $b_{12}$, and the coefficients to the right of Y2 are $b_{21}$ and $b_{22}$, respectively. For each case, estimates of $a_{11}, a_{12}, a_{21},$ and $a_{22}$ relate to estimates of $b_{11}, b_{12}, b_{21},$ and $b_{22}$ according to Equations 30, 31, 34, and 35 (within rounding error).

*p < .05. **p < .01.
First, consider the relationship between $L_X$ and $L_Y$. For Cases 1, 2, and 3, the coefficients linking $L_X$ to $L_Y$ is positive and significant. However, as noted earlier, these coefficients do not reveal whether $X_1$ and $X_2$ have equal effects on both $Y_1$ and $Y_2$, as implied by $L_X$ and $L_Y$. This pattern was evident only for Case 2, for which the coefficients relating $X_1$ and $X_2$ to $Y_1$ and $Y_2$ were similar in magnitude. Constraining these coefficients to be equal did not significantly reduce the fit of the model, $\Delta \chi^2(3) = 1.32, p > .05$, indicating that the conditions implied by using $L_X$ to predict $L_Y$ were tenable. In contrast, imposing these constraints reduced model fit for Case 1, $\Delta \chi^2(3) = 90.22, p < .05$, and Case 3, $\Delta \chi^2(3) = 68.40, p < .057$. Moreover, for Case 1, $X_1$ was not significantly related to $Y_2$, whereas for Case 3, the only significant relationship was between $X_1$ and $Y_2$. The differences among these results would go undetected by focusing on the relationship between $L_X$ and $L_Y$.

We now turn to the relationship between $C_X$ and $L_Y$. For Cases 3, 4, and 5, the coefficient relating $C_X$ to $L_Y$ was negative and significant. Again, these coefficients are insensitive to the conditions implied by $C_X$ and $L_Y$, whereby $X_1$ and $X_2$ have equal but opposite effects on both $Y_1$ and $Y_2$, and the magnitudes of these effects are equal across $Y_1$ and $Y_2$. This pattern was consistent with Case 4, as shown by the results for $X_1$ and $X_2$ as predictors of $Y_1$ and $Y_2$. Imposing this pattern of constraints did not significantly reduce the fit of the model for Case 4, $\Delta \chi^2(3) = 0.91, p > .05$. However, these constraints were rejected for Case 3, $\Delta \chi^2(3) = 81.29, p < .05$, and Case 5, $\Delta \chi^2(3) = 116.60, p < .05$. Furthermore, the coefficients relating $X_1$ and $X_2$ to $Y_1$ and $Y_2$ were markedly different across Cases 3, 4, and 5, as seen by inspecting Table 3.

Concerning the relationship between $L_X$ and $C_Y$, results for Cases 5, 6, and 7 each yielded significant negative coefficients. However, only Case 6 evidenced the pattern implied by $L_X$ and $C_Y$ in which the coefficients on $X_1$ and $X_2$ were equal within each equation for $Y_1$ and $Y_2$ but opposite in sign across the $Y_1$ and $Y_2$ equations. Constraining the coefficients to follow this pattern did not significantly reduce model fit for Case 6, $\Delta \chi^2(3) = 2.63, p > .05$, but worsened model fit for Case 5, $\Delta \chi^2(3) = 68.66, p < .05$, and Case 7, $\Delta \chi^2(3) = 39.92, p < .05$. Again, the pattern of coefficients relating $X_1$ and $X_2$ to $Y_1$ and $Y_2$ differed considerably across Cases 5, 6, and 7, differences that are obscured by the similar results for the coefficient relating $L_X$ to $C_Y$.

Finally, the coefficient linking $C_X$ to $C_Y$ was positive and significant for Cases 7, 8, and 9. However, results for $X_1$ and $X_2$ as predictors of $Y_1$ and $Y_2$ revealed that only the coefficients for Case 8 were consistent with the pattern implied by $C_X$ and $C_Y$, when $X_1$ and $X_2$ have equal but opposite relationships for both $Y_1$ and $Y_2$, and the relationships for $X_1$ and $X_2$ are equal but opposite across $Y_1$ and $Y_2$. For Case 8, imposing this pattern of coefficients on the model did not significantly reduce model fit, $\Delta \chi^2(3) = 1.34, p > .05$. However, this pattern of constraints was rejected for Case 7, $\Delta \chi^2(3) = 55.82, p < .05$, and Case 9, $\Delta \chi^2(3) = 69.96, p < .05$. Moreover, for Case 7, the only significant relationship was between $X_1$ and $Y_2$, whereas for Case 9, the only nonsignificant relationship was between $X_1$ and $Y_2$. These and other differences in the results for Cases 7, 8, and 9 are not evident from the coefficient relating $C_X$ to $C_Y$.

Combining the results for $L_X$, $C_X$, $L_Y$, and $C_Y$ highlights several key points. First, consistent with the derivations that follow Equations 30 and 31, evidence that supports any one relationship linking $L_X$ and $C_X$ to $L_Y$ and $C_Y$ simultaneously refutes the other three relationships. Therefore, only one of the four possible relationships can exist for a given data set.
Allowing for more than one relationship denies the definitions of $L_X$, $C_X$, $L_Y$, and $C_Y$ in Equations 22 through 25 and invites conclusions that are more complex than justified by the data. For example, Cases 3 and 7 suggested that all four relationships between $L_X$, $C_X$, $L_Y$, and $C_Y$ were supported when, in fact, the only significant relationship was between $X_1$ and $Y_2$. It hardly seems worthwhile to draw inferences about level and congruence predicting level and congruence based on a single bivariate relationship. Cases 1 and 9 also yielded four significant relationships between $L_X$, $C_X$, $L_Y$, and $C_Y$. However, for these cases, $Y_1$ was related to both $X_1$ and $X_2$, whereas $Y_2$ was only related to $X_2$. For Case 1, the coefficients relating $X_1$ and $X_2$ to $Y_1$ did not significantly differ, as implied by $L_X \Delta \chi^2(3) = 0.06$, $p > .05$, whereas for Case 9, the coefficients linking $X_1$ and $X_2$ to $Y_1$ were opposite in sign and not significantly different in magnitude, as implied by $C_X \Delta \chi^2(3) = 0.20$, $p > .05$. Hence, results for Cases 1 and 9 indicate a simple bivariate relationship between $X_2$ and $Y_2$, along with support for level and congruence, respectively, for $X_1$ and $X_2$ predicting $Y_1$. These patterns are obscured by results based on $L_X$, $C_X$, $L_Y$, and $C_Y$.

Reanalysis of Data From Cheung (2007)

The data used in the preceding examples demonstrate problems that occur when level and congruence are used as predictors, outcomes, or both. These problems can also be illustrated using data analyzed by Cheung (2007), as shown below. The Cheung (2007) data produce a limited pattern of relationships, which makes it less effective than the artificial data used earlier to illustrate the problems under consideration here. Nonetheless, the following reanalyses show that the problems demonstrated up to this point are not limited to the particular data generated for illustration.

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The Cheung (2007) data contained responses from 220 managers and their supervisors on 24 items. These items were assigned to eight factors, with 3 items per factor. The eight factors represented perceptions of managers and supervisors on four aspects of the manager’s behavior, including leadership, communication, planning and organizing, and customer service. The model examined by Cheung contained eight first-order factors representing manager and supervisor perceptions of leadership, communication, planning and organizing, and customer service, along with eight second-order factors representing level and congruence for the same four dimensions. The loadings of the first-order factors on the second-order factors were fixed according to Equations 1 and 2, such that the level and congruence factors represented the mean and difference, respectively, of their corresponding first-order factors. Cheung (2007) analyzed two versions of this model, one that treated level and congruence on leadership, communication, planning and organizing as predictors of level and congruence on customer service, and a second model that specified manager and supervisor perceptions of leadership, communication, planning and organizing as predictors of manager and supervisor perceptions of customer service.

The following reanalysis retained the eight first-order factors used by Cheung (2007) but dropped the eight second-order factors. The resulting model treated manager and supervisor perceptions of leadership, communication, and planning and organizing as correlated exogenous variables and manager and supervisor perceptions of customer service as endogenous variables with correlated residuals. All paths relating the six exogenous variables to
the two endogenous variables were freely estimated. These paths were also used to compute paths that would result from analyzing level and congruence rather than manager and supervisor perceptions, based on Equations 34 and 35. These computations were conducted using the additional parameters function of LISREL, which also yields standard errors of the computed values. Hence, the model used to reanalyze the Cheung (2007) data yields the same information as that produced by the model examined by Cheung but is simpler, in that it does not require the specification of second-order level and congruence factors. LISREL syntax for the model analyzed here is given in Appendix A.

Results from the model used for reanalysis are reported in Table 4. The top panel contains coefficients directly estimated by the model, and the bottom panel gives coefficients computed by the additional parameters function. Several aspects of these results are worth noting. First, as expected, the results in Table 4 match those reported by Cheung (2007), confirming that results produced by the LCM can be obtained without second-order factors representing level and congruence. Second, as indicated by the top panel, only three coefficients relating manager and supervisor perceptions were significant, each of which involved a relationship between perceptions reported by the same person. Third, none of the coefficient patterns in the upper panel indicate support for level or congruence as predictors of level or congruence. The closest instance involved planning and organizing, for which the pattern of coefficients was consistent with level predicting level. In line with this pattern, the bottom panel shows that the only significant coefficient for planning and organizing involved level predicting level, as expected when the four coefficients in the upper panel do not differ from one another. However, the upper panel also indicates that the four coefficients for planning and organizing did not differ from zero, which invalidates the apparent evidence for level in the bottom panel. Finally, for communication, the bottom panel suggests support for level predicting level and congruence predicting congruence, even though both relationships cannot coexist. This apparent contradiction is resolved by the results in the upper panel, which show that the results for level and congruence represent nothing more than relationships between perceptions reported by the same person.

Tests of the patterns of coefficients for level and congruence predicting level and congruence are reported in Table 5. These tests were conducted by imposing constraints on the coefficients in the upper panel of Table 4 and testing the deterioration in model fit using the chi-square difference test. These constraints were tested separately for leadership, communication, and planning and organizing. For level predicting level, the four coefficients for each dimension in the upper panel of Table 4 were constrained to be equal. For congruence predicting level, the four coefficients were constrained to be opposite across the columns and equal across the rows. For level predicting congruence, the four coefficients were constrained to be equal across the columns and opposite across the rows. Finally, for congruence predicting congruence, the four coefficients were constrained to be opposite across the columns and opposite across the rows. These patterns are illustrated by Cases 2, 4, 6, and 8, respectively, in the columns of Table 3 reporting results for X1 and X2 as predictors of Y1 and Y2.

Table 5 shows that, for leadership, the constraints for congruence predicting level were not rejected. This result is consistent with the pattern of coefficients for leadership in the upper panel of Table 4, for which the coefficients on manager perceptions of leadership
were both positive and the coefficients on supervisor perceptions of leadership were both negative. However, only one of the four coefficients was significant, meaning that the coefficients for leadership should not be interpreted in terms of congruence predicting level but instead as a single relationship between manager perceptions of leadership and

| Table 4 | Coefficient Estimates for Cheung (2007) Data |
|-----------------|------------------|------------------|------------------|
| Manager and Supervisor Perceptions as Predictors and Outcomes | | | |
| | Leadership | Communication | Planning & Organizing |
| | Manager | Supervisor | Manager | Supervisor | Manager | Supervisor |
| Manager | 0.40* | −0.23 | 0.43* | 0.12 | 0.10 | 0.14 |
| Supervisor | 0.04 | −0.27 | −0.12 | 1.18** | 0.22 | 0.15 |
| Level and Congruence as Predictors and Outcomes | | | |
| | Leadership | Communication | Planning & Organizing |
| | Level | Congruence | Level | Congruence | Level | Congruence |
| Level | −0.03 | −0.24 | 0.81** | 0.25 | 0.31* | −0.01 |
| Congruence | −0.39 | 0.16 | 0.52 | 0.81** | 0.14 | −0.06 |

Note: N = 220. Table entries are unstandardized coefficients. Coefficients in the upper panel were estimated from a structural equation model that specified manager and supervisor perceptions of leadership, communication, and planning and organizing as latent exogenous variables and manager and supervisor perceptions of customer service as latent endogenous variables. Coefficients in the lower panel were computed and tested using the additional parameters feature of LISREL.

| Table 5 | Chi-Square Difference Tests for Cheung (2007) Data |
|-----------------|------------------|------------------|------------------|
| Coefficient Pattern | Leadership | Communication | Planning & Organizing |
| Level predicting level | 8.69* | 19.42** | 0.71 |
| Congruence predicting level | 4.21 | 25.25** | 7.48 |
| Level predicting congruence | 8.75* | 46.43** | 6.96 |
| Congruence predicting congruence | 9.03* | 140.58** | 6.92 |

Note: N = 220. Table entries are chi-square difference tests for constraints associated with each coefficient pattern listed in the left column. Each test involved three constraints and therefore had 3 degrees of freedom. For level predicting level, the four coefficients for each dimension in the upper panel of Table 4 were constrained to be equal. For congruence predicting level, the four coefficients were constrained to be opposite across the columns and equal across the rows. For level predicting congruence, the four coefficients were constrained to be equal across the columns and opposite across the rows. Finally, for congruence predicting congruence, the four coefficients were constrained to be opposite across the columns and opposite across the rows. These patterns are illustrated by Cases 2, 4, 6, and 8, respectively, in the columns of Table 3 reporting results for X1 and X2 as predictors of Y1 and Y2.

*p < .05. **p < .01.
customer service. For communication, all four patterns of coefficients were rejected. Finally, for planning and organizing, none of the patterns was rejected, which is symptomatic of the fact that none of the four coefficients for planning and organizing differed significantly from zero.

The scarcity of significant relationships in Table 4 was striking given that, from a substantive perspective, perceptions of leadership, communication, planning and organizing, and customer service from the same source should be related. These relationships were further examined by conducting a confirmatory factor analysis of the eight manager and supervisor factors. Results revealed that the correlations among the four manager factors were high, averaging .88 and ranging from .84 to .92. The correlations among the four supervisor factors were also high, averaging .80 and ranging from .72 to .89. Hence, the absence of significant relationships in Table 4 was due to high correlations among the leadership, communication, planning and organizing factors used as predictors, which introduced multicollinearity into the structural equations of the model. Moreover, each of the eight factors failed to achieve discriminant validity with at least one other factor, as evidenced by correlations that included 1.0 in their 95% confidence intervals. The lack of discriminant validity among these factors undermines the utility of the Cheung (2007) data for illustrating the LCM and partly explains the anomalous results reported here and by Cheung.

Future Directions for Latent Variable Modeling in Congruence Research

The preceding analyses show that the LCM conceals information needed to test level and congruence hypotheses and yields results that invite erroneous conclusions. Nonetheless, the LCM has two important strengths in that it takes measurement error into account and permits tests of measurement equivalence for the component variables. Of course, these strengths are not unique to the LCM, because they derive from the use of structural equation modeling with latent variables, not the use of level and congruence as second-order factors. These factors were dropped from the model used to reanalyze the Cheung (2007) data, yet this model controlled for measurement error and included the measurement invariance constraints embedded in the LCM. Hence, moving to structural equation modeling with latent variables is an important and logical step for congruence research, but this step does not require the LCM.

Other approaches to incorporating structural equation modeling into congruence research are available. The linear model used here to reanalyze the Cheung (2007) data can be applied to linear congruence relationships, as when satisfaction increases as rewards approach needs and continues to increase as rewards exceed needs. However, the linear model cannot verify that the hypothesized relationship is actually linear rather than curvilinear, a conceptually plausible alternative in many domains of congruence research (Edwards, Caplan, & Harrison, 1998; Locke, 1976; Rice, McFarlin, Hunt, & Near, 1985). Moreover, linear models are insufficient when congruence is conceptualized as the fit, similarity, match, or agreement between two constructs (Chatman, 1989; Dawis, 1992; Edwards, 1994; Judge & Ferris, 1992; Kristof, 1996; Muchinsky & Monahan, 1987). For
example, person-organization fit is defined as the match between person and organization attributes (e.g., values) and is hypothesized to produce various positive outcomes, such as job satisfaction and organizational commitment (Kristof-Brown et al., 2005). The notion that fit generates positive outcomes implies a curvilinear (i.e., inverted-U) relationship, such that outcomes are maximized when the person and organization are equal and decrease as the person and organization differ in either direction.

Curvilinear relationships that characterize much congruence research require analytical approaches that go beyond linear structural equation models. One approach is to translate the quadratic regression equation typically used in polynomial regression (Edwards & Parry, 1993) into a quadratic structural equation with latent variables (Edwards & Kim, 2002). This approach requires extending methods for testing interactions in structural equation modeling (Cortina, Chen, & Dunlap, 2001; Jöreskog, 1998; Li et al., 1998) to include curve components for the two variables involved in the interaction (Cohen, 1978; Cortina, 1993; MacCallum & Mar, 1995). Like the LCM, quadratic structural equation modeling takes measurement error into account and allows tests of measurement equivalence. However, unlike the LCM, quadratic structural equation modeling accommodates curvilinear relationships. Furthermore, results from quadratic structural equations can be used to conduct response surface analyses (Edwards, 2002; Edwards & Parry, 1993), yielding rigorous and comprehensive tests of congruence hypotheses in terms of latent variables.

Quadratic structural equations treat congruence as a predictor. When congruence is an outcome, multivariate regression procedures outlined by Edwards (1995) can be applied to multiple-group structural equation models in which groups are defined based on whether scores on one latent component variable are greater than or less than the other. Latent variable scores can be obtained using procedures described by Jöreskog (2000), such that the classification of cases into subgroups takes into account measurement error in the component variables. Models for each subgroup are linear, such that their specification and estimation are straightforward, and conditions for assessing congruence described by Edwards (1995) can be directly applied.

Level and Congruence as Constructs

The procedures recommended in this article enable the study of congruence with latent variable structural equation modeling without creating variables that signify level or congruence. This notion might create objections among researchers who view congruence as distinct from its components (Cheung, 2007; Tisak & Smith, 1994). Although congruence is distinct from either component taken separately, it is not distinct from the two components considered jointly. This fact is evident in Equation 4, which shows that congruence is defined as the difference between the component variables. As such, any meaning ascribed to congruence cannot go beyond the two component variables that define congruence. This point also applies to level, which is defined as the mean of the component variables, as indicated by Equation 3. Defined in this manner, congruence and level are redundant with their component variables, and any attempt to distinguish congruence and
level from their components is futile. For instance, a basic prerequisite for establishing the unique existence of a construct is discriminant validity, such that the construct does not overlap with other similar constructs (Campbell & Fiske, 1959). Because level and congruence are defined in terms of their components, the multiple correlations relating the components to level and congruence are 1.00, and the matrix of correlations among level, congruence, and the two component variables is singular, such that four distinct factors cannot be extracted.

Rather than conceptualizing congruence and level as constructs, they should be viewed as statements about the relative standing of the component variables to one another. For instance, if congruence is conceptualized as the fit, match, or similarity between two component variables (Edwards, 1994), then congruence is indicated when the component variables are equal. Saying that the component variables are equal does not invoke some new construct, no more than saying that a single variable equals some low or high score creates constructs we would call “low” or “high.” Likewise, hypotheses concerning the effects of congruence can be viewed as statements about the joint effects of the component variables. For instance, predicting that congruence leads to positive outcomes is tantamount to predicting that outcomes are maximized when the component variables are equal. Predicting a congruence effect does not invoke a “congruence” construct any more than predicting an interaction calls forth an “interaction” construct. Similar arguments apply to level, which can be conceptualized in terms of the component variables taken jointly.

Some researchers operationalize congruence not by subtracting component variables, as in Equation 4, but instead by asking respondents to directly report the difference or similarity between the component variables (e.g., Cable & DeRue, 2002). When congruence is measured in this manner, it is arguably distinct from the component measures taken jointly, given that comparative judgments are susceptible to influences other than the elements being compared (Chambers & Windschitl, 2004; Mussweiler, 2003; Tversky, 1977). The mapping of component variables onto judgments of their difference and congruence is worth studying in its own right (Edwards, Cable, Williamson, Lambert, & Shipp, 2006), but this research is meaningful only when congruence is operationalized not by subtracting component variables, as in Equation 4, but by measuring perceived differences and congruence directly.

**Conclusion**

Congruence research has been marked by various methodological developments, such as the movement from difference scores and profile similarity indices to polynomial regression, and an important next step is to translate polynomial regression into structural equation models with latent variables (Edwards & Kim, 2002). The LCM proposed by Cheung (2007) capitalizes on the advantages of structural equation modeling but takes a step backward by focusing analyses on the mean and difference between components variables. This article highlights the problems associated with the LCM and shows how the questions that the LCM is intended to address can be answered using structural equation models with latent component variables. Additional work is needed to move beyond linear
models to incorporate quadratic equations into structural equation models (Edwards & Kim, 2002), which are required for testing theories in congruence research.

Appendix A

**LISREL Syntax for Reanalysis of Cheung (2007) Data**

The following LISREL syntax revises the syntax reported by Cheung (2007) by dropping the second-order level and congruence factors and adding 12 additional parameters to compute the coefficients that would be produced by the level and congruence factors. Below the syntax are lines of code that impose constraints implied by level and congruence predicting level and congruence. Each set of constraints can be tested by inserting the relevant lines of code above the OU line.

Leadership and Teams: Modification of Cheung (2007) syntax

DA NI=24 NO=220
LA
SI L1 SI L2 SI L3 SI C1 SI C2 SI C3
SI P01 SI P02 SI P03 SI C1 SI C2 SI C3
SupL1 SupL2 SupL3 SupC1 SupC2 SupC3
SupP01 SupP02 SupP03 SupC1 SupC2 SupC3
CM=CSB.CM RE
ME=CSB.ME RE
SELECT
SI C1 SI C2 SI C3 SupC1 SupC2 SupC3
SI P01 SI P02 SI P03 SupP01 SupP02 SupP03/
MO NY=6 NE=2 LY=FI TY=FI TE=FR NX=18 NK=6 LX=FI TX=FI TD=FI TD = FR C
PS=SY,FR PS = SY,FR AL=FR KA=FR GA=FR AP=12
! Measurement model for leadership
VA 1 LX(1,1) LX(4,2)
FR LX(2,1) LX(3,1) LX(5,2) LX(6,2)
FR TX 2 TX 3 TX 5 TX 6
EQ LX(2,1) LX(5,2) /* Syntax for metric equivalence
EQ LX(3,1) LX(6,2) /* Syntax for metric equivalence
EQ TX 2 TX 5 /* Syntax for scalar equivalence
EQ TX 3 TX 6 /* Syntax for scalar equivalence
! Measurement model for communication
VA 1 LX(7,3) LX(10,4)
FR LX(8,3) LX(9,3) LX(11,4) LX(12,4)
FR TX 8 TX 9 TX 11 TX 12
EQ LX(8,3) LX(11,4) /* Syntax for metric equivalence
EQ LX(9,3) LX(12,4) /* Syntax for metric equivalence
EQ TX 8 TX 11 /* Syntax for scalar equivalence
!EQ TX 9 TX 12 /* Item with nonequivalent intercepts
! Measurement model for planning and organizing
VA 1 LX(13,5) LX(16,6)
FR LX(14,5) LX(15,5) LX(17,6) LX(18,6)
FR TX 14 TX 15 TX 17 TX 18
EQ LX(14,5) LX(17,6) /* Syntax for metric equivalence
EQ LX(15,5) LX(18,6) /* Syntax for metric equivalence
!EQ TX 14 TX 17 /* Item with nonequivalent intercepts
EQ TX 15 TX 18 /* Syntax for scalar equivalence

! Measurement model for customer service
VA 1 LY(1,1) LY(4,2)
FR LY(2,1) LY(3,1) LY(5,2) LY(6,2)
FR TY 2 TY 3 TY 5 TY 6
EQ LY(2,1) LY(5,2) /* Syntax for metric equivalence
EQ LY(3,1) LY(6,2) /* Syntax for metric equivalence
EQ TY 2 TY 5 /* Syntax for scalar equivalence
EQ TY 3 TY 6 /* Syntax for scalar equivalence

LK
SlfLdr SupLdr SlfCom SupCom SlfPO SupPO
LE
SlfCust SupCus
! Coefficients for level predicting level
CO PA(1)=5*GA(1,1)+.5*GA(1,2)+.5*GA(2,1)+.5*GA(2,2)
CO PA(2)=.5*GA(1,3)+.5*GA(1,4)+.5*GA(2,3)+.5*GA(2,4)
CO PA(3)=.5*GA(1,5)+.5*GA(1,6)+.5*GA(2,5)+.5*GA(2,6)
! Coefficients for congruence predicting level
CO PA(4)=−.25*GA(1,1)+.25*GA(1,2)−.25*GA(2,1)+.25*GA(2,2)
CO PA(5)=−.25*GA(1,3)+.25*GA(1,4)−.25*GA(2,3)+.25*GA(2,4)
CO PA(6)=−.25*GA(1,5)+.25*GA(1,6)−.25*GA(2,5)+.25*GA(2,6)
! Coefficients for level predicting congruence
CO PA(7)=−1*GA(1,1)−GA(1,2)+GA(2,1)+GA(2,2)
CO PA(8)=−1*GA(1,3)−GA(1,4)+GA(2,3)+GA(2,4)
CO PA(9)=−1*GA(1,5)−GA(1,6)+GA(2,5)+GA(2,6)
! Coefficients for congruence predicting congruence
CO PA(10)=.5*GA(1,1)−.5*GA(1,2)−.5*GA(2,1)+.5*GA(2,2)
CO PA(11)=.5*GA(1,3)−.5*GA(1,4)−.5*GA(2,3)+.5*GA(2,4)
CO PA(12)=.5*GA(1,5)−.5*GA(1,6)−.5*GA(2,5)+.5*GA(2,6)
OU AD=OFF ND=4
! Constraints for level predicting level
! Leadership
CO GA(1,2)=GA(1,1)
CO GA(2,1)=GA(1,1)
CO GA(2,2)=GA(1,1)
! Constraints for level predicting level
! Communication
CO GA(1,4)=GA(1,3)
CO GA(2,3)=GA(1,3)
CO GA(2,4)=GA(1,3)
! Constraints for level predicting level
! Planning and organizing
CO GA(1,6)=GA(1,5)
CO GA(2,5)=GA(1,5)
CO GA(2,6)=GA(1,5)


! Constraints for congruence predicting level
! Leadership
CO GA(1,2)=−1*GA(1,1)
CO GA(2,1)=GA(1,1)
CO GA(2,2)=−1*GA(1,1)

! Constraints for congruence predicting level
! Communication
CO GA(1,4)=−1*GA(1,3)
CO GA(2,3)=GA(1,3)
CO GA(2,4)=−1*GA(1,3)

! Constraints for congruence predicting level
! Planning and organizing
CO GA(1,6)=−1*GA(1,5)
CO GA(2,5)=GA(1,5)
CO GA(2,6)=−1*GA(1,5)

! Constraints for level predicting congruence
! Leadership
CO GA(1,2)=GA(1,1)
CO GA(2,1)=−1*GA(1,1)
CO GA(2,2)=−1*GA(1,1)

! Constraints for level predicting congruence
! Communication
CO GA(1,4)=GA(1,3)
CO GA(2,3)=−1*GA(1,3)
CO GA(2,4)=−1*GA(1,3)

! Constraints for level predicting congruence
! Planning and organizing
CO GA(1,6)=GA(1,5)
CO GA(2,5)=−1*GA(1,5)
CO GA(2,6)=−1*GA(1,5)

! Constraints for congruence predicting congruence
! Leadership
CO GA(1,2)=−1*GA(1,1)
CO GA(2,1)=−1*GA(1,1)
CO GA(2,2)=GA(1,1)

! Constraints for congruence predicting congruence
! Communication
CO GA(1,4)=−1*GA(1,3)
CO GA(2,3)=−1*GA(1,3)
CO GA(2,4)=GA(1,3)

! Constraints for congruence predicting congruence
! Planning and organizing
CO GA(1,6)=−1*GA(1,5)
CO GA(2,5)=−1*GA(1,5)
CO GA(2,6)=GA(1,5)
Appendix B

Solving for Coefficients Relating $L_X$ and $C_X$ to $L_Y$ and $C_Y$ in Terms of $b_{11}$, $b_{12}$, $b_{21}$, and $b_{22}$

To write the coefficients relating $L_X$ and $C_X$ to $L_Y$ and $C_Y$ in terms of $b_{11}$, $b_{12}$, $b_{21}$, and $b_{22}$, we begin by solving Equations 22, 23, 24, and 25 for $X_1$, $X_2$, $Y_1$, and $Y_2$. First, we solve Equations 22 and 23 for $X_1$ and $X_2$, which yields:

\[
X_1 = L_X - .5C_X \
X_2 = L_X + .5C_X.
\]  

(A1)  
(A2)

Next, we solve Equations 24 and 25 for $Y_1$ and $Y_2$, which produce:

\[
Y_1 = L_Y - .5C_Y \
Y_2 = L_Y + .5C_Y.
\]  

(A3)  
(A4)

We then substitute these expressions into Equations 32 and 33,

\[
L_Y - .5C_Y = b_{10} + b_{11}(L_X - .5C_X) + b_{12}(L_X + .5C_X) + e_1
\]

(A5)

\[
L_Y + .5C_Y = b_{20} + b_{21}(L_X - .5C_X) + b_{22}(L_X + .5C_X) + e_2.
\]

(A6)

Adding Equations A5 and A6 and simplifying gives the equation for $L_Y$:

\[
L_Y - .5C_Y + L_Y + .5C_Y = b_{10} + b_{11}(L_X - .5C_X) + b_{12}(L_X + .5C_X) + e_1
+ b_{20} + b_{21}(L_X - .5C_X) + b_{22}(L_X + .5C_X) + e_2
\]

\[
2L_Y = b_{10} + b_{20} + b_{11}L_X - .5b_{11}C_X + b_{12}L_X + .5b_{12}C_X
+ b_{21}L_X - .5b_{21}C_X + b_{22}L_X + .5b_{22}C_X + e_1 + e_2
\]

\[
2LY = b_{10} + b_{20} + (b_{11} + b_{12} + b_{21} + b_{22})L_X
+ .5(b_{12} - b_{11} + b_{22} - b_{21})C_X + e_1 + e_2
\]

\[
L_Y = .5(b_{10} + b_{20}) + .5(b_{11} + b_{12} + b_{21} + b_{22})L_X
+ .25(b_{12} - b_{11} + b_{22} - b_{21})C_X + .5(e_1 + e_2).
\]

(A7)

Subtracting Equation A5 from Equation A6 and simplifying gives the equation for $C_Y$:

\[
L_Y + .5C_Y - L_Y + .5C_Y = b_{20} - b_{10} + b_{21}(L_X - .5C_X) - b_{11}(L_X - .5C_X)
+ b_{22}(L_X + .5C_X) - b_{12}(L_X + .5C_X) + e_2 - e_1
\]

\[
C_Y = b_{20} - b_{10} + b_{21}L_X - .5b_{21}C_X - b_{11}L_X + .5b_{11}C_X
+ b_{22}L_X + .5b_{22}C_X - b_{12}L_X - .5b_{12}C_X + e_2 - e_1
\]

\[
C_Y = b_{20} - b_{10} + (b_{21} - b_{11} + b_{22} - b_{12})L_X
+ .5(b_{11} - b_{12} - b_{21} + b_{22})C_X + e_2 - e_1
\]

(A8)
Notes

1. The erroneous interpretation yielded by Equation 6 can also be seen by noting that, in general, regression coefficients, such as $a_1$, indicate the effect of the associated predictor holding all other predictors constant. However, this interpretation does not apply to Equation 6, in that a change in the term associated with $a_1$ also entails a change in the term associated with $a_2$, given that $Y_1$ and $Y_2$ appear in both terms. Thus, the conclusion that $a_1$ implies equal effects of $Y_1$ and $Y_2$ is correct only when $a_2 = 0$. This perspective applies to the erroneous interpretations yielded by other equations that use L and C as predictors. I am indebted to an anonymous reviewer for articulating this perspective.

2. In total, the five levels of $b_1$ and nine levels of $b_2$ used here could yield 45 possible combinations. However, the nine combinations selected for illustration are sufficient to demonstrate the problems that result from using L and C as predictors. Examples presented later in this article also use subsets of the possible combinations of parameters chosen for illustration, thereby demonstrating problems with L and C while keeping the illustrations manageable in size.

References


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