CHAPTER 11

Alternatives to Difference Scores

Polynomial Regression Analysis and Response Surface Methodology

Jeffrey R. Edwards

For decades, difference scores have been widely used in industrial/organizational (I/O) psychology research. Difference scores typically consist of the algebraic, absolute, or squared difference between two measures or the sum of squared or absolute differences between profiles of measures. Difference scores are widely used in research on the congruence (that is, fit, similarity, or agreement) between two constructs as a predictor of outcomes. Examples of such research include person-environment fit as a predictor of attitudes, behavior, and well-being (Chatman, 1989; Edwards, 1991; French, Caplan, & Harrison, 1982; Kristof, 1996), value fulfillment as a predictor of satisfaction (Dawis & Lofquist, 1984; Locke, 1976; Rice, McFarlin, Hunt, & Near, 1985), met expectations as a predictor of absenteeism, turnover, and organizational commitment (Porter & Steers, 1973; Wanous, Poland, Premack, & Davis, 1992), and self-other agreement as a predictor of managerial effectiveness (Atwater & Yammarino, 1997; Fleenor, McCauley, & Brutus, 1996).

Despite their widespread use, difference scores are prone to numerous methodological problems (Cronbach, 1958; Edwards, 1994; Johns, 1981). For instance, difference scores are often less reliable than either of their component measures. Difference scores are also inherently ambiguous, given that they combine measures of conceptually distinct constructs into a single score. Furthermore, they confound the effects of their component measures on outcomes and impose constraints on these effects that are rarely tested empirically. Finally, they reduce an inherently three-dimensional relationship between their component measures and the outcome to two dimensions.

Problems with difference scores can be avoided by using polynomial regression analysis (Edwards, 1994; Edwards & Parry, 1993). In essence, polynomial regression replaces difference scores with the component measures that constitute the difference and higher-order terms such as the squares and product of these measures. This approach provides comprehensive tests of relationships that motivate the use of difference scores, as well as relationships that are more complex than difference scores can represent. The polynomial regression approach also creates new opportunities for theory development by encouraging researchers to conceptualize the joint effects of the components on an outcome not as a two-dimensional function, but instead as a three-dimensional surface. Studies using this approach have shown that difference scores often severely distort the joint effects of their components on various outcomes (Atwater, Ostroff, Yammarino, & Fleenor, 1998; Edwards, 1996; Edwards & Harrison, 1993; Edwards & Rothbard, 1999; Elsaas & Veiga, 1997; Finegan, 2000; Hesketh & Gardner, 1993; Hom, Griffeth, Palich, & Bracker, 1999; Irving & Meyer, 1994, 1995; Johnson & Fersht, 1999; Kallith, Blueford, & Strube, 1999; Livingstone, Nelson, & Barr, 1997; Slocombe & Blueford, 1999; Van Vianen, 2000; Westman & Eden, 1996). Moreover, the three-dimensional surfaces examined in these studies often reveal complexities that were anticipated in theories of congruence (French et al., 1982; Kulka, 1979; Naylor, Prichard, & Ilgen, 1980; Rice et al., 1985) but have eluded empirical investigation due to the use of difference scores.
This chapter provides an overview of polynomial regression analysis as an alternative to difference scores. The chapter begins with a review of major problems with difference scores. The polynomial regression procedure is then discussed, followed by an empirical example that compares this procedure to the use of difference scores. Next, response surface methodology is presented, which provides a comprehensive framework for analyzing features of surfaces relating two components to an outcome. The chapter concludes with a discussion of the strengths and limitations of the polynomial regression approach and directions for future methodological developments in congruence research.

Problems with Difference Scores

Difference scores are prone to numerous methodological problems. This section summarizes these problems, focusing on situations in which a difference score is used as a predictor of an outcome. Many of these problems also pertain to difference scores as dependent variables (Edwards, 1995) and as measures of change (Cronbach & Furby, 1970; Werts & Linn, 1970), although methods that avoid these problems differ from those that apply when difference scores are used as predictors. Alternatives to difference scores as dependent variables and measures of change are summarized later in this chapter.

Reduced Reliability

Perhaps the most widely known problem with difference scores is low reliability. Although difference scores are not necessarily unreliable (Rogosa & Willett, 1983; Zimmerman & Williams, 1982), they are often less reliable than their component measures (Johns, 1981). The source of this problem is evidenced by the formula for the reliability of an algebraic difference between two measures (Johns, 1981; Nunnally, 1978),

\[
a_{(X-Y)} = \frac{\sigma_X^2 a_X + \sigma_Y^2 a_Y - 2\sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}
\]

(11.1)

where \(X\) and \(Y\) are the two measures constituting the difference, \(a_X\) and \(a_Y\) are the reliabilities of the measures, \(\sigma_X^2\) and \(\sigma_Y^2\) are the variances of the measures, and \(\sigma_{XY}\) is the covariance between the measures. When \(X\) and \(Y\) are positively correlated (as is usually the case in congruence research), the reliability of the algebraic difference between \(X\) and \(Y\) is often less than the reliability of either \(X\) or \(Y\). For example, if \(X\) and \(Y\) have unit variances, reliabilities of .75, and are correlated .40, the reliability of their difference is .58. Reliabilities of other difference scores can be derived using principles for the reliabilities of squares, products, and weighted linear combinations of measures (Bohrnstedt & Marwell, 1978; Nunnally, 1978). For example, a squared difference between two normally distributed measures with zero means, unit variances, reliabilities of .75, and a correlation of .40 has a reliability of .09. Reliabilities of profile similarity indices tend to be higher due to the large number of dimensions on which differences are calculated (O’Reilly, Chatman, & Caldwell, 1991). However, these dimensions are often conceptually heterogeneous, which creates interpretational problems regarding the meaning of the “true score” to which the reliability estimate refers (Hattie, 1985).

Ambiguous Interpretation

Difference scores collapse measures of conceptually distinct constructs into a single score that is inherently ambiguous. It may be tempting to interpret difference scores based on the weights implicitly assigned to the component measures when the difference is calculated. For example, an algebraic difference may seem to represent equal but opposite contributions of its component measures. However, the variance of a difference score depends not only on the weights assigned to the component measures, but also on the variances and covariance of these measures. This can be seen by the following formula for the variance of an algebraic difference:

\[
\sigma_{(X-Y)}^2 = \sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}.
\]

(11.2)

As equation 11.2 shows, \(X\) and \(Y\) account for equal amounts of the variance in their difference only when \(\sigma_X^2\) and \(\sigma_Y^2\) are equal. In
practice, component measure variances are likely to differ, as when person-organization fit is assessed for different people within the same organization (Chatman, 1991; O’Reilly et al., 1991) or supervisor-subordinate agreement is assessed for different subordinates who share the same supervisor (Meglino, Ravlin, & Adkins, 1989). In extreme cases, one component is a constant and therefore has no variance, as when multiple employees are compared to a single job profile (Caldwell & O’Reilly, 1990). In such cases, a difference score is simply a rescaled version of the component measure with nonzero variance.

Additional ambiguities pertain to the interpretation of absolute and squared difference scores. As with algebraic differences, the variances of absolute and squared differences give greater weight to the component measure with larger variance, as can be seen by applying formulas for the variances and covariances of squares and products (Bohmnstedt & Goldberger, 1969; Goodman, 1960). However, the interpretation of absolute and squared difference scores also depends on the joint distribution of the component measures. Typically, absolute and squared differences are interpreted as symmetric measures of congruence, given that they treat positive and negative differences the same. However, this interpretation implies that both positive and negative scores contributed to the difference. If scores are predominantly positive or negative, then an absolute or squared difference effectively reduces to a unidirectional measure of congruence, analogous to an algebraic difference. For example, studies of need fulfillment show that people often receive less than what they want of various intrinsic and extrinsic rewards (Wanous & Lawler, 1972). Therefore, need fulfillment scores calculated by subtracting wanted amount from received amount are predominantly negative, and absolute or squared differences based on these scores should not be interpreted as symmetric indices of need fulfillment.

**Confounded Effects**

The coefficient relating a difference score to an outcome is typically viewed as the effect of congruence, not the effects of the difference score components. However, because a difference score is calculated from its component measures, it captures nothing more than the combined effects of its components, and these effects are confounded when they are reduced to a single coefficient. In many cases, this coefficient conceals substantial differences in the effects of the components. For example, studies of the relationship between job satisfaction and the difference between actual and wanted job attributes have found that this relationship is markedly reduced when actual job attributes are statistically controlled (Sweeney, McFarlin, & Inderrieden, 1990; Wall & Payne, 1973). Because controlling for actual job attributes transforms the algebraic difference score into a partialed measure of wanted job attributes (Wall & Payne, 1973; Werts & Linn, 1970), these findings indicate that the relationship between the algebraic difference score and satisfaction primarily represents the influence of actual job attributes.

Some researchers have attempted to disentangle absolute and squared difference scores from their components by statistically controlling both component measures (French et al., 1982; O’Brien & Dowling, 1980; Rice, McFarlin, & Bennett, 1989; Tsui & O’Reilly, 1989). However, when the component measures are controlled, the coefficient on the difference score cannot be interpreted separately from the coefficients on the component measures. For example, a positive coefficient on an absolute difference is typically interpreted as a V-shaped relationship between congruence and an outcome. Controlling for the component measures can substantially alter the shape of this relationship, depending on the coefficients obtained for the component measures. For example, if the coefficient on |X − Y| is positive, the coefficient on X equals the coefficient on |X − Y|, and the coefficient on Y is the opposite of the coefficient on |X − Y|, then the left side of the V-shaped relationship implied by the positive coefficient on |X − Y| is flat, meaning that only positive differences are related to the outcome. Similarly, controlling for the components of a squared difference shifts the turning point of the implied U-shaped relationship to the left or right, such that the coefficient on the squared difference no longer represents the effects of deviations from perfect congruence.

**Untested Constraints**

Difference scores impose constraints on the relationship between the component measures and the outcome. These constraints can be identified by writing an equation using a difference score as a
predictor, distributing the coefficient on the difference score through the equation, and comparing the resulting expression to an equation that uses the components of the difference as separate predictors. For example, an equation using an algebraic difference score as a predictor is as follows:

\[ Z = b_0 + b_1(X - Y) + e \]  \quad (11.3)

where \(X\) and \(Y\) are component measures and \(Z\) is the outcome. Distributing \(b_1\) through the difference score yields

\[ Z = b_0 + b_1X - b_1Y + e. \]  \quad (11.4)

Now consider an equation that uses \(X\) and \(Y\) as separate predictors of \(Z\)

\[ Z = b_0 + b_1X + b_2Y + e. \]  \quad (11.5)

Comparing equation 11.5 to equation 11.4 shows that using an algebraic difference score as a predictor is equivalent to constraining the coefficients on \(X\) and \(Y\) in equation 11.5 to be equal in magnitude but opposite in sign (\(b_1 = -b_2\)).

The constraints imposed by a squared difference score can be identified in a similar manner. The following equation uses a squared difference score as a predictor:

\[ Z = b_0 + b_1(X - Y)^2 + e. \]  \quad (11.6)

Expanding this equation yields the following:

\[ Z = b_0 + b_1X^2 - 2b_1XY + b_1Y^2 + e. \]  \quad (11.7)

Thus, a squared difference score effectively uses \(X^2\), \(XY\), and \(Y^2\) as predictors. The corresponding unconstrained equation uses these three terms supplemented by \(X\) and \(Y\), given that unbiased estimation of coefficients on squared and product terms requires the inclusion of their constituent terms (Aiken & West, 1991; Cohen, 1978):

\[ Z = b_0 + b_1X + b_2Y + b_3X^2 + b_4XY + b_5Y^2 + e. \]  \quad (11.8)

Comparing equation 11.7 to equation 11.8 shows that using a squared difference score as a predictor imposes the following constraints on equation 11.8: (1) \(b_1 = 0\); (2) \(b_2 = 0\); (3) \(b_3 = b_5\); and (4) \(b_2 + b_4 + b_5 = 0\).\(^1\)

The constraints imposed by an absolute difference score are somewhat more difficult to identify, given that an absolute difference is a logical rather than a mathematical transformation. However, this transformation can be expressed in equation form by introducing a dummy variable (here labeled \(W\)) that equals 0 when \(X \geq Y\) and equals 1 when \(X < Y\), as follows:

\[ Z = b_0 + b_1(1 - 2W)(X - Y) + e. \]  \quad (11.9)

The term \((1 - 2W)\) reduces to 1 when \(X\) is greater than or equal to \(Y\) and -1 when \(X\) is less than \(Y\). Therefore, when \((X - Y)\) is positive or zero, its sign is unaltered, whereas when \((X - Y)\) is negative, its sign is reversed. Thus, equation 11.9 is equivalent to using an absolute difference score as a predictor. Expanding equation 11.9 yields

\[ Z = b_0 + b_1X - b_1Y - 2b_1WX + 2b_1WY + e. \]  \quad (11.10)

Now consider an unconstrained piecewise linear equation containing the same terms as those in equation 11.10:

\[ Z = b_0 + b_1X + b_2Y + b_3WX + b_4WY + e. \]  \quad (11.11)

Equation 11.11 includes \(W\) as a separate predictor to obtain unbiased estimates of the coefficients on the product terms \(WX\) and \(WY\) (Aiken & West, 1991; Cohen, 1978). Comparing equation 11.10 to equation 11.11 reveals that using an absolute difference score as a predictor imposes the following constraints on equation 11.11: (a) \(b_1 = -b_2\); (b) \(b_4 = -b_5\); (c) \(b_3 = 0\); and (d) \(b_4 = -2b_1\).

**Dimensional Reduction**

Finally, difference scores reduce an inherently three-dimensional relationship between the component measures and the outcome to two dimensions. This phenomenon is illustrated by Figures 11.1
and 11.2, which depict two-dimensional functions for algebraic, absolute, and squared difference scores and their three-dimensional counterparts. A comparison of these figures shows that the two-dimensional straight line implied by an algebraic difference score corresponds to a three-dimensional plane with equal but opposite slopes with respect to the X- and Y-axes. Similarly, the two-dimensional V-shaped function associated with an absolute difference score corresponds to a three-dimensional V-shaped surface with its minimum along the Y = X line. Finally, the two-dimensional U-shaped function for a squared difference score corresponds to a three-dimensional U-shaped surface with its minimum along the Y = X line. By reducing these inherently three-dimensional surfaces to two-dimensional functions, difference scores discard information and oversimplify the relationship of the components with the outcome.

Figure 11.1. Two-Dimensional Difference Score Functions.

Figure 11.2. Three-Dimensional Difference Score Surfaces.
Polynomial Regression as an Alternative to Difference Scores

Problems with difference scores may be addressed by using polynomial regression analysis. This section outlines the fundamental principles of polynomial regression, followed by a discussion of the mechanics of the approach and an empirical example that compares polynomial regression to the use of difference scores.

Basic Principles and Assumptions

The polynomial regression approach is based on three principles. First, congruence should be viewed not as a single score, but instead as the correspondence between the component measures in a two-dimensional space. From this perspective, perfect congruence is not a point, but instead is a line along which the component measures are equal. Incongruence is represented by the perpendicular distance of the component scores from the line of congruence. Viewing congruence in this manner captures the magnitude and direction of incongruence between the components as well as the absolute levels of the components. Difference scores embody the assumption that the absolute levels of the components can be disregarded, and this assumption carries a burden of proof that is readily evaluated by viewing congruence in a two-dimensional space. More fundamentally, component measures used to calculate difference scores invariably represent conceptually distinct constructs (such as expected and actual work experiences) or the same construct from different perspectives (say, supervisor and subordinate values), and these distinctions should be maintained in data analysis and interpretation.

Second, the effect of congruence on an outcome should be treated not as a two-dimensional function, but instead as a three-dimensional surface relating the two components to the outcome. These surfaces may be used to test simple congruence hypotheses associated with difference scores (see Figure 11.2) as well as complex congruence hypotheses that difference scores cannot represent. These surfaces invite researchers to develop and test hypotheses regarding the effects of congruence that take into account the full range of both component measures. For example, person-environment fit theory suggests that outcomes may differ depending on whether perfect fit refers to low versus high levels of person and environment constructs (Edwards, Caplan, & Harrison, 1998). Hypotheses such as these are necessarily overlooked when the three-dimensional relationship between congruence and an outcome is reduced to two dimensions.

Third, the constraints associated with difference scores should not be imposed on the data, but instead should be treated as hypotheses to be tested empirically. For example, the constraint imposed by an algebraic difference score \( b_1 = -b_2 \) in equation 11.5) represents a hypothesis that the components have equal but opposite effects on the outcome. Similarly, the constraints imposed by absolute and squared difference scores constitute compound hypotheses regarding the joint effects of the components on the outcome. Testing these constraints generates evidence to evaluate the conceptual model on which the difference score is based. Without testing these constraints, the conceptual model underlying a difference score evades empirical scrutiny and therefore cannot be falsified. Different sets of constraints may be tested to compare alternative models, thereby obtaining strong inference tests of congruence effects (Platt, 1964).

The polynomial regression approach is based on the following assumptions. First, the component measures should be commensurate, meaning that they express the components in terms of the same content dimension (Caplan, 1987; Graham, 1976). Examples of commensurate measures are actual and desired challenge, expected and received pay, and supervisor and subordinate reports of performance. Commensurate measurement is required to ensure the conceptual relevance of the component measures to one another and is necessary to meaningfully interpret results in terms of congruence. Second, it is assumed that the component measures use the same numeric scale. Scale equivalence is required to determine the degree of correspondence between the component measures and compare coefficient estimates. Third, like any application of regression analysis, it is assumed that all measures are at the interval or ratio level and that the component measures contain no measurement error (Kennedy, 1992; Pedhazur, 1997). This latter assumption is rarely satisfied, given that most measures in the social sciences contain some degree of error.
The implications of violating this assumption are considered later in this chapter.

**Application of Polynomial Regression Analysis**

As with most methods of analysis, polynomial regression may be applied in either a confirmatory or exploratory manner. Some researchers have mistaken polynomial regression as inherently exploratory (Tinsley, 2000), which is clearly at odds with how it has been presented and applied (Edwards, 1994; Edwards & Harrison, 1993; Edwards & Rothbard, 1999). Given that it frames difference score constraints as hypotheses to be tested, the polynomial regression procedure is first and foremost confirmatory. Polynomial regression analyses should be exploratory only if theory is not sufficiently developed to derive hypotheses for the joint effects of the components on the outcome. Moreover, results from exploratory analyses are subject to cross-validation and conceptual scrutiny. As forcefully stated elsewhere, “It is folly to construct elaborate post hoc interpretations of complex surfaces that are not both generalizable and conceptually meaningful” (Edwards, 1994, p. 74).

**Confirmatory Approach**

The confirmatory procedure begins by selecting a conceptual model of congruence and identifying the corresponding regression equation. The asymmetric congruence model implied by an algebraic difference score requires a linear equation that uses both component measures as predictors (equation 11.5). The symmetric congruence model implied by a squared difference requires a quadratic equation (equation 11.8), and the symmetric model corresponding to an absolute difference requires a piece linear equation (equation 11.11). These two models are similar, in that both predict symmetric effects of incongruence and no slope along the line of perfect congruence. However, the quadratic equation can capture curvilinearity, whereas the piecewise linear equation can depict abrupt changes in slope. Moreover, the quadratic equation allows a test of the hypothesis that the surface changes shape along the line of perfect congruence, whereas the piecewise linear equation incorporates this hypothesis as an assumption through the coding of W and provides no means to verify this assumption. A conservative approach is to use both equations to test symmetric congruence hypotheses and determine whether their results yield the same substantive conclusions.

After the appropriate equation is identified and estimated, analyses should be conducted to evaluate the model of interest. Support for the model rests on four conditions: (1) the variance explained by the equation differs from zero; (2) the coefficients follow the appropriate pattern, meaning that coefficients expected to have nonzero values differ from zero and have the correct sign; (3) the constraints corresponding to the model are satisfied; and (4) the variance explained by the set of terms one order higher than those in the equation does not differ from zero. The first condition is a simple omnibus test to establish that the equation explains variance in the outcome. The second condition verifies the general form of the model (for example, satisfaction is maximized rather than minimized along the line of perfect congruence) and rules out situations in which constraints are satisfied because all coefficients are near zero. The third condition determines whether the relative magnitudes of the coefficients correspond to the model of interest. Finally, the fourth condition ensures that the model does not underestimate the complexity of the joint effects of the components on the outcome. The third and fourth conditions provide support for the model when their associated tests are not statistically significant. Therefore, it is important to establish that these tests have adequate statistical power (Cohen, 1988).

**Exploratory Approach**

If no model is hypothesized a priori, polynomial regression may be applied in an exploratory manner. This approach involves estimating equations of progressively higher order (linear, quadratic, cubic, and so on) by adding the required terms in sets until the increment in variance explained does not differ from zero. These analyses should be supplemented by diagnostic procedures to detect outliers and influential cases (Belsley, Kuh, & Welsch, 1980), which can dramatically affect the variance explained by higher-order terms. Moreover, results from the exploratory approach should be cross-validated to ensure that the obtained results do not merely reflect sampling variability. If the results are replicated across samples and are conceptually meaningful, they may be used
to develop hypotheses to test in subsequent confirmatory studies (Runkel & McGrath, 1972).

**Empirical Illustration**

The polynomial regression procedure is illustrated using data from 366 M.B.A. students engaged in the job search process. All respondents completed commensurate scale-equivalent measures of actual and desired levels of various attributes of a job they were actively pursuing and their anticipated satisfaction with the job. Actual and desired job attribute measures contained three items and used seven-point scales with anchors ranging from "none at all" to "a very great amount." To avoid ceiling effects, measures of desires asked respondents to indicate job attribute levels that were adequate, not ideal (Locke, 1969). Measures were created by averaging the relevant items and subtracting the scale midpoint (4), producing scores that could range from −3 to +3. Scale centering reduces multicollinearity between the component measures and their associated higher-order terms (Gronbach, 1987) and facilitates the interpretation of coefficients on first-order terms when higher-order terms are in the equation (for a quadratic equation, the coefficients on X and Y represent the slope of the surface at the midpoint of X and Y scales, and for the piecewise linear equation, the coefficient on W represents the vertical shift in the surface at the midpoint of the X and Y scales). This illustration uses measures of actual and desired autonomy, prestige, span of control, and travel. Reliabilities were estimated using coefficient alpha and produced values ranging from .826 to .945 for measures of actual and desired job attributes and a value of .980 for the satisfaction measure. For all analyses, X represents actual amount, Y represents desired amount, and Z represents satisfaction.

Prior to analysis, data were screened for outliers and influential cases, using leverage, Cook’s D statistic, and standardized residuals from quadratic regression equations (Belsley et al., 1980; Fox, 1991) as criteria. Cases that exceeded the minimum cutoff on all three criteria (Bollen & Jackman, 1990) and were clearly discrepant from other cases on plots that combined these criteria were dropped. This procedure was conservative, affecting no more than five cases per job attribute. Removing outliers, influential cases, and cases with missing data yielded sample sizes ranging from 358 to 360. With alpha at .05 and power at .80, these sample sizes were able to detect a reduction in $R^2$ of about .02 for tests of the algebraic difference constraint and .03 for tests of the absolute and squared difference constraints. Previous research indicates that the reduction in $R^2$ produced by difference score constraints is often much larger than these values (Edwards, 1994; Edwards & Harrison, 1993). Therefore, the statistical power available to test difference score constraints was deemed adequate. In addition, using the same alpha and power criteria, these sample sizes were able to detect increases in $R^2$ about .03 for the sets of terms one order higher than those in the unconstrained algebraic, absolute, and squared difference equations. This increase in $R^2$ represents a small effect size (Cohen, 1988), and therefore the statistical power for tests of higher-order terms was also considered adequate.

**Confirmatory Approach**

Results from confirmatory analyses of the algebraic difference model are shown in Table 11.1, and surfaces corresponding to the constrained and unconstrained equations are displayed in Figures 11.3 and 11.4. For all four job attributes, the constrained equations indicate that satisfaction increased as actual amount approached desired amount and continued to increase as actual amount exceeded desired amount. The unconstrained equations yielded significant $R^2$ values and coefficients on X and Y that were significant and in the appropriate direction. The constraint imposed by the algebraic difference score ($b_1 = -b_2$; see equation 11.4) was rejected for autonomy, prestige, and travel but was not rejected for span of control, as indicated by F-ratios reported in the column labeled $F_C$ in Table 11.1. However, the higher-order terms were significant for all four job attributes, as shown by F-ratios in the column labeled $F_H$ in Table 11.1. Thus, the first two conditions of the confirmatory approach were satisfied for all four job attributes, but the third condition was satisfied only for span of control, and the fourth condition was not satisfied for any job attribute.
### Table 11.1. Algebraic Difference Model

<table>
<thead>
<tr>
<th>Job Attribute</th>
<th>Estimated Beta</th>
<th>R²</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autonomy</td>
<td>0.399**</td>
<td>0.115**</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Prestige</td>
<td>0.249**</td>
<td>0.097**</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Span of Control</td>
<td>0.157**</td>
<td>0.086**</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Travel</td>
<td>0.118**</td>
<td>0.050**</td>
<td>&lt; .001</td>
</tr>
</tbody>
</table>

**Note:** Sample sizes for autonomy, prestige, span of control, and travel were 350, 358, 356, and 359, respectively. For columns labeled (b), y' is the desired amount and the dependent variable (Z) is satisfaction. The column labeled p contains p-values. If the constrained and unconstrained equations are equivalent at the 5% level, then β = 0. If the constrained and unconstrained equations are equivalent at the 5% level, then β = 0.

### Figure 11.3. Constrained Linear Surfaces

- **a. Constrained Linear Surface for Autonomy**
- **b. Constrained Linear Surface for Prestige**
- **c. Constrained Linear Surface for Span of Control**
- **d. Constrained Linear Surface for Travel**
Results for the absolute difference model are shown in Table 11.2 and Figures 11.5 and 11.6. For autonomy, prestige, and travel, the constrained equations indicated that satisfaction was greatest when actual and desired amounts were equal and decreased as actual amount deviated from desired amount in either direction. Although similar results were obtained for span of control, the coefficient on the absolute difference score was not significant. The unconstrained equations yielded significant $R^2$ values for all four job attributes. Coefficients were consistent with the expected pattern (positive coefficients on $X$ and $Y$, negative coefficients on $Y$ and $WX$, a coefficient of zero on $W$) for autonomy, prestige, and travel but not for span of control, which yielded a single negative coefficient on desired amount. The constraints imposed by the absolute difference score were rejected for all four job attributes, as indicated by the $F$-ratios in the $F_C$ column in Table 11.2. In addition, significant higher-order terms were found for prestige, span of control, and travel, as evidenced by $F$-ratios in the $F_H$ column of Table 11.2. Thus, the first two conditions of the confirmatory approach were satisfied for autonomy, prestige, and travel, but the third condition was not satisfied for any job attribute, and the fourth condition was satisfied only for autonomy.

Results for the squared difference model are provided in Table 11.3 and Figures 11.7 and 11.8. For all four job attributes, the constrained equations indicated that satisfaction was maximized when actual and desired amounts were equal and decreased as actual amount deviated from desired amount in either direction. The unconstrained equations yielded significant $R^2$ values for all four job attributes, but the expected pattern of coefficients (coefficients of zero on $X$ and $Y$, positive coefficients on $X^2$ and $Y^2$, and a negative coefficient on $XY$) was not supported for any job attribute. The constraint imposed by the squared difference score was rejected for all four job attributes, but significant higher-order terms were not found for any job attribute, as shown by the $F$-ratios reported in the $F_C$ and $F_H$ columns of Table 11.3, respectively. Hence, for all four job attributes, the first and fourth conditions of the confirmatory approach were satisfied, but the second and third conditions were not satisfied.
Table 11.2. Absolute Difference Model.

<table>
<thead>
<tr>
<th>Job Attribute</th>
<th>Constrained Equation</th>
<th>Unconstrained Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IX - Yi</td>
<td>R²</td>
</tr>
<tr>
<td>Autonomy</td>
<td>-0.531**</td>
<td>.105**</td>
</tr>
<tr>
<td>Prestige</td>
<td>-0.372**</td>
<td>.050**</td>
</tr>
<tr>
<td>Span of Control</td>
<td>-0.136</td>
<td>.009</td>
</tr>
<tr>
<td>Travel</td>
<td>-0.231**</td>
<td>.038**</td>
</tr>
</tbody>
</table>

Note: Sample sizes for autonomy, prestige, span of control, and travel were 360, 358, 358, and 359, respectively. For columns labeled |X - Yi|, X, Y, W, WX, and WY, table entries are unstandardized regression coefficients from equations in which X is actual amount, Y is desired amount, W is a dummy variable that equals 0 when X ≥ Y and equals 1 when X < Y, and the dependent variable (Z) is satisfaction. The column labeled F_c contains F-ratios for the test of constraints imposed by the absolute difference score, which is equivalent to the test of difference in R² values for the constrained and unconstrained equations (degrees of freedom for these F-ratios are 4 and N - 6). The column labeled F_u contains F-ratios for the test of higher-order terms, which for the piecewise linear equation include the six quadratic terms X², XY, Y², WX, WY, and WY² (degrees of freedom for these F-ratios are 6 and N - 12).

*p < .05. **p < .01.
Table 11.3. Squared Difference Model.

<table>
<thead>
<tr>
<th>Job Attribute</th>
<th>Constrained Equation</th>
<th>Unconstrained Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(X - Y)^2$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>Autonomy</td>
<td>$-0.183^{**}$</td>
<td>.096^{**}</td>
</tr>
<tr>
<td>Prestige</td>
<td>$-0.140^{**}$</td>
<td>.040^{**}</td>
</tr>
<tr>
<td>Span of Control</td>
<td>$-0.067^{**}$</td>
<td>.015^{**}</td>
</tr>
<tr>
<td>Travel</td>
<td>$-0.083^{**}$</td>
<td>.051^{**}</td>
</tr>
</tbody>
</table>

Note: Sample sizes for autonomy, prestige, span of control, and travel were 960, 358, 358, and 359, respectively. For columns labeled $(X - Y)^2$, $X$, $Y$, $X^2$, $XY$, and $Y^2$, table entries are unstandardized regression coefficients from equations in which $X$ is the actual amount, $Y$ is the desired amount, and the dependent variable ($Z$) is satisfaction. The column labeled $F_c$ contains $F$-ratios for the test of constraints imposed by the squared difference score, which is equivalent to the test of difference in $R^2$ values for the constrained and unconstrained equations (degrees of freedom for these $F$-ratios are 4 and $N-6$). The column labeled $F_h$ contains $F$-ratios for the test of higher-order terms, which for the quadratic equation include the four cubic terms $X^3$, $X^2Y$, $XY^2$, $Y^3$ (degrees of freedom for these $F$-ratios are 4 and $N-10$).

* $p < .05$. ** $p < .01$. 
Figure 11.7. Constrained Quadratic Surfaces.

a. Constrained Quadratic Surface for Autonomy
b. Constrained Quadratic Surface for Prestige
c. Constrained Quadratic Surface for Span of Control
d. Constrained Quadratic Surface for Travel

Figure 11.8. Unconstrained Quadratic Surfaces.
a. Unconstrained Quadratic Surface for Autonomy
b. Unconstrained Quadratic Surface for Prestige
c. Unconstrained Quadratic Surface for Span of Control
d. Unconstrained Quadratic Surface for Travel
Exploratory Approach

Application of the exploratory approach indicated that for all four job attributes, sets of linear and quadratic terms were significant, whereas the set of cubic terms was not significant. Results from these analyses can be extracted from Tables 11.1 and 11.3, as follows: (1) tests of the two linear terms as a set correspond to tests of the $R^2$ values for the linear equations in Table 11.1; (2) tests of the three quadratic terms as a set are provided by the $F$-ratios in the $F_H$ column of Table 11.1; and (3) tests of the four cubic terms as a set are provided by the $F$-ratios in the $F_H$ column of Table 11.3. Thus, the final exploratory equations correspond to the quadratic equations shown in Table 11.3. These equations indicate an interaction between actual and desired amounts for autonomy and span of control, an interaction and curvilinearity with regard to actual amount for prestige, and an interaction and curvilinearity for both actual and desired amount for travel. It should be noted that with the outliers and influential cases retained in the data, exploratory analyses yielded cubic equations for autonomy, prestige, and travel and a sextic (sixth-order) equation for span of control. Comparing surfaces for these equations to those for the quadratic equations showed that the additional higher-order terms added minor curvatures to capture a few discrepant cases but did not alter the overall shapes of the surfaces.

Response Surface Methodology as a General Analytic Framework

The preceding analyses indicate that the effects of congruence between actual and desired job attributes on satisfaction are captured by quadratic regression equations. To formally analyze and interpret the surfaces implied by these equations, it is useful to apply response surface methodology (Box & Draper, 1987; Khuri & Cornell, 1987; Myers, 1971). Response surface methodology comprises a collection of procedures for estimating and interpreting three-dimensional surfaces relating to variables to an outcome. Response surface methodology is relevant to the study of congruence for two reasons. First, empirical applications of polynomial regression analysis show that difference scores rarely survive confirmatory analyses (Edwards, 1991, 1994; Edwards & Harrison, 1993; Edwards & Parry, 1993). Consequently, substantive interpretation is usually based on three-dimensional surfaces such as those shown in Figures 11.4, 11.6, and 11.8. When the surfaces are planar (as in Figure 11.4), interpretation is relatively straightforward, whereas when surfaces are curvilinear (as in Figure 11.8), interpretation can be more difficult. Response surface methodology provides a formal means to analyze and interpret these surfaces. Second, a central premise of polynomial regression is that the effects of components on the outcome should be conceptualized in three dimensions. Accordingly, theory building and hypothesis testing should focus on surfaces as whole entities, which requires the use of response surface methodology. This section discusses the fundamentals of response surface methodology for quadratic regression equations and applies this methodology to data from the preceding empirical example.

Key Features of Response Surfaces

Response surface methodology involves the analysis of various features of surfaces corresponding to polynomial regression equations. For a quadratic equation, the surface can be one of three types: (1) concave, meaning the surface is dome shaped; (2) convex, meaning the surface is bowl shaped; and (3) saddle, which combines upward and downward curvature to produce a saddle-shaped surface. For each of these surfaces, response surface methodology involves the analysis of three key features.

The first feature is the stationary point of the surface, which is the point at which the slope of the surface is zero in all directions. For a concave surface, the stationary point is at the overall maximum of the surface. For a convex surface, the stationary point represents the overall minimum of the surface. Finally, for a saddle surface, the stationary point lies at the intersection of the lines along which the upward and downward curvatures of the surface are greatest. The location of the stationary point can be calculated using the coefficients from a quadratic regression equation (equation 11.8) using the following formulas,

$$X_0 = \frac{b_2b_4 - 2b_1b_3}{4b_3^2 - b_4^2}$$ (11.12)
\[ Y_0 = \frac{b_4 b_5 - 2b_2 b_3}{4b_5 b_3 - b_4^2} \]  

where \( X_0 \) and \( Y_0 \) represent the coordinates of the stationary point with respect to the \( X \) and \( Y \)-axes. For example, using coefficients from the quadratic equation for autonomy (see Table 11.3) yields the following values for \( X_0 \) and \( Y_0 \):

\[
X_0 = \frac{(-0.293)(0.276) - 2(0.197)(-0.035)}{4(-0.056)(-0.035) - 0.276^2} = 0.982
\]

\[
Y_0 = \frac{(0.197)(0.276) - 2(-0.293)(-0.056)}{4(-0.056)(-0.035) - 0.276^2} = 0.315.
\]

The second key feature is the principal axes of the surface, which are perpendicular to one another and intersect at the stationary point. The principal axes describe the overall orientation of the surface with respect to the \( X,Y \)-plane. For a concave surface, the first principal axis is the line along which the downward curvature of the surface is minimized, and the second principal axis is the line along which the downward curvature of the surface is maximized. For a convex surface, the first principal axis is the line along which the upward curvature of the surface is maximized, and the second principal axis is the line along which the upward curvature of the surface is minimized. Finally, for a saddle surface, the first principal axis is the line along which the downward curvature of the surface is maximized, and the second principal axis is the line along which the downward curvature of the surface is maximized.

The principal axes can be expressed using equations that describe a line in the \( X,Y \)-plane. An equation for the first principal axis is as follows:

\[ Y = p_{10} + p_{11} X. \]  

(11.14)

The equation for the slope of the first principal axis (\( p_{11} \)) is as follows:

\[
p_{11} = \frac{b_2 - b_3 + \sqrt{(b_3 - b_5)^2 + b_4^2}}{b_4}.
\]  

(11.15)

Two properties of equation 11.15 should be noted. First, if \( b_3 \) and \( b_5 \) are equal (corresponding to the constraints imposed by a squared difference score), equation 11.15 reduces to \( |b_4|/b_5 \). Consequently, the slope of the first principal axis is either \(-1\) or \(+1\), depending on whether \( b_4 \) is negative or positive, respectively. Second, if \( b_4 \) equals \( 0 \), both the numerator and denominator of equation 11.15 become \( 0 \), rendering it undefined. In this case, one of three conclusions may be drawn regarding the first principal axis, depending on the relative magnitudes of \( b_3 \) and \( b_5 \): (1) if \( b_3 \) is greater than \( b_5 \), the first principal axis has a slope of \( 0 \), meaning it runs parallel to the \( X \)-axis; (b) if \( b_3 \) is less than \( b_5 \), the first principal axis has a slope of infinity, meaning it runs parallel to the \( Y \)-axis; (3) if \( b_4 \) and \( b_5 \) are equal, the surface is a symmetric dome or bowl (depending on whether \( b_3 \) and \( b_5 \) are negative or positive, respectively) and therefore has no unique set of principal axes.

Once \( X_0 \), \( Y_0 \), and \( p_{11} \) have been obtained, \( p_{10} \) can be calculated as follows:

\[
p_{10} = Y_0 - p_{11} X_0.
\]  

(11.16)

Coefficients from the quadratic equation for autonomy yield the following values for \( p_{11} \) and \( p_{10} \):

\[
p_{11} = \frac{-0.035 - (-0.056) + \sqrt{(-0.056 - (-0.035))^2 + 0.276^2}}{0.276} = 1.079
\]

\[
p_{10} = -0.315 - (1.079)(0.982) = -1.375.
\]

An equation for the second principal axis can be written as

\[ Y = p_{20} + p_{21} X. \]  

(11.17)

The equation for the slope of the second principal axis, \( p_{21} \), is as follows:

\[
p_{21} = \frac{b_2 - b_3 - \sqrt{(b_3 - b_5)^2 + b_4^2}}{b_4}.
\]  

(11.18)
Note that equation 11.18 is identical to equation 11.15, except that the sign preceding the expression $\sqrt{(b_7-b_6)^2+b_6^2}$ is reversed. Thus, if $b_3$ and $b_6$ are equal, equation 11.18 reduces to $-b_4/b_4$, and the slope of the second principal axis is either $-1$ or $+1$, depending on whether $b_4$ is positive or negative, respectively. Analogously, if $b_4$ equals 0, equation 11.18 is undefined, and one of three conclusions may be drawn regarding the second principal axis: (1) if $b_3$ is greater than $b_6$, the second principal axis has a slope of infinity, running parallel to the $Y$-axis; (2) if $b_3$ is less than $b_6$, the second principal axis has a slope of 0, running parallel to the $X$-axis; (3) if $b_3$ and $b_6$ are equal, the surface is a symmetric dome or bowl, depending on whether $b_3$ and $b_6$ are negative or positive, respectively, and no unique set of principal axes exists.

Using $X_0$, $Y_0$, and $p_{21}$, the following equation may be used to calculate $p_{20}$:

$$p_{20} = Y_0 - p_{21}X_0. \hspace{1cm} (11.19)$$

Again using coefficients from the quadratic equation for autonomy, the following values for $p_{21}$ and $p_{20}$ are obtained:

$$p_{21} = \frac{-0.035-(-0.056) - \sqrt{(-0.056-(-0.035))^2 + 0.276^2}}{0.276} = -0.927$$

$$p_{20} = -0.315-(-0.927)(0.982) = 0.594.$$

In congruence research, it is often useful to locate the principal axes relative to lines other than the $X$ and $Y$-axes. For instance, studies of congruence often hypothesize that an outcome is maximized along the line of perfect congruence. This hypothesis implies a first principal axis that runs along the $Y = X$ line, such that $p_{10} = 0$ and $p_{11} = 1$. Rotation of the first principal axis off the $Y = X$ line is indicated by deviation of $p_{11}$ from 1. The lateral shift of the axis from the $Y = X$ line can be gauged by the point at which the axis crosses the $Y = -X$ line. This point is obtained by substituting $-X$ for $Y$ in the equation for the first principal axis and solving for $X$, which yields $-p_{10}/(p_{11} + 1)$. Analogously, if a hypothesis predicts that an outcome is minimized along the line of perfect congruence, the second principal axis should run along the $Y = X$ line, meaning that $p_{20} = 0$ and $p_{21} = 1$. The rotation of the axis from the $Y = X$ line is indicated by the deviation of $p_{21}$ from 1, and the lateral shift of the axis along the $Y = -X$ line is given by $-p_{20}/(p_{21} + 1)$.

The third response surface feature involves the shape of the surface along lines in the $X, Y$ plane. The shape of the surface along any line can be calculated by substituting the expression for the line into equation 11.8. For example, studies of congruence often hypothesize that an outcome is minimized or maximized along the line of perfect fit. This hypothesis implies that the surface is flat along the $Y = X$ line. The shape of the surface along the $Y = X$ line can be analyzed by substituting $X$ for $Y$ in equation 11.8, which yields the following:

$$Z = b_0 + b_1 X + b_2 X + b_3 X^2 + b_4 X^2 + b_5 X^2 + e. \hspace{1cm} (11.20)$$

Equation 11.20 shows that along the $Y = X$ line, the slope of the surface at the point $X = 0$ (and, by construction, $Y = 0$) equals $(b_1 + b_0)$, and the curvature of the surface equals $(b_2 + b_3)$. If these sums differ from zero, the hypothesis that the surface is flat along the $Y = X$ line is rejected. For autonomy, the shape of the surface along the $Y = X$ line is

$$Z = 5.825 + [0.197 + (-0.293)] X + [-0.056 + 0.276 + (-0.035)] X^2 + e.$$

$$= 5.825 - 0.096 X + 0.185 X^2 + e.$$

Studies of congruence are also concerned with the slope of the surface along the $Y = -X$ line, which runs perpendicular to the $Y = X$ line. In particular, if a hypothesis states that an outcome is maximized along the line of perfect congruence, then the surface should be curved downward along the $Y = -X$ line and flat at the point $X = 0, Y = 0$ (where the $Y = -X$ line intersects the $Y = X$ line). The shape of the surface along the $Y = -X$ line can be obtained by substituting $-X$ for $Y$ in equation 11.8, which produces the following:

$$Z = b_0 + b_1 X - b_2 X + b_3 X^2 - b_4 X^2 + b_5 X^2 + e. \hspace{1cm} (11.21)$$

$$= b_0 + (b_1 - b_2) X + (b_3 - b_4 + b_5) X^2 + e.$$
The quantity \((b_3 - b_4 + b_5)\) may be used to analyze the curvature of the surface along the \(Y = -X\) line. If this quantity is negative, the surface is curved downward along the \(Y = -X\) line, whereas if this quantity is positive, the surface is curved upward along the \(Y = -X\) line. If the quantity \((b_1 - b_2)\) equals zero, the surface is flat along the line of perfect congruence at the point \(X = 0, Y = 0\). In conjunction, these quantities may be used to test the hypothesis that the outcome is maximized or minimized along the line of perfect congruence. The shape of the surface for autonomy along the \(Y = -X\) line is:

\[
Z = 5.825 + [0.197 - (-0.293)]X + [-0.056 - 0.276 + (-0.035)]X^2 + e.
\]

\[= 5.825 + 0.490X - 0.367X^2 + e.\]

The shape of a surface along its principal axes can be analyzed in a similar manner. For example, the shape of the surface along the first principal axis can be derived by substituting the expression for the axis \(X = p_{10} + p_{11}X\) into equation 11.8:

\[
Z = b_0 + b_1X + b_2(p_{10} + p_{11}X) + b_3X^2 + b_4X(p_{10} + p_{11}X)
+ b_5(p_{10} + p_{11}X)^2 + e
\]
\[= 5.825 + b_1X + b_2p_{10} + b_2p_{11} + b_4p_{10} + b_4p_{11} + b_5p_{10}p_{11} + e.
\]

As equation 11.22 shows, the slope of the surface along the first principal axis at the point \(X = 0\) (where the first principal axis crosses the \(Y\)-axis) is given by \((b_1 + b_2p_{11} + b_4p_{10} + 2b_5p_{10}p_{11})\), and the curvature of the surface is \((b_3 + b_4p_{11} + b_5p_{11})\). For autonomy, the shape of the surface along the first principal axis is:

\[
Z = 5.825 + (-0.293)(-1.375) + (-0.035)(-1.375^2)
+ [0.197 + (-0.293)(1.079) + (0.276)(-1.375)]X
+ [2(-0.035)(-1.375)(1.079)]X^2
+ [-0.056 + (0.276)(-1.079) + (-0.035)(1.079^2)]X^2 + e.
\]
\[= 6.162 - 0.395X + 0.201X^2.
\]

Analogously, the shape of the surface along the second principal axis is obtained by substituting the expression for this axis \((Y = p_{20} + p_{21}X)\) into equation 11.8:

\[
Z = b_0 + b_1X + b_2(p_{20} + p_{21}X) + b_3X^2 + b_4X(p_{20} + p_{21}X)
+ b_5(p_{20} + p_{21}X)^2 + e
\]
\[= 5.825 + b_1X + b_2p_{20} + b_2p_{21} + b_4p_{20} + b_4p_{21} + 2b_5p_{20}p_{21} + e.
\]

For autonomy, the shape of the surface along the second principal axis is

\[
Z = 5.825 + (-0.293)(0.594) + (-0.035)(0.594^2)
+ [0.197 + (-0.293)(-0.927) + (0.276)(0.594)]X
+ [2(-0.035)(0.594)(-0.927)]X^2
+ [-0.056 + (0.276)(-0.927) + (-0.035)(-0.927^2)]X^2 + e.
\]

\[= 5.639 + 0.671X - 0.342X^2.
\]

**Confidence Intervals and Tests of Significance**

The expressions for response surface features introduce complications for significance testing and the construction of confidence intervals. For expressions involving linear combinations of regression coefficients, such as those preceding \(X\) and \(X^2\) in equations 11.20 and 11.21, standard errors can be calculated using ordinary rules for variances of linear combinations of random variables (DeGroot, 1975). However, these rules do not apply to expressions that contain nonlinear combinations of regression coefficients, such as the formulas for the stationary point and principal axes and the shape of the surface along the principal axes. For these expressions, sampling distributions can be derived empirically using the jackknife or bootstrap (Efron & Tibshirani, 1993; Mooney & Duval, 1993). In general, the bootstrap is superior to the jackknife in terms of bias and efficiency (Efron & Tibshirani, 1993) and is therefore preferred for response surface analysis. Bootstrap sampling distributions of
Empirical Example

Response surface analysis is illustrated using results from the quadratic regression equations estimated for the sample of M.B.A. and

students described earlier (see Table 11.3). Stationary points and principal axes are reported in Table 11.4 and shapes of these surfaces along

diagonal lines of interest are reported in Figure 11.15. The surfaces represented by these results are shown in Figure 11.15. For each surface represented by the diagonal lines of interest, the X−Y plane displays information useful for interpreting the surface.

Table 11.4. Stationary Points and Principal Axes.

<table>
<thead>
<tr>
<th>Job Attribute</th>
<th>Stationary Point</th>
<th>First Principal Axis</th>
<th>Second Principal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X₀</td>
<td>Y₀</td>
<td>P₁₀, P₁₁</td>
</tr>
<tr>
<td>Autonomy</td>
<td>0.982*</td>
<td>-0.315</td>
<td>-1.375*, 1.070**</td>
</tr>
<tr>
<td>Prestige</td>
<td>0.919</td>
<td>-0.651</td>
<td>-2.168**, 1.651***</td>
</tr>
<tr>
<td>Span of Control</td>
<td>-0.719</td>
<td>-1.464</td>
<td>-1.097</td>
</tr>
<tr>
<td>Travel</td>
<td>29.867</td>
<td>31.985</td>
<td>-0.873**, 1.119**</td>
</tr>
</tbody>
</table>

Note: Sample sizes for autonomy, prestige, span of control, and travel were 360, 358, 358, and 359, respectively. Columns labeled X₀ and Y₀ contain stationary point coordinates in the X,Y plane. Columns labeled P₁₀ and P₁₁ contain intercepts and slopes, respectively, of the first principal axis. Columns labeled P₂₀ and P₂₁ contain intercepts and slopes, respectively, of the second principal axis. Significance levels are based on confidence intervals constructed from coefficients from ten thousand bootstrap samples, using the percentile method to determine critical values.

*The 95 percent confidence interval for the slope of the first principal axis excluded 1.00.

**The 95 percent confidence interval for the slope of the second principal axis excluded -1.00.

*p < .05, **p < .01.
Table 11.5. Slopes Along Lines of Interest.

<table>
<thead>
<tr>
<th>Job Attribute</th>
<th>Y = X</th>
<th>Y = -X</th>
<th>First Principal Axis</th>
<th>Second Principal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_x$</td>
<td>$a_x^2$</td>
<td>$a_x$</td>
<td>$a_x^2$</td>
</tr>
<tr>
<td>Autonomy</td>
<td>-0.096</td>
<td>0.185**</td>
<td>0.490**</td>
<td>-0.367*</td>
</tr>
<tr>
<td>Prestige</td>
<td>0.217**</td>
<td>0.069</td>
<td>0.833**</td>
<td>-0.505**</td>
</tr>
<tr>
<td>Span of Control</td>
<td>0.170**</td>
<td>0.114**</td>
<td>0.364**</td>
<td>-0.178</td>
</tr>
<tr>
<td>Travel</td>
<td>0.116*</td>
<td>-0.003</td>
<td>0.378**</td>
<td>-0.465**</td>
</tr>
</tbody>
</table>

Note: Sample sizes for autonomy, prestige, span of control, and travel were 360, 358, 358, and 359, respectively. For each line (Y = X, Y = -X, first principal axis, second principal axis), $a_x$ represents the curvature of the surface along the line, and $a_x$ represents the slope of the surface along the line at $X = 0$. For slopes along the $Y = X$, $Y = -X$ lines, significance levels are based on confidence intervals for linear combinations of regression coefficients. For slopes along the first and second principal axes, significance levels are based on confidence intervals constructed from coefficients from ten thousand bootstrap samples, using the percentile method to determine critical values.

*p < .05. **p < .01.

Figure 11.9. Response Surface Analyses.
used to estimate the surface are plotted in the $X, Y$ plane to indicate the region of the surface on which interpretation should be focused (portions of the surface that extend beyond the data are extrapolations that should be disregarded).

The power of response surface methodology invites more thorough development of hypotheses regarding the effects of components on outcomes. For the data presented here, theories of job satisfaction (Locke, 1976; Rice et al., 1985) and person-environment fit (Edwards et al., 1998; French et al., 1982) indicate that for most job attributes, satisfaction should decrease as actual amount falls short of desired amount. When actual amount exceeds desired amount, satisfaction may increase, decrease, or remain constant, depending on how excess amounts influence fulfillment regarding other job attributes (Edwards, 1996; Edwards & Rothbard, 1999; Harrison, 1978). For autonomy, excess amounts may provide the person with influence to achieve fulfillment on other job attributes. However, at high levels, excess autonomy can bring a burden of responsibility, which may reduce satisfaction. Therefore, satisfaction should increase as actual autonomy exceeds desired autonomy, perhaps tapering off when excess autonomy is substantial. Similar arguments apply to prestige, in that excess prestige may provide influence and, at the same time, signify increased responsibility. Excess span of control may imply higher status and therefore increase satisfaction, although these benefits are probably outweighed by the increased workload of managing others. Therefore, satisfaction is likely to decrease for excess span of control. Similarly, excess travel may convey the status of representing an organization externally, but these benefits are probably overwhelmed by feelings of disruption and fatigue. In summary, for all four job attributes, satisfaction should increase as actual amount increases toward desired amount. For autonomy and prestige, satisfaction should continue to increase as actual amount exceeds desired amount, tapering off when excess amounts are large. For span of control and travel, satisfaction should decrease as actual amount exceeds desired amount.

Theories of person-environment fit (Edwards et al. 1998; French et al., 1982) also suggest that satisfaction may be higher when actual and desired amounts are both high than when both are low. Achieving high desired levels of job attributes conveys a sense of competence and self-efficacy, given that high aspirations have been successfully achieved (White, 1959). In contrast, achieving low desired levels merely signifies that a modest goal has been met. Therefore, for all four job attributes, satisfaction should be higher when actual and desired job attributes are both high than when both are low.

These predictions regarding the effects of actual and desired job attributes on satisfaction can be comprehensively assessed using response surface methodology, as illustrated below. For autonomy, the surface was saddle-shaped, with its stationary point just to the right of the $Y = X$ line. The first principal axis was nearly parallel to the $Y = X$ line, as indicated by a $p_{11}$ value that did not differ from 1.00. The quantity $-p_{00}/(1 + p_{11})$ was 0.661, and its 95 percent confidence interval excluded zero, indicating that the first principal axis was shifted to the right of the $Y = X$ line. In contrast, the second principal axis did not differ from the $Y = -X$ line, as evidenced by a slope and intercept that did not differ from $-1.00$ and 0.00, respectively. The surface was curved upward along the $Y = X$ line, and its slope at the point $X = 0, Y = 0$ did not differ from zero. The surface was also curved upward along the first principal axis but was negatively sloped where the surface crossed the $Y$-axis ($Y = -1.375$). Because few respondents reported low levels of actual and desired autonomy (only six respondents had scores below the line running parallel to the $Y = X$ line and intersecting the $Y = X$ line at $X = -1, Y = -1$), these results indicate that along the $Y = X$ line and first principal axis, satisfaction increased at an increasing rate. Along the $Y = -X$ line and second principal axis, the surface was curved downward and positively sloped where either line crossed the $Y$-axis. Substantively, these results indicate that satisfaction was maximized not along the line of perfect congruence, but instead along a line indicating that actual amount slightly exceeded desired amount. Thus, satisfaction was higher when respondents had slightly more autonomy than they considered adequate. Satisfaction was also higher when actual and desired amounts of autonomy were both high than when both were moderate, indicating that wanting and attaining a great deal of autonomy may itself lead to satisfaction.

The surface for prestige was also saddle-shaped, with its stationary point about one unit to the right of the point $X = 0, Y = 0$.
along the \( Y = -X \) line. The first principal axis was rotated counterclockwise from the \( Y = X \) line, as evidenced by a \( p_{11} \) value that was significantly greater than 1.00. The quantity \(-p_{10}/(1 + p_{11})\) was 0.818, and its 95 percent confidence interval excluded zero, indicating that the axis was shifted to the right of the \( Y = X \) line. Correspondingly, the second principal axis was rotated counterclockwise from the \( Y = -X \) line, as shown by a \( p_{21} \) value that was significantly greater than −1.00. The surface had a positive linear shape along the \( Y = X \) line, as indicated by a positive slope at the point \( X = 0, Y = 0 \) and curvature that did not differ from zero. Along the \( Y = -X \) line, the surface had a downward curvature and a positive slope at the point \( X = 0, Y = 0 \). The surface displayed significant upward curvature along the first principal axis and downward curvature along the second principal axis, and the slope of the surface along either axis at the point \( X = 0 \) did not differ from zero. In conjunction, these results indicate that satisfaction increased as actual prestige approached desired prestige and continued to increase as actual prestige exceeded desired prestige, although at a decreasing rate. In addition, satisfaction was higher when actual and desired prestige were both high than when both were low. Finally, the rotation of the surface indicated that when actual prestige was moderate (near the midpoint of the scale), satisfaction was greatest when actual amount exceed desired amount by about two units, whereas when actual prestige was high, satisfaction was greatest when actual amount and desired amount were approximately equal.

The surface for span of control was also saddle-shaped, with its stationary point to the right of the \( Y = X \) line where \( X \) and \( Y \) were both negative. The first principal axis was rotated clockwise from the \( Y = X \) line, as shown by a \( p_{11} \) value that was significantly less than 1.00. The quantity \(-p_{10}/(1 + p_{11})\) was 0.727, but its 95 percent confidence interval included zero, thereby failing to reject the null hypothesis of no lateral shift along the \( Y = -X \) line. As would be expected, the second principal axis also indicated a clockwise rotation, as evidenced by a \( p_{21} \) value that was significantly less than −1.00. Along the \( Y = X \) line, the surface was curved upward and had a positive slope at the point \( X = 0, Y = 0 \), whereas along the \( Y = -X \) line, the surface had a positive linear slope. In contrast, the surface had an upward curvature along the first principal axis and was essentially flat along the second principal axis. Substantively, these results indicated that satisfaction increased as actual span of control increased toward desired span of control and continued to increase as actual span of control exceeded desired span of control, although at a decreasing rate. Moreover, when actual span of control was high, satisfaction was highest when actual span of control was greater than desired span of control, perhaps due to the increased rewards brought by managing large numbers of subordinates.

Finally, the surface for travel was concave with its stationary point near the \( Y = X \) line but well beyond the range of the data. The first principal axis was nearly parallel to the \( Y = X \) line, as indicated by a \( p_{11} \) value that did not differ from 1.00. The quantity \(-p_{10}/(1 + p_{11})\) was 0.412, and its 95 percent confidence interval excluded zero, meaning that the first principal axis was shifted to the right of the \( Y = X \) line. As would be expected, the second principal axis was essentially parallel to the \( Y = -X \) line, as shown by a \( p_{21} \) value that did not differ from −1.00. However, because the second principal axis was far outside the range of the data, it should be disregarded when interpreting the surface. The surface had a positive linear slope along the \( Y = X \) line and had a downward curvature along the \( Y = -X \) line with a positive slope at \( X = 0, Y = 0 \). The shape of the surface along the first principal axis was similar to the shape along the \( Y = X \) line, although the positive linear slope along the axis produced a 95 percent confidence interval that included zero. Overall, these results indicate that satisfaction was maximized along a line where actual travel slightly exceeded desired travel. In addition, satisfaction was slightly higher when actual and desired travel were both high than when both were low, suggesting that wanting and attaining a great deal of travel may produce a marginal increase in satisfaction.

**Discussion**

**Advantages of Polynomial Regression Analysis**

The polynomial regression procedure offers several advantages over the use of difference scores. First, polynomial regression circumvents problems of reduced reliability created when component measures are subtracted from one another. Second, by using com-
ponent measures in their original form, polynomial regression avoids ambiguities that result when the component measures are reduced to a single score. Third, whereas difference scores confound the effects of their components, polynomial regression allows comprehensive assessment of the separate and joint effects of the components. Fourth, polynomial regression offers tests of constraints imposed by difference scores, treating these constraints as hypotheses regarding the combined effects of the components on the outcome. Finally, polynomial regression preserves the inherently three-dimensional relationship between the components and the outcome, thereby enabling researchers to develop and test congruence hypotheses that are more comprehensive and complex than the simplified models implied by difference scores.

The empirical illustration of the polynomial regression procedure demonstrated its advantages over difference scores. For all four job attributes, algebraic difference scores indicated that satisfaction increased as actual amount increased toward desired amount and continued to increase as actual amount exceeded desired amount. In contrast, absolute and squared difference scores suggested that satisfaction was maximized when actual and desired amounts were equal and decreased as actual amount deviated from desired amount in either direction. Although these results are conceptually plausible when taken individually, they are logically inconsistent when considered jointly, given that they indicate opposite effects on satisfaction when actual amounts exceeded desired amounts. In contrast, polynomial regression analysis rejected the models implied by the algebraic, absolute, and squared difference scores for each job attribute. Response surface analyses produced results that were largely consistent with hypotheses that considered asymmetric effects of incongruence and variation in satisfaction along the line of congruence. These results add to a growing body of research that consistently rejects constraints imposed by difference scores and yields conceptually meaningful surfaces relating components to outcomes.

Limitations and Areas for Further Development

Despite its advantages, polynomial regression analysis has several limitations that provide avenues for further methodological development. First, it adopts the standard regression assumption that independent variables are measured without error. As component measure reliability decreases, coefficient estimates may be biased upward or downward. This problem can be particularly pronounced for higher-order terms used in quadratic equations. For example, if two component measures have zero means and unit variances and are uncorrelated, the reliability of their product equals the product of their reliabilities (Bohrnstedt & Marwell, 1978). Measurement error was not severe for the empirical example presented earlier, for which reliabilities of all squared and product terms exceeded .823. Nonetheless, even modest amounts of measurement error can affect tests of constraints and response surface features. Problems created by measurement error may be addressed using structural equation modeling with latent variables (Bollen, 1989; Jöreskog & Sörbom, 1996). Structural equation modeling is straightforward for linear equations such as equation 11.5, and multiple groups structural equation models can be applied to piecewise linear equations such as equation 11.11. Methods for estimating quadratic equations may be derived from procedures for testing interactions between latent variables (Jaccard & Wan, 1996; Jöreskog & Yang, 1996). Chi-square difference tests may be used to test constraints imposed by difference scores, and structural equation parameters may be used to calculate and test response surface features.

Second, to examine the effects of congruence on multiple dimensions simultaneously, polynomial regression equations require a large number of terms. For instance, if the effects of congruence on the four job attributes from the empirical example were analyzed simultaneously, a regression equation with twenty independent variables would be required. With this approach, the effects of congruence on each job attribute are analyzed using the five quadratic terms for that attribute, and the quadratic terms for the remaining job attributes would be treated as covariates. This approach provides estimates of the effects of congruence on each job attribute, taking into account congruence on all other job attributes. Alternately, separate equations can be used to examine the effects of congruence for each dimension, and type I error rate can be controlled using the Bonferroni correction. Although using separate equations is relatively simple, it may introduce bias due to omitted variables (James, 1980). This potential bias may be taken into account by recognizing that the effects of congruence for a
particular dimension also reflect the effects of congruence for other dimensions excluded from the equation.

Third, polynomial regression analysis applies only to congruence as a predictor. When congruence is an outcome, different analytical procedures are required. Edwards (1995) presents multivariate regression procedures that provide alternatives to algebraic and absolute difference scores as dependent variables. Alternatives to squared difference scores as dependent variables are more difficult to derive, but hypotheses that motivate the use of squared difference scores can usually be tested using procedures that apply to absolute difference scores. However, these procedures incorporate the assumption that the effects on the components change in slope where the two components are equal, which may not be the case. This assumption can be tested using alternatives to squared difference scores as dependent variables, although these procedures await further development.

Finally, although polynomial regression can be applied to change scores as independent variables, it does not apply to change scores as dependent variables. Although some researchers advocate using change scores as dependent variables (Allison, 1990; Liker, Augustyniak, & Duncan, 1985), doing so is tantamount to using the initial value of the dependent variable as a covariate and constraining its coefficient to unity (Cronbach & Furby, 1970). This constraint may be tested by estimating the coefficient and determining whether its confidence interval includes unity. When change involves multiple waves, growth curve modeling may be applied (Rogosa, Brandt, & Zimowski, 1982). Although this procedure has advantages over change scores, it transforms successive values of the dependent variable into intercept and slope coefficients, thereby discarding information regarding deviations of the dependent variable from the growth curve. Research has yet to explore fully the comparative advantages of growth curve modeling, change scores, and using lagged values of dependent variables as covariates.

Conclusion

For decades, researchers have recognized that difference scores present numerous methodological problems. These problems can be avoided with polynomial regression analysis, which provides comprehensive tests of hypotheses that difference scores are intended to capture. Moreover, polynomial regression allows researchers to pursue research questions that cannot be addressed using difference scores. These questions involve fundamental issues in congruence research, such as asymmetries in the effects of incongruence and differences in congruence effects depending on the absolute levels of the components. Questions such as these have been raised in theories of congruence (French et al., 1982; Kulka, 1979; Naylor et al., 1980; Rice et al., 1985), but research into these questions has languished due to the limitations of difference scores. Polynomial regression matches the inherent complexity of congruence theories and allows researchers to comprehensively test predictions from these theories and pursue new theoretical questions that previously could not be addressed.

Notes
1. Given the constraint \( b_3 = b_6 \), the constraint \( b_3 + b_4 + b_5 = 0 \) is equivalent to \( b_4 = -2b_5 \) or \( b_4 = -2b_6 \) (Edwards, 1994).
2. Given the constraints \( b_1 = -b_7 \) and \( b_4 = -b_5 \), the constraint \( b_4 = -2b_1 \) is equivalent to \( b_4 = 2b_6 \), \( b_7 = 2b_5 \), or \( b_5 = -2b_6 \).
3. To allow meaningful interpretation of the slope of the surface at the point \( X = 0, Y = 0 \), it is assumed that the component measures are scale centered, as in the empirical example used here.

References


