Asymmetric information and the pecking (dis)order*

Paolo Fulghieri† Diego García‡ Dirk Hackbarth§

March 18, 2013

Abstract

In this paper we show that when growth options represent a significant component of overall firm value, equity financing can dominate (i.e., be less dilutive than) debt financing under asymmetric information. In particular, we find that equity is more likely to dominate debt for younger firms with larger investment needs and with riskier growth opportunities. Thus, our model can explain why high-growth firms may prefer equity over debt, and then switch to debt financing as they mature. We also find that equity financing can be relatively more attractive when a firm already has debt in its capital structure. Finally, we show that equity can dominate debt in multidivisional firms, and we provide new predictions on the cross-sectional variation of capital structures.

*We would like to thank Miguel Cantillo, David Dicks, Nick Gantchev, Nicolae Garleanu, Günter Strobl, Geoff Tate, and Ed Van Wesep for comments on a very early draft, as well as seminar participants at Duke University, HKUST, Leland-Rubinstein conference, University of Lausanne, and York University.

†Paolo Fulghieri, Kenan-Flagler Business School, University of North Carolina at Chapel Hill, McColl Building, C.B. 3490, Chapel Hill, NC 27599-3490. Tel: 1-919-962-3202; Fax: 1-919-962-2068; Email: paolo_fulghieri@unc.edu; Webpage: http://public.kenan-flagler.unc.edu/faculty/fulghiep

‡Diego García, Kenan-Flagler Business School, University of North Carolina at Chapel Hill, McColl Building, C.B. 3490, Chapel Hill, NC, 27599-3490. Tel: 1-919-962-8404; Fax: 1-919-962-2068; Email: diego_garcia@unc.edu; Webpage: http://www.unc.edu/~garcia4d

§Dirk Hackbarth, University of Illinois at Urbana-Champaign, 515 East Gregory Drive, MC-520, Mailbox 35, Champaign, IL 61820. Tel: 1-217-333-7343; Fax: 1-217-244-3102; Email: dhackbar@illinois.edu; Webpage: http://www.business.illinois.edu/dhackbar
1 Introduction

Raising capital under asymmetric information exposes firms to potential value dilution. When insiders have better information than investors on firm value, firms of better-than-average quality will find that investors price their securities below the value perceived by their insiders. Under these circumstances, Myers and Majluf (1984) suggest that firms can reduce dilution (i.e., mispricing) by issuing debt rather than equity, an intuition known as the pecking order theory. The reason for the pecking order preference, as Myers (1984) argues, is that the value of debt, by virtue of being a more senior security than equity, is less sensitive to private information.

As a description of capital structure choice, the pecking order theory is typically interpreted as implying that debt should be more desirable when asymmetric information is more severe. This conclusion is at odds with several stylized facts. For example, a well established empirical regularity is that small, high-growth firms, a class of firms which is presumably more exposed to the effects of asymmetric information, typically rely heavily on financing through outside equity, rather than debt (Frank and Goyal, 2003; Fama and French, 2005).

Violations of the pecking order preference could arise because asymmetric information is not a first-order determinant of corporate capital structures, as suggested by Fama and French (2002). But the preference of equity over debt financing could also mean that the conditions under which asymmetric information makes equity more dilutive than debt are not met. To generate properly specified empirical tests, it is critical to assess the conditions under which the pecking order theory holds and, more importantly, what to expect when they are not satisfied.

The precise conditions under which the pecking order theory holds have been the object of considerable research. In a seminal paper, Nachman and Noe (1994) show that the original Myers and Majluf conjecture holds only under very special conditions. An important class of distributions that satisfies these conditions is the lognormal distribution (or, equivalently, a Geometric Brownian Motion, GBM), where the mean of the distribution (or the drift) is private information.

While the circumstances under which the pecking order theory holds are now well understood, considerably less is known for the cases where these conditions are not met. In this paper, we argue that the Myers and Majluf conjecture can be violated in several economically important situations. We show that when firms have both assets in place and a

\[1\text{In particular, they show that debt emerges as the solution of an optimal security design problem (and, thus, is less dilutive than equity) if and only if the private information held by firm insiders orders the distribution of firm value by Conditional Stochastic Dominance (CSD).}\]
growth option, equity financing can dominate debt financing under asymmetric information. In particular, we find that equity is more likely to dominate debt for young firms with large investment needs and with risky growth opportunities. Thus, our model can explain why high-growth firms may initially prefer equity over debt, and then switch to debt financing as they mature and exploit their growth opportunities.

We study two simple economic environments in which the pecking order is violated. In the first one, we adopt a real options approach and we model firm value as an exchange (or “rainbow”) option. We adopt the exchange option framework because it is a well understood paradigm in option pricing theory (Rubinstein, 1991), and it is a natural description of an investment decision in a firm (Stulz, 1982). By making an early capital expenditure, a firm acquires the option to exchange, at a later date, its existing assets in place with new assets that embed the growth opportunity. The value of the assets in place and of the new assets (embedding the growth opportunity) are both characterized by a lognormal distribution (or, equivalently, by a GBM). The firm must finance the early investment by raising funds in capital markets characterized by asymmetric information. We model asymmetric information by assuming that the firm insiders have private information on the means (or the drifts) of the distributions, while their second moments are common knowledge.

We show that even if the distribution of the value of assets in place and the new assets both individually satisfy the conditions for the pecking order theory, the distribution of firm value may fail such conditions. This happens for the simple reason that a random variable defined as the maximum of two random variables with lognormal distributions does not have itself a lognormal distribution. Equivalently, the maximum of two GBMs is not itself a GBM. This means that even if the pecking order theory may hold for each individual asset separately, it may not hold for a combination of the same two assets.2

Our paper shows that equity can be less dilutive than debt when the asset that has greater risk (say, the growth opportunity) has also less exposure to asymmetric information, and when it represents a greater proportion of overall firm value. We also show that equity can be less dilutive than debt when the firm has larger investment needs and when the exchange option has longer time to expiration.

In the second environment, we consider a multidivisional firm composed by two segments. The distribution of the value of individual divisions is described again by lognormal distributions (or by two GBMs). Since the (weighted) average of two lognormal distributions (or GBMs) is not itself a lognormal distribution, again the pecking order may fail for the overall firm, even if it would hold for the two divisions individually. We show that, again, equity can be

2Thus, the rainbow option can generate a distribution that violates the CSD condition of Nachman and Noe (1994), even when the individual underlying assets satisfy that same condition.
dominate debt when the division that has lower exposure to asymmetric information has also a lower variance. Thus our model generates new predictions on the cross-sectional variation of firm capital structures of multidivisional firms.

Technically, our results reflect the rather limited closure properties of the hazard rate ordering. In particular, they depend on the fact that the right-tail properties of the distribution of overall firm value (as given, for example, by the exchange option) are determined by the asset with higher volatility and/or greater mean. Thus, when the asset that is less affected by asymmetric information also has greater volatility, a security with exposure to payoffs in the right tail of the firm-value distribution, such as equity, can be less dilutive than a security with proportionately more sensitive to payoffs in its left tail, such as debt.

An example with the potential for a reversal of the pecking order is a firm with assets in place that has the option to acquire another firm. Firm insiders have private information on both assets in place and the value of the target firm’s assets. In this setting, it is plausible to expect that the firm has relatively better information on the assets in place, that are already under the firm’s control, than on the new assets that still have to be acquired. The volatility of the assets in place may be lower than the volatility of the target firm’s assets, a situation that we show can generate a reversal of the pecking order.

A second example is a firm with assets in place that have been acquired by the exploitation of past growth opportunities (for example, the outcome of R&D activities, or the development of oil fields), and new growth opportunities (for example, new R&D activities, or the rights to develop new oil fields). In these situations it is again plausible that the firm has relatively more accurate information on its assets in place, on which more information is privately available to its insiders, rather than on the new growth opportunities, where critical information on their true value still has to be revealed. If the new growth opportunities have greater volatility, our model shows that the original Myers and Majluf conjecture may not hold.

We argue that the desirability of debt relative to equity reduces to the comparison of their relative dilution costs in three different regions. These regions are identified by the relative payouts of debt and equity to investors, and by the relative likelihood of the payouts as a function of the firm’s type. The first region is for low realizations of firm value. Because of first-order stochastic dominance, good firms have a lower probability to have payouts in this region. Thus, debt has an informational benefit over equity for good firms since it gives as large a payout as possible in these states of the world. The second region is for high realizations of firm value. In this case, payouts to outside investors are greater under equity than debt precisely in those states that are more likely to occur for more valuable firms.

3See section 1.B.3 of Shaked and Shanthikumar (2007).
making equity relatively more expensive than debt. All else equal, these two regions make
debt more desirable than equity.

The third and last region is the intermediate range in which payouts from debt are still
higher than equity, but now firm value realizations are sufficiently large to be relatively more
likely to occur for a good firm than a bad one. It is precisely the presence of this intermediate
region that can make equity more desirable than debt. Technically, the reversal of the pecking
order occurs when the distribution of firm value for good firms, relative to bad firms, loads
in this intermediate region with sufficiently large probability, and the difference in likelihood
in the upper tail of the distribution for the two types of firms is sufficiently small to make
the second region (discussed above) relatively unimportant — a condition that will refer
to as “low-information-cost-in-the-right-tail.” Note that, crucially, this latter property holds
when the asset with lower exposure to asymmetric information has also greater variance, as
discussed earlier.

The second contribution of the paper is to study the case when the firm has pre-existing
debt in its capital structure. We show that, interestingly, firms that already have debt
outstanding are relatively more likely to prefer equity over debt. This happens because low
realizations of future firm value (i.e. the first region above) have already been pledged to the
existing bondholders, making new debt financing less attractive than equity. This feature has
the implication that the presence of asymmetric information, by promoting equity financing,
has the surprising effect of alleviating the debt overhang problem. This prediction is novel
within models based on informational frictions, and invites for further research.

We conclude the paper with considering an explicit optimal security design problem, where
the firm can issue other securities than equity and debt. Feasible securities include convertible
bonds, warrants, as well as equity and debt, among others. Our main conclusions extend to
the more general security design problem: we show that when the “low-information-cost-in-
the-right-tail” condition holds, straight (but risky) debt is optimal when the firm needs to
raise low levels of capital, but “equity-like securities” — such as convertible debt — emerge
as the optimal securities when the firm must raise larger amounts of capital. Furthermore,
we find that warrants can be optimal securities in the presence of pre-existing debt.

Tests of pecking order use a variety of empirical methodologies, with rather mixed findings.
For example, Leary and Roberts (2010) conclude that “the pecking order is never able to accu-

\[4\] Note that our model is essentially a “convex combination” of Black and Scholes (1973)/Merton (1974)
and Myers and Majluf (1984). As such, it has a static capital structure choice, even if the model lends itself
to a dynamic specification (note that, in a similar framework, also Leland, 1994, allows for a static financing
decision). Further research focusing on dynamic capital structure choices is suggested by the fact that the
existing set of securities in a firm’s balance sheet affects the optimal financing choice.
rately classify more than half of the observed financing decisions.”5 This literature typically interprets violations of the pecking order as implying that asymmetric information and dilution through external financing are not a major determinant of corporate capital choice, as opposed to other factors that typically appear in trade-off models (such as bankruptcy costs, taxes and agency costs). The paper shows that greater information asymmetries need not make firms more inclined to follow the prescription of the pecking order in several economically important environments. This means that failure of empirical tests of the pecking order theory may just be a sign that the statistical conditions under which the theory holds are not met.

Our paper is also linked to several other papers belonging to the ongoing research on the pecking order and, more generally, the security design literature. In addition to Nachman and Noe (1994), subsequent research has focused on different aspects of the security design problem. DeMarzo and Duffie (1999) consider the ex-ante security design problem faced by a firm before learning its private information, rather than the interim security design problem (that is, after becoming informed) studied by Nachman and Noe (1994). DeMarzo (2005) considers both the ex-ante and the interim security design problems, and examines the question of whether (or not) to pool multiple assets in a single firm (pooling), and the priority structure of the securities issued by the firm (tranching). DeMarzo, Kremer, and Skrzypacz (2005) examine the security design problem in the context of auctions. Chakraborty and Yilmaz (2009) show that when investors have access to noisy public information on the firm’s private information, the dilution problem can be costlessly avoided by issuing securities having the structure of callable, convertible bonds. Chemmanur and Fulghieri (1997) and Chakraborty, Gervais, and Yilmaz (2011) argue that warrants may be part of the optimal security structure. Finally, a growing literature considers dynamic capital structure choice (Fischer, Heinkel, and Zechner, 1989; Hennessy and Whited, 2005; Strebulaev, 2007; Hennessy, Livdan, and Miranda, 2010; Morellec and Schürhoff, 2011). We conjecture that the economic forces of our static framework will play a first-order role in a dynamic version of the model.

There are other papers that challenge Myers and Majluf (1984) and Myers (1984) by extending their framework. A series of papers shows that a wider range of financing choices, which allow for signaling with costless separation, can invalidate the pecking order (see, e.g., Brennan and Kraus, 1987; Noe, 1988; Constantinides and Grundy, 1989). However, Admati...
and Pfleiderer (1994) point out that the conditions for a fully revealing signaling equilibrium identified in these papers are rather restrictive. Cooney and Kalay (1993) relax the assumption that projects have a positive net present value (NPV). Fulghieri and Lukin (2001) relax the assumption that the informational asymmetry between a firm’s insiders and outside investors is exogenous, and allow for endogenous information production. Finally, Dybvig and Zender (1991) study the effect of optimally designed managerial compensation schemes, and Edmans and Mann (2012) look at the possibility of asset sales for financing purposes. Unlike these papers, our framework is closest to the original one in Myers and Majluf (1984), in that the only friction is the asymmetric information between insiders and financiers.

The remainder of the paper is organized as follows. Section 2 presents the basic model and Section 3 studies the debt-equity choice. Section 4 considers the real options model that illustrates the main results. Section 5 extends the analysis to pre-existing debt. Section 6 considers the security design problem, where we provide conditions under which convertible debt and warrants are the optimal securities. Section 7 discusses robustness issues, and Section 8 discusses empirical implications. All the proofs are in the Appendix.

2 The basic model

An all equity-financed firm with no cash has a one-period investment project. The project requires a capital outlay $I$ at the beginning of the period. Conditional on making the investment, the firm’s value at the end of the period is given by a random variable $Z_\theta$. There are two types of firms: “good” firms, $\theta = G$, and “bad” firms, $\theta = B$, which are present in the economy with probabilities $p$ and $1 - p$, respectively. A firm of type $\theta$ is characterized by its density function $f_\theta(z)$ and by the corresponding cumulative distribution function $F_\theta(z)$, with $\theta \in \{G, B\}$. Because of limited liability, we assume that $Z_\theta$ takes values of the positive real line. For ease of exposition, we shall also assume that the density function of $Z_\theta$ satisfies $f_\theta(z) > 0$ for all $z \in \mathbb{R}_+$. In addition, we assume type $G$ firms dominate type $B$ ones by first-order stochastic dominance, defined as follows.

**Definition 1 (FOSD).** We will say that the distribution $F_G$ dominates the distribution $F_B$ by (strong) first-order stochastic dominance if $F_G(z) \leq (<) F_B(z)$ for all $z \in \mathbb{R}_{++}$.

The stronger property of Conditional Stochastic Dominance, CSD, plays a crucial role in the security design problem, as argued in Nachman and Noe (1994).

**Definition 2 (CSD).** We will say that the distribution $F_G$ dominates the distribution $F_B$ by
conditional stochastic dominance if \( F_G(z|z') \leq F_B(z|z') \) for all \( z' \in \mathbb{R}_+ \), where

\[
F_\theta(z|z') = \frac{F_\theta(z + z') - F_\theta(z')}{1 - F_\theta(z')}
\]

Note that, by setting \( z' = 0 \), CSD implies FOSD. In addition, Nachman and Noe (1994) show that CSD is equivalent to the condition that the ratio \( (1 - F_G(z))/(1 - F_B(z)) \) is non-decreasing in \( z \) for all \( z \in \mathbb{R}_+ \) (see their Proposition 4). Thus, loosely speaking, CSD implies that the set of payoffs in the right tail of the firm-value distribution are progressively more likely to occur for a firm of type \( G \) relatively to a firm of type \( B \).

We note that CSD can equivalently be defined by requiring that the truncated random variables \( [Z_\theta|Z_\theta \geq \bar{z}] \), with distribution functions \( (F_\theta(z) - F_\theta(\bar{z}))/ (1 - F(\bar{z})) \), satisfy FOSD for all \( \bar{z} \), that is \( [Z_G|Z_G \geq \bar{z}] \) \( \geq \) \( [Z_B|Z_B \geq \bar{z}] \) for all \( \bar{z} > 0 \). This ordering is weaker than the Monotone Likelihood Ratio order, which requires \( [Z_G|Z_G \in (z, \bar{z})] \) \( \geq \) \( [Z_G|Z_G \in (\bar{z}, \bar{z})] \) for all \( z \) and \( \bar{z} \).

Firms raise the amount \( I \) to fund the investment project by seeking financing in capital markets populated by a large number of competitive, risk-neutral investors. Capital markets are characterized by asymmetric information in that a firm’s type \( \theta \in \{G, B\} \) is private information to its insiders. We also assume that the NPV of the project is sufficiently large that firms will always find it optimal to issue securities and invest, rather than not issuing any security and abandon the project.

When insiders have private information, firms will typically issue securities at prices that diverge from their symmetric information values. Under these circumstances, firms will find it desirable to raise capital by issuing securities that reduce the adverse impact of asymmetric information. To fix ideas, let \( S \) be the set of admissible securities that the firm can issue to raise the required capital \( I \). As is common in this literature (see, for example, Nachman and Noe (1994)), we let the set \( S \) be the set of functions satisfying the following conditions:

1. \( 0 \leq s(z) \leq z \), for all \( z \geq 0 \).
2. \( s(z) \) is non-decreasing in \( z \), for all \( z \geq 0 \).
3. \( z - s(z) \) is non-decreasing in \( z \), for all \( z \geq 0 \).

Condition (1) ensures limited liability for both the firm and investors, while (2) and (3) are monotonicity conditions that ensure absence of risk-less arbitrage.\(^6\) We define \( S \equiv \{s(z) : \).

\(^6\)See equation (1.B.7) and Theorem 1.C.5 in Shaked and Shanthikumar (2007).

\(^7\)See, for example, the discussion in Innes (1990). Note that, as pointed out in Nachman and Noe (1994), condition (2) is critical to obtain debt as an optimal security. In absence of (2), the optimal contract may
\[ \mathbb{R}_+ \to \mathbb{R}_+ : s(z) \text{ satisfies } (1), (2), \text{ and } (3) \} \text{ as the set of admissible securities.} \]

In this paper we will consider the following capital raising game. The firm moves first, and chooses a security \( s(z) \) from the set of admissible securities \( \mathbb{S} \), that is \( s \in \mathbb{S} \). After observing the security \( s(z) \) issued by the firm, investors update their beliefs on firm type \( \theta \), and form posterior beliefs \( p(s) : \mathbb{S} \to [0, 1] \). Given their posterior beliefs on firm type, investors purchase the security issued by the firm at a price \( V(s) \). The value \( V(s) \) that investors are willing to pay for the security \( s(z) \) issued by the firm is equal to the expected value of the security, conditional on the posterior beliefs \( p(s) \), that is

\[
V(s) = p(s)\mathbb{E}[s(Z_G)] + (1 - p(s))\mathbb{E}[s(Z_B)].
\]

Condition (4) implies that securities are fairly priced, given investors’ beliefs. If security \( s \) is issued, capital \( V(s) \) is raised, and the investment project is undertaken, the payoff to the initial shareholders for a firm of a type \( \theta \) is given by

\[
W(\theta, s, V(s)) \equiv \mathbb{E}[Z_\theta - s(Z_\theta)] + V(s) - I.
\]

The firm will choose the security issued to finance the investment project by maximizing its payoff (5), subject to the constraint that the security is admissible and that it raises at least the required funds \( I \). Let \( s_0(z) \in \mathbb{S} \) be the security issued by a firm of type \( \theta \).

In this paper, following the literature, we will adopt the notion of Perfect Bayesian Equilibrium, PBE, as follows.

**Definition 3 (Equilibrium).** A PBE equilibrium of the capital raising game is a collection \( \{ s^*_G(z), s^*_B(z), p^*(s), V^*(s) \} \) such that: (i) \( s^*_\theta(z) \) maximizes \( W(\theta, s, V^*(s)) \) subject to the constraint that \( s \in \mathbb{S} \) and \( V^*(s) \geq I \), for \( \theta \in \{G, B\} \), (ii) securities are fairly priced, that is

\[
V^*(s) = p^*(s)\mathbb{E}[s(Z_G)] + (1 - p^*(s))\mathbb{E}[s(Z_B)] \text{ for all } s \in \mathbb{S}, \text{ and (iii) posterior beliefs } p^*(s) \text{ satisfy Bayes Rules whenever possible.}
\]

We start with a characterization of the possible equilibrium sets that will be quite useful in simplifying our exposition.\(^8\)

**Proposition 1.** (Nachman and Noe, 1994) Let \( F_\theta \) satisfy strict FOSD. No separating equilibrium exists in the capital raising game. In addition, in any pooling equilibrium, with have a “do or die” component, whereby outside investors obtain all of the firm cash flow when it falls below a certain threshold, and nothing otherwise.

\(^8\)We note that the strong form of FOSD is only necessary for Proposition 1. Our main results go through assuming only FOSD. At the same time, all the parametric examples we consider satisfy the strong version of FOSD.
\( s^*_G = s^*_B = s^* \), the capital raising game is uninformative, \( p(s^*) = p \), and the financing constraint is met with equality, \( \mathbb{E}[s^*(Z)] = I \).

Proposition 1 derives from the fact that, with two types of firms only, a type B firm has always the incentive to mimic the behavior of a type G firm (i.e. to issue the same security). This happens because (2) and strict FOSD together imply that securities issued by a type G firms are always priced better by investors than those issued by a type B firm, and type B firm is always better-off by mimicking a type G one. This also implies that, in equilibrium, the type G firm is exposed to dilution due to the pooling with a type B firm, and the corresponding loss of value can be limited by issuing only the securities needed to raise the capital outlay \( I \).

Proposition 1 allows us to simplify the exposition as follows. If both type of firms pool and issue the same security \( s \) and the capital constraint is met as equality, we have that

\[
I = p\mathbb{E}[s(Z_G)] + (1 - p)\mathbb{E}[s(Z_B)],
\]

Combining (5) and (6), it is easy to see that the payoff to the original shareholders of firm type G becomes

\[
W(G, s, V(s)) = \mathbb{E}[Z_G] - I - (1 - p)D_s,
\]

where the term

\[
D_s \equiv \mathbb{E}[s(Z_G)] - \mathbb{E}[s(Z_B)]
\]

represents the dilution suffered by a firm of type G when security \( s \in S \) is used.

Under these circumstances, firms of type G will find it optimal to finance the project by issuing a security that minimizes dilution \( D_s \), that is

\[
\min_{s \in S} D_s
\]

subject to the financing constraint (6). Defining the function \( c(z) \equiv f_G(z) - f_B(z) \), the dilution costs of security \( s(z) \) can be expressed as:

\[
D_s = \int_0^\infty s(z)c(z)dz.
\]

Note that the density function \( f_{\theta}(z) \) measures, loosely speaking, the (implicit) private valuation of a $1 claim made by the insiders of a firm of type \( \theta \in \{G, B\} \) if the final payoff of the firm is \( z \). Thus, we can interpret the term \( c(z) \) as representing the private “asymmetric information cost” for a firm of type G, relative to a type B firm, of issuing a security that
has a payoff of $1 if the final firm value is \( z \). In particular, if \( c(z) > 0 \) we will say that the information costs for a type \( G \) are “positive,” and that these costs are “negative” if \( c(z) < 0 \). More formally, the asymmetric information costs of a security that pays $1 if and only if the final payoff is in the interval \( z \in [z_L, z_H] \) is equal to \( \int_{z_L}^{z_H} c(z) dz \).

In what follows we will be concerned on the asymmetric information costs in the upper tail of the value distribution \( F_\theta(z) \) for a firm of type \( G \) relative to a firm of type \( B \). These asymmetric information costs are related to the function \( H(z) \) defined as:

\[
H(z) \equiv \frac{F_B(z) - F_G(z)}{1 - F(z)},
\]

where \( F(z) \) denotes the mixture of the distribution of the good and bad types, that is,

\[
F(z) = pF_G(z) + (1 - p)F_B(z).
\]

The function \( H(z) \) plays a critical role in our analysis. First note that FOSD implies that \( H(z) > 0 \) for all \( z \in \mathbb{R}_+ \). In addition, and more importantly, monotonicity of \( H(z) \) is equivalent to requiring that the hazard rates \( h_\theta(z) \equiv f_\theta(z)/(1 - F_\theta(z)) \) satisfy \( h_G(z) \leq (\leq) h_B(z) \) for all \( z \in \mathbb{R}_+ \).

The function \( H(z) \) provides a measure of the extent of asymmetries of information in the right tail of the distribution of firm value, which for monotonic securities is closely linked to the cost to a type \( G \) firm of promising to investors an extra dollar in state \( z \).\(^9\)

In what follows, it will be important to characterize properties the right tail of the firm-value distribution that are stronger than FOSD, but at the same time weaker than CSD. Note first that \( H(0) = 0 \) and that, from FOSD, we have \( H(z) > 0 \) for \( z \) in a right neighborhood of \( z = 0 \), which together imply that \( H'(0) > 0 \). It is important to note that, while the monotonicity properties of \( H(z) \) on the left-tail of the distribution of \( z \) are dictated by FOSD, this is not the case for the right-tail of the distribution. To characterize the behavior of the information costs in the right-tail of the distribution, we introduce the following definition, which will play a key role in our analysis.

\(^9\)This happens because, for monotonic securities, an extra dollar paid in state \( z \) means that investors will be paid an extra dollar also in all states \( z' > z \). This interpretation will become apparent in Section 6 (see equation (34)).
**Definition 4 (h-ICRT).** We will say that distribution \( F_G \) has information costs in the right tail of degree \( h \) (h-ICRT) over distribution \( F_B \) if \( \lim_{z \to \infty} H(z) \leq h \).

We will use the term NICRT (no-information-costs-in-the-right-tail) to denote the case \( h = 0 \). The relationship between FOSD, CSD and h-ICRT may be seen by noting that for two distributions \( \{F_G, F_B\} \) that satisfy FOSD, there may exist a sufficiently low \( h \in \mathbb{R}_+ \) such that the h-ICRT property holds, while conditional stochastic dominance fails. Thus, intuitively, distributions that satisfy the h-ICRT condition “fill” part of the space of distributions that satisfy FOSD but do not satisfy the CSD condition. In particular, all distributions that satisfy Definition 4 for \( h = 0 \) (NICRT) will fail to satisfy the CSD condition.

We conclude this section by introducing an additional regularity condition that will simplify the analysis and greatly streamline the presentation of some of the results.

**Definition 5 (SCDP).** The distributions \( F_{\theta}(z) \), for \( \theta = G, B \), satisfy the single-crossing density property (SCDP) if \( F_G \) strictly first-order stochastically dominates \( F_B \), and there exists a unique \( \hat{z} \in \mathbb{R}_+ \) such that \( f_G(\hat{z}) = f_B(\hat{z}) \).

Note that the SCDP condition implies that for all \( z \leq \hat{z} \) we have \( f_B(z) \geq f_G(z) \), and for all \( z \geq \hat{z} \) we have \( f_B(z) \leq f_G(z) \). Intuitively, this means that cash flows above the critical cutoff \( \hat{z} \) have a positive information cost for type \( G \) firms, \( c(z) > 0 \), whereas cash flows below that cutoff have negative information costs, \( c(z) < 0 \). Note that FOSD alone only implies that there exists \( z_1 \) and \( z_2 \) such that \( c(z) < 0 \) for all \( z < z_1 \) and \( c(z) > 0 \) for all \( z > z_2 \), but it does not rule out other interior crossings; in contrast, SCDP ensures that \( z_1 = z_2 \).

## 3 The debt-equity choice

We start the analysis by restricting our attention in this section to two classes of securities, debt and equity. From (7), the dilution costs associated with equity are given by

\[
D_E = \lambda (\mathbb{E}[Z_G] - \mathbb{E}[Z_B]),
\]

with \( \lambda = I/\mathbb{E}[Z] \), whereas those associated with debt

\[
D_D = \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)],
\]

where \( K \) represents the (smallest) face value of debt that satisfies the financing constraint

\[
I = p\mathbb{E}[\min(Z_G, K)] + (1 - p)\mathbb{E}[\min(Z_B, K)].
\]

In what follows we will say the pecking order

\[\text{We are assuming SCDP for ease of exposition. The discussion below could be adapted to take into account the presence of multiple crossings.}\]
(PO) is satisfied if $D_E > D_D$, and that the model generates a “reverse pecking order” (RPO) if $D_D > D_E$.

We begin our analysis by providing a necessary and sufficient condition for RPO to hold.

**Proposition 3.** The dilution costs of equity will be strictly smaller than those of debt, i.e. $D_E < D_D$, if and only if

$$\frac{\mathbb{E}[Z_G]}{\mathbb{E}[Z_B]} < \frac{\mathbb{E}[\min(Z_G, K)]}{\mathbb{E}[\min(Z_B, K)]}. \quad (14)$$

The proposition implies that when the security choice is restricted between equity and debt, the security that generates lowest dilution in dollar terms is the one with the lowest relative valuation between the good and bad type. This feature supports the traditional intuition that debt dominates equity precisely because debt valuation is less sensitive to the underlying asymmetries in information, limiting dilution.\(^{11}\)

We next determine the restrictions on the distribution functions $F_\theta(z)$ for $\theta \in \{G, B\}$ at which (14) holds or fails, that is when PO or RPO obtain. The next proposition shows dominance of debt over equity under the CSD condition of Nachman and Noe (1994).

**Proposition 4.** Condition (14) cannot hold if $F_G$ dominates $F_B$ in the conditional stochastic dominance sense.

The CSD condition is a rather strong requirement that may fail in many economically interesting cases, opening the possibility for equity financing to dominate debt financing. In this paper we will focus on such cases.

To obtain the conditions under which equity financing dominates debt, note RPO holds if and only if the dilution costs of debt, $D_D$, are greater than the dilution costs of equity, $D_E$. From (9), (12) and (13) this happens when:

$$D_D - D_E = \int_0^\infty (\min(z, K) - \lambda z) c(z) dz > 0. \quad (15)$$

Define $\tilde{z}(K, \lambda) \equiv K/\lambda$ and note that for $z < \tilde{z}(K, \lambda)$ we have that $\min(z, K) > \lambda z$, which implies that the payoffs to debtholders are greater than those to equity holders; the converse holds for $z > \tilde{z}(K, \lambda)$. Note that the cutoff point $\tilde{z}(K, \lambda)$ depends positively on the face value of debt $K$, and negatively on fraction of equity issued to outside investors, $\lambda$.

The factors that drive of the relative dilution of debt and equity can be seen by decomposing (15) in more fundamental components. To this aim, it is useful to rank the point where

\(^{11}\)A similar condition was obtained, in the context of unit IPOs, in Chakraborty, Gervais, and Yilmaz (2011), see their Propositions 1 and 2.
equity payouts are equal to debt payouts, denoted by \( \bar{z}(K, \lambda) \), with respect to the critical point in the SCDP condition, given by \( \hat{z} \), as follows. As we will show below, this condition is necessary to reverse the pecking order, that is for an “unpecking” order to arise.

**Definition 6 (UNC).** The “unpecking” necessary condition (UNC) is satisfied if \( \bar{z} > \hat{z} \).

Under SCDP, the point \( \hat{z} \) divides the positive real line into two disjoint sets: a first set at the lower end of the positive real line, \([0, \hat{z})\) where \( c(z) < 0 \), that is where a type-G firm enjoys “negative information costs” (that is, effectively an information benefit), and a second set \([\hat{z}, \infty)\) where \( c(z) \geq 0 \), that is where a type-G firm faces “positive information costs.” The point \( \bar{z}(K, \lambda) \) divides the positive real line in two other subsets, depending on whether or not equity yield higher payoffs than debt to investors. Thus, SCDP and UNC together divide the positive real line into three regions: (i) a low-value region where \( z < \hat{z} \) and \( z \leq \bar{z} \); (ii) an intermediate region where \( [\hat{z}, \bar{z}] \); and (iii) a high-value region where \( z > \bar{z}(K, \lambda) \) (see Figure 1).

In the first region we have that \( c(z) < 0 \) and \( \lambda z < K \), which means that type-G firms enjoy “negative information costs” and the payoff to equity is strictly less than the payoff to debt. In the second region we have that \( c(z) \geq 0 \) and \( \lambda z \leq K \), which means that type-G firms are exposed to “positive information costs” and the payoff to equity is no greater than the payoff to debt. In the third region we have that \( c(z) \geq 0 \) and \( \lambda z > K \), which means that type-G firms are exposed to “positive information costs” and the payoff to equity is strictly greater than the payoff to debt. The relative dilution costs of equity and debt, \( \mathcal{D}_D - \mathcal{D}_E \), depend on the comparison of the information costs and relative payoffs of debt and equity, as follows.

**Proposition 5.** Assume the SCDP holds. Then a necessary and sufficient condition for the reverse pecking order is that (i) UNC holds, and (ii)

\[
\mathcal{D}_D - \mathcal{D}_E = \int_{\hat{z}}^{\bar{z}} (\min(z, K) - \lambda z)c(z)dz - \int_{\hat{z}}^{\bar{z}} (\lambda z - \min(z, K))c(z)dz - \int_{\bar{z}}^{\infty} (\lambda z - K)c(z)dz > 0. 
\]

(16)

Under UNC and the maintained assumptions the three integrals in (16) are all positive and they represent the decomposition of the dilution costs of debt relative to equity in three components that are determined by the regions we have identified.

The first term of the r.h.s. of (16) measures the dilution cost of debt relative to equity in the *intermediate-value region* \([\hat{z}, \bar{z}]\), where debt has higher payouts than equity and type-G firms suffer a positive information cost, \( c(z) > 0 \). In this region dilution costs of equity are lower than those of debt because equity has lower payoff than debt precisely in those states.
in which type-$G$ firms are exposed to positive information cost (since $c(z) > 0$). Note that existence of this region is guaranteed by UNC. It is the presence of this term that makes equity potentially less dilutive than debt.

The second term of the r.h.s. of (16) measures the benefits of debt financing for low realizations of firm value (i.e. for $z < \hat{z}$). In this **low-value region**, dilution costs are lower for debt than equity because debt gives a higher payoff than equity, but such payoff has negative information costs (i.e. $c(z) < 0$). The third and last term measures the dilution costs of equity relative to debt for high realizations of firm value (i.e. for $z > \bar{z}$). In this **high-value region**, equity payoffs are greater than debt in those states that are more likely to occur to a type-$G$ firm, and thus carry positive information costs (i.e. $c(z) > 0$).

The relative importance of these three regions determines the optimality of debt versus equity choice. In particular, equity financing dominates debt financing when the advantages of equity financing originating from the intermediate region of firm value (for $z \in [\hat{z}, \bar{z}]$), that is, the first term on the r.h.s. of (16) dominate the disadvantages in the low (for $z < \hat{z}$) and the high (for $z > \bar{z}$) regions of firm value, that is, the second and the third term on the r.h.s. of (16). Note that if UNC does not hold (so that $\bar{z}(K, \lambda) < \hat{z}$), equity has negative information costs (that is, $c(z) < 0$) precisely in the states where the payouts to equityholders are greater than those to debtholders, making it impossible for the inequality (16) to be satisfied. Thus, UNC is a necessary condition to reverse the pecking order.

### 4 A real options model

In this section, we present a real options specification of our basic model. This approach presents two main advantages. First, it draws on well established option pricing techniques that provide analytical tractability. Second, it provides sufficient flexibility for modeling asymmetric information between firm owners and outside investors. In particular, this specification allows us to have both first-order stochastic dominance and the right-tail behavior of firm-value distribution that generates a reverse pecking order.

We model the real options problem as follows. By paying the investment cost $I$ at the beginning of the period, $t = 0$, the firm generates a new growth opportunity that is exercisable at date $T$. We model the growth opportunity as an exchange (or “rainbow”) option. That is, the firm holds an option to exchange the existing assets in place, with a value of $X_{\theta T}$ at a future date $T$, for the new assets with value $Y_{\theta T}$, for $\theta \in \{G, B\}$. We interpret the new assets $Y_{\theta T}$ as embedding the incremental firm value of the new investment project; therefore, we refer to $X_{\theta T}$ as the “assets in place” and to $Y_{\theta T}$ as the “growth opportunity.” Thus, the firm has a European exchange option on non-dividend paying assets. We adopt the exchange
option framework also because it is quite common in the real options literature (see, for instance, Stulz, 1982).

We assume that, after the initial investment \( I \) is made, at the end of the period \( T \) the value of a firm of type \( \theta \) is given by \( Z_{\theta T} \equiv \max(X_{\theta T}, Y_{\theta T}) = X_{\theta T} + \max(Y_{\theta T} - X_{\theta T}, 0) \), for \( \theta \in \{B, G\} \). We assume that both \( X_{\theta T} \) and \( Y_{\theta T} \) follow a lognormal process, that is, both \( \log(X_{\theta T}) \) and \( \log(Y_{\theta T}) \) are normally distributed with means \( \mu_{\theta x} \) and \( \mu_{\theta y} \) and with variances \( \sigma_{\theta x}^2 \) and \( \sigma_{\theta y}^2 \). Let \( \rho_{\theta} \) be the correlation coefficient between \( \log(X_{\theta T}) \) and \( \log(Y_{\theta T}) \). Thus, the real option specification is isomorphic to a model where time flows continuously, that is \( t \in [0, T] \), and where asset values \( X_{\theta T} \) and \( Y_{\theta T} \) follow two geometric Brownian motions with drifts \( \mu_{\theta x} \) and \( \mu_{\theta y} \), variances \( \sigma_{\theta x}^2 \) and \( \sigma_{\theta y}^2 \), respectively, and correlation coefficient \( \rho \).

In the spirit of Myers and Majluf (1984), we model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while their variances are common knowledge. We let \( \mathbb{E}[X_{\theta T}] = X_{\theta} \) and \( \mathbb{E}[Y_{\theta T}] = Y_{\theta} \), and we assume \( X_G \geq X_B \) and \( Y_G \geq Y_B \), with at least one strict inequality. We define the average value of the assets in place and of the growth opportunity as \( \bar{X} = pX_G + (1 - p)X_B \) and \( \bar{Y} = pY_G + (1 - p)Y_B \), respectively, and let \( c_x = X_G - X_B \) and \( c_y = Y_G - Y_B \). Thus, \( c_x \) and \( c_y \) measure the exposure to asymmetric information of the assets in place and the growth opportunity. Finally, to ensure FOSD we assume that \( \sigma_{Gx} = \sigma_{Bx} = \sigma_x \), \( \sigma_{Gy} = \sigma_{By} = \sigma_y \), \( \rho_G = \rho_B = \rho \), and, without loss of generality, that \( \sigma_y \geq \sigma_x \). Thus, we will say that the growth opportunity has greater volatility than the assets in place.

Throughout most of the discussion we will assume that the payoff to the firm before the investment is \( X_{\theta T} \), so that the incremental cash flow that accrues to the investment is the rainbow option \( \max(X_{\theta T}, Y_{\theta T}) - X_{\theta T} = \max(Y_{\theta T} - X_{\theta T}, 0) \).

We can now proceed to explicitly characterize the choice of financing, where the choice is exogenously limited to equity and debt. The value of a firm of type \( \theta \) is given by the value of the exchange option, denoted by \( A_{\theta} \equiv \mathbb{E}[Z_{\theta T}] \). Following Margrabe (1978), we know that, at the beginning of the period, \( t = 0 \), the value of this option for a firm of type \( \theta \) is given by

\[
A_{\theta} = X_{\theta} \hat{\Delta}_{x\theta} + Y_{\theta} \hat{\Delta}_{y\theta},
\]

where \( \hat{\Delta}_{x\theta} \equiv N(a_{x\theta}) \), \( \hat{\Delta}_{y\theta} \equiv N(a_{y\theta}) \), \( \Sigma^2 = \sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y\rho \), and

\[
a_{x\theta} = \frac{\log (X_{\theta}/Y_{\theta})}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T},
\]

\[
a_{y\theta} = \frac{\log (Y_{\theta}/X_{\theta})}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T},
\]

\(12\)Recall that we assume that the project’s NPV is sufficiently large for investment to be optimal. Thus, the assumption \( \sigma_y \geq \sigma_x \) is without loss of generality.
where \( N(\cdot) \) denotes the cumulative distribution function of a standard normal random variable. Note that (17) is equal to the value of the replicating portfolio of the exchange option, and the terms \( \Delta_{x\theta} \) and \( \Delta_{y\theta} \) represent the deltas of the option, that is, the sensitivity of the value of the exchange option with respect to the value of the underlying assets, \( X_\theta \) and \( Y_\theta \), respectively.\(^{13}\)

If the firm raises the required capital by issuing equity, existing shareholders will have to sell to outside investors a fraction \( \lambda \) of the firm to satisfy the financing constraint, that is

\[
\lambda = \frac{I}{p\Delta A + (1 - p)\Delta B}.
\]

If the firm raises the required capital by issuing debt, we denote by \( V_\theta(K) \) as the value of risky debt with face value of \( K \) when issued by a firm of type \( \theta \). Note that the debt value \( V_\theta(K) \) can be written as

\[
V_\theta(K) = E[\min(Z_{\theta T}, K)] = K - P_\theta,
\]

that is, as the value of the default-free debt, \( K \), minus the value of the option to default, which is equal to \( P_\theta = E[\max(K - Z_{\theta T}, 0)] \). The option to default for a firm of type \( \theta \) is given by the compound put option given by \( \max(K - Z_{\theta T}, 0) \), where in turn \( Z_{\theta T} \) is given by the exchange option \( \max(X_{\theta T}, Y_{\theta T}) \). Following Stulz (1982) and Rubinstein (1991), the value of this put option is given by

\[
P_\theta = K \Gamma_{\theta} - X_\theta \Delta_{x\theta}^* - Y_\theta \Delta_{y\theta}^*,
\]

(21)

where \( \Delta_{x\theta}^* \equiv \Gamma(b_{x\theta}, a_{x\theta}, \rho_x) \), \( \Delta_{y\theta}^* \equiv \Gamma(b_{y\theta}, a_{y\theta}, \rho_y) \), \( \Gamma_{\theta} \equiv \Gamma(b_{x\theta} + \sigma_x \sqrt{T}, b_{y\theta} + \sigma_y \sqrt{T}, \rho) \), \( \rho_x = (\sigma_x - \rho \sigma_y)/\Sigma \), and \( \rho_y = (\sigma_y - \rho \sigma_x)/\Sigma \). The variables \( a_{x\theta} \) and \( a_{y\theta} \) are given in (18)–(19), and the variables \( b_{x\theta} \) and \( b_{y\theta} \) are given by

\[
b_{x\theta} = \frac{\log(K/X_\theta)}{\sigma_x \sqrt{T}} - \frac{1}{2} \sigma_x \sqrt{T},
\]

(22)

\[
b_{y\theta} = \frac{\log(K/Y_\theta)}{\sigma_y \sqrt{T}} - \frac{1}{2} \sigma_y \sqrt{T},
\]

(23)

where the function \( \Gamma(\cdot) \) denotes the cumulative distribution function of a bivariate standard normal random variable.\(^{14}\) Note that (21) is equal to the value of the replicating portfolio

\[^{13}\]In addition, we can interpret the term \( X_\theta \Delta_{x\theta} \) as the expectation of the value of asset in place conditional on being larger than \( Y_\theta \), that is, as the product of the value of of the asset, \( X_\theta \), times the probability that \( X_\theta \) is greater than \( Y_\theta \). Thus, \( \Delta_{x\theta} \equiv N(a_{x\theta}) \) can be interpreted (loosely speaking) as the probability that the assets in place are more valuable than the growth opportunity, \( Y_\theta \). By symmetry, the second-term in (17) has a similar interpretation.

\[^{14}\]Namely \( \Gamma(a, b, c) \) is the area under a bivariate standard normal distribution function with correlation \( c \) from \( -\infty \) to \( a \), \( -\infty \) to \( b \). Thus, if \( f(x_1, x_2) \) is the density of a standard normal bivariate vector \( x = (x_1, x_2) \) with correlation \( c \), then \( \Gamma(a, b, c) = \int_{-\infty}^{a} \int_{-\infty}^{b} f(x_1, x_2) dx_1 dx_2 \).
of the put option, where the terms $\Delta^*_x$ and $\Delta^*_y$ in (21) are the deltas of the compound put option with respect to the value of underlying assets $X_\theta$ and $Y_\theta$, respectively, and $K\Gamma_\theta$ represents the investment in the riskless asset in the replicating portfolio. Finally, the face value of the debt, $K$, has to satisfy the financing constraint, which is given by

$$K - (pP_G + (1 - p)P_B) = I. \quad (24)$$

We can now characterize the conditions under which the reverse pecking order holds in the real options model. From (15) we obtain that equity financing is less dilutive than debt financing, so that RPO obtains, if and only if

$$\lambda (A_G - A_B) < P_B - P_G; \quad (25)$$

where $A_\theta$ and $P_\theta$ are given in (17) and (21), respectively. It is important to note that while condition (25) gives a closed-form solution for the preference of equity over debt financing, the left hand side of (25) includes the term $\lambda$, which depends on the model’s primitives via the financing constraint (20), and the right hand side depends on $K$ which is determined by the financing constraint (24). Analytical tractability is further hindered by the presence of the bivariate normal cumulative distribution function $\Gamma$ in the valuation equation (21) for the put option. This means that in what follows we will characterize (25) mainly by numerical means.

We start the analysis by considering a perturbation of the parameter values around the case without asymmetric information, i.e. when $Y_G = Y_B$ and $X_G = X_B$. In the perturbation, only the assets in place are exposed to (a small amount of) asymmetric information: $X_G = \bar{X} + \epsilon$ and $X_B = \bar{X} - \epsilon$. For $\epsilon$ sufficiently close to zero, it is easy to see that condition (25) reduces to

$$\lambda \Delta_x < \Delta^*_x, \quad (26)$$

where we have dropped the type $\theta$ subscript. This “delta” condition has the intuitive interpretation that equity is less dilutive than debt if the sensitivity to $X$ of the value of the equity sold to outside investors (which depends on the delta of the exchange option, $\Delta_x$) is smaller than the corresponding sensitivity of debt, measured by $\Delta^*_x$ (which depends on the corresponding delta of the compound put option, $\Delta^*_x$). The deltas of the two options measure the sensitivity of the value equity and debt to the value of the underlying asset(s),

\[15\] Note also that the term $\Gamma^P$ can be interpreted as the probability, for a firm of type $\theta$, that the put option in (21) is in the money. In a similar way, the term $\Delta^P_\theta$ can be interpreted as the probability of the underlying asset $X_\theta$ to be larger than $K$ and $Y_\theta$ for a firm of type $\theta$; similarly for the term $\Delta^*_\theta$. 

17
and therefore their exposure to asymmetric information and potential mispricing.

Condition (26) can be further simplified in terms of univariate cumulative normal distributions when \( \sigma_x = \rho \sigma_y \). In this case, it is easy to see that \( \Delta_x^* \equiv \Gamma(b_x, a_x, 0) = N(b_x) \times N(a_x) = \Delta_x \times \hat{\Delta}_x \), where \( \Delta_x \) is the delta of a “plain vanilla” put option written on the assets in place only, namely \( \Delta_x = N(b_x) \), and \( \hat{\Delta}_x \) is again the delta of the exchange option in (17). This means that the delta of the compound put option (i.e., the default option) that can be decomposed into the product of the delta of a “simple” put option written only on the assets in place, \( X \), with a strike price equal to the face value of the debt, \( K \), times the delta with respect to the assets in place \( X \) of the underlying exchange option. Substituting \( \Delta_x^* = \Delta_x \times \hat{\Delta}_x \) into (26) and using (20), we obtain that (25) reduces to

\[
\lambda = \frac{I}{X \Delta_x + \bar{Y} \hat{\Delta}_y} < \Delta_x. \tag{27}
\]

The next Proposition allows us to characterize the reverse pecking order under these parametric assumptions.\(^{16}\)

**Proposition 6.** Consider the case where there is no informational asymmetry on \( Y \), \( Y_G = Y_B = \bar{Y} \), but there is on \( X \), namely, \( X_G = \bar{X} + \epsilon \) and \( X_B = \bar{X} - \epsilon \). Further assume that \( \rho \sigma_y = \sigma_x \). Then, as we let \( \epsilon \downarrow 0 \), we have that: (i) condition (25) holds for sufficiently large values of \( \bar{Y} \), where it can never hold for small values of \( Y \); (ii) as \( \sigma_x \downarrow 0 \), condition (25) holds if \( \bar{X} < K \), but cannot hold if \( \bar{X} > K \).

Part (i) of Proposition 6 can be seen as follows. First, note that (27) is more likely to be satisfied when \( \bar{Y} \) is large compared to \( \bar{X} \). This happens because, in this case, the exchange option is sufficiently in-the-money with respect to \( Y \) to make \( \hat{\Delta}_y \) relatively large. This feature, combined with a large value of the growth opportunity itself, \( \bar{Y} \), leads to a low value of \( \lambda \) on the l.h.s. of (27), while the r.h.s. is independent of \( \bar{Y} \). This implies the firm must issue to outside investors a relatively small equity share \( \lambda \), while the sensitivity of the option to default with respect to \( X \) may still be rather significant.

Second, note that for \( \bar{Y} = 0 \), (27) can never be satisfied. This happens because, when \( \bar{Y} = 0 \), the exchange option is always equal to the value of the assets in place (since the growth opportunity has no value). This means that \( \hat{\Delta}_x = 1 \), and (27) requires that \( I < \Delta_x \bar{X} \), which violates the financing constraint (24).\(^{17}\)

\(^{16}\)We conjecture that the statements in the following Proposition are more general, as we verify in the numerical analysis. Analytical proofs in the general case are much more demanding due to the presence of the bivariate normal cumulative distribution function \( \Gamma \) in the valuation equation (21) for the put option.

\(^{17}\)To see this note that the financing constraint (24) can be written in this case as \( I = (1 - \Gamma)K + X \Delta_x \).
Part (ii) of Proposition 6 stresses the role of the option to default under debt financing to generate the reverse pecking order. When the volatility of the assets in place, $\sigma_x$, is sufficiently small, the parameters $\bar{X}$ and $K$ identify two separate regions that will arise with very high probability. The first region occurs for $\bar{X} > K$ and is a “safety region” (with respect to $X$), whereby the put option is exercised with very low probability. In this case, the value of delta of the put option, $\Delta_x$, is very small, and (27) cannot be verified. Thus, a reversal of the pecking order cannot arise.

The second region occurs for $\bar{X} < K$ and is a “bankruptcy region” (with respect to $X$), whereby the put option is exercised with very high probability. In this case, the value of delta of the put option, $\Delta_x$, is large (i.e. close to one). Thus, the debt security is highly sensitive to changes in value for the assets in place $X$, whereas the exchange option still gets a significant value from the growth opportunity component $Y$. This means that $\lambda$ is small (i.e. smaller one) and that (27) is always verified, generating a reverse pecking order.

More generally (and outside the parametric restrictions of Proposition 6) condition (26) is more likely to be satisfied when $\bar{Y}$ is relatively large, so that the exchange option is deep in-the-money with respect to $Y$, but the option to default is still rather dependent on the value of $X$ (which happens when $K > \bar{X}$). The first condition ensures that the exchange option is not too sensitive to changes in $\bar{X}$, while the second condition makes the (default) put option particularly sensitive to $\bar{X}$.

So far we have considered perturbations where only the assets in place (i.e. the assets with the lower volatility) are exposed to a small amount of asymmetric information. These perturbations control the probability mass in the intermediate region of Proposition 5. The symmetric case occurs when there is no asymmetric information on the assets in place, $X$, but the growth opportunity $Y$ is exposed to a small amount of asymmetric information. This corresponds to the case where $X_G = X_B = \bar{X}$ and $Y_G = \bar{Y} + \epsilon$ with $Y_B = \bar{Y} - \epsilon$, for $\epsilon > 0$ arbitrarily small.\(^\text{18}\) We will show in Section 6 that in the case where the asymmetric information loads only on the growth-option $Y$, debt financing is always optimal, and the standard pecking order holds. More generally, we will show that a reverse pecking order will occur when the assets in place are more exposed to asymmetric information than the growth opportunity. Thus, the fact that asymmetric information characterizes the assets with lower volatility is a key feature to generate the reversal of the pecking order.

We conclude this section by conducting a series of numerical examples of the general

\(^{18}\)Note that (26) does not simplify as in the case with $X$, since the correlation term $\rho_y = (\sigma_y - \rho \sigma_x)/\Sigma$ defined after (21) satisfies $\rho_y > 0$ when $\sigma_y > \sigma_x$. Its implicit term, the face value of the debt $K$, makes it analytically challenging. We note how none of the volatility limits in Proposition 6 apply under our stated condition $\rho \sigma_y = \sigma_x$. 

condition (25), all centered on a base case reported in Table 1. In order to keep the parameters as parsimonious as possible, we will assume that \( Y_G = Y_B = 175, \sigma_x = 0.3, \sigma_y = 0.6, T = 10 \) and \( \rho = 0 \). The asymmetric information corresponds to the assets in place, namely \( X_G = 125 \) and \( X_B = 75 \). We let both types be equally likely, \( p = 0.5 \). The value of the firm post-investment for the two types is given by \( \mathbb{E}[Z_{GT}] = 257.6 \) and \( \mathbb{E}[Z_{BT}] = 218.3 \), so that \( p \mathbb{E}[Z_{GT}] + (1 - p) \mathbb{E}[Z_{BT}] = 237.9 \). In the base case specification, we let the investment amount \( I = 110 \).

Without the project, the status-quo firm value is the value of asset \( X \), which is equal to \( X = 100 \). Since the value of the firm post investment is 237.9, and the investment is \( I = 110 \), the project has an (unconditional) positive NPV of 27.9. Note also that the efficient outcome is for both types of firms would be to finance the project, since for a type-\( G \) we have that \( \mathbb{E}[Z_{GT}] - I = 257.6 - 110 = 147.6 > 125 = X_G \), and for a type-\( B \) we have that \( \mathbb{E}[Z_{BT}] - I = 218.3 - 110 = 108.3 > 75 = X_B \).

It is easy to verify that issuing equity will require that the equity holders give up a stake of \( \lambda = 0.462 = 110/237.9 \). This means that residual equity value for a type-\( G \) type firm is equal to \( (1 - 0.462) \times 257.6 = 138.6 > 125 = X_G \), and for a type-\( B \) type firm is equal to \( (1 - 0.462) \times 218.3 = 117.4 > 75 = X_B \). In order to finance the project with debt, the firm needs to promise bondholders a face value of \( K = 198.3 \) at maturity. Using (21), one can readily check that the values of debt for the good type and the bad type are \( \mathcal{V}_G(K) = \mathbb{E}[^\text{min}(Z_{GT}, K)] = 120.6 \) and \( \mathcal{V}_B(K) = \mathbb{E}[^\text{min}(Z_{BT}, K)] = 99.4 \), respectively. This means that the residual equity value for a type-\( G \) and a type-\( B \) firm are equal to \( \mathbb{E}[Z_{GT}] - \mathcal{V}_G(K) = 137.0 > 125 \), and \( \mathbb{E}[Z_{BT}] - \mathcal{V}_B(K) = 118.9 > 75 \), respectively. Note that \( \mathbb{E}[Z_{GT}] - \mathcal{V}_G(K) = 137.0 < (1 - \lambda) \mathbb{E}[Z_{GT}] = 138.6 \), which means that equity dominates debt as a financing instrument. Note also that the dilution costs of equity are \( D_E = 0.462 \times (257.6 - 218.4) = 18.2 \), whereas those of debt are \( D_D = 120.6 - 99.4 = 21.2 \); thus, the type-\( G \) firm is exposed to lower dilution by raising capital with equity rather than debt.

For the parameter values in Table 1, Figure 1 displays the plots of the function \( c(z) \) (top panel, solid line) and of the densities of firm value for both type of firms and their average, \( \{f_G(z), f_B(z), f(z)\} \) (bottom panel). Note that in this numerical example the region in which debt has a disadvantage over equity (i.e. the intermediate region of (16)) is large, namely for values of \( z \) that lie in the [79, 429] interval.

Figures 2 and 3 present comparative static exercises around the base case of Table 1. The top graph in Figure 2 displays indifference lines of \( D_D = D_E \), as a function of the exposure to asymmetric information of the assets in place, \( c_z \), and the growth opportunity, \( c_y \), for three levels of the volatility of the growth opportunity, \( \sigma_y \in \{.6, .7, .8\} \). In the region above the lines, we have that \( D_D > D_E \) and hence equity is less dilutive than debt and RPO obtains.
In the region below the lines, we have that $D_D < D_E$ and hence equity is more dilutive than debt, and the usual PO obtains. Note that the slope of the indifference lines declines when the volatility of the growth opportunity rises. These graphs reveal that equity is more likely to be less dilutive than debt when less volatile assets in place are more exposed to asymmetric information than more volatile assets.

The bottom graph in Figure 2 sets the exposure to asymmetric information back to the values of the base case of Table 1 (i.e. $c_x = 25$ and $c_y = 0$) and charts indifference lines of $D_D = D_E$, as a function of the time horizon, $T$, and the investment cost, $I$, for three levels of the average value of assets in place $\bar{X} \in \{95, 100, 105\}$. For pairs of $(I, T)$ below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt for lower values of assets in place (i.e. smaller firms), higher investment costs, and longer time horizons (i.e. for younger firms).

The top graph of Figure 3 displays the pairs of the average value of assets in place and the average value of the growth option, $(\bar{X}, \bar{Y})$, for which the dilution costs of equity and debt are the same, i.e. $D_E = D_D$, for different level of asymmetric information on asset $c_X \in \{10, 25, 40\}$. For pairs of $(\bar{X}, \bar{Y})$ below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the average value of assets in place is low relative to the average value of growth opportunities, and when the exposure to asymmetric information of assets in place increases.

Finally, the bottom graph of Figure 3 sets $\bar{X} = 100$ and $\bar{Y} = 175$, as in the base case of Table 1, and plots the pairs of volatilities, $(\sigma_x, \sigma_y)$, such that the dilution costs of equity and debt are the same, i.e. $D_E = D_D$, for three levels of the investment cost $I \in \{100, 110, 120\}$. For pairs of volatilities, $(\sigma_x, \sigma_y)$, below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when investment cost is large, when the volatility of assets in place is low, and when the volatility of growth opportunities is large.

Figures 2 and 3 reveal a very consistent pattern. If growth opportunities are characterized by greater volatility and lower exposure to asymmetric information, equity is more likely to be less dilutive than debt for small and younger firms that have large investment needs. In addition, equity is more likely to be less dilutive than debt when growth opportunities represent a greater proportion of firm value, when these growth opportunities are riskier, and when assets in place are more exposed to asymmetric information than growth opportunities. These predictions help to explain the stylized fact that small and young firms with large financing needs tend to often prefer equity financing over debt financing in circumstance where asymmetric information is potentially severe.
5 Optimal financing with existing debt

We have considered so far a firm that is all equity-financed ex-ante. In this section, we study the effect of prior financing on the debt-equity choice. In particular, we assume the firm has already issued straight debt with face value $K_0$ prior to the beginning of the period, $t = 0$, which is due at the end of the period, $T$. In accordance to anti-dilutive “me-first” rules that may be included in the debt covenants, we assume that this pre-existing debt is senior to all new debt that the firm may issue in order to finance the new project. We maintain the assumption that the new investment is sufficiently profitable that all firms want to raise external capital to finance it.\(^\text{19}\)

As in the previous analysis, we restrict the choice of security to equity or junior debt. This implies that the firm can raise the necessary capital either by sale of junior debt with face value $K$, or by sale of a fraction $\lambda$ of total (levered) equity of the firm to outside investors. Following an argument similar to the one in Section 3, the relative dilution of debt versus equity is now given by:

$$D_D - D_E = \int_{K_0}^{\infty} [(1 - \lambda) \max(z - K_0, 0) - \max(z - (K_0 + K), 0)] c(z)dz. \quad (28)$$

Note that the main difference of (28) relative to the corresponding expression in (15) is the fact that all payoffs below $K_0$ are allocated to the pre-existing senior debt. This implies that only the probability mass located in the interval $[K_0, \infty)$ is relevant for the determination of the relative dilution costs of debt and equity and, thus, for the choice of financing of the new project. Recall from (16) that the two regions located at the left and the right tails of the probability distribution favor debt financing, while the intermediate region favors equity financing. This means that the presence of pre-existing debt in a firm’s capital structure, by reducing the importance of the left-tail region, makes equity more likely to be the less dilutive source of financing.

The presence of pre-existing debt affects the financing choice in the context of the real options model presented in Section 4 as follows. Similar to the previous case, $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$ represents the exchange option between the assets in place and the growth opportunity. At the beginning of the period, $t = 0$, the value of levered equity for a firm of type $\theta$ with a face value of debt $K_0$ is given by

$$C_\theta(K_0) \equiv E[\max(Z_{\theta T} - K_0, 0)] \quad (29)$$

\(^{19}\)This assumption allows us to ignore a possible debt overhang problem in the sense of Myers (1977), whereby the presence of pre-existing debt may induce a firm not to undertake a positive-NPV project.
where $C_\theta(K_0)$ represents the value of a call option on the with strike price $K_0$, written on the exchange option $Z_{\theta T}$. Similarly, the value of the junior debt with face value $K$, $J_\theta(K)$, is given by

$$J_\theta(K) = C_\theta(K_0) - C_\theta(K_0 + K), \quad (30)$$

From Stulz (1982) and Rubinstein (1991), we know that the value of these call options is given by

$$C_\theta(\hat{K}) = X_\theta \Delta^*_x \theta + Y_\theta \Delta^*_y \theta - \hat{K}(1 - \Gamma_\theta), \quad (31)$$

where $\hat{K} \in \{K, K_0 + K\}$ represents the investment in the risk-free asset in the corresponding duplicating portfolios, and $\Delta^*_x \theta$, $\Delta^*_y \theta$, and $\Gamma_\theta$ are defined in the previous section.

The analysis of the relative dilution costs of debt versus equity follows an argument similar to the one developed in Section 4. Given an investment amount $I$, the new equity holders require a fraction of the outstanding equity $\lambda$ such that the financing constraint $I = \lambda[pC_G(K_0) + (1 - p)C_B(K_0)]$ holds. Similarly, the new debt holders will ask for a face value $K$ such that the financing constraint $I = pJ_G(K) + (1 - p)J_B(K)$ holds. We have the following Proposition.

**Proposition 7.** The capital raising game exhibits a reverse pecking order, in which equity is preferred to debt, if and only if

$$C_G(K_0 + K) - C_B(K_0 + K) < (1 - \lambda)(C_G(K_0) - C_B(K_0)).$$

The example introduced in Table 2 allows us to illustrate this case. For simplicity, we assume again there is information asymmetry on $X$, but not $Y$, namely let $Y_G = Y_B = 120$, and $X_G = 110$ and $X_B = 90$ and we set the volatilities at $\sigma_x = 0.25$ and $\sigma_y = 0.50$. Furthermore, we assume that $I = 50$, and that the project’s payoffs are realized at $T = 10$. Finally, we assume that type-$G$ and type-$B$ firms are again equally likely, $p = 1/2$. We consider the case where the firm haspre-existing debt outstanding maturing in $T = 10$ with a face value of $K_0 = 50$. Equity financing of the project requires setting $\lambda = 0.38$, with associated dilution costs of $D_E = 5.6$. Junior debt financing requires a promised payment of $K = 91.3$, with has associated dilution of $D_D = 6$. Thus, under these parameter values, equity is less dilutive than debt. It is easy to verify that in the absence of pre-existing debt (i.e. for $K_0 = 0$) the project can be financed by selling a fraction $\lambda = 0.28$ of the firm, or promising bond holders a face value $K = 53.7$. It is straightforward to check that in this case
the dilution of the new debt is $D_D = 1.2$, while the dilution of new equity is $D_E = 4.4$, which means that debt dominates equity. Thus, due to the existence of senior debt, equity becomes a better financing instrument than junior debt. This happens because when $K_0 = 0$ the firm can finance the project by issuing close to risk-free debt, which makes the dilution costs of debt very small. In contrast, the presence of pre-existing debt forces the firm to issue new debt that is riskier, and thus more information sensitive, creating the potential for greater dilution.

6 Optimal security design

We now consider the general optimal security design problem in a setting where probability distributions of firm value satisfy only FOSD. Following Nachman and Noe (1994), the optimal security design problem in (8) can be expressed as:

$$\min_{s \in S} \int_0^{\infty} s'(z)(F_B(z) - F_G(z))dz,$$

subject to

$$\int_0^{\infty} s'(z)(1 - F(z))dz = I.$$  

The Lagrangian to the above problem is

$$\mathcal{L}(s', \gamma) = \int_0^{\infty} s'(F_B(z) - F_G(z) - \gamma(1 - F(z)))dz.$$  

The determination of the optimal security depends critically on the information costs in the right tail of the payoff distribution. This can be seen by noting that the function $H(z)$ in (10) is a transformation of the Lagrangian expression (34). The following is an immediate consequence of the linearity of the security design problem.

**Proposition 8.** (Nachman and Noe, 1994) A solution $s^*$ must satisfy, for some $\gamma \in \mathbb{R}_+$,

$$\left( s^* \right)'(z) = \begin{cases} 1 & \text{if } H(z) < \gamma; \\ [0, 1] & \text{if } H(z) = \gamma; \\ 0 & \text{if } H(z) > \gamma. \end{cases}$$  

Proposition 8 proposes an algorithm to solve the problem: (a) identify the set of $z$ such that $H(z) = \gamma$ for a given $\gamma > 0$, (b) construct the piecewise linear security $s^*$, and (c)
find the value of $\gamma$ such a security $E[s^*(Z)] = I^*$. The resulting security is the optimal one for the problem for a given $I^*$. Because of FOSD, Proposition 8 implies that the optimal security must satisfy $(s^*)'(0) = 1$, i.e. it must yield maximum payoff to outside investors in a neighborhood of $z = 0$. This means that an optimal security will always have a (possibly small) straight debt component.$^{22}$

The following Proposition characterizes the optimal security design problem when the firm value distribution satisfies FOSD but not CSD.

**Proposition 9.** Consider the security design problem (32) - (33) with no pre-existing debt, $K_0 = 0$.

(a) (Nachman and Noe, 1994) If the distribution $F_G$ conditionally stochastically dominates $F_B$, then $H'(z) \geq 0$ for all $z$, and straight debt is the optimal security.

(b) If the problem satisfies the NICRT condition, and $H'(z^*) = 0$ for a unique $z^* \in \mathbb{R}_+$, then convertible bonds are optimal for all investment levels $I$.

(c) If $\lim_{z \to \infty} H(z) = \tilde{h} > 0$ and there exists a unique $z^* \in \mathbb{R}_+$ such that $H'(z^*) = 0$, then there exists $\tilde{I}$ such that for all $I \leq \tilde{I}$ straight debt is optimal, whereas for all $I \geq \tilde{I}$ convertible bonds are optimal.

Part (a) of Proposition 9 assumes CSD. As discussed in Nachman and Noe (1994), CSD requires that the ratio of the measure of the upper tails of the probability distribution for the two types, $(1 - F_G(z))/(1 - F_B(z))$, is monotonically increasing in $z$.$^{23}$ Note also that, for monotonic securities, this ratio can be interpreted as measuring the marginal cost of increasing the payouts to investors by $\$1$ for a type-$G$ relative to a type-$B$. The proposition shows that the optimal security design is debt when the relative incremental cost of increasing a payout for a better type is non-decreasing in the realization of $z$. This happens because, in this case, firms of better types prefer to increase the payout to investors for low realizations of $z$ and to limit the payout to investors for high realizations of $z$, that are relatively more expensive to them. These considerations, together with the requirement that the security is monotonic, lead to the optimality of debt contracts.

The cases considered in parts (b) and (c) of Proposition 9 provide the conditions under which convertible bonds can be the optimal financing instruments. The key driver of the optimal security choice is, as in the debt-equity case discussed in the earlier sections, the

---

$^{22}$Note, however, that as Proposition 10 shows, this property hinges critically on the assumption that the firm has no pre-existing debt (i.e. $K_0 = 0$).

$^{23}$CSD also requires that the hazard rate of the payoff distribution for a type-$G$ is smaller than that for a type-$B$ for all values of $z$. 

25
size of the informational costs in the right-tail of the payoff distribution. In part (b) we establish that if there are no information costs in the right tail, that is, when the NICRT holds, then convertible bonds will always be optimal security. This happens because convertible bonds allow type-$G$ firms to load the payoff to investors in the right tail of the distribution, where information costs are low because of NICRT (in addition to straight debt component, as discussed above). In part (c), neither CSD nor NICRT hold, since we have both a non-monotone function $H$ and the $\bar{h}$-ICRT condition holds for $\bar{h} > 0$. The proposition shows that the size of a project affects the financing choices of a firm. Specifically, convertible debt is optimal for large levels of the investment $I$, while straight debt is optimal for low levels of $I$. This happens because when investment needs are low, the firm can finance the project by issuing only straight debt, that is a security that loads only in the left-tail of the distribution, where the information costs are the lowest. For greater investment needs, under $\bar{h}$-ICRT the firm finds it optimal to increase its payout to investor by issuing a security that also loads on the right-tail of the distribution, that is by issuing convertible debt.

We now turn to the optimal security design problem when the firm has already issued a security to outside investors and we focus on the more empirically relevant case in which the firm has already issued straight debt (as discussed in Section 5). We assume again that pre-existing debt is senior with respect to any of the new securities that the firm may issue in order to finance the project. We also continue to assume that the project is sufficiently profitable, so the firm always seeks external finance to undertake the project (rather than not issuing any security and passing on the new investment opportunity).24

The presence of pre-existing debt changes the structure of information costs in a non-trivial way, because cash flows in the left tail of the distribution cannot be pledged any longer to new investors. As we argued earlier, this makes equity-like securities relatively more attractive.

**Proposition 10.** Consider the optimal security design problem in (32)–(33). Assume that $F_\theta(z)$ satisfies the NICRT condition, and that there exists a unique $z^*$ such that $H'(z^*) = 0$.

(a) If $H'(K_0) > 0$, then there exists $\bar{I}$ such that: (i) warrants are optimal for $I < \bar{I}$, and
   (b) convertible bonds are optimal for $I \geq \bar{I}$.

---

24The optimal security design problem with pre-existing debt can re-interpreted in the context of our previous setup by introducing a new distribution on firm value, $F_\theta(z)$, that is induced by the original distribution $F_\theta(z)$ after defining a new random variable $\tilde{Z} = \max(0, Z - K_0)$, where $K_0$ denotes the face value of the existing senior debt. The new security design problem can be solved as before, with the difference that now only claims on $\tilde{Z}$, rather than on the original random variable $Z$, are allowed, because the firm has already pledged payoffs in the $[0, K_0]$ interval to the pre-existing senior bondholders. Note however that FOSD of $F_\theta(z)$ does not guarantees FOSD of $\tilde{F}_\theta(z)$. 

26
(b) If $H'(K_0) < 0$, then the optimal securities are warrants.

Proposition 10 provides conditions under which warrants arise as optimal financing instruments, in contrast to the case in which only straight debt or convertible bonds are solution to the optimal security design problem that we discussed in Proposition 9. Intuitively, warrants are optimal securities when pre-existing debt has absorbed the information benefits in the left tail (which generate the optimality of debt when $K_0 = 0$). Note that the value of the warrants will depend on their exercise price. Specifically, when $K_0$ is moderate, so that $H'(K_0) > 0$, NICRT implies that the optimal security design is one that always loads in the right-tail, where information costs are the now the lowest (since now the left tail is already committed). In addition, when the financing needs are low, the firm is able to raise the required capital by issuing only warrants; when the financing needs are high, the firm raises the additional capital by issuing also (junior) debt, that is by using convertible debt. When $K_0$ is large, so that $H'(K_0) < 0$, the firm will always find it optimal to issue only warrants (since the firm now faces decreasing information costs).

We conclude this section by illustrating the characterizations of the optimal securities in Propositions 9 and 10 in the real options model of Section 4. We start by highlighting that the distribution of the random variable $Z_{\theta T}$ does not satisfy CSD, even if the individual random variables $X_{\theta T}$ and $Y_{\theta T}$ satisfy CSD. The next Proposition gives conditions under which the real options model satisfies the NICRT condition, as well as the CSD condition of Nachman and Noe (1994).

**Proposition 11.** The model satisfies the NICRT condition if there is no information asymmetry on $y$, $c_y = 0$, and the volatility of the growth opportunity is higher than that of the assets in place, $\sigma_y > \sigma_x$. Furthermore, in the lognormal case CSD condition holds if $Y_G = Y_B = 0$.

The first part of Proposition 11 gives a sharp parametric example in which CSD fails. Since second moments dominate tail behavior under Gaussian assumptions, the proposition shows that conditions that $Y$ has no information costs and $\sigma_y > \sigma_x$ is sufficient to generate a non-monotonic $H(z)$ function. Furthermore, Proposition 11 shows the limits of using the standard lognormal specification in models of asymmetric information, since it shows that in this case the model satisfies the CSD ordering of Nachman and Noe (1994).

Figure 5 plots the $H(z)$ function in the left panels, and the optimal security in the right panels, for three different scenarios, summarized in Table 3. In all cases we assume that $p = 1/2$, $\sigma_x = 0.2$ and $\sigma_y = 0.5$.

The first scenario (Panel A) presents the case where the asymmetric information is concentrated in the high volatility asset, namely we let $X_G = X_B = 120$, $Y_G = 110$, $Y_B = 90$.
and we set the investment to be $I = 100$. In this case straight debt will be optimal, as the $H(z)$ function is monotone over its whole domain (see the top left graph in Figure 5). In particular, a standard bond with a face value of $K = 101.6$ is sufficient to finance the project and minimize information costs.

The second case (Panel B) is closer to the examples from Section 4, in that the asymmetric information is concentrated in the low-volatility asset. Namely, we set $Y_G = Y_B = 175$, $X_G = 120$, $X_B = 100$, and the investment amount to $I = 90$. The optimal security in this case is a convertible debt contract with $K = 88.9$, and conversion trigger at $z_c = 309.4$. As in the case of Proposition 11 the intuition is that, under $h$-ICRT, securities should load in the lower end of the payoffs, due to the usual Myers and Majluf (1984) intuition, but also on upper end of the payoff distribution.

The last case (Panel C of Figure 5) provide an illustration of case (b) in Proposition 10. We use the same parameter values as in previous example, but we now assume that the firm has debt outstanding with $K_0 = 100$. Further, let the investment amount be $I = 15$. In this case, Proposition 10 shows that the optimal security is warrants, with an exercise price of $\kappa = 174.8$.

## 7 Other specifications and robustness

Section 3 and Section 6 of this paper characterize the optimal financing choices under asymmetric information for general distributions that satisfy FOSD but not necessarily CSD. We have linked the possibility of generating a reversal of the pecking order to the non monotonic behavior the function $H(z)$ that describes the information costs of a type-$G$ firm when pooling with a type-$B$ firms. The parametric examples that we use to generate the reverse pecking order have been based on an exchange option specification. In this section, we provide another specification of firm value which also generates the reverse pecking order.

We consider the following modification of the payoff structure. The firm is now endowed by two assets, $X_{\theta T}$ and $Y_{\theta T}$. For example, we can interpret $X_{\theta T}$ and $Y_{\theta T}$ as being the value of the assets of the two divisions of a multi-divisional firm. If the firm makes the capital expenditure $I$ at the beginning of the period, then the end-of period value of the firm is given by the random variable $Z_{\theta T} = X_{\theta T} + Y_{\theta T}$. Again the random variables $X_{\theta T}$ and $Y_{\theta T}$ are characterized by lognormal distribution, as in Section 4. Under the lognormality assumption, the model does not admit closed-form solutions, but it is straightforward to be solved numerically.\(^\text{25}\)

\(^\text{25}\)In the analysis that follows we approximate the relevant integrals by simulations, with sample sizes that guarantee accuracy on the order of four significant digits.
In Table 4 we consider an example that uses parameter values close to those in Table 1. The firm can finance the new project, at a cost of $I = 110$ by either selling a fraction $\lambda = 0.40$ of the equity of the firm, or by issuing straight debt with a face value equal to $K = 213.2$, which carries a credit spread of 451 basis points. As in the base case, the parameter values are such that the NICRT condition is satisfied. It is easy to verify that the dilution costs associated with equity are $D_E = 20.1$, whereas those associated with debt are $D_D = 21.2$.

We generate comparative static results around the base case of Table 4 that correspond to Figure 2 and 3. The simulated results for the additive case, which are omitted here, are available from the authors on request) mimic the results we obtained in the exchange-option case. Equity financing is again more likely to dominate debt financing when the asset with less exposure to asymmetric information has also greater volatility, when the level of investment is greater, when the asset with greater volatility is a greater proportion of firm value and is relatively more volatile.

This specification of our model generates novel predictions on the cross-sectional variation of the capital structures of multidivisional firms. Specifically, we predict that firms are more likely to be equity financed when they consist of relatively heterogenous divisions, and when the relatively more transparent division - that is the one that is less likely to be afflicted by asymmetric information - is also riskier (i.e. it is characterized by greater volatility). In addition, in these situations, firms are more likely to raise new capital by using equity, especially to meet larger investment needs.\footnote{Note also that multidivisional firms may arise as the outcome of a merger. Thus, our model also generate new predictions on why certain mergers are equity financed while others are financed by cash (where firm raises the cash preemptively by borrowing).}

The additive structure inherits many of the properties of the model solved in closed-form in Section 4. The critical properties of the model are driven, again, by the relative probability mass on the right-tail of the payoff distribution. Other specifications that potentially yield similar results are mixtures of binomial distributions or normal distributions (Leland, 2007). All these specifications can generate situations where the h-ICRT or the NICRT property are satisfied and, as a consequence, produce a reversal of the pecking order. Thus, while the reason for considering the exchange option specification is its analytical tractability, its modeling flexibility regarding right-tail behavior can be achieved in other specifications too.

Throughout the discussion we have assumed that the payoff to the firm before the investment is $X_{\theta_T}$, so that the incremental cash flow from investment is the rainbow option $\max(X_{\theta_T}, Y_{\theta_T}) - X_{\theta_T} = \max(Y_{\theta_T} - X_{\theta_T}, 0)$. But the model is more general. The analysis, under the positive NPV assumption, goes through as stated if the payoff of the firm without the investment is $w \max(X_{\theta_T}, Y_{\theta_T})$ for some $w \in (0, 1)$, so the incremental value of the
investment at maturity is \((1 - w) \max(X_{\theta T}, Y_{\theta T})\).\(^{27}\)

We conclude this section by noting another implication of the reverse pecking order. There are scenarios in which firms are willing to raise capital and invest in a positive NPV project if the project is financed by equity, but are not willing to do so if the project is financed by debt. This happens when the dilution costs under debt financing are sufficiently large to make the type-\(G\) to prefer the no-investment payoff, while the dilution costs under equity financing are sufficiently low to make the firm willing to issue equity and invest in the project.

Consider a modification of the example of Table 1, where \(Y_G = Y_B = 130\), \(I = 95\), \(T = 15\) and all other parameters are the same. The value of the firm post-investment is now \(208.2\). Equity financing now involves selling a fraction \(\lambda = 0.456\), whereas debt financing involves a promised payment of \(K = 215\), with a credit spread of 559 basis points. Equity financing is still optimal, as the dilution costs are given by \(D_E = 20.2\) and \(D_D = 21.9\).

The original shareholders of a type-\(G\) firm have a status-quo of \(X_G = 125\). Under the new set of parameter values, the payoff of the old equity holders is \((1 - 0.456) \times 208.2 = 125.2\). With the new parameter values, debt financing yields a total value for existing shareholders of type-\(G\) of \(124.3\), which is less than the value of their existing shareholders under no investment, \(125\). Thus, if firms were restricted to issue debt, type-\(G\) firms will prefer not issue and invest, preventing the pooling equilibrium we discussed in the paper to exist. This also means that the social optimum, which is to invest in the new project, cannot be achieved under debt financing, while this is possible under equity financing.

8 Empirical implications

In this section, we discuss the empirical implications from the real options specification. Given the highly non-linear nature of the exchange option model, we now follow a different route to better understand the forces behind it. Specifically, we first produce a simulated dataset that is generated by our model and we obtain a large panel of observations. We then use the simulated sample to conduct traditional empirical tests with the aim of characterizing the regularities that an econometrician would estimate in our randomly generated economy.

Let \(U_i, i = 1, \ldots, 10\), denote a set of independent uniformly distributed random variables in \([0, 1]\). We set \(\sigma_x = \min(0.2 + 0.8 U_1, 0.2 + 0.8 U_2)\), and \(\sigma_y = \max(0.2 + 0.8 U_1, 0.2 + 0.8 U_2)\), so that \(Y\) maps into the higher volatility asset component, which we previously referred to as the firm’s “growth opportunity.” Note how the volatilities are bounded in the set \([0.2, 0.8]\). We

\(^{27}\)This remark is important in order to formally nest the lognormal specification. In our framework we can take the limit \(Y_{\theta T} \downarrow 0\) (a.s.) but still have a non-trivial asymmetric information problem with a positive NPV if \(w < 1\) and we do not interpret \(Y_{\theta T}\) as the incremental cash flow.
let $\rho = -0.5 + 1.5 U_3$, so that the correlation parameter is uniformly distributed in $[-0.5, 1]$. We set the time to maturity to be $T = 5 + 25U_4$, with support in $[5,30]$. We further let $\mu_x = U_5$ and $\mu_y = U_6$. We then set $\mu_{xG} = \mu_x + kU_7$ and $\mu_{yB} = \mu_x - kU_7$, and similarly $\mu_{yG} = \mu_y + kU_8$ and $\mu_{yB} = \mu_y - kU_8$, where we set arbitrarily $k = 0.3$. We let $X_0 = e^{\mu_x \phi}$ and $Y_0 = e^{\mu_y \phi}$. Note how the information asymmetry is parametrized by a uniformly distributed random variable that spreads the means of the type-$G$ and type-$B$ by at most a log-return of 60%. We set $p = 0.2 + 0.6 U_7$ as the probability of the type-$G$ firm, with support in $[0.2,0.8]$. We set the value of the existing senior debt at $K_0 = (0.2 + 0.6 U_9)A$, where $A$ denotes the value of the (total) assets post-investment. Thus the principal of the old debt will be between 20-80% of the total firm value. Finally, we let $\bar{C} = pC_G(K_0) + (1-p)C_B(K_0)$ denote the value of the equity of the firm (net of the senior debt), and set the investment amount at $I = (0.3 + 0.5U_{10})\bar{C}$ (this guarantees that the problem has a solution).\footnote{Note that the actual generation of the parameter values is rather irrelevant for our purposes, in the sense that we can condition on different subsets of the parameter space in the analysis that follows. In general, the above simulation procedure will generate scenarios where (quasi) risk-less debt is feasible, and thus optimal. But it will also generate parameter values for which the trade-off at the financing choice satisfies conditions that are close to those associated with the $h$-ICRT for low values of $h$.}

We simulate the model one million times, solving it numerically, using the closed-form solutions from Section 5 at each iteration, and save only the results for which the relative dilution of equity is within 20% of that of debt.\footnote{Namely, if we let the relative dilution of equity (over debt) to be defined as $R = D_E/D_D$, we only consider those cases where $R \in (0.8,1.2)$. About 46.1% of the simulated parameters satisfy this constraint. For 45.5% debt’s dilution is less than 20% that of equity, mostly when debt is (close to) riskfree. For 5.9% of the cases studied, equity’s dilution is less than 20% than that of debt. We focus on parameter values for which there is some tension in the debt-equity choice.} We then run sets of standard regressions for models of the form $Y_i = \beta X_i + \epsilon_i$, where $Y_i$ is either (i) the ratio of the dilution costs of equity over the dilution costs of debt, $R_i = D_{Ei}/D_{Di}$, or (ii) a dummy that equals to 1 if the firm finds it optimal to issue debt, i.e. $D_{Ei} > D_{Di}$. In the later case we estimate a logit model, whereas in the former case we shall report ordinary-least-squares (OLS) coefficients. As the set of explanatory variables $X_i$ we shall include: a constant; two metrics of the information asymmetry faced by investors, $c_x = X_G - X_B$ and $c_y = Y_G - Y_B$; the level of the payoffs $\bar{X}$ and $\bar{Y}$; the volatilities of each of the components of the assets, $\sigma_x$ and $\sigma_y$, as well as the correlation $\rho$; the probability of a good type $p$, and the face value of senior debt $K_0$, as well as the investment amount $I$ and the time to maturity $T$. When giving point estimates of a regression, we normalize all independent variables to zero mean and unit variance, so the intercept of the OLS regression can be interpreted as an unconditional mean, and the OLS coefficients as the marginal effect of a one standard deviation change in the independent variables. In the logit results, where the point estimates do not have marginal interpretations,
the relative size of the estimates do give a sense of the relative importance of each of the
explanatory variables.

Panel A of Table 5 gives the estimates of the logit specification, whereas Panel B presents
the results where the relative dilution \( R_i \) is the dependent variable. Each set of pair of columns
contains the estimated coefficients, and the related comparative static. For example, the point
estimate on \( c_x \) of \(-4.8\) (Panel B, second column) means that a one standard deviation increase
in \( c_x \), decreases the relative dilution of equity, i.e. it makes debt relatively more expensive by
4.8%. The first column presents the OLS estimates over all the cases that comprise the main
sample. Columns 3–8 present the results for different sub-samples, depending on whether the
observations are in the top or bottom quintiles of the variables that measure existing debt,
\( K_0 \), the information asymmetry on the assets in place, \( c_x \), and the information asymmetry on
the assets in place, \( c_y \). Given the size of the sample, all point estimates are highly significant,
so \( t \)-statistics are omitted.

The regression suggests that the relative dilution costs of equity increase as \( c_y, \sigma_x, \) and \( \mu_x \)
increase, but decrease if \( c_x, \mu_y, \sigma_y, \rho, K_0, I, p \) and \( T \) increase. It is remarkable that across all
seven subsets of the parameter space considered, and both the logit and OLS specifications,
the comparative statics with respect to ten primitives of the model, out of eleven, do not
change signs. Only for the parameter \( I \) does the coefficient flip signs in the OLS specification,
albeit with economically small magnitudes, which hints at the non-linearities of the model.

These comparative statics reinforce the intuition behind the reverse pecking order from
the previous sections. The information asymmetry needs to be concentrated on the low-
volatility asset: the higher the parameter \( c_x \) is, the more likely equity will be issued. The
information asymmetry on the high-volatility asset, \( c_y \), which governs the behavior of the
information costs on the right-tail, has the opposite effect. Rather intuitively, if the right-tail
cash flows become more expensive in terms of information costs, then the firm is more likely
to issue straight debt. The volatility parameters play a dual role — amplifying/reducing the
information asymmetry costs. The higher the existing assets volatility (\( \sigma_x \)) is, the more likely
straight debt is optimal, whereas higher volatility (\( \sigma_y \)) for the new assets favors equity.

Table 5 also shows how the size of the assets (existing and new) favor debt over equity.
The mechanism is simple: the higher the asset value, all else equal, the closer the debt contract
is to be risk-free. Furthermore, the higher the probability of the good type \( p \), the more likely
equity becomes optimal. Table 5 further confirms that the presence of existing debt is an
important determinant of the debt/equity choice. In particular, equity is more likely to be
optimal if the firm already has some debt in its capital structure. Finally, the larger the
investment \( I \), the less likely it is that the firm will issue debt.

These regression results for simulated datasets suggest that the critical drivers of the re-
verse pecking order are low information asymmetry on the right tail of the payoff distribution, large investment needs, and existing debt in the capital structure. Under such conditions, a debt security will be more sensitive to private information than an equity security. As such, equity financing can be less dilutive than debt financing under asymmetric information.

9 Conclusion

In this paper, we revisit the pecking order of Myers and Majluf (1984) and Myers (1984) in the context of a simple real options problem. We model firm value as an exchange option between two risky assets, and show that even if the distribution of each individual assets satisfies the conditional stochastic dominance condition, the distribution of the exchange option may not. This means that, contrary to common intuition, equity financing can dominate debt financing under asymmetric information, even in cases where individual assets would be financed by debt when taken in isolation. We also show that the presence of existing debt makes equity less dilutive that debt. Finally, our model also predicts the optimality of convertible debt and warrants. Taken together, these results suggest that the relationship between asymmetric information and choice of financing is more subtle than previously believed.
Appendix


Proof of Proposition 2. From the definition of $H(z)$ in (10), we have:

\[
\frac{dH(z)}{dz} = \frac{(f_B(z) - f_G(z))(1 - F(z)) + (pf_G(z) + (1 - p)f_B(z))(F_B(z) - F_G(z))}{(1 - F(z))^2}
\]

\[
= \frac{f_B(z) - f_G(z) + F_B(z)f_G(z) - F_G(z)f_B(z)}{(1 - F(z))^2}
\]

\[
= \frac{f_B(z)(1 - F_G(z)) - f_G(z)(1 - F_B(z))}{(1 - F(z))^2}.
\]

Thus $H'(z) > 0$ if and only if $f_B(z)(1 - F_G(z)) > f_G(z)(1 - F_B(z))$, which reduces to the CSD condition.

Proof of Proposition 3. From the budget constraint for equity and debt securities, one has that

\[
\lambda = \frac{pE[\min(Z_G,K)] + (1 - p)E[\min(Z_B,K)]}{pE[Z_G] + (1 - p)E[Z_B]}
\]

(36)

Using (36) in (12) and comparing this to (13) one easily arrives at (14).

Proof of Proposition 4. The following result from Shaked and Shanthikumar (2007) is useful.

Lemma 1 (Theorem 1.B.12 from Shaked and Shanthikumar (2007)). Given two distribution functions $F_G$ and $F_B$, the following two statements are equivalent: (a) $F_G$ conditionally stochastic dominates $F_B$; (b) $E[\alpha(X_B)]E[\beta(X_G)] \leq E[\alpha(X_G)]E[\beta(X_B)]$, for all functions $\alpha$ and $\beta$ such that $\beta$ is non-negative and $\alpha/\beta$ and $\beta$ are non-decreasing.

Let $\alpha(z) = z$ and $\beta(z) = \min(z,K)$ for some $K \geq 0$. Clearly $\beta$ is non-decreasing and non-negative for $x \geq 0$. Furthermore, $\alpha(z)/\beta(z) = z/\min(z,K)$ is non-decreasing. Thus, if $F_G$ conditionally stochastically dominates $F_B$ it must be that

\[
E[Z_B]E[\min(Z_G,K)] \leq E[Z_G]E[\min(Z_B,K)]
\]

which clearly rules out (14).
Proof of Proposition 5. It is clear than in order for the UPO to hold, it is necessary that $D_D > D_E$. This condition, if $\hat{z} > \bar{z}$ (i.e. the UNC does not hold), can be written as

$$\int_0^{\hat{z}} (\min(K,z) - \lambda z) c(z) \, dz + \int_{\hat{z}}^{\bar{z}} (K - \lambda z) c(z) \, dz + \int_{\bar{z}}^{\infty} (K - \lambda z) c(z) \, dz > 0 \quad (37)$$

We note that since $g$ is the difference of two densities, it must be the case that

$$\int_0^{\infty} c(z) \, dz = 0; \quad \Rightarrow -\int_0^{\hat{z}} c(z) \, dz = \int_{\hat{z}}^{\infty} c(z) \, dz$$

Further, we have

$$\int_{\hat{z}}^{\infty} (\lambda z - K) c(z) \, dz > \int_{\hat{z}}^{\infty} (\lambda \hat{z} - K) c(z) \, dz = (\lambda \hat{z} - K) \int_{\hat{z}}^{\infty} c(z) \, dz = (K - \lambda \hat{z}) \int_0^{\hat{z}} c(z) \, dz > (K - \lambda \hat{z}) \int_{\hat{z}}^{\bar{z}} c(z) \, dz$$

Therefore, the sum of the last two terms in (37) are negative, and since the first one is negative as well it is clear that $D_D - D_E < 0$, i.e. UPO cannot hold if UNC is not true.

The statement in the Proposition is immediate from (15), the definitions of $\hat{z}$ and $\bar{z}$, and the discussion in the text.

Proof of Proposition 6. We first note that letting $\epsilon \downarrow 0$ the statements in the Proposition boil down to the delta condition given in (27). Equation (27) can be expressed more explicitly as

$$\bar{X} N \left( \frac{\log(\bar{X}/\bar{Y})}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T} \right) + \bar{Y} N \left( \frac{\log(\bar{Y}/\bar{X})}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T} \right) < N \left( \frac{\log(K/\bar{X})}{\sigma \sqrt{T}} - \frac{1}{2} \sigma \sqrt{T} \right) \quad (38)$$

We note that the right-hand side is independent of $\bar{Y}$. The left-hand side of this condition tends to zero for $I$ sufficiently large, so (27) holds in this case. For $\bar{Y}$ sufficiently small, the
financing constraint for debt reduces to

\[ K(1 - N(b_x + \sigma_x \sqrt{T})) + XN(b_x) = I \]

so that

\[ \frac{I}{X} = N(b_x) + \frac{K}{X} (1 - N(b_x + \sigma_x \sqrt{T})). \]  \hspace{1cm} (39)

As \( \bar{Y} \) goes to zero, (27) reduces to \( I/\bar{X} < N(b_x) \), which is impossible from the financing constraint (39). This proves (i).

In order to prove (ii), we note that the limit of the left-hand side of (38) as \( \sigma^2_x \downarrow 0 \) is finite, and strictly greater than zero. On the other hand, the argument of \( N(\cdot) \) in the right-hand side of the condition tends to either positive (or negative) infinity, depending on whether \( X < K \) (or \( X > K \)). In the former case (38) always holds, whereas if \( X > K \) it can never hold. This completes the proof.

**Proof of Proposition 7.** Immediate from the discussion in the text.

**Proof of Proposition 8.** See Theorem 8 in Nachman and Noe (1994).

**Proof of Proposition 9.** Since \( H \) is increasing in (a), there is a single crossing point \( z \) such that \( H(z) = \gamma \), for any \( \gamma \in \mathbb{R}_+ \). The claim in (a) follows immediately from Proposition 8. Assuming the NICRT, and that \( H'(z^*) = 0 \) at most once, it is immediate that there are two crossing points for \( H(z^*) = \gamma \), for any \( \gamma \in \mathbb{R}_+ \). The claim is immediate from Proposition 8. Case (c) is analogous, but noting that for \( \gamma \leq \bar{\gamma} \) there is a single point satisfying \( H(z^*) = \gamma \), but two such points for \( \gamma \) sufficiently large.

**Proof of Proposition 10.** The proof is analogous to that of Proposition 8. The first-order conditions require \( s'(z) \) to be either one (or zero) at points for which \( H(z) < \gamma \) (or \( H(z) > \gamma \)). Under the conditions in (b), and the initial assumptions, there is only one crossing, and all mass of the security is concentrated in the right tail. This occurs for low values of \( \gamma \), or equivalently of the investment \( I \). The claim in (a) mirrors case (b) from Proposition 9.

**Proof of Proposition 11.** Using l’Hopital’s rule, one has

\[ \lim_{z \uparrow \infty} H(z) = \lim_{z \uparrow \infty} \frac{F_B(z) - F_G(z)}{1 - F(z)}. \]  \hspace{1cm} (40)

\[ = \lim_{z \uparrow \infty} \frac{f_G(z) - f_B(z)}{pf_G(z) + (1 - p)f_B(z)}. \]  \hspace{1cm} (41)
From basic principles it is clear that:

\[ P(Z_{\theta T} = z) = f_\theta(z) = f_{x\theta}(z) + f_{y\theta}(z) \]

with

\[
f_{x\theta}(z) = \frac{1}{z \sigma_x \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{x\theta}}{\sigma_x} \right)^2} N \left( \frac{\log(z) - \mu_{y\theta}}{\sigma_y \sqrt{1 - \rho^2}} - \frac{\rho (\log(z) - \mu_{x\theta})}{\sigma_x \sqrt{1 - \rho^2}} \right)
\]

\[
f_{y\theta}(z) = \frac{1}{z \sigma_y \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{y\theta}}{\sigma_y} \right)^2} N \left( \frac{\log(z) - \mu_{x\theta}}{\sigma_x \sqrt{1 - \rho^2}} - \frac{\rho (\log(z) - \mu_{y\theta})}{\sigma_y \sqrt{1 - \rho^2}} \right)
\]

where \( \mu_{x\theta} = \log(X_{\theta}) \) and \( \mu_{y\theta} = \log(Y_{\theta}) \).

The limit in (41) is easy to compute by factoring out leading terms. We note that when \( \sigma_y > \sigma_x \) the right-tail behavior is determined by the piece of the densities \( f_\theta(z) \) that corresponds to the density of \( Y \). When \( c_y = 0 \), the limit of these densities is zero.

Next consider the case where \( Y_G = Y_B = 0 \). The good type distribution is then given by a lognormal distribution with log-mean \( \mu_{xG} \) and variance \( \sigma^2_x \), whereas the bad type follows a lognormal law with log-mean \( \mu_{xB} \) and variance \( \sigma^2_x \). We argue next that in this case the distribution of the good type dominates the distribution of the bad type in the likelihood ratio sense, namely \( f_G(z)/f_B(z) \) is monotonically non-decreasing for all \( z \in \mathbb{R}_+ \). From basic principles we have:

\[
\frac{f_G(z)}{f_B(z)} = \frac{1}{z \sigma_x \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{xG}}{\sigma_x} \right)^2} \frac{1}{z \sigma_x \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{xB}}{\sigma_x} \right)^2} = e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{xG}}{\sigma_x} \right)^2 + \frac{1}{2} \left( \frac{\log(z) - \mu_{xB}}{\sigma_x} \right)^2}
\]

\[
= e^{-\frac{1}{2} \left( \frac{\mu^2_{xG} - \mu^2_{xB}}{\sigma^2_x} + \log(z) \left( \frac{\mu_{xG} - \mu_{xB}}{\sigma_x} \right) \right)}
\]

\[
= e^{-\frac{1}{2} \left( \frac{\mu^2_{xG} - \mu^2_{xB}}{\sigma^2_x} \right) z \left( \frac{\mu_{xG} - \mu_{xB}}{\sigma_x} \right)}
\]

which is monotonically increasing in \( z \) when \( \mu_{xG} > \mu_{xB} \), as we set to prove. Since the likelihood ratio order implies conditional stochastic dominance (Shaked and Shanthikumar, 2007), we conclude that the lognormal specification yields debt financing as the optimal security. This completes the proof. \( \blacksquare \)
References


Table 1: Optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 4. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $\mathbb{E}[X_{\theta T}] = X_\theta$, $\mathbb{E}[Y_{\theta T}] = Y_\theta$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$, $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$. Figure 1 plots the equilibrium debt and equity securities, as well as the densities of the good and bad types.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_G$</td>
<td>125</td>
</tr>
<tr>
<td>$X_B$</td>
<td>75</td>
</tr>
<tr>
<td>$Y_G$</td>
<td>175</td>
</tr>
<tr>
<td>$Y_B$</td>
<td>175</td>
</tr>
<tr>
<td>$T$</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.60</td>
</tr>
<tr>
<td>$p$</td>
<td>0.50</td>
</tr>
<tr>
<td>$I$</td>
<td>110</td>
</tr>
<tr>
<td>$p\mathbb{E}[Z_{GT}] + (1 - p)\mathbb{E}[Z_{BT}]$</td>
<td>237.9</td>
</tr>
<tr>
<td>$\mathbb{E}[Z_{GT}]$</td>
<td>257.6</td>
</tr>
<tr>
<td>$\mathbb{E}[Z_{BT}]$</td>
<td>218.3</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.462</td>
</tr>
<tr>
<td>$K$</td>
<td>198.3</td>
</tr>
<tr>
<td>$r_D = (K/D)^{1/T} - 1$</td>
<td>6.06%</td>
</tr>
<tr>
<td>$\mathbb{E}[\min(Z_{GT}, K)]$</td>
<td>120.6</td>
</tr>
<tr>
<td>$\mathbb{E}[\min(Z_{BT}, K)]$</td>
<td>99.4</td>
</tr>
<tr>
<td>$\mathcal{D}<em>E = \lambda(\mathbb{E}[Z</em>{GT}] - \mathbb{E}[Z_{BT}])$</td>
<td>18.2</td>
</tr>
<tr>
<td>$\mathcal{D}<em>D = \mathbb{E}[\min(Z</em>{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$</td>
<td>21.1</td>
</tr>
</tbody>
</table>
Table 2: Optimal debt-equity choice with existing debt

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $E[X_{\theta T}] = X_\theta$, $E[Y_{\theta T}] = Y_\theta$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$, $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$. The firm already has debt outstanding with principal payment of $K_0$.

The dilution costs of equity are defined as

$$D_E = \lambda(E[\max(Z_{GT} - K_0, 0)] - E[\max(Z_{BT} - K_0, 0)])$$

where $\lambda$ satisfies $I = \lambda(pE[\max(Z_{GT} - K_0, 0)] + (1 - p)E[\max(Z_{BT} - K_0, 0)])$. The dilution costs of equity are defined as

$$D_D = E[\max(\min(K, Z_{GT} - K_0), 0)] - E[\max(\min(K, Z_{BT} - K_0), 0)]$$

where $K$ satisfies $I = pE[\max(\min(K, Z_{GT} - K_0), 0)] + (1 - p)E[\max(\min(K, Z_{BT} - K_0), 0)]$. Figure 4 plots the equilibrium debt and equity securities, as well as the densities of the good and bad types.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primitives</td>
<td></td>
</tr>
<tr>
<td>Value of assets in place for the good type</td>
<td>$X_G$</td>
</tr>
<tr>
<td>Value of assets in place for the bad type</td>
<td>$X_B$</td>
</tr>
<tr>
<td>Value of new assets for the good type</td>
<td>$Y_G$</td>
</tr>
<tr>
<td>Value of new assets for the bad type</td>
<td>$Y_B$</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>$T$</td>
</tr>
<tr>
<td>Volatility of assets in place</td>
<td>$\sigma_x$</td>
</tr>
<tr>
<td>Volatility of new assets</td>
<td>$\sigma_y$</td>
</tr>
<tr>
<td>Probability of the good type</td>
<td>$p$</td>
</tr>
<tr>
<td>Correlation between assets</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Investment amount</td>
<td>$I$</td>
</tr>
<tr>
<td>Existing debt</td>
<td>$K_0$</td>
</tr>
<tr>
<td>Equilibrium outcomes</td>
<td></td>
</tr>
<tr>
<td>Value of firm post-investment</td>
<td>$pE[Z_{GT}] + (1 - p)E[Z_{BT}]$</td>
</tr>
<tr>
<td>Value of good-type firm</td>
<td>$E[Z_{GT}]$</td>
</tr>
<tr>
<td>Value of bad-type firm</td>
<td>$E[Z_{BT}]$</td>
</tr>
<tr>
<td>Equity fraction issued</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Face value of debt</td>
<td>$K$</td>
</tr>
<tr>
<td>Credit spread</td>
<td>$r_D = (K/D)^{1/T} - 1$</td>
</tr>
<tr>
<td>Value of good-type new equity</td>
<td>$\lambda E[\max(Z_{GT} - K_0, 0)]$</td>
</tr>
<tr>
<td>Value of bad-type new equity</td>
<td>$\lambda E[\max(Z_{BT} - K_0, 0)]$</td>
</tr>
<tr>
<td>Value of good-type new debt</td>
<td>$E[\max(\min(K, Z_{GT} - K_0), 0)]$</td>
</tr>
<tr>
<td>Value of bad-type new debt</td>
<td>$E[\max(\min(K, Z_{BT} - K_0), 0)]$</td>
</tr>
<tr>
<td>Dilution costs of equity</td>
<td>$D_E$</td>
</tr>
<tr>
<td>Dilution costs of debt</td>
<td>$D_D$</td>
</tr>
</tbody>
</table>
Table 3: Optimal security design problem

The table presents the parameter values and equilibrium outcomes of the security design problem discussed in Section 6. The payoff of the firm for type \( \theta \) is given by \( Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T}) \), where both \( X_{\theta T} \) and \( Y_{\theta T} \) are lognormal, with \( \mathbb{E}[X_{\theta T}] = X_\theta \), \( \mathbb{E}[Y_{\theta T}] = Y_\theta \). We further denote \( \text{var}(\log(X_{\theta T})) = \sigma^2_x T \), \( \text{var}(\log(Y_{\theta T})) = \sigma^2_y T \), and \( \text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T \). The labels “Straight debt,” “Convertibles,” and “Warrants” refer to the functions \( s(z) = \min(K, z) \), \( s(z) = \min(K, z) + \max(z - \kappa, 0) \), and \( s(z) = \max(z - \kappa, 0) \) respectively.

<table>
<thead>
<tr>
<th>Primitives</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of assets in place type G</td>
<td>( X_G )</td>
<td>150</td>
<td>120</td>
</tr>
<tr>
<td>Value of assets in place type B</td>
<td>( X_B )</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>Value of new assets type G</td>
<td>( Y_G )</td>
<td>120</td>
<td>175</td>
</tr>
<tr>
<td>Value of new assets type B</td>
<td>( Y_B )</td>
<td>100</td>
<td>175</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>( T )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Volatility of assets in place</td>
<td>( \sigma_x )</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>Volatility of new assets</td>
<td>( \sigma_y )</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>Probability of the good type</td>
<td>( p )</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Correlation between assets</td>
<td>( \rho )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Investment amount</td>
<td>( I )</td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td>Existing debt face value</td>
<td>( K_0 )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| Equilibrium outcomes                            | \( p\mathbb{E}[Z_{GT}] + (1 - p)\mathbb{E}[Z_{BT}] \) | 218.9 | 186.3 | 150.5 |
| Optimal security                                | \( s(z) \) | Straight debt | Convertibles | Warrants |
| Face value                                      | \( K \) | 101.9 | 88.9   | –      |
| Conversion trigger/exercise price               | \( \kappa \) | –     | 309.4  | 167.9  |
Table 4: Robustness, additive cash-flows and the optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 7. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = X_{\theta T} + Y_{\theta T}$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $E[X_{\theta T}] = X_\theta$, $E[Y_{\theta T}] = Y_\theta$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$, $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primitives</td>
<td></td>
</tr>
<tr>
<td>Value of assets in place for the good type</td>
<td>$X_G$</td>
</tr>
<tr>
<td>Value of assets in place for the bad type</td>
<td>$X_B$</td>
</tr>
<tr>
<td>Value of new assets for the good type</td>
<td>$Y_G$</td>
</tr>
<tr>
<td>Value of new assets for the bad type</td>
<td>$Y_B$</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>$T$</td>
</tr>
<tr>
<td>Volatility of assets in place</td>
<td>$\sigma_x$</td>
</tr>
<tr>
<td>Volatility of new assets</td>
<td>$\sigma_y$</td>
</tr>
<tr>
<td>Probability of the good type</td>
<td>$p$</td>
</tr>
<tr>
<td>Correlation between assets</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Investment amount</td>
<td>$I$</td>
</tr>
<tr>
<td>Equilibrium outcomes</td>
<td></td>
</tr>
<tr>
<td>Value of firm post-investment</td>
<td>$E[Z_T]$</td>
</tr>
<tr>
<td>Equity fraction issued</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Face value of debt</td>
<td>$K$</td>
</tr>
<tr>
<td>Credit spread</td>
<td>$r_D = (K/D)^{1/T} - 1$</td>
</tr>
<tr>
<td>Dilution costs of equity</td>
<td>$D_E = \lambda(E[Z_{GT}] - E[Z_{BT}])$</td>
</tr>
<tr>
<td>Dilution costs of debt</td>
<td>$D_D = E[\min(Z_{GT}, K)] - E[\min(Z_{BT}, K)]$</td>
</tr>
</tbody>
</table>
Table 5: Comparative statics via regression

The table presents estimates of: (a) a logit regression model where the dependent variable is a dummy that equals to one if the firm optimal chooses debt, zero if the firm prefers equity, as a function of a set of explanatory variables from the model (Panel A); (b) a classical regression model of the form \( R_i = \beta \top X_i + \epsilon_i \) in Panel B, where \( R_i \) denotes the relative dilution of debt versus equity, \( R_i = D_{E_i}/D_{D_i} \), and \( X_i \) denotes a the set of explanatory variables (Panel B). The set of explanatory variables include: measures of asymmetric information on the assets in place and the new assets (\( c_x \) and \( c_y \)), the two parameters on volatility (\( \sigma_x \) and \( \sigma_y \)), the level of the cash flows (\( \mu_x = \log(X) \) and \( \mu_y = \log(Y) \)), the probability of the good type (\( p \)), the amount of existing debt (\( K_0 \)), as well as the investment amount (\( I \)). Details on the construction of the simulated dataset are given in Section 8.

### A. Logit regressions (success if straight debt issued)

<table>
<thead>
<tr>
<th></th>
<th>Base case</th>
<th>Existing debt</th>
<th>Info. asy. X</th>
<th>Info. asy. Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low ( K_0 )</td>
<td>High ( K_0 )</td>
<td>Low ( c_x )</td>
<td>High ( c_x )</td>
</tr>
<tr>
<td>Asy. info on existing assets ( c_x )</td>
<td>-1.8</td>
<td>-2.0</td>
<td>-1.8</td>
<td>-6.3</td>
</tr>
<tr>
<td>Asy. info on new assets ( c_y )</td>
<td>3.2</td>
<td>3.7</td>
<td>2.9</td>
<td>8.7</td>
</tr>
<tr>
<td>Size existing assets ( \mu_x )</td>
<td>1.5</td>
<td>2.1</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Size new assets ( \mu_y )</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-1.2</td>
<td>-1.1</td>
</tr>
<tr>
<td>Volatility existing assets ( \sigma_x )</td>
<td>2.5</td>
<td>2.5</td>
<td>2.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Volatility new assets ( \sigma_y )</td>
<td>-1.9</td>
<td>-2.0</td>
<td>-2.0</td>
<td>-2.0</td>
</tr>
<tr>
<td>Correlation ( \rho )</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-0.0</td>
<td>-0.1</td>
</tr>
<tr>
<td>Prob. high type ( p )</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.1</td>
<td>-0.0</td>
</tr>
<tr>
<td>Debt’s principal ( K_0 )</td>
<td>-0.3</td>
<td>-0.7</td>
<td>-0.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>Investment ( I )</td>
<td>-0.2</td>
<td>-0.5</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Time to maturity ( T )</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

### B. Relative dilution regressions (dilution equity/dilution debt)

<table>
<thead>
<tr>
<th></th>
<th>Base case</th>
<th>Existing debt</th>
<th>Info. asy. X</th>
<th>Info. asy. Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low ( K_0 )</td>
<td>High ( K_0 )</td>
<td>Low ( c_x )</td>
<td>High ( c_x )</td>
</tr>
<tr>
<td>Asy. info on existing assets ( c_x )</td>
<td>-4.8</td>
<td>-5.3</td>
<td>-4.4</td>
<td>-7.3</td>
</tr>
<tr>
<td>Asy. info on new assets ( c_y )</td>
<td>7.4</td>
<td>8.9</td>
<td>6.4</td>
<td>7.5</td>
</tr>
<tr>
<td>Size existing assets ( \mu_x )</td>
<td>4.3</td>
<td>5.9</td>
<td>3.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Size new assets ( \mu_y )</td>
<td>-2.4</td>
<td>-2.6</td>
<td>-2.4</td>
<td>-1.5</td>
</tr>
<tr>
<td>Volatility existing assets ( \sigma_x )</td>
<td>6.7</td>
<td>6.7</td>
<td>6.9</td>
<td>3.4</td>
</tr>
<tr>
<td>Volatility new assets ( \sigma_y )</td>
<td>-5.0</td>
<td>-5.6</td>
<td>-4.7</td>
<td>-2.7</td>
</tr>
<tr>
<td>Correlation ( \rho )</td>
<td>-0.5</td>
<td>-1.1</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>Prob. high type ( p )</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Debt’s principal ( K_0 )</td>
<td>-1.0</td>
<td>-2.3</td>
<td>-0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>Investment ( I )</td>
<td>-1.8</td>
<td>-2.6</td>
<td>-1.6</td>
<td>-0.9</td>
</tr>
<tr>
<td>Time to maturity ( T )</td>
<td>-0.9</td>
<td>-1.4</td>
<td>-0.5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

45
Figure 1: The top graph plots on the $x$-axis the payoffs from the firm at maturity, and in the $y$-axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ ($y$-axis labels on the left), and as dotted lines the payoffs from debt and equity ($y$-axis labels on the right). The left-most vertical dashed line is the point $\hat{z}$ for which $f_G(\hat{z}) = f_B(\hat{z})$, so points to the right of that line have positive information costs. The right-most vertical dashed line is the point $\bar{z}$ for which $K = \lambda \bar{z}$, so for payoffs to the right of that line equityholders receive more than debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The parameter values correspond to the case summarized in Table 1.
Figure 2: The top graph plots the set of points \((c_x, c_y)\) for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\). We consider the following parameter values: \(X = 100, \ Y = 175, \ \sigma_x = 0.3, \ \sigma_y = 0.6, \ I = 110, \ T = 10, \ \rho = 0\) and \(p = 0.5\). Recall we set \(X_G = X + c_x\) and \(X_B = X - c_x\), and similarly for \(Y_G\) and \(Y_B\). The dashed line corresponds to the base case parameters from Table 1, namely sets \(\sigma_y = 0.6\), whereas the other two lines correspond to \(\sigma_y = 0.7\) and \(\sigma_y = 0.8\). For pairs of \((c_x, c_y)\) below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph fixes \(c_x = 25\) and \(c_y = 0\), as in Table 1, and plots the set of points \((I, T)\) for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\). The solid line corresponds to the base case of Table 1, where \(X = 100\), whereas the dashed and dotted lines consider the cases \(X = 105\) and \(X = 95\). For pairs of \((I, T)\) below the lines debt is optimal, whereas equity is optimal above the lines.
Figure 3: The top graph plots the set of points \((\bar{X}, \bar{Y})\) for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\). We consider the following parameter values: \(c_x = 25\), \(c_y = 0\), \(\sigma_x = 0.3\), \(\sigma_y = 0.6\), \(I = 110\), \(T = 10\), \(\rho = 0\) and \(p = 0.5\). Recall we set \(X_G = \bar{X} + c_x\) and \(X_B = \bar{X} - c_x\), and similarly for \(Y_G\) and \(Y_B\). The solid line corresponds to the base case parameters from Table 1, namely sets \(c_x = 25\), whereas the other two lines correspond to \(c_x = 10\) and \(c_x = 40\). For pairs of \((\bar{X}, \bar{Y})\) below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph fixes \(\bar{X} = 100\) and \(\bar{Y} = 175\), as in Table 1, and plots the set of points \((\sigma_x, \sigma_y)\) for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\). The solid line corresponds to the base case of 1, where \(I = 110\), whereas the dashed and dotted lines consider the cases \(I = 100\) and \(I = 120\). For pairs of \((\sigma_x, \sigma_y)\) below the lines debt is optimal, whereas equity is optimal above the lines.
Figure 4: The top graph plots on the $x$-axis the payoffs from the firm at maturity, and in the $y$-axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ ($y$-axis labels on the left), and as dotted lines the payoffs from debt and equity ($y$-axis labels on the right). The left-most vertical dashed line is the point $\hat{z}$ for which $f_G(\hat{z}) = f_B(\hat{z})$, so points to the right of that line have positive information costs. The right-most vertical dashed line is the point $\bar{z}$ for which $K = \lambda(\bar{z} - K_0)$, so for payoffs to the right of that line equityholders receive more than the new debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The parameter values correspond to the case summarized in Table 2.
Figure 5: The left panels plot the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$, whereas the right panels plot the optimal securities. The parameter values correspond to the cases listed in Table 3.