

Asymmetric information and the pecking (dis)order*

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Abstract

Firms endowed with a portfolio of heterogeneous assets seek external financing under asymmetric information. Deviations from the pecking order theory can occur when the assets with relatively lower volatility are more affected by information asymmetries. Thus, the preference for debt over equity is not driven by the overall level of asymmetric information affecting a firm, but by the composition of the firm's assets and their relative exposure to private information. In a real options specification of the model, we show that equity is more likely to dominate debt for younger firms with larger investment needs and riskier, more valuable growth opportunities. Our model can explain why high-growth firms may initially prefer equity, and then switch to debt as they mature.

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1 Introduction

Raising capital under asymmetric information exposes existing shareholders to potential value dilution. When insiders have better information than investors on the value of their firm’s assets, firms of better-than-average quality may find that the market price of their securities are below the fundamental value perceived by the insiders, exposing existing shareholders to dilution. In a classic paper, Myers and Majluf (1984) suggest that, under these circumstances, higher-quality firms can reduce mispricing and thus dilution by issuing debt rather than equity, an intuition known as the pecking order theory. The rationale behind this theory, as argued in Myers (1984), is that debt, by virtue of being senior to equity, is less sensitive to private information, thus limiting dilution.

Important deviations from the pecking order theory have emerged in recent empirical studies. For example, Frank and Goyal (2003) and Fama and French (2005) document that small, high-growth firms, a class of firms which is presumably more exposed to asymmetric information, typically rely heavily on financing through outside equity, rather than debt. Leary and Roberts (2010) conclude that “the pecking order is never able to accurately classify more than half of the observed financing decisions.”¹ This evidence has led researchers to conclude that asymmetric information may not be a first-order determinant of corporate capital structures.²

In this paper, we reconsider the question that is at the heart of the pecking order theory of Myers and Majluf (1984): what is the relative mispricing of debt and equity under asymmetric information? We argue that Myers and Majluf’s pecking order can be violated in economically relevant scenarios. We view a firm as a portfolio of assets, where firm insiders have varying degrees of private information on each asset. We show that the dilution of debt versus equity depends critically on the composition of the firm’s assets and their relative exposure to asymmetric information.

¹Leary and Roberts (2010) also note that most of the empirical evidence is inconclusive, and write: “Shyam-Sunder and Myers (1999) conclude that the pecking order is a good descriptor of broad financing patterns; Frank and Goyal (2003) conclude the opposite. Lemmon and Zender (2010) conclude that a ‘modified’ pecking order—which takes into account financial distress costs—is a good descriptor of financing behavior; Fama and French (2005) conclude the opposite. Frank and Goyal (2003) conclude that the pecking order better describes the behavior of large firms, as opposed to small firms; Fama and French (2005) conclude the opposite. Finally, Bharath, Pasquariello, and Wu (2010) argue that firms facing low information asymmetry account for the bulk of the pecking order’s failings; Jung, Kim, and Stulz (1996) conclude the opposite.”

²For example, Fama and French (2005) suggest that violations of the pecking order theory imply that “asymmetric information problems are not the sole (or perhaps even an important) determinant of capital structures.”

In a real options specification of our model, we consider a firm endowed with both assets in place and a growth option, where the growth option is riskier than the assets in place.³ The firm must finance the new investment opportunity by raising funds in capital markets characterized by asymmetric information.⁴ We show that when insiders are relatively better informed on the assets in place of their firm, rather than on the growth opportunities, equity financing can dominate (i.e., be less dilutive than) debt financing. In addition, we show that equity is more likely to dominate debt for young firms with greater investment needs and that have access to riskier and more valuable growth opportunities. Thus, our model can explain why high-growth firms may initially prefer equity over debt and then switch to debt financing as they mature. This means that the pecking order theory can be expected to hold primarily for more mature firms, while violations of the pecking order preference can be observed primarily for younger firms endowed with significant growth opportunities.

Intuitively, our results depend on the different sensitivities of debt and equity to the value of a firm's assets. Debt gives investors no exposure to a firm's upside potential and, because of seniority, gives maximum exposure to the downside risk, when bondholders collect the entirety of the firm's assets. Thus, debt's dilution is driven by asymmetric information about low realizations of the firm-value distribution (that is, in its "left tail"). In contrast, equity offerings, which give outside investors exposure to the upside potential of the firm, will cause greater dilution to a firm's existing shareholders when the asymmetric information is relatively more severe for high realizations of the firm-value distribution (that is, in its "right tail"). This implies that when the asset that has greater risk—the growth opportunity—is relatively less affected by asymmetric information than the firm's assets in place, issuing a security that gives investors more exposure to the firm's upside, such as equity, can be less dilutive than a security that is only exposed to the firm's downside, such as debt. A key implication of our paper is that, contrary to common intuition, the preference of debt versus equity financing for a firm is not driven by its overall "level" of asymmetric information,

³The idea of the firm as a collection of tangible and intangible assets as in Myers (1984) is a common one in the literature, see Berk, Green, and Naik (1999) for an example of a model of a firm with assets in place and growth options.

⁴Note that, in the spirit of Myers and Majluf (1984), we rule out of the possibility that firms finance their growth opportunities separately from the assets in place, i.e., by "project financing."

but rather, by the composition of its assets and especially by the “location” of the asymmetric information across assets.

Greater information asymmetry on a firm’s assets in place relative to its growth opportunities may emerge, for example, in cases where a firm is exposed to substantial “learning-by-doing,” as in Berk, Green, and Naik (2004). Consider a firm which obtained its assets in place through the exploitation of past investment opportunities, and which also has untapped growth options. In this situation it is plausible that the firm has accumulated more accurate information on its assets in place relative to the still undeveloped growth opportunities. This is because more information on assets in place has become privately available to insiders over time (perhaps as the result of past R&D activities), rather than on new potential investments, where critical information has yet to be revealed.⁵ Further, if the new growth opportunities have greater volatility than assets in place, our model shows that the original Myers and Majluf’s result may not hold.

More generally, we argue that the pecking order can be violated in the case of multidivisional firms. Our paper implies that equity can dominate debt when the division that has lower exposure to asymmetric information also has greater risk. Thus, our model generates new predictions on the cross-sectional variation of firm capital structures of multidivisional firms.

We also consider the case where the firm has pre-existing debt in its capital structure. We show that firms with debt outstanding are, all else equal, relatively more likely to prefer equity over debt financing for reasons solely driven by information asymmetry. This feature of our model suggests that pre-existing high leverage may lead to more equity financing, and vice versa. It also suggests that, in a dynamic model of securities offering, asymmetric information may in fact lead to a “mean reversion” in leverage levels, as is often documented in the empirical literature on capital structure (see Leary and Roberts, 2005). These predictions are novel within models based on informational frictions, and invite further research.⁶

⁵An example of such situation is provided by a pharmaceutical company whose assets are formed by fully developed drugs as well as new drugs where substantial additional R&D is necessary to obtain a commercially exploitable product. The new R&D will privately reveal to the company valuable information to assess the true commercial value of the drug, thus increasing the extent of asymmetric information with outside investors with respect to the initial patent stage.

⁶Our model features a static capital structure choice, but it lends itself to a dynamic specification, as in Leland (1994). Further research focusing on dynamic capital structure choices is suggested by the fact that the existing set of securities on a firm’s balance sheet affects the optimal financing choice at later dates.

We conclude the paper with an explicit optimal security design problem, where the firm can issue securities other than equity and debt. Feasible securities include convertible bonds and warrants, as well as equity and debt, among others. We show that the optimal security depends on the “location” of the information asymmetry in the firm-value distribution. In particular, we show that when a certain “low-information-cost-in-the-right-tail” condition holds, straight (but risky) debt is optimal when the firm needs to raise low levels of capital, but “equity-like” securities — such as convertible debt — emerge as the optimal securities when the firm must raise larger amounts of capital. Furthermore, we find that warrants can be optimal in the presence of pre-existing debt.

Our paper is linked to several papers belonging to the ongoing research on firm financing under asymmetric information. First and foremost, we build on the pecking order theory of Myers and Majluf (1984) and Myers (1984). In this paper, we focus on the very fundamental question behind the pecking order theory: if firms of heterogeneous quality wish to raise capital by issuing the *same* security, are firms of better-than-average quality less exposed to value dilution with a debt or an equity issue? In contrast from Myers (1984), we do not consider internal financing, which in our model would dominate external financing. In addition, by research design, in our model there are no (partially) separating equilibria which can generate “announcement effects.” As argued in Myers and Majluf (1984), a firm’s equity price reacts negatively to the announcement of its intention to issue new equity, a situation that may lead firms to avoid new equity offers and reject new valuable investment opportunities. In our model, firms always pool by raising capital either through equity or debt.

Directly relevant for our work is the seminal paper by Nachman and Noe (1994), showing that the original Myers and Majluf pecking order intuition obtains only under very special conditions regarding how the insiders’ private information affects firm-value distributions. Specifically, they show that debt emerges as the solution of an optimal security design problem if and only if the private information held by firm insiders orders the distribution of firm value by Conditional Stochastic Dominance (CSD), a condition that is considerably stronger than First Order Stochastic Dominance (FOSD).⁷ We show that CSD may not hold (and, thus the pecking order be violated)

⁷Loosely speaking, CSD requires that private information orders the conditional distributions in the right tails by FOSD, for all possible truncations. The Statistics and Economics literature also frequently uses the term Hazard

when firms hold a portfolio of assets, even in cases where CSD holds for each individual asset. This observation implies that optimal financing of a portfolio of assets may be quite different from the financing of each asset in isolation, even in cases where asymmetric information is the only friction.⁸

Subsequent research focuses on different aspects of the security design problem. DeMarzo and Duffie (1999) consider the *ex-ante* security design problem faced by a firm before learning its private information, rather than the *interim* security design problem (that is, after becoming informed) studied by Nachman and Noe (1994). DeMarzo (2005) considers both the *ex-ante* and the *interim* security design problems, and examines both the question of whether to keep multiple assets in a single firm (pooling) and the priority structure of the securities issued by the firm (tranching). DeMarzo, Kremer, and Skrzypacz (2005a) examine the security design problem in the context of auctions. Biais and Mariotti (2005) build on DeMarzo and Duffie (1999) and study the interaction between adverse selection and liquidity provision.

Our paper differs from this literature in several ways. First, and most importantly, in our paper we only require FOSD and, thus, our distributions can violate conditions posited in the previously mentioned literature, e.g. the uniform worst case condition of DeMarzo and Duffie (1999), or MLRP in DeMarzo, Kremer, and Skrzypacz (2005a), or perfect observation of true firm value at the time the security is issued, as in Biais and Mariotti (2005). Second, as in Myers and Majluf (1984) and Nachman and Noe (1994), we constrain the firm to raise a fixed amount of capital, which typically leads to pooling rather than separating equilibria. In contrast, in DeMarzo and Duffie (1999) issuers can separate in the interim security issuance stage by using retention as a signal (in the spirit of Leland and Pyle, 1977). We stress that in our paper, by research design, we focus on pooling equilibria because we want to study the core issue of the pecking order theory, namely the relative dilution of debt versus equity when firms of heterogeneous quality pool, raising capital by issuing the same security.

Other closely related papers include Chakraborty and Yilmaz (2009), which shows that if investors have access over time to noisy public information on the firm's private value, the dilution

Rate Ordering to refer to CSD.

⁸A different optimal financial structure for a portfolio of assets may also emerge in the context of a traditional taxes versus bankruptcy trade-off (see, for example, Lewellen, 1971; Kim and McConnell, 1977; Hackbarth, Hennessy, and Leland, 2007, among others).

problem can be costlessly avoided by issuing securities having the structure of callable, convertible bonds. Chemmanur and Fulghieri (1997) and Chakraborty, Gervais, and Yilmaz (2011) argue that warrants may be part of the optimal security structure. Finally, a growing literature considers dynamic capital structure choice (Fischer, Heinkel, and Zechner, 1989; Hennessy and Whited, 2005; Strebulaev, 2007; Morellec and Schürhoff, 2011). We conjecture that the economic forces of our static framework will play a first-order role in a dynamic version of our model.

There are several other papers that challenge Myers and Majluf (1984) and Myers (1984) by extending their framework in various ways.⁹ These papers derive a wide range of financing choices, which allow for signaling with costless separation that can invalidate the pecking order (e.g., Brennan and Kraus, 1987; Noe, 1988; Constantinides and Grundy, 1989).¹⁰ Cooney and Kalay (1993) relax the assumption that projects have a positive net present value. Fulghieri and Lukin (2001) relax the assumption that the informational asymmetry between a firm’s insiders and outside investors is exogenous, and allow for endogenous information production. Dybvig and Zender (1991) study the effect of optimally designed managerial compensation schemes, Hackbarth (2008) shows that managers with risk perception bias or “overconfidence” have a reverse pecking order preference, and Edmans and Mann (2012) look at the possibility of asset sales for financing purposes. Hennessy, Livdan, and Miranda (2010) consider a dynamic model with asymmetric information and bankruptcy costs, with endogenous investment, dividends and share repurchases, where the choice of leverage generates separating equilibria. Bond and Zhong (2014) show that stock issues and repurchases are part of an equilibrium in a dynamic setting. Strebulaev, Zhu, and Zryumov (2016) consider a dynamic model of the issuance decision, where information asymmetry is reduced over time. In contrast to these papers, but in the spirit of Myers and Majluf (1984), we consider a pooling equilibrium of a static model where the only friction is asymmetric information between insiders and outsiders.

The remainder of this paper is organized as follows. We begin in Section 2 by providing a simple example that illustrates the basic results of our paper and its underlying intuition. Section

⁹While we focus only on papers that study informational frictions, moral hazard considerations are also important drivers of capital structure choices, i.e, DeMarzo and Fishman (2007), Biais, Mariotti, Plantin, and Rochet (2007).

¹⁰Admati and Pfleiderer (1994) points out, however, that the conditions for a fully revealing signaling equilibrium identified in these papers are rather restrictive.

3 presents the basic model. Section 4 studies the drivers of the debt-equity choice. Section 5 considers the real options specification of our model. Section 6 considers the security design problem, where we provide conditions under which convertible debt and warrants are the optimal securities. Section 7 concludes. All proofs are in the Appendix.

2 A simple example

The core intuition of the pecking order theory is typically illustrated via a pooling equilibrium with two types of firms and a discrete state space. The basic results of our paper, and their intuition, can be shown with such a simple numerical example, summarized in Table 1.¹¹ We consider two types of firms: good type, $\theta = G$, and bad type, $\theta = B$, where a firm's type is private information to its insiders. We assume that the two types of firms are equally likely in the eyes of investors. At the beginning of the period, firms have assets in place and wish to raise capital I to invest in a new growth opportunity. We focus on a pooling equilibrium such that, when raising capital, the two types of firms issue the same security, so that investors do not update their priors on the firms' type when seeing the security issuance decision.

For reasons that will become apparent below, we will assume that a firm's end-of-period firm value, Z , is characterized by a *trinomial* distribution with three possible outcomes $Z \in \{z_1, z_2, z_3\}$. To fix ideas, we assume that states z_1 and z_2 are relevant for the value of assets in place, while state z_3 is relevant for the value of the growth opportunity. In particular, we assume that the end-of-period value of the assets in place is given by $z_1 = 10$, $z_2 = z_3 = 100$. If the growth opportunity is exercised, firm value becomes $z_1 = 10$, $z_2 = 100$, $z_3 = 300$. Thus, exploitation of the growth opportunity adds value to the firm only in state z_3 , increasing the end-of-period firm value in that state from 100 to 300. The firm's capital requirements are set to be equal to $I = 60$.

The probability of the three possible outcomes of Z depends on private information held by the firm's insiders, and is given by $f_\theta \equiv \{f_{\theta 1}, f_{\theta 2}, f_{\theta 3}\}$ for a firm of type θ . In our examples below, we will assume that $f_G = \{0.2, 0.4, 0.4\}$ and $f_B = \{0.3, 0.4 - x, 0.3 + x\}$, and we will focus in the

¹¹The numerical example of this section builds on the discussion in Nachman and Noe (1994), Section 4.3.

cases $x = 0.02$ and $x = 0.08$ for the discussion.¹² Note that the presence of the growth opportunity has the effect of changing the distribution of firm value in its right tail, and that the parameter x affects the probability on the high state, z_3 , relative to the middle state, z_2 , for the type- B firm.

Consider first the case where $x = 0.08$, so that $f_B = \{0.3, 0.32, 0.38\}$. Firm values for the good and bad types are given by $\mathbb{E}[Z_G] = 162$ and $\mathbb{E}[Z_B] = 149$, with a pooled value equal to 155.5. Firms can raise the investment of 60 to finance the growth opportunity by issuing a fraction of equity equal to $\lambda = 0.386 = 60/155.5$. Hence, under equity financing, the initial shareholders of a firm of type- G retain a residual equity value equal to $(1 - 0.386)162 = 99.5$. The firm could also raise the required capital by issuing debt, with face value equal to $K = 76.7$. In this case debt is risky, with payoffs equal to $\{10, 76.7, 76.7\}$, and it will default only in state z_1 . The value of the debt security when issued by a type G firm is $D_G = 63.3$, and when issued by a type- B firm is $D_B = 56.7$, with a pooled value of 60, since the two types are equally likely. This implies that under debt financing the shareholders of a type- G firm will retain a residual equity value equal to $\mathbb{E}[Z_G] - D_G = 98.7 < 99.5$, and equity is *less dilutive* than debt. Thus, the pecking order preference is reversed.

The role of the growth opportunity in reversing the pecking order can be seen by considering the following perturbation of the basic example. Now set $x = 0.02$, so that $f_B = \{0.30, 0.38, 0.32\}$. In the new example the growth opportunity is relatively less important for a type- B firm than in the base case. Note that this change does not affect debt financing, because debt is in default only in state z_1 . Therefore the change in x only affects equity dilution. In the new case, $\mathbb{E}[Z_B] = 137$, lowering the pooled value to 149.5. Now the firm must issue a larger equity stake, $\lambda = 0.401 = 60/149.5$, and thus existing shareholders' value is now equal to $(1 - 0.401)162 = 97.0 < 98.7$. Thus, equity financing is now more dilutive than debt financing, restoring the pecking order.

The reason for the change in the relative dilution of debt and equity rests on the impact of asymmetric information on the right tail of the firm-value distribution. In the base case, for $x = 0.08$, asymmetric information has a modest impact on the growth opportunity (since $f_{G3} - f_{B3} = 0.02$) relative to the middle of the distribution (since $f_{G2} - f_{B2} = 0.08$), which impact is

¹²Table 1 considers all cases $x \in (0, 0.1)$. We remark that $x \leq 0.1$ is necessary to maintain first-order stochastic dominance.

determined by the exposure of the assets in place to asymmetric information. Thus, firms of type G can reduce dilution by issuing a security that has greater exposure to the right tail of the firm-value distribution, such as equity, rather than debt, which lacks such exposure. In contrast, in the case of $x = 0.02$, asymmetric information has a more substantial impact on the growth opportunity and, thus, on the right tail relative to the middle of the distribution (since now we have $f_{G3} - f_{B3} = 0.08$ and $f_{G2} - f_{B2} = 0.02$) making equity more mispriced.

A second key ingredient of our example is that the firm is issuing (sufficiently) risky debt to make dilution a concern. If debt is riskless, or nearly riskless, the pecking order would hold. To illustrate this in our example we can assume $z_1 = 10$ and set $I = 10$. At the lower level of investment, the firm can issue riskless debt and avoid any dilution altogether. Similarly, for investment needs sufficiently close to $I = 10$, debt has little default risk and the potential mispricing will be small. In contrast, for sufficiently large investment needs the firm will need to issue debt with non-trivial default risk, creating the potential for a reversal of the pecking order.

Finally, note that in the special case in which $f_B \equiv \{0.3, 0.3, 0.4\}$ there is no asymmetric information at all in the right tail (that is, for $z_3 = 300$). In this case, type- G firms would in fact be able to avoid dilution altogether by issuing securities that load only on cash flows in the right tail, such as warrants. We will exploit this feature in Section 6, where we study the security design problem, proving the optimality of securities with equity-like features.

In the remainder of the paper, we build models that generate a reversal of the pecking order and show that a reversal can emerge in many economically relevant situations. In Section 3 we introduce a condition, which we refer to as “low-information-costs-in-the-right-tail,” that generalizes the parametric assumptions in the previous example. This condition is novel in the literature and it is critical to generate reversals of the pecking order. The decomposition of the firm-value distribution into three regions in Section 4 establishes formally that the trinomial structure of our example is necessary for our results, and it provides its key drivers. Section 5 considers a simple real options model which generates new cross-sectional predictions that can be used to test asymmetric information theories, with and without a pecking order.

3 The basic model

3.1 The capital raising game

We study a one-period model with two dates, $t \in \{0, 1\}$. At the beginning of the period, $t = 0$, a firm with no cash wishes to raise a certain amount of capital, I , that needs to be invested in the firm immediately.¹³ We initially assume that the firm is all equity financed, we will later study the effect of the presence of pre-existing debt in the firm capital structure at $t = 0$. Firm value at the end of the period, $t = 1$, is given by a random variable Z_θ . There are two types of firms: “good” firms, $\theta = G$, and “bad” firms, $\theta = B$, which are present in the economy with probabilities p and $1 - p$, respectively. A firm of type θ is characterized by its density function $f_\theta(z)$ and by the corresponding cumulative distribution function $F_\theta(z)$, with $\theta \in \{G, B\}$. Because of limited liability, we assume that Z_θ takes values on the positive real line. For ease of exposition, we will also assume that the density function of Z_θ satisfies $f_\theta(z) > 0$ for all $z \in \mathbb{R}_+$. In addition, we assume type- G firms dominate type- B firms by first-order stochastic dominance.

Definition 1 (FOSD). *The distribution F_G dominates the distribution F_B by (strong) first-order stochastic dominance if $F_G(z) \leq (<)F_B(z)$ for all $z \in \mathbb{R}_{++}$.*

The stronger property of Conditional Stochastic Dominance, CSD, plays a crucial role in the security design problem, as argued in Nachman and Noe (1994).

Definition 2 (CSD). *We will say that the distribution F_G dominates the distribution F_B by conditional stochastic dominance if $F_G(z|z') \leq F_B(z|z')$ for all $z' \in \mathbb{R}_+$ and $z \geq z'$, where*

$$F_\theta(z|z') \equiv \frac{F_\theta(z + z') - F_\theta(z')}{1 - F_\theta(z')}.$$

By setting $z' = 0$, it is easy to see that CSD implies FOSD. Note that CSD can equivalently be defined by requiring that the truncated random variables $[Z_\theta|Z_\theta \geq \bar{z}]$, with distribution functions

¹³Following Nachman and Noe (1994), we initially do not explicitly model the reason for this capital requirement. The investment requirement of the firm I may reflect, for example, a new investment project that the firm wishes to undertake, as discussed in Section 4. Also note that, in the spirit of Myers and Majluf (1984), we rule out the possibility that firms finance their growth opportunities separately from the assets in place, i.e., by “project financing.”

$(F_\theta(z) - F_\theta(\bar{z})) / (1 - F(\bar{z}))$, satisfy FOSD for all \bar{z} .¹⁴ In addition, Nachman and Noe (1994) show that CSD is equivalent to the condition that the ratio $(1 - F_G(z)) / (1 - F_B(z))$ is non-decreasing in z for all $z \in \mathbb{R}_+$ (see their Proposition 4). Thus, loosely speaking, CSD implies that the set of payoffs in the right tail of the firm-value distribution are always more likely to occur for a type- G firm relatively to a type- B firm.¹⁵

Firms raise the amount I by seeking financing in capital markets populated by a large number of competitive, risk-neutral investors. Capital markets are characterized by asymmetric information in that a firm's type, $\theta \in \{G, B\}$ is private information to its insiders. We assume (and verify in our numerical examples) that firms always find it optimal to issue securities and raise I , rather than foregoing the investment opportunity. We make this assumption to rule out the possibility of separating equilibria where type- B firms raise capital, I , while type- G firms separate by not issuing any security. As stated earlier, we make this assumption because, by design, we study equilibria where both types of firms pool and raise capital by issuing the same security.

When insiders have private information, firms will typically issue securities at prices that diverge from their symmetric information values. Under these circumstances, better-quality firms will find it desirable to raise capital by issuing securities that reduce the adverse impact of asymmetric information. To fix ideas, let \mathbb{S} be the set of admissible securities that the firm can issue to raise the required capital I . As is common in this literature (see, for example, DeMarzo, Kremer, and Skrzypacz, 2005b), we let the set \mathbb{S} be the set of functions satisfying the following conditions:

$$0 \leq s(z) \leq z, \quad \text{for all } z \geq 0, \quad (1)$$

$$s(z) \text{ is non-decreasing in } z \quad \text{for all } z \geq 0, \quad (2)$$

$$z - s(z) \text{ is non-decreasing in } z \quad \text{for all } z \geq 0. \quad (3)$$

Condition (1) ensures limited liability for both the firm and investors, while (2) and (3) are mono-

¹⁴We remark that the CSD (hazard-rate) ordering is weaker than the Monotone Likelihood Ratio order, which requires $[Z_G | Z_G \in (\underline{z}, \bar{z})] \stackrel{\text{fosd}}{\geq} [Z_B | Z_B \in (\underline{z}, \bar{z})]$ for all \underline{z} and \bar{z} ; see equation (1.B.7) and Theorem 1.C.5 in Shaked and Shanthikumar (2007).

¹⁵Referring back to the example in Section 2, it is easy to verify that if $x \leq 0.05$ the type- G distribution not only dominates the type- B in the first-order sense, but *also* in the CSD sense. A necessary condition for the distributions in the example to *not* satisfy CSD is that $x > 0.05$.

tonicity conditions that ensure absence of risk-less arbitrage.¹⁶ We define $\mathbb{S} \equiv \{s(z) : \mathbb{R}_+ \rightarrow \mathbb{R}_+ : s(z) \text{ satisfies (1), (2), and (3)}\}$ as the set of *admissible securities*.

We consider the following capital raising game. The firm moves first, and chooses a security, $s(z)$, from the set of admissible securities \mathbb{S} . After observing the security, $s(z)$, issued by the firm, investors update their beliefs on firm type, θ , and form posterior beliefs, $p(s) : \mathbb{S} \rightarrow [0, 1]$. Given their posterior beliefs on firm type, investors purchase the security issued by the firm at a price, $V(s)$. The value, $V(s)$, that investors are willing to pay for the security, $s(z)$, issued by the firm is equal to the expected value of the security, conditional on the posterior beliefs, $p(s)$. That is

$$V(s) = p(s)\mathbb{E}[s(Z_G)] + (1 - p(s))\mathbb{E}[s(Z_B)]. \quad (4)$$

Condition (4) implies that securities are fairly priced, given investors' beliefs. If security s is issued, capital $V(s)$ is raised, and the investment project is undertaken. The payoff to the initial shareholders for a firm of a type θ is given by

$$W(\theta, s, V(s)) \equiv \mathbb{E}[Z_\theta - s(Z_\theta)] + V(s) - I. \quad (5)$$

The firm will choose the security to issue to finance the investment project by maximizing its payoff, (5), subject to the constraint that the security is admissible and that it raises at least the required funds, I . Let $s_\theta(z) \in \mathbb{S}$ be the security issued by a firm of type θ .

3.2 Equilibrium

Following the literature, we will adopt the notion of Perfect Bayesian Equilibrium, PBE, as the solution concept for the capital raising game.

Definition 3 (Equilibrium). *A Perfect Bayesian Equilibrium (PBE) of the capital raising game is a collection $\{s_G^*(z), s_B^*(z), p^*(s), V^*(s)\}$ such that: (i) $s_\theta^*(z)$ maximizes $W(\theta, s, V^*(s))$ subject to*

¹⁶See, for example, the discussion in Innes (1990). As pointed out in Nachman and Noe (1994), condition (2) is critical to obtain debt as an optimal security. In absence of (2), the optimal contract may have a “do or die” component, whereby outside investors obtain all of the firm cash flow when it falls below a certain threshold, and nothing otherwise.

the constraint that $s \in \mathbb{S}$ and $V^*(s) \geq I$, for $\theta \in \{G, B\}$, (ii) securities are fairly priced, that is $V^*(s) = p^*(s)\mathbb{E}[s(Z_G)] + (1 - p^*(s))\mathbb{E}[s(Z_B)]$ for all $s \in \mathbb{S}$, and (iii) posterior beliefs $p^*(s)$ satisfy Bayes rule whenever possible.

We start with a characterization of the possible equilibria in our capital raising game. The following proposition mimics Proposition 1 of Nachman and Noe (1994).

Proposition 1. *Let F_θ satisfy strong FOSD. No separating equilibrium exists in the capital raising game. In addition, in a pooling equilibrium with $s_G^* = s_B^* = s^*$, the capital raising game is uninformative, $p(s^*) = p$, and the financing constraint is met with equality*

$$I = p\mathbb{E}[s(Z_G)] + (1 - p)\mathbb{E}[s(Z_B)]. \quad (6)$$

This equilibrium is supported by the out-of-equilibrium belief that if investors observe the firm issuing a security $s' \neq s^$ they believe that $p(s') = p$ (passive conjectures).*

Proposition 1 follows from the fact that, with two types of firms only, a type- B firm always has the incentive to mimic the behavior of a type- G firm (i.e., to issue the same security). This happens because condition (2) and strong FOSD together imply that securities issued by a type- G firm are always priced better by investors than those issued by a type- B firm, and a type- B firm is always better-off by mimicking type- G actions. This also implies that, in equilibrium, the type- G firm is exposed to dilution due to the pooling with a type- B firm, and the corresponding loss of value can be limited by issuing only the securities needed to raise the capital outlay I .

Proposition 1 allows us to simplify the exposition as follows. Since both types of firms pool and issue the same security, $s(z)$, and the capital constraint is met as equality, (5) and (6) imply that the payoff to the original shareholders of a firm of type G becomes

$$W(G, s, V(s)) = \mathbb{E}[Z_G] - I - (1 - p)\mathcal{D}_s,$$

where the term

$$\mathcal{D}_s \equiv \mathbb{E}[s(Z_G)] - \mathbb{E}[s(Z_B)] \quad (7)$$

represents the mispricing when security $s \in S$ is used, which is the cause of the dilution suffered by a firm of type G .

Since, from Proposition 1, type- B firms will always pool with type- G firms, firms of type G will find it optimal to finance the project by issuing a security that minimizes dilution \mathcal{D}_s , that is

$$\min_{s \in \mathbb{S}} \mathcal{D}_s \tag{8}$$

subject to the financing constraint (6).

3.3 Asymmetric information in the right tail and the pecking order

Nachman and Noe (1994) show that if (and only if) the distribution F_G dominates F_B by (strong) CSD, the solution to the optimal security design problem (8) is debt and, thus, the standard pecking order obtains. The aim of our paper is to characterize economic environments where private information orders firm-value distributions F_θ by FOSD as in Myers and Majluf (1984), but not by CSD, i.e. specifications where the pecking order theory may fail.

We characterize the effect of asymmetric information on firm-value distributions by the function $H(z)$, defined as

$$H(z) \equiv \frac{F_B(z) - F_G(z)}{1 - F(z)}, \tag{9}$$

where $F(z) = pF_G(z) + (1 - p)F_B(z)$ denotes the mixture of the distributions of the good and bad types. Note that CSD is equivalent to monotonicity of $H(z)$: the function $H(z)$ is (strictly) increasing in z for all $z \in R_+$ if and only if the distribution F_G dominates F_B by (strong) CSD.¹⁷

The function $H(z)$ plays a critical role in our analysis (see Section 6). By rearranging terms, and noting that $F_B(z) - F_G(z) = 1 - F_G(z) - (1 - F_B(z))$, it is easy to see that, for monotonic securities, $H(z)$ determines the cost to a type- G firm, relative to a type- B firm, of promising to investors an extra dollar in state z .¹⁸ Thus, the function $H(z)$ reflects the “cost,” due to asymmetric information, for a firm of type G to pay cash flows in the right tail of the firm-value distribution.

¹⁷For distributions endowed with density functions, monotonicity of $H(z)$ is equivalent to requiring that the hazard rates $h_\theta(z) \equiv f_\theta(z)/(1 - F_\theta(z))$ satisfy $h_G(z) \leq (<)h_B(z)$ for all $z \in \mathbb{R}_+$; see Ross (1983) for further discussion.

¹⁸This happens because, for monotonic securities, an extra dollar paid in state z means that investors will be paid an extra dollar also in all states $z' > z$.

The properties of the function $H(z)$ depend on the impact of private information on the firm-value distributions $F_\theta(z)$. Note first that (strong) FOSD implies that $H(z)$ is (strictly) increasing at $z = 0$. This can be seen by noting that $H(0) = 0$ and that, from (strong) FOSD, we have $H(z) \geq (>)0$ and $H'(z) \geq (>)0$ for z in a right neighborhood of $z = 0$.

While FOSD dictates the monotonicity properties of $H(z)$ on the left tail of firm-value distributions, this is not the case for the right tail. In particular, as noted above, the function $H(z)$ is (strictly) increasing in z , for all $z \in \mathbb{R}_+$, if and only if CSD holds. Non-monotonicity of $H(z)$, in contrast, implies that asymmetric information is relatively less severe in the right tail of the firm-value distributions. This implies that it may be relatively cheaper for firms of type G to pay cash flows at high realizations of z , leading to potential violations of the pecking order.

To characterize the impact of the information cost in the right tail of the firm-value distributions, F_θ , we introduce the following definition, which will play a key role in our analysis.

Definition 4 (*h-ICRT*). *We will say that distribution F_G has information costs in the right tail of degree h (*h-ICRT*) over distribution F_B if $\lim_{z \uparrow \infty} H(z) \leq h$.*

We denote NICRT (no-information-costs-in-the-right-tail) as the case where $h = 0$. The relationship between FOSD, CSD and *h-ICRT* may be seen by noting that for two distributions, $\{F_G, F_B\}$, that satisfy FOSD, there may exist a sufficiently low $h \in \mathbb{R}_+$ such that the *h-ICRT* property holds, while CSD fails. Thus, intuitively, distributions that satisfy the *h-ICRT* condition “fill” part of the space of distributions that satisfy FOSD but do not satisfy the CSD condition. In particular, all distributions that satisfy Definition 4 for $h = 0$ (NICRT) will fail to satisfy the CSD condition.

In the remainder of this section, we provide numerical examples that shed light on the *h-ICRT* condition, and we present parametric specifications that can generate such cases. We begin with an extension of the simple example presented in Section 2 that illustrates the conditions under which the NICRT condition arises. We now assume a continuous distribution rather than the discrete one

we used in Section 2. Specifically, we assume that type- G and type- B densities are now given by:

$$f_G(z) = \begin{cases} 0.2k & \text{for } z \in [0, 100] \\ 0.4k & \text{for } z \in [100, 200] \\ 0.4k & \text{for } z \in [200, 300] \end{cases}, \quad f_B(z) = \begin{cases} 0.3k & \text{for } z \in [0, 100] \\ (0.4 - x)k & \text{for } z \in [100, 200] \\ (0.3 + x)k & \text{for } z \in [200, 300] \end{cases}, \quad (10)$$

where $k = 0.01$ is a normalizing constant, and $x \in [0, 0.10]$ in order that a type- G firm dominates a type- B firm by FOSD. It is important to note that this example mirrors the one from Section 2 in that the type- B distribution, $f_B(z)$, has a higher probability in the left tail of the payoff distribution, $z \in [0, 100]$, and the parameter x moves mass from the intermediate set of payoffs, $z \in [100, 200]$ to the right tail, $z \in [200, 300]$.

Figure 1 plots the densities for the type- G and type- B firms, as well as the $H(z)$ function, for three different parameter specifications. When $x = 0.05$ the $H(z)$ function is monotonically increasing, and the CSD condition is satisfied. Making $x = 0.1$ moves mass from the middle of the distribution to the right tail. In this case, the densities of firms of type G and type B , $\{f_G(z), f_B(z)\}$, coincide on the range $[200, 300]$, i.e. they have the same probability distribution over that range, even while the type- G firm first-order stochastically dominates the type- B firm. For $x = 0.1$ we have the case that we refer to as NICRT, since the $H(x)$ function is non-monotonic and has a zero limit. The bottom two graphs plot the densities and $H(z)$ for the intermediate case where $x = 0.09$, which shows how one can have cases where the $H(z)$ function is non-monotonic, and thus CSD does not hold, even when the NICRT condition is not satisfied.

The common feature of our next set of examples is to represent the firm as a collection (a portfolio) of assets, where the end-of-period firm value Z_θ is the combination of two lognormal random variables, X_θ and Y_θ .¹⁹ We will consider three alternative specifications: (a) $Z_\theta = \max(X_\theta, Y_\theta)$, where the firm has the option to exchange two assets, X_θ and Y_θ at the end of the period (“rainbow” or exchange option case);²⁰ (b) $Z_\theta = X_\theta + Y_\theta$, that is the firm is a multi-division firm, where firm value is the sum of the value of its divisions, X_θ and Y_θ (“multi-division firm” case);

¹⁹Equivalently, following common practice, the lognormal random variables X_θ and Y_θ could be characterized in terms of geometric brownian motions.

²⁰Stulz (1982) uses the exchange option specification in a real options framework, and motivates its appeal in a corporate context.

(c) $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, whereby the firm has the option at the future date T to invest in a “growth opportunity” by making the additional capital expenditure I_T (“real option” case), in addition to the initial investment I . The next proposition summarizes the right tail behavior of parametric constellations for these specifications.

Proposition 2. *Let X_θ and Y_θ be two lognormal random variables with $\mathbb{E}[\log(X_\theta)] = \mu_{\theta x}$, $\mathbb{E}[\log(Y_\theta)] = \mu_{\theta y}$, $\text{var}(\log(X_\theta)) = \sigma_x^2$, $\text{var}(\log(Y_\theta)) = \sigma_y^2$, $\text{cov}[\log(X_\theta), \log(Y_\theta)] = \rho\sigma_x\sigma_y$. Without loss of generality, assume that $\sigma_y > \sigma_x$. Then:*

1. *If $Z_\theta = X_\theta$, that is, Z_θ has a lognormal distribution, then the distribution F_G dominates the distribution F_B by CSD if and only if $\mu_{Gx} > \mu_{Bx}$.*
2. *If $\mu_{Gy} = \mu_{By}$, $\mu_{Gx} > \mu_{Bx}$ and the payoff from the project Z_θ satisfies either (a) $Z_\theta = \max(X_\theta, Y_\theta)$, (b) $Z_\theta = X_\theta + Y_\theta$, or (c) $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, then NICRT holds.*

Proposition 2 provides one of the main results of our paper. It shows that simple deviations from the standard lognormal model can generate economically relevant environments where the CSD condition does not hold and, thus, violations of the pecking order theory can arise. We emphasize that the (stronger) monotone likelihood ratio property holds in the standard one-dimensional lognormal model, as shown in part 1 of Proposition 2. In addition, Proposition 2 suggests that second moments of firm-value distributions play a critical role in generating violations of the pecking order. Specifically, such violations may occur when the asset that has relatively lower exposure to asymmetric information also has greater volatility. This property follows from the fact that for Gaussian random variables (such as lognormal distributions) second moments of the distributions characterize tail behavior. This implies that the joint assumptions that Y has higher volatility, $\sigma_y > \sigma_x$, and it suffers no information costs, $\mu_{Gy} = \mu_{By}$, are sufficient to guarantee that the NICRT condition holds. While the NICRT condition (and in particular that $\mu_{Gy} = \mu_{By}$) is sufficient to generate non-monotonic $H(z)$ functions, it is, however not necessary, as the numerical simulations we provide below demonstrate.

In the remainder of the paper we will characterize economic environments where violations of the pecking order occur. We start our analysis by restricting our attention to two classes of

securities, debt and equity. We study this case explicitly because the debt-equity choice problem has attracted so much attention in both the theoretical and empirical corporate finance literature. We will then study the more general security design problem in Section 6.

4 The drivers of the debt versus equity choice

In this section, firms are restricted to seeking external financing in the form of either debt or equity only. From (7) the dilution costs associated with equity are given by

$$\mathcal{D}_E = \lambda (\mathbb{E}[Z_G] - \mathbb{E}[Z_B]), \quad (11)$$

with $\lambda = I/\mathbb{E}[Z]$. In contrast, the dilution costs associated with debt are

$$\mathcal{D}_D = \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)], \quad (12)$$

where the parameter K represents the (smallest) face value of debt that satisfies the financing constraint

$$I = p\mathbb{E}[\min(Z_G, K)] + (1 - p)\mathbb{E}[\min(Z_B, K)]. \quad (13)$$

In what follows, we say the pecking order obtains if $\mathcal{D}_E > \mathcal{D}_D$, and a reversal of the pecking order holds if $\mathcal{D}_D > \mathcal{D}_E$. We begin our analysis by providing a necessary and sufficient condition for a reversal of the pecking order to hold.

Proposition 3. *The dilution cost of equity will be strictly smaller than that of debt, i.e., $\mathcal{D}_E < \mathcal{D}_D$, if and only if*

$$\frac{\mathbb{E}[Z_G]}{\mathbb{E}[Z_B]} < \frac{\mathbb{E}[\min(Z_G, K)]}{\mathbb{E}[\min(Z_B, K)]}. \quad (14)$$

The proposition implies that when the security choice is restricted between equity and debt, the security that generates lowest dilution in dollar terms is the one with the lowest *relative* valuation between firms of good and bad type. This supports the traditional intuition that debt dominates equity precisely because debt valuation is less sensitive to the underlying asymmetries in informa-

tion, limiting dilution.²¹ The next proposition shows that equity is always more sensitive to private information than debt under CSD.

Proposition 4. *Condition (14) cannot hold if F_G dominates F_B in the conditional stochastic dominance sense.*

To obtain further insights on the factors that drive the relative dilution of debt and equity, note that the difference in the dilution costs of debt and equity can be written as

$$\mathcal{D}_D - \mathcal{D}_E = \int_0^{\infty} (\min(z, K) - \lambda z) c(z) dz, \quad (15)$$

where $c(z) \equiv f_G(z) - f_B(z)$. Loosely speaking, the density function, $f_{\theta}(z)$, measures the private valuation of a \$1 claim made by the insiders of a firm of type $\theta \in \{G, B\}$ if the final payoff of the firm is z . Thus, the function, $c(z)$, can be interpreted as representing the private cost due to asymmetric information for a firm of type- G , relative to a firm of type- B , of issuing a security that has a payoff of \$1 if the final firm value is z . In particular, if $c(z) > 0$ we will say that the “information costs” for a type- G are positive, and that these costs are negative if $c(z) < 0$.

Next, we introduce an additional regularity condition that will simplify the analysis and streamline the presentation of some of the results of our paper.

Definition 5 (SCDP). *The distributions $F_{\theta}(z)$ satisfy the single-crossing density property (SCDP) if F_G strictly first-order stochastically dominates F_B , and there exists a unique $\hat{z} \in \mathbb{R}_+$ such that $f_G(\hat{z}) = f_B(\hat{z})$.*

Note that the SCDP condition implies that for all $z \leq \hat{z}$ we have $f_B(z) \geq f_G(z)$, and for all $z \geq \hat{z}$ we have $f_B(z) \leq f_G(z)$. Intuitively, this means that cash flows above the critical cutoff \hat{z} have a positive information cost for type- G firms, $c(z) > 0$, whereas cash flows below that cutoff have negative information costs, $c(z) < 0$.²²

²¹A similar condition was obtained, in the context of unit IPOs, in Chakraborty, Gervais, and Yılmaz (2011), see their Propositions 1 and 2.

²²Note that FOSD alone only implies that there exists z_1 and z_2 such that $c(z) < 0$ for all $z < z_1$ and $c(z) > 0$ for all $z > z_2$, but it does not rule out other interior crossings. In contrast, SCDP ensures that $z_1 = z_2$. We will assume SCDP for ease of exposition. The discussion below could be adapted to take into account the presence of multiple crossings.

Expression (15) can be further decomposed as follows (see Figure 2, top panel, for an illustration). Define $\bar{z}(K, \lambda) \equiv K/\lambda$ and note that for $z < \bar{z}(K, \lambda)$ we have that $\min(z, K) > \lambda z$, which implies that the payoffs to debtholders are greater than those to equity holders; the converse holds for $z > \bar{z}(K, \lambda)$. Under SCDP, the point \hat{z} divides the positive real line into two disjoint sets: a first set at the lower end of the positive real line, $[0, \hat{z})$ where $c(z) < 0$, and a type- G firm enjoys “negative information costs” (effectively an information benefit), and a second set $[\hat{z}, \infty]$ where $c(z) \geq 0$, that is where a type- G firm faces “positive information costs.” The point $\bar{z}(K, \lambda)$ divides the positive real line into two other subsets, depending on whether or not equity yields higher payoffs to investors than does debt. We have the following.

Proposition 5. *Assume the SCDP holds. Then a necessary condition for the reverse pecking order is that $\bar{z} > \hat{z}$.*

If $\bar{z} > \hat{z}$, which we will refer to as the “un-pecking necessary condition” (or UC), from (15) a reversal of the pecking order obtains if and only if

$$\mathcal{D}_D - \mathcal{D}_E = - \int_0^{\hat{z}} (\lambda z - \min(z, K)) c(z) dz + \int_{\hat{z}}^{\bar{z}} (\min(z, K) - \lambda z) c(z) dz - \int_{\bar{z}}^{\infty} (\lambda z - K) c(z) dz > 0. \quad (16)$$

Under UC and the maintained assumptions, the three integrals in (16) are all positive. The first term of the r.h.s. of (16) measures the benefits of debt financing for low realizations of firm value (i.e., for $z < \hat{z}$), that is, in the left tail of the firm-value distributions. In this *low-value region*, dilution costs are lower for debt than equity, because debt gives a higher payoff than equity, but such payoff has negative information costs (i.e., $c(z) \leq 0$). Note that (strong) FOSD guarantees that $c(z) \leq 0$ and, thus, that this term is always negative.

The second term of the r.h.s. of (16) measures the dilution cost of debt relative to equity in the *intermediate-value region* $[\hat{z}, \bar{z}]$, where debt has higher payouts than equity and type- G firms suffer a positive information cost, $c(z) > 0$. In this region dilution costs of equity are lower than those of debt because equity has a lower payoff than debt and information cost are positive (i.e., $c(z) > 0$). Note that existence of this region is guaranteed by UC. It is the presence of this term that makes

equity potentially less dilutive than debt.²³

The third and last term measures the dilution costs of equity relative to debt for high realizations of firm value (i.e., for $z > \bar{z}$), that is in the right tail of the firm-value distributions. In this *high-value region*, equity payoffs are greater than debt in those states that are more likely to occur to a type- G firm, and thus carry positive information costs (i.e., $c(z) > 0$).

The preference of debt over equity financing (the pecking order) depends on the relative importance of these three regions.²⁴ In particular, equity dominates debt when the advantages of equity financing originating from the intermediate region of firm value (for $z \in [\hat{z}, \bar{z}]$), the second term on the r.h.s. of (16) dominate the disadvantages in the low (for $z < \hat{z}$) and the high (for $z > \bar{z}$) regions of firm value, the first and the third term on the r.h.s. of (16).

The relative importance of the three regions depends crucially on the term $c(z)$ and, thus, on how information asymmetries affect the firm-value distributions (i.e., the location of the information asymmetry in the domain of the firm-value distribution). Intuitively, equity can be less dilutive than debt when asymmetric information is more pronounced in the center of the firm-value distribution, generating large values of $c(z)$ in the intermediate region, while it has a relatively smaller impact on either the left tail of the distribution (the low-value region) or its right tail (the high-value region). While the first term in (16), which represents the left tail of the distribution, is always (strictly) positive by (strong) FOSD, the magnitude of the third term in (16) depends on the impact of asymmetric information on the right tail of the distribution. If this third term is very small, or zero, which can happen when the NICRT condition holds, a reversal of the pecking order may occur.

We conclude by examining the effect of prior financing on the debt-equity choice. In particular, we allow for the possibility that, at the beginning of the period, $t = 0$, the firm has already issued straight debt with face value K_0 which is due at the end of the period, T . In accordance to anti-dilutive “me-first” rules that may be included in the debt covenants, we assume that this pre-existing debt is senior to all new debt that the firm may issue at $t = 0$. We maintain the

²³Note that if UC does not hold (so that $\bar{z}(K, \lambda) < \hat{z}$), equity has negative information costs (that is, $c(z) < 0$) precisely in the states where the payouts to equityholders are greater than those to debtholders, making it impossible for the inequality (16) to be satisfied. Thus, UC is a necessary condition to reverse the pecking order.

²⁴Note that these three regions are reminiscent of the trinomial distribution considered in Section 2.

assumption that the firm always finds it optimal to raise external capital, I , through either an equity or a debt offer.²⁵

Similar to the previous analysis, we restrict the choice of security to equity or (junior) debt. We assume that the firm can raise the necessary capital either by sale of junior debt with face value K , or by sale of a fraction λ of total (levered) equity of the firm to outside investors. Following an argument similar to the one in Section 4, the relative dilution of debt versus equity is now given by:

$$\mathcal{D}_D - \mathcal{D}_E = \int_{K_0}^{\infty} [(1 - \lambda) \max(z - K_0, 0) - \max(z - (K_0 + K), 0)] c(z) dz. \quad (17)$$

The main difference of (17) relative to the corresponding expression (15) is the fact that all payoffs below K_0 are now allocated to the pre-existing senior debt. This implies that only the probability mass located in the interval $[K_0, \infty)$ is relevant for the determination of the relative dilution costs of debt and equity and, thus, for the choice of financing of the new project. Recall from (16) that the two regions located at the left and the right tails of the probability distribution favor debt financing, while the intermediate region favors equity financing. This observation implies that the presence of pre-existing debt in a firm's capital structure, by reducing the importance of the left tail region, makes equity more likely to be the less dilutive source of financing (all else equal), and therefore for the pecking order to be reversed.

5 Cross-sectional predictions

In this section we study a real options specification of our model that is based on case 2(c) of Proposition 2, which will serve as the basis for our main cross-sectional predictions.²⁶ As in Section 4, we focus on the pecking order of Myers and Majluf (1984), and we restrict ourselves to the choice between debt and equity.

We assume that a firm of type $\theta \in \{G, B\}$ is endowed at the beginning of the period, $t = 0$, with both assets in place and a growth opportunity. By making the investment I at $t = 0$, the

²⁵This assumption allows us to ignore a possible debt overhang problem in the sense of Myers (1977).

²⁶ Case (a) in Proposition 2 is nested in case (c). An earlier draft of the paper focused our cross-sectional predictions on case (b), which yields the same qualitative predictions. We note that this is a rather standard specification of a real option framework, see, for example, Berk, Green, and Naik (1999).

firm generates a new “growth option” that can be exercised at the future date T . To exercise the growth opportunity, the firm must make an additional investment I_T at date T . Thus, the end of period firm value, Z_θ , is given by

$$Z_\theta \equiv X_\theta + \max(Y_\theta - I_T, 0),$$

where X_θ represents the value of the firm’s assets in place at time T , and $\max(Y_\theta - I_T, 0)$ represents the value of the growth opportunity. Note that, by setting $I_T = 0$, this specification nests the case of a multidivisional firm, where $Z_\theta \equiv X_\theta + Y_\theta$. We assume (and verify) that the NPV of the growth opportunity is sufficiently large that firms will always find it optimal to issue either debt or equity and invest I in the new growth opportunity (rather than not issuing any security and thus abandon the project). For simplicity, we also assume that the growth option is of the European type.²⁷ We assume that both X_θ and Y_θ follow lognormal processes, that is, both $\log(X_\theta)$ and $\log(Y_\theta)$ are normally distributed with means $\mu_{\theta x}T$ and $\mu_{\theta y}T$ and with variances σ_x^2T and σ_y^2T . We let ρ denote the correlation coefficient between $\log(X_\theta)$ and $\log(Y_\theta)$.

In the spirit of Myers and Majluf (1984), we model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while their variances are common knowledge. We let $\mathbb{E}[X_\theta] = \bar{X}_\theta$ and $\mathbb{E}[Y_\theta] = \bar{Y}_\theta$, and we assume $\bar{X}_G \geq \bar{X}_B$ and $\bar{Y}_G \geq \bar{Y}_B$, with at least one strict inequality. We define the average value of the assets in place and of the growth opportunity as $\bar{X} = p\bar{X}_G + (1-p)\bar{X}_B$ and $\bar{Y} = p\bar{Y}_G + (1-p)\bar{Y}_B$, respectively, and let $c_x = \bar{X}_G - \bar{X}_B$ and $c_y = \bar{Y}_G - \bar{Y}_B$. Thus, c_x and c_y measure the exposure to asymmetric information of the assets in place and the growth opportunity, respectively. The assumptions ensure FOSD, and allow for scenarios in which the NICRT condition holds, as discussed in Proposition 2.

It is well known that real option models like the one presented in this section do not admit closed-form solutions. Therefore, following existing literature (see, for example, Childs, Mauer, and Ott, 2005; Gamba and Triantis, 2008, among others) we conduct a series of numerical simulations of our base model, and derive numerical comparative statics results. Our numerical examples are

²⁷This assumption allows us to abstract from issues related to the optimal exercise time of the growth option, which we leave for future research (see, e.g., Morellec and Schürhoff, 2011, for a model with endogenous investment timing).

centered on the base case reported in Table 2. In this base case, asymmetric information is more severe on assets in place, where $\bar{X}_G = 125$ and $\bar{X}_B = 75$, rather than the growth opportunity, where $\bar{Y}_G = 205$ and $\bar{Y}_B = 195$. In addition, we assume that assets in place have lower return volatility than the growth opportunities, as in Berk, Green, and Naik (2004), and we set $\sigma_x = 0.3$, $\sigma_y = 0.6$, $T = 15$ and $\rho = 0$. We let both types be equally likely, $p = 0.5$. In this base case specification, we set the initial investment amount to be $I = 100$, and the investment at exercise of the growth option to be $I_T = 50$.

The bottom panel of Figure 2 plots the distributions, $f_\theta(z)$, of $Z_\theta \equiv X_\theta + \max(Y_\theta - I_T, 0)$ as well as the unconditional distribution $f(z)$. By direct inspection, it is easy to verify that the distribution of firm value Z_θ closely resembles a lognormal distribution, with the important difference that the asymmetric information loads in the middle of the distribution, and to a lesser extent in its right tail. The top panel of Figure 2 displays the function $c(z)$, together with the payoffs of the debt and equity securities, when the initial investment is $I = 100$. In this case, the region in which debt has a disadvantage over equity, the intermediate region of (16) is relatively large, and is equal to the interval $[76.4, 470.7]$.

We now derive the relative dilution of debt and equity for the base-case scenario. Note first that, if the investment opportunity is taken, the value of the firm for the two types is given by $\mathbb{E}[Z_G] = 307.9$ and $\mathbb{E}[Z_B] = 248.2$, so that $p\mathbb{E}[Z_G] + (1 - p)\mathbb{E}[Z_B] = 278.1$. Without the project, the (average) status-quo firm value is the value of assets in place X , which is equal to $\bar{X} = 100$. Since the value of the post-investment is firm 278.1, and the investment is $I = 100$, the project has an (unconditional) positive NPV of 178.1. Note also that the efficient outcome is for *both* types of firms to finance the project, since for a type- G firm $\mathbb{E}[Z_G] - I = 307.9 - 100 = 207.9 > 125 = \bar{X}_G$, and for a type- B firm $\mathbb{E}[Z_B] - I = 248.2 - 100 = 148.2 > 75 = \bar{X}_B$.

It is easy to verify that issuing equity will require that the equity holders give up a stake of $\lambda = 0.360 = 100/278.1$. In order to finance the project with debt, the firm needs to promise bondholders a face value at maturity of $K = 218.4$. The dilution costs of equity are $\mathcal{D}_E = 0.36 \times (307.9 - 248.2) = 21.5$, whereas those of debt are $\mathcal{D}_D = 111.9 - 88.1 = 23.7$, with a relative dilution $\mathcal{D}_D/\mathcal{D}_E = 23.7/21.5 = 1.10$. Thus, the type- G firm is exposed to lower dilution by raising

capital with equity rather than debt.²⁸ Note that the implied credit spread for the risky debt is 5.3%, which in the recent years was roughly equivalent to the spread on non-investment grade bonds, such as BB-rated debt.

The bottom portion of Table 2 examines the impact of changes of some of the key parameters in the base case on the relative dilution of debt and equity. The first set of examples focus on the exposure to asymmetric information of the assets in place relative to the growth opportunity. Specifically, a decrease of the exposure to asymmetric information in the growth opportunity, by setting $Y_G = Y_B = 200$, has the effect of increasing the dilution of debt relative to equity to 1.27. Conversely, an increase of the exposure to asymmetric information in the growth opportunity, by setting now $Y_G = 225$ and $Y_B = 175$, has the effect of reducing the dilution of debt relative to equity to 0.76, and making debt less dilutive than equity. Similarly, an increase of the exposure to asymmetric information in the assets in place, by setting $\bar{X}_G = 150$ and $\bar{X}_B = 50$, has the effect of increasing the dilution of debt relative to equity to 1.26, and a decrease of their exposure to asymmetric information, by setting $\bar{X}_G = \bar{X}_B = 100$, has the effect of reducing the dilution of debt relative to equity to 0.21, again making debt less dilutive than equity. These results conform with the notion that reversals of the pecking order preference can occur when the asset with greater volatility is less exposed to asymmetric information relative to the asset with lower volatility.

We consider next the effect of the volatility parameters, σ_x and σ_y . An increase of the volatility of the assets in place to $\sigma_x = 0.40$ has the effect of reducing the dilution of debt relative to equity from $D_D/D_E = 1.10$ to 1.01, while an increase of the volatility of the growth opportunity to $\sigma_y = 0.80$ has the opposite effect of increasing the relative dilution of debt and equity to 1.53. These examples show that equity is less dilutive than debt when the volatility of the growth opportunities is sufficiently large relative to the volatility of the assets in place, and the asymmetric information is concentrated in the growth opportunities. The critical role played by the relative volatilities of growth opportunities and assets in place was highlighted in Proposition 2.

²⁸It is worthwhile to remark that the investment choices are individually rational when using either debt or equity. To see this, note that in the case of equity financing the residual equity value for a type- G firm is equal to $(1 - 0.36) \times 307.9 = 197.1 > 125 = \bar{X}_G$, and for a type- B firm it is equal to $(1 - 0.36) \times 248.2 = 158.9 > 75 = \bar{X}_B$. In the case of debt financing, the residual equity value for a type- G firm is equal to $307.9 - 119.9 = 188 > 125 = \bar{X}_G$, and for a type- B firm it is equal to $248.2 - 88.1 = 160.1 > 75 = \bar{X}_B$.

The impact of the subsequent investment, I_T , is as follows. A decrease of the future investment requirement, from $I_T = 50$ to $I_T = 0$, reduces the dilution of debt relative to equity to 0.88, which makes debt overall less dilutive than equity, restoring the pecking order. In contrast, an increase of the subsequent investment to $I_T = 100$ worsens the relative dilution of debt and equity, which is now equal to 1.18. An increase of the subsequent investment requirements I_T , increasing the “exercise price” of the growth option, has the same effect as an increase of the volatility σ_y .

In the last set of examples, we examine the impact of pre-existing debt on the relative dilution of debt and equity. The presence of pre-existing debt with face value $K_0 = 20$ in our base-case parameter constellation has the effect of increasing the relative dilution of debt to equity to 1.28, raising the advantage to equity relative to debt financing. This effect is further reinforced at greater levels of pre-existing debt, where for $K_0 = 40$ the relative dilution of debt to equity becomes 1.47. The default spreads implicit in these cases, in which equity is less dilutive than debt, range from 5.3% to 9.8%. These default spreads are associated with bonds with credit ratings ranging from BB to C.

The dilution effects presented in Table 2 are further studied in Figures 3, 4 and 5, which present more general comparative static exercises. The top graph in Figure 3 displays indifference lines of $\mathcal{D}_D = \mathcal{D}_E$, as a function of the exposure to asymmetric information of the assets in place, c_x , and the growth opportunity, c_y , for three levels of the volatility of the growth opportunity, $\sigma_y \in \{0.6, 0.7, 0.8\}$. In the region above the lines, we have that $\mathcal{D}_D > \mathcal{D}_E$ and, hence, equity is less dilutive than debt and the reverse pecking order obtains. In the region below the lines, we have that $\mathcal{D}_D < \mathcal{D}_E$ and, hence, equity is more dilutive than debt, and the usual pecking order obtains. Note that the slope of the indifference lines declines as the volatility of the growth opportunity rises. These graphs reveal that equity is more likely to be less dilutive than debt when the exposure to asymmetric information on the less volatile assets in place, c_x , is large and when the exposure to asymmetric information of the more volatile growth opportunities, c_y , is small. In addition, the parameter region where equity dominates debt becomes larger when the volatility of the growth opportunity increases.

The bottom graph in Figure 3 charts indifference lines of $\mathcal{D}_D = \mathcal{D}_E$, as a function of the time

horizon, T , and the investment cost, I , for three levels of the average value of assets in place, $\bar{X} \in \{95, 100, 105\}$. For pairs of (I, T) below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is less dilutive than debt for higher investment costs I , and longer time horizons T (i.e., for younger firms). In addition, the parameter region where equity dominates debt becomes larger when the (average) values of assets in place, X , is lower (i.e., smaller firms).

The top graph of Figure 4 displays the pairs of the average value of assets in place and the average value of the growth option, (\bar{X}, \bar{Y}) , for which the dilution costs of equity and debt are the same (i.e., $\mathcal{D}_E = \mathcal{D}_D$) for different level of asymmetric information on asset $c_x \in \{10, 25, 40\}$. For pairs of (\bar{X}, \bar{Y}) below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the growth opportunities represent a larger component of firm value. In addition, the parameter region where equity dominates debt becomes larger when the exposure to asymmetric information of assets in place, c_x , increases.

The bottom graph of Figure 4 plots the pairs of volatilities, (σ_x, σ_y) , such that the dilution costs of equity and debt are the same (i.e., $\mathcal{D}_E = \mathcal{D}_D$) for three levels of the investment cost $I \in \{100, 110, 120\}$. For pairs of volatilities, (σ_x, σ_y) , below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the volatility of assets in place is low, and when the volatility of growth opportunities is large. In addition, the parameter region where equity dominates debt becomes larger when the firm's investment need, I , increases.

The top graph of Figure 5 examines the impact of the size of the investments needs on the form of financing. The graph reveals that equity financing is more likely to be less dilutive than debt when the firm has greater investment needs either at the time of the initial investment, $t = 0$, or at the time the growth option is exercised, $t = T$. These observations imply that future capital needs of the firm will have an independent effect on the financing decisions.

Finally, the bottom graph of Figure 5 examines the impact of pre-existing debt on the form of financing. The graph reveals that, for a given level of assets in place, \bar{X} , equity financing is less

dilutive than debt when the firm has a greater amount of pre-existing debt, K_0 . In addition, the graph suggests that firms are likely to switch from equity to debt financing as they accumulate assets in place, that is as \bar{X} becomes larger. At the same time, firms that finance asset acquisitions through debt financing are likely to switch to equity financing as they increase the amount of debt in their capital structure, K_0 . These considerations suggest that asymmetric information may in fact lead to “mean reversion” in leverage levels, as is often documented in the empirical literature on capital structure (see Frank and Goyal, 2003; Fama and French, 2005; Leary and Roberts, 2005).

In summary, the examples in Table 2, as well as Figures 3, 4, and 5 reveal a very consistent pattern: violations of the pecking order are likely to be optimal for young firms, endowed with valuable and risky growth opportunities and with large investment needs. In addition, equity is more likely to be less dilutive than debt when growth opportunities represent a greater proportion of firm value, when these growth opportunities are riskier, and when the firm has greater financing needs. Thus, our model can help explain the stylized fact that small and young firms with large financing needs and valuable growth opportunities (i.e. high-growth firms) often prefer equity over debt financing, even in circumstances where asymmetric information is potentially severe.

6 Optimal security design

We now consider explicitly the optimal security design problem. Following Nachman and Noe (1994), the optimal security design problem (8) can be expressed as

$$\min_{s \in \mathcal{S}} \int_0^\infty s'(z)(F_B(z) - F_G(z))dz, \quad (18)$$

subject to

$$\int_0^\infty s'(z)(1 - F(z))dz = I. \quad (19)$$

The Lagrangian to the above problem is

$$\mathcal{L}(s', \gamma) = \int_0^\infty s'(z)(F_B(z) - F_G(z) - \gamma(1 - F(z)))dz \quad (20)$$

$$= \int_0^\infty s'(z)(1 - F(z))(H(z) - \gamma)dz, \quad (21)$$

where $H(z)$ is defined in (9). Remember that the unction $H(z)$ measures, for any value z , the extent of the asymmetric information costs in the right tail of the firm-value distribution. The following is an immediate consequence of the linearity of the security design problem.

Proposition 6. *A solution s^* must satisfy, for some $\gamma \in \mathbb{R}_+$,*

$$(s^*)'(z) = \begin{cases} 1 & \text{if } H(z) < \gamma; \\ [0, 1] & \text{if } H(z) = \gamma; \\ 0 & \text{if } H(z) > \gamma. \end{cases} \quad (22)$$

Note that the value of the Lagrangian multiplier γ depends on the tightness of the financing constraint (19) and, thus, on the level of the required investment, I , with $\partial\gamma/\partial I > 0$. From $H(0) = 0$ and FOSD we have that $H(z) < \gamma$, which implies that the optimal security must satisfy $(s^*)' = 1$ in a right neighborhood of $z = 0$. This means that an optimal security will always have a (possibly small) straight-debt component.²⁹ The importance of this straight-debt component (that is, the face value of the debt) will depend on the size of the investment, I (since it affects the Lagrangian multiplier γ), as well as on the particular functional form for $H(z)$.

The overall shape of the optimal security for a greater value of z depends on the monotonicity properties of the function $H(z)$ (and, thus, on the extent of asymmetric information in the right tail of the firm-value distribution). It is characterized in the following proposition.

Proposition 7. *Consider the security design problem in (18)–(19).*

- (a) *If the distribution F_G conditionally stochastically dominates F_B , then straight debt is the optimal security (Nachman and Noe, 1994).*

²⁹Note, however, that as Proposition 8 below shows, this property hinges critically on the assumption that the firm has no pre-existing debt.

- (b) *If the problem satisfies the NICRT condition, and $H'(z^*) = 0$ for a unique $z^* \in \mathbb{R}_+$, then convertible debt is optimal for all investment levels I .*
- (c) *Assume that there exists \bar{z} such that $f_G(z) = f_B(z)$ for all $z \geq \bar{z}$, and $H'(0) > 0$. Further assume that $H'(z^*) = 0$ for a unique $z^* \in [0, \bar{z}]$. Then NICRT holds, and there exists an $\bar{I} > 0$ such that for all $I \leq \bar{I}$ warrants are optimal, whereas for all $I \geq \bar{I}$ convertible debt is optimal.*
- (d) *If $\lim_{z \uparrow \infty} H(z) = \bar{h} > 0$ and there exists a unique $z^* \in \mathbb{R}_+$ such that $H'(z^*) = 0$, then there exists an \bar{I} such that for all $I \leq \bar{I}$ straight debt is optimal, whereas for all $I \geq \bar{I}$ convertible debt is optimal.*

Part (a) of Proposition 7 assumes CSD. In this case, monotonicity of the function $H(z)$ implies that there is a z^* below which $(s^*)'(z) = 1$, for all $z \leq z^*$, with $(s^*)'(z) = 0$ otherwise, yielding straight debt as an optimal security. The intuition for the optimality of straight debt can be seen as follows. As discussed in Nachman and Noe (1994), CSD (and thus monotonicity of $H(z)$) requires that the ratio of the measure of the right tails of the probability distribution for the two types, $(1 - F_G(z))/(1 - F_B(z))$, is monotonically increasing in z .³⁰ For monotonic securities, this ratio can be interpreted as measuring the marginal cost of increasing the payouts to investors by \$1 for a type G firm relative to a type- B firm. When it becomes relatively more expensive for a firm of type G to increase payouts to investors as the firm value z becomes larger (that is, when information costs faced by a type- G firm are increasing in z) the proposition shows that the optimal security is debt. In this case, firms of better types prefer to have the maximum payout to investors for low realizations of z , that is in the (right) neighborhood of $z = 0$, and then to limit the payout to investors for high realizations of z . These considerations, together with the requirement that the security is monotonic, lead to the optimality of debt contracts.

The cases considered in parts (b) and (c) of Proposition 7 provide the conditions under which securities with equity-like components are optimal. The key driver of the optimal security choice is the size of the informational costs in the right tail of the payoff distribution, measured by $H(z)$.

³⁰Equivalently, CSD requires that the hazard rate of the payoff distribution for a type- G is smaller than that for a type- B for all values of z .

Under NICRT, we have that, in the limit, $H(z) = 0$ and, thus, that the information costs suffered by a type- G firm becomes progressively smaller as the firm value z increases. Part (b) of Proposition (7) shows that, in this case, type- G firms can reduce their overall dilution by maximizing the payout to investors in the right tail of the distribution, in addition to a neighborhood of $z = 0$. This happens because by increasing the payoffs in the right tail, where information costs are now low because of NICRT, allows the firm to correspondingly reduce the (fixed) payout in the middle of the distribution, where the information costs are now relatively high. This implies that the optimal security will initially have a unit slope, then a fixed payout, and then again a unit slope. Thus the optimal security will have the shape of a convertible bond, where the bond is convertible into 100% of equity with lump-sum payment to original shareholders equal to κ , which we will refer to as the “*conversion price*.”

Part (c) of the Proposition considers a particular (strong) form of NICRT. Specifically, in this case we have $H(z) = 0$ for all $z \geq \bar{z}$, and NICRT holds. Furthermore, the absence of information costs for $z \geq \bar{z}$ implies that, when the capital requirements are low, that is, for $I \leq \bar{I} \equiv \mathbb{E}[\max(z - \bar{z}, 0)]$, a type- G firm can raise financing without incurring any dilution. For $I \geq \bar{I}$, the firm cannot fully finance the project by pledging the right tail of the payoff distribution and, in that case, the proposition shows that convertible debt is optimal.

In part (d) of Proposition (7), neither CSD nor NICRT hold, since we have both a non-monotone function H and the h -ICRT condition holds for $\bar{h} > 0$. The proposition shows that the size of a project affects the financing choices of a firm: straight debt is optimal for low levels of I , while convertible debt becomes optimal for large levels of the investment I . This happens because when investment needs are low, the firm can finance the project by issuing only straight debt, a security that loads only in the left tail of the distribution, where the information costs are the lowest. For greater investment needs, under h -ICRT the firm again finds it optimal to maximize its payout to investors in the right tail of the distribution, as discussed for part (b) of the proposition, by issuing convertible debt.

We now consider the case in which the firm has pre-existing debt, as discussed in the last part of Section 4. The security design game is modified as follows. At the beginning of the period, the

firm chooses a security $s \in \mathbb{S}$, where the set \mathbb{S} satisfies (2)-(3), with the added constraints $s(z) = 0$ for all $z < K_0$, and

$$0 \leq s(z) \leq z - K_0, \text{ for all } z \geq K_0. \quad (23)$$

The presence of pre-existing debt changes the structure of information costs in a non-trivial way, because cash flows in the left tail of the distribution cannot any longer be pledged to new investors. This makes equity-like securities relatively more attractive.

Proposition 8. *Consider the optimal security design problem when the firm has a senior debt security with face value K_0 outstanding. Assume that the NICRT condition holds, and that there exists a unique z^* such that $H'(z^*) = 0$.*

(a) *If $H'(K_0) > 0$, then there exists \bar{I} such that: (i) warrants are optimal for $I < \bar{I}$, and (ii) convertible debt is optimal for $I \geq \bar{I}$.*

(b) *If $H'(K_0) < 0$, then the optimal securities are warrants.*

Proposition 8 provides conditions under which warrants arise as optimal financing instruments, in contrast to the case in which only straight debt or convertible debt solves the optimal security design problem that we discussed in Proposition 7. Intuitively, warrants are optimal securities when pre-existing debt has absorbed the information benefits in the left tail of the distribution in a right neighborhood of $z = 0$ (which generates, as discussed in the previous section, the optimality of debt when $K_0 = 0$). When K_0 is moderate, so that $H'(K_0) > 0$, NICRT implies that the optimal security design is one that always loads in the right tail, where information costs are the now the lowest (since now the left tail is already committed). In addition, when the financing needs are low, the firm is able to raise the required capital by issuing only warrants. When the financing needs are high, the firm raises the additional capital by also issuing (junior) debt, that is, by using convertible debt. When K_0 is large, so that $H'(K_0) < 0$, the firm will always find it optimal to issue only warrants (since the firm now faces decreasing information costs). We note that warrants can emerge as optimal securities when the firm has pre-existing debt in its capital structure, even when the asymmetric information environment is such that straight debt would be optimal in the absence of pre-existing debt.

We conclude this section with several numerical examples that shed light on the drivers of the optimal security design problem (18)-(19) and on the optimal securities characterized in Propositions 7 and 8.

We begin with examples based on the continuous distribution case introduced in Section 3.3. In these examples the firm has no pre-existing debt, $K_0 = 0$, and the initial investment is $I = 75$. When $x \leq 0.05$ the function $H(z)$ is monotonic, and straight debt is the optimal security (see the top right graph in Figure 1). For example, at $x = 0.05$, a straight debt security with a face value of $F = 83.8$ is optimal. When $x = 0.10$, the function $H(z)$ achieves a maximum at $z = 100$ and for $z \in [200, 300]$ we have $H(z) = 0$ (see middle right graph in Figure 1). In the $x = 0.10$ case, there is no asymmetric information in the right tail, so the NICRT condition holds. It is possible to verify that a convertible bond with a face value of $F = 24.9$ and with a conversion price at $\kappa = 193.8$ is the optimal security that finances the investment of $I = 75$. In the first case, when $x \leq 0.05$ and the CSD condition holds, the firm issues optimally risky debt, a security that loads primarily on the left tail of the distribution. In the second case, when $x = 0.10$ and NICRT holds, the firm issues first a debt tranche with a lower face value (and thus lower risk), and a warrants component that loads payouts to investors in the right tail, where there is no asymmetric information.

Next, consider the intermediate case in the bottom of Figure 1, where $x = 0.09$, so that neither NICRT nor CSD hold. If $I = 40$, the optimal security is standard debt, with $F = 42.2$. For larger investment levels, namely for $I \geq 42$, the optimal security includes a convertibility provision. For example, for $I = 45$, the optimal security is convertible debt with face value of $F = 44.44$ and a conversion price $\kappa = 238.2$. Furthermore, it is easy to show that for values of $I \in (42.0, 82.1)$ the optimal security will involve a straight debt component with $F = 44.4$, and a warrants component with a conversion trigger, κ , that is a decreasing function of the investment requirements I .

Finally, note that when $x = 0.10$, condition (c) in Proposition 7 is satisfied and issuing only warrants may be optimal. For example, when $I = 40$ a warrant with a conversion price $\kappa = 223.5$ is an optimal security.³¹ When $x = 0.10$, a type- G firm does not suffer any dilution for cash flows in the $[200, 300]$ range and the asymmetric information problem can be entirely avoided by restricting

³¹Using the notation in Proposition 7, one can verify that $\bar{I} = 50$ in our example, that is, warrants are optimal for all $I \leq 50$, whereas for $I > 50$ convertible bonds are the optimal securities.

payouts to this range. When the required investment, I , is sufficiently small, the firm’s cash needs can be entirely satisfied by issuing a security that loads *only* in the right tail, and warrants become the optimal security. For larger investment requirements (as was the case for $I = 75$), the firm exhausts the capacity to make payouts to investors out of the right tail, inducing the firm to also issue (straight) debt, making convertible bonds the optimal security.

The main feature of the examples we discussed so far is to stress the key role of the exposure to asymmetric information in the right tail of the payoff distribution. In particular, these examples show that once CSD is violated, it may be “cheaper” to issue a security that concentrates payouts in the right tail of the distribution, in contrast to the standard pecking order intuition.

We conclude by revisiting the real options model from Section 5, and study three different constellations of parameter values that numerically illustrate the results of Proposition 7. Table 3 presents three different scenarios where, respectively, standard debt (Case A), convertible debt (Case B) and warrants (Case C) are optimal securities. For each of these cases, Figure 6 plots the $H(z)$ function in the left panels, and the optimal security in the right panels, each row corresponding to each of the cases in Table 3. In all cases, we assume that $p = 0.5$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\rho = 0.5$, $T = 5$, and that $I_T = 50$.

The first scenario (Case A) presents the case where the asymmetric information is concentrated entirely in the high volatility asset, Y . Namely, we set $\bar{X}_G = \bar{X}_B = 100$, $\bar{Y}_G = 250$, $\bar{Y}_B = 150$ and $I = 100$. In this case, the $H(z)$ function is monotone over its whole domain (see the top left graph in Figure 6). This implies that the optimal security will have unit slope when $H(z) < \gamma$ (where γ is represented by the horizontal dotted line) and will have zero slope when $H(z) \geq \gamma$. Thus, the optimal security is standard debt with a face value K , determined by $H(K) = \gamma$ (which is equal to $K = 138.8$).

In the second scenario, (Case B), the NICRT condition holds since the asymmetric information is concentrated entirely in the low-volatility asset, X . Namely, we set $\bar{Y}_G = \bar{Y}_B = 200$, $\bar{X}_G = 150$, $\bar{X}_B = 50$, and $I = 120$. In this case, the $H(z)$ function is “hump-shaped.” It is first an increasing and then a decreasing function of firm value z (see the middle left graph in Figure 6). This implies that the optimal security will have unit slope when $z < K$ (where $H(z) < \gamma$) and will have zero

slope for $K \leq z \leq \kappa$ (where $H(z) \geq \gamma$) and will have unit slope when for $z \geq \kappa$ (where again $H(z) < \gamma$). Thus, the optimal security is a convertible debt contract with face value $K = 69.5$ and conversion price $\kappa = 593.4$, where the values $\{K, \kappa\}$ satisfy $H(K) = H(\kappa) = \gamma$. As shown in Proposition 2 securities load in the lower-end of the payoffs, due to the usual Myers and Majluf (1984) intuition, but also on upper-end of the payoff distribution, because of the NICRT property introduced in our paper.

In the last numerical example, (Case C), we consider the effect of pre-existing debt on the security design problem discussed in Proposition 8. We modify Case B by assuming that the firm has debt outstanding with $K_0 = 100$, and that the initial investment is 70 (see the lower left graph in Figure 6). It can be calculated that the value of the pre-existing debt is 79.3, while total firm value (debt plus equity) is equal to 259.2. In this case, the $H(z)$ function is the same as in Case B, but now $H(K_0) \geq \gamma$, which means that the optimal security will have zero slope for $K_0 \leq z \leq \kappa$ (where $H(z) \geq \gamma$) and will again have unit slope for $z \geq \kappa$ (where $H(z) < \gamma$). Thus, the optimal security design is a warrant with an exercise price of $\kappa = 502.5$ (where $H(\kappa) = \gamma$) as in the case (i) of part (a) of Proposition 8.

Finally, it can be shown that if we change the initial investment from 70 to 120, as in the previous Case B, the optimal security will again be convertible debt, where the face value of the new (junior) debt is $K = 135.7$, and the conversion price becomes $\kappa = 317.2$, as in case (ii) of part (a) of Proposition 8. It can also be shown that, given the parameters values of Case C, warrants will always be optimal if the pre-existing debt has a face value of $K_0 \geq 199$, as in part (b) of Proposition 8. This happens because, in this case, $H'(K_0) < 0$, and $H(K_0) \geq \gamma$, which means that the optimal security has zero slope for $K_0 \leq z \leq \kappa$ (where $H(z) \geq \gamma$) and will have unit slope when for $z \geq \kappa$ (where $H(z) < \gamma$).

7 Conclusion

In this paper, we revisit the pecking order of Myers and Majluf (1984) and Myers (1984) in the context of a real options model. We show that when insiders are relatively better informed on the assets in place of their firm, rather than on its growth opportunities, equity financing can dominate

(i.e., be less dilutive than) debt financing, reversing the pecking order. We find that equity is more likely to dominate debt for younger firms that have larger investment needs and with riskier, more valuable growth, opportunities. Thus, our model can explain why high-growth firms may prefer equity over debt, and then switch to debt financing as they mature.

More generally, we consider firms endowed with a portfolio of heterogeneous assets with different exposure to asymmetric information. We argue that deviations from the pecking order theory can occur when the asset with relatively lower volatility has greater exposure to asymmetric information. This means that a firm's preference for debt versus equity financing is not driven by its overall *level* of asymmetric information but, rather, by the *composition* of its assets and by the *location* of the asymmetric information across assets. It also means that, contrary to common intuition, equity financing can dominate debt financing under asymmetric information, even in cases where individual assets would be financed by debt when taken in isolation. Our results suggest that the relationship between asymmetric information and the choice of financing is more subtle than previously believed. Finally, allowing for pre-existing debt in the firm's capital structure makes a preference for equity over debt relatively more likely, all else equal. In particular, pre-existing high leverage implies a higher propensity of equity financing, which suggests that in a dynamic model of securities offering, asymmetric information may lead to mean reversion in leverage ratios. Overall, these predictions are novel within models featuring only informational frictions and invite further research.

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Appendix

Proof of Proposition 1. In a separating equilibrium $\{s_G^*, s_B^*\}$ where we have that $s_G^* \neq s_B^*$, $p(s_G^*) = 1$, and $p(s_B^*) = 0$, which implies that $V^*(s_\theta^*) = \mathbb{E}[s_\theta^*(Z_\theta)]$ and that $W(\theta, s_\theta^*, V^*(s_\theta^*)) = \mathbb{E}[Z_\theta] - I$. This implies that $W(B, s_G^*, V(s_G^*)) - W(B, s_B^*, V(s_B^*)) = V(s_G^*) - \mathbb{E}[s_G^*(Z_B)] = \mathbb{E}[s_G^*(Z_G)] - \mathbb{E}[s_G^*(Z_B)] > 0$ by FOSD. Thus, the pair $\{s_G^*, s_B^*\}$ cannot be an equilibrium. Furthermore, if in a candidate pooling equilibrium where the security s^* is offered by both types of firms, we have that $V^*(s^*) > I$, consider the scaled down contract γs^* for $\gamma \in (0, 1)$. Then, there is at least one value of $\gamma \in (0, 1)$ such that $p(\gamma s^*) = p$, by passive beliefs, $V^*(\gamma s^*) \geq I$ and $W(G, \gamma s^*, V^*(\gamma s^*)) = \mathbb{E}[Z_G] - \gamma(\mathbb{E}[s^*(Z_G)] - V(s^*(Z_G))) - I > \mathbb{E}[Z_G] - (\mathbb{E}[s^*(Z_G)] - V(s^*(Z_G))) - I = W(G, s^*, V^*(s^*))$, a contradiction. Thus, any pooling equilibrium must satisfy the budget constraint with equality, $V(s) = I$. ■

Proof of Proposition 2. In order to prove the first statement, we argue that the distribution of the good type dominates the distribution of the bad type in the likelihood ratio sense, namely $f_G(z)/f_B(z)$ is monotonically non-decreasing for all $z \in \mathbb{R}_+$. From basic principles we have:

$$\begin{aligned} \frac{f_G(z)}{f_B(z)} &= \frac{\frac{1}{z\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_G}{\sigma}\right)^2}}{\frac{1}{z\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_B}{\sigma}\right)^2}} \\ &= e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_G}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{\log(z)-\mu_B}{\sigma}\right)^2} \\ &= e^{-\frac{1}{2}\frac{(\mu_G^2-\mu_B^2)}{\sigma^2} + \log(z)\left(\frac{\mu_G-\mu_B}{\sigma^2}\right)} \\ &= e^{-\frac{1}{2}\frac{(\mu_G^2-\mu_B^2)}{\sigma^2}} z^{\left(\frac{\mu_G-\mu_B}{\sigma^2}\right)}; \end{aligned}$$

which is monotonically increasing in z when $\mu_G > \mu_B$, as we set to prove. Since the likelihood ratio order implies conditional stochastic dominance (Shaked and Shanthikumar, 2007), this concludes the proof.

In order to prove the second statement, we start with case (a). Using l'Hopital's rule, one has

$$\lim_{z \uparrow \infty} H(z) = \lim_{z \uparrow \infty} \frac{F_B(z) - F_G(z)}{1 - F(z)} \quad (24)$$

$$= \lim_{z \uparrow \infty} \frac{f_G(z) - f_B(z)}{pf_G(z) + (1-p)f_B(z)}. \quad (25)$$

From basic principles it is clear that:

$$\mathbb{P}(Z_\theta = z) \equiv f_\theta(z) = f_{x\theta}(z) + f_{y\theta}(z)$$

with

$$\begin{aligned} f_{x\theta}(z) &= \frac{1}{z\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_{x\theta}}{\sigma_x}\right)^2} N\left(\frac{\log(z)-\mu_{y\theta}}{\sigma_y\sqrt{1-\rho^2}} - \rho\frac{\log(z)-\mu_{x\theta}}{\sigma_x\sqrt{1-\rho^2}}\right) \\ f_{y\theta}(z) &= \frac{1}{z\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_{y\theta}}{\sigma_y}\right)^2} N\left(\frac{\log(z)-\mu_{x\theta}}{\sigma_x\sqrt{1-\rho^2}} - \rho\frac{\log(z)-\mu_{y\theta}}{\sigma_y\sqrt{1-\rho^2}}\right) \end{aligned}$$

where $\mu_{x\theta} = \log(X_\theta)$ and $\mu_{y\theta} = \log(Y_\theta)$. The limit in (25) is easy to compute by factoring out leading terms. We note that when $\sigma_y > \sigma_x$ the right tail behavior is determined by the piece of the densities $f_\theta(z)$ that corresponds to the density of Y . When $c_y = 0$, the limit of these densities is zero, as we set to prove.

Consider next case (b), in which $Z_\theta = X_\theta + Y_\theta$. Let $F_m(z)$ denote the distribution function of a lognormal random variable with log-mean μ_{Gy} and log-variance σ_y^2 . Since $1 - F(z) = p(1 - F_B(z)) + (1 - p)(1 - F_G(z))$, we have that

$$\lim_{z \uparrow \infty} \frac{1 - F(z)}{1 - F_m(z)} = \lim_{z \uparrow \infty} p \frac{1 - F_B(z)}{1 - F_m(z)} + (1 - p) \frac{1 - F_G(z)}{1 - F_m(z)}. \quad (26)$$

Using Theorem 1 from Asmussen and Rojas-Nandayapa (2008), we have that

$$\lim_{z \uparrow \infty} \frac{1 - F_G(z)}{1 - F_m(z)} = 1, \quad (27)$$

and that

$$\lim_{z \uparrow \infty} \frac{1 - F_B(z)}{1 - F_m(z)} = \begin{cases} 1 & \text{if } \mu_{yB} = \mu_{yG}, \\ 0 & \text{if } \mu_{yB} < \mu_{yG}. \end{cases} \quad (28)$$

Further note that

$$H(z) = \left(\frac{1 - F_G(z)}{1 - F_m(z)}\right) \left(\frac{1 - F(z)}{1 - F_m(z)}\right)^{-1} - \left(\frac{1 - F_B(z)}{1 - F_m(z)}\right) \left(\frac{1 - F(z)}{1 - F_m(z)}\right)^{-1}. \quad (29)$$

Using this last expression together with (26)-(28), we conclude that

$$\lim_{z \uparrow \infty} H(z) = \begin{cases} 0 & \text{if } \mu_{yB} = \mu_{yG}, \\ (1 - p)^{-1} & \text{if } \mu_{yB} < \mu_{yG}. \end{cases} \quad (30)$$

This completes the proof of case (b).

In order to see case (c), note that

$$\mathbb{P}(X_\theta + \max(Y_\theta - I_T, 0) > z) > \mathbb{P}(X_\theta + Y_\theta > z + I_T) \quad (31)$$

and

$$\mathbb{P}(X_\theta + \max(Y_\theta - I_T, 0) > z) < \mathbb{P}(X_\theta + Y_\theta > z). \quad (32)$$

These two inequalities serve as a bound for the limit of the function $H(z)$ for the random variable $X_\theta + \max(Y_\theta - I_T, 0)$. The two bounds fall within the scope of the proof of case (b) of the Proposition, and therefore have the same limits, which coincide with those of case (c). This completes the proof. \blacksquare

Proof of Proposition 3. From the budget constraint for equity and debt securities, one has that

$$\lambda = \frac{p\mathbb{E}[\min(Z_G, K)] + (1-p)\mathbb{E}[\min(Z_B, K)]}{p\mathbb{E}[Z_G] + (1-p)\mathbb{E}[Z_B]} \quad (33)$$

Using (33) in (11) and comparing this to (12) one easily arrives at (14). \blacksquare

Proof of Proposition 4. The following result from Shaked and Shanthikumar (2007) is useful.

Lemma 1 (Theorem 1.B.12 from Shaked and Shanthikumar (2007)). *Given two distribution functions F_G and F_B , the following two statements are equivalent: (a) F_G conditionally stochastically dominates F_B ; (b) $\mathbb{E}[\alpha(X_B)]\mathbb{E}[\beta(X_G)] \leq \mathbb{E}[\alpha(X_G)]\mathbb{E}[\beta(X_B)]$, for all functions α and β such that β is non-negative and α/β and β are non-decreasing.*

Let $\alpha(z) = z$ and $\beta(z) = \min(z, K)$ for some $K \geq 0$. Clearly β is non-decreasing and non-negative for $x \geq 0$. Furthermore, $\alpha(z)/\beta(z) = z/\min(z, K)$ is non-decreasing. Thus, if F_G conditionally stochastically dominates F_B it must be that

$$\mathbb{E}[Z_B]\mathbb{E}[\min(Z_G, K)] \leq \mathbb{E}[Z_G]\mathbb{E}[\min(Z_B, K)]$$

which clearly rules out (14). \blacksquare

Proof of Proposition 5. From the definition of the reverse pecking order, we are set to prove, by contradiction, that $\mathcal{D}_D > \mathcal{D}_E$ cannot hold if $\hat{z} > \bar{z}$, i.e., if UC does not hold. The reverse pecking order condition, if $\hat{z} > \bar{z}$, can be written as

$$\int_0^{\bar{z}} \underbrace{(\min(K, z) - \lambda z)}_{>0} \underbrace{c(z)}_{<0} dz + \int_{\bar{z}}^{\hat{z}} \underbrace{(K - \lambda z)}_{<0} \underbrace{c(z)}_{<0} dz + \int_{\hat{z}}^\infty \underbrace{(K - \lambda z)}_{<0} \underbrace{c(z)}_{>0} dz > 0. \quad (34)$$

We note that since g is the difference of two densities, it must be the case that

$$\int_0^\infty c(z)dz = 0; \quad \Rightarrow \quad - \int_0^{\hat{z}} c(z)dz = \int_{\hat{z}}^\infty c(z)dz$$

Further, we have

$$\begin{aligned}
\int_{\hat{z}}^{\infty} (\lambda z - K)c(z)dz &> \int_{\hat{z}}^{\infty} (\lambda \hat{z} - K)c(z)dz \\
&= (\lambda \hat{z} - K) \int_{\hat{z}}^{\infty} c(z)dz \\
&= (K - \lambda \hat{z}) \int_0^{\hat{z}} c(z)dz \\
&> (K - \lambda \hat{z}) \int_{\bar{z}}^{\hat{z}} c(z)dz \\
&> \int_{\bar{z}}^{\hat{z}} (K - \lambda z)c(z)dz.
\end{aligned}$$

We note that the last two inequalities follow from the fact that $\int_0^{\bar{z}} c(z)dz < 0$, and by our conjecture that $\bar{z} < \hat{z}$, which implies that $K - \lambda \hat{z} < 0$. Therefore, these inequalities imply that the sum of the last two terms in (34) is negative, and since the first one is negative as well, it follows that (34) cannot hold, and thus $\mathcal{D}_D - \mathcal{D}_E < 0$, i.e., a reversal of the pecking order cannot obtain if UC is not true. ■

Proof of Proposition 6. See Theorem 8 in Nachman and Noe (1994). From the Lagrangian in (21), we note that the objective function is linear in the choice variable $s'(z)$. Thus, only corner solutions are optimal. When $H(z) < \gamma$ the Lagrangian is minimized making $s'(z)$ be equal to its upper bound, $s'(z) = 1$, whereas for $H(z) > \gamma$, the minimization calls for setting $s'(z)$ to its lower bound, $s'(z) = 0$. ■

Proof of Proposition 7. Since H is increasing in (a), there is a single crossing point z such that $H(z) = \gamma$, for any $\gamma \in \mathbb{R}_+$. The claim in (a) follows immediately from Proposition 6. Assuming NICRT, and that $H'(z^*) = 0$ at most once, it is immediate that there are two unique crossing points for $H(z) = \gamma$, for any $\gamma \in \mathbb{R}_+$. The claim in (b) is immediate from Proposition 6. Under the conditions of case (c), we have that NICRT holds. Since $H'(0) > 0$, for a sufficiently low \bar{I} all investment levels $I \leq \bar{I}$ are associated with $s'(z) = 0$ for all $z \leq z^*$. This will be true up to the level \bar{I} that is possible to finance pledging all residual cash flows above z^* , namely $\bar{I} = \mathbb{E}[\max(Z - z^*, 0)]$. For $I > \bar{I}$, we have that the condition $H(z) = \gamma > 0$ defines two crossings, and the optimal securities are convertible bonds, as in (b). Case (d) is analogous to case (b), but noting that for $\gamma \leq \bar{\gamma}$ there is a single point satisfying $H(z^*) = \gamma$, but two such points for γ sufficiently large. ■

Proof of Proposition 8. The proof is analogous to that of Proposition 6. The first-order conditions require $s'(z)$ to be either one (or zero) at points for which $H(z) < \gamma$ (or $H(z) > \gamma$). Under the conditions in (b), and the initial assumptions, there is only one crossing, and all mass of the security is concentrated in the right tail. This occurs for low values of γ , or equivalently of the investment I . The claim in (a) mirrors case (b) from Proposition 7. ■

Table 1: A simple example

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 2. The payoff of the firm is given by a trinomial random variable $Z \in \{z_1, z_2, z_3\}$. The growth opportunity requires an investment of $I = 60$, and generates an extra cash flow of 200 in the high state. The payoff and the state probabilities are summarized below.

Payoffs	z_1	z_2	z_3
Assets in place	10	100	100
Growth opportunity	0	0	200
Total payoff	10	100	300
Distributions	z_1	z_2	z_3
Good-type, f_G	0.2	0.4	0.4
Bad-type, f_B	0.3	$0.4 - x$	$0.3 + x$

The column labelled “Pooled value” below computes the expected value of the firm, $\mathbb{E}[Z]$, where each type is assumed equally likely. The variable x can take values in $[0, 0.10]$, to guarantee that the distribution f_G first-order stochastically dominates f_B . The variable λ denotes the fraction of equity the firm needs to issue to finance the investment of $I = 60$. The column labelled \mathcal{D}_E denotes the dilution costs of equity, namely $\lambda(\mathbb{E}[Z_G] - \mathbb{E}[Z_B])$. For all values of x , the firm can also finance the project with a debt security with a face value $K = 76.7$, for which the dilution costs, $\mathcal{D}_D \equiv \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)]$, are 6.7 (last column).

x	$\mathbb{E}[Z_G]$	$\mathbb{E}[Z_B]$	Pooled value	λ	\mathcal{D}_E	\mathcal{D}_D
0.00	162	133	147.5	0.407	11.8	6.7
0.01	162	135	148.5	0.404	10.9	6.7
0.02	162	137	149.5	0.401	10.0	6.7
0.03	162	139	150.5	0.399	9.2	6.7
0.04	162	141	151.5	0.396	8.3	6.7
0.05	162	143	152.5	0.393	7.5	6.7
0.06	162	145	153.5	0.391	6.6	6.7
0.07	162	147	154.5	0.388	5.8	6.7
0.08	162	149	155.5	0.386	5.0	6.7
0.09	162	151	156.5	0.383	4.2	6.7
0.10	162	153	157.5	0.381	3.4	6.7

Table 2: Optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5. The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, with $\mathbb{E}[X_\theta] = X_\theta$, $\mathbb{E}[Y_\theta] = Y_\theta$. We further denote $\text{var}(\log(X_\theta)) = \sigma_x^2 T$, $\text{var}(\log(Y_\theta)) = \sigma_y^2 T$, and $\text{cov}(\log(X_\theta), \log(Y_\theta)) = \rho \sigma_x \sigma_y T$.

	Symbol	Value				
Base case						
Value of assets in place for the good type	\bar{X}_G	125				
Value of assets in place for the bad type	\bar{X}_B	75				
Value of new assets for the good type	\bar{Y}_G	205				
Value of new assets for the bad type	\bar{Y}_B	195				
Good type firm value	$\mathbb{E}[Z_G]$	307.9				
Bad type firm value	$\mathbb{E}[Z_B]$	248.2				
Time to maturity	T	15				
Volatility of assets in place	σ_x	0.30				
Volatility of new assets	σ_y	0.60				
Probability of the good type	p	0.50				
Correlation between assets	ρ	0				
Investment amount	I	100				
Investment at maturity	I_T	50				
Equilibrium outcomes						
Value of firm post-investment	$\mathbb{E}[Z_T]$	278.1				
Equity fraction issued	λ	0.360				
Face value of debt	K	218.4				
Credit spread	$r_D = (K/D)^{1/T} - 1$	5.3%				
Dilution costs of debt	$\mathcal{D}_D = \mathbb{E}[\min(Z_{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$	23.7				
Dilution costs of equity	$\mathcal{D}_E = \lambda(\mathbb{E}[Z_{GT}] - \mathbb{E}[Z_{BT}])$	21.5				
Relative dilution	$\mathcal{D}_D/\mathcal{D}_E$	1.10				
Comparative statics						
New parameter(s)	Equity share λ	Face value K	Spread r_D	Debt dilution \mathcal{D}_D	Equity dilution \mathcal{D}_E	Relative dilution $\mathcal{D}_D/\mathcal{D}_E$
$\bar{Y}_G = \bar{Y}_B = 200$	0.360	218.3	5.3%	22.9	18.0	1.27
$\bar{Y}_G = 225, \bar{Y}_B = 175$	0.359	219.4	5.4%	26.9	35.3	0.76
$\bar{X}_G = \bar{X}_B = 100$	0.360	213.9	5.2%	0.7	3.5	0.21
$\bar{X}_G = 150, \bar{X}_B = 50$	0.360	233.4	5.8%	49.7	39.5	1.26
$\sigma_x = 0.4$	0.360	290.6	7.4%	21.6	21.5	1.01
$\sigma_y = 0.8$	0.344	316.6	8.0%	31.6	20.6	1.53
$I_T = 0$	0.333	169.8	3.6%	17.6	20.0	0.88
$I_T = 100$	0.375	247.9	6.2%	26.5	22.3	1.19
$K_0 = 20$	0.386	303.3	7.7%	29.5	23.0	1.28
$K_0 = 40$	0.410	406.1	9.8%	34.5	23.4	1.47

Table 3: Optimal security design problem

The table presents the parameter values and equilibrium outcomes of the security design problem discussed in Section 6. The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, with $\mathbb{E}[X_\theta] = X_\theta$, $\mathbb{E}[Y_\theta] = Y_\theta$. We further denote $\text{var}(\log(X_\theta)) = \sigma_x^2 T$, $\text{var}(\log(Y_\theta)) = \sigma_y^2 T$, and $\text{cov}(\log(X_\theta), \log(Y_\theta)) = \rho \sigma_x \sigma_y T$. The labels “Straight debt,” “Convertibles,” and “Warrants” refer to the functions $s(z) = \min(K, z)$, $s(z) = \min(K, z) + \max(z - \kappa, 0)$, and $s(z) = \max(z - \kappa, 0)$ respectively.

	Symbol	Case A	Case B	Case C
Primitives				
Value of assets in place type G	\bar{X}_G	100	150	150
Value of assets in place type B	\bar{X}_B	100	50	50
Value of new assets type G	\bar{Y}_G	250	200	200
Value of new assets type B	\bar{Y}_B	150	200	200
Time to maturity	T	5	5	5
Volatility of assets in place	σ_x	0.3	0.3	0.3
Volatility of new assets	σ_y	0.6	0.6	0.6
Probability of the good type	p	0.5	0.5	0.5
Correlation between assets	ρ	0.5	0.5	0.5
Pre-existing debt face value	K_0	0	0	100
Initial investment	I	100	120	70
Investment at exercise	I_T	50	50	50
Equilibrium outcomes				
Optimal security	$s(z)$	Straight debt	Convertibles	Warrants
Face value	K	138.8	69.5	—
Conversion trigger/exercise price	κ	—	593.4	502.5

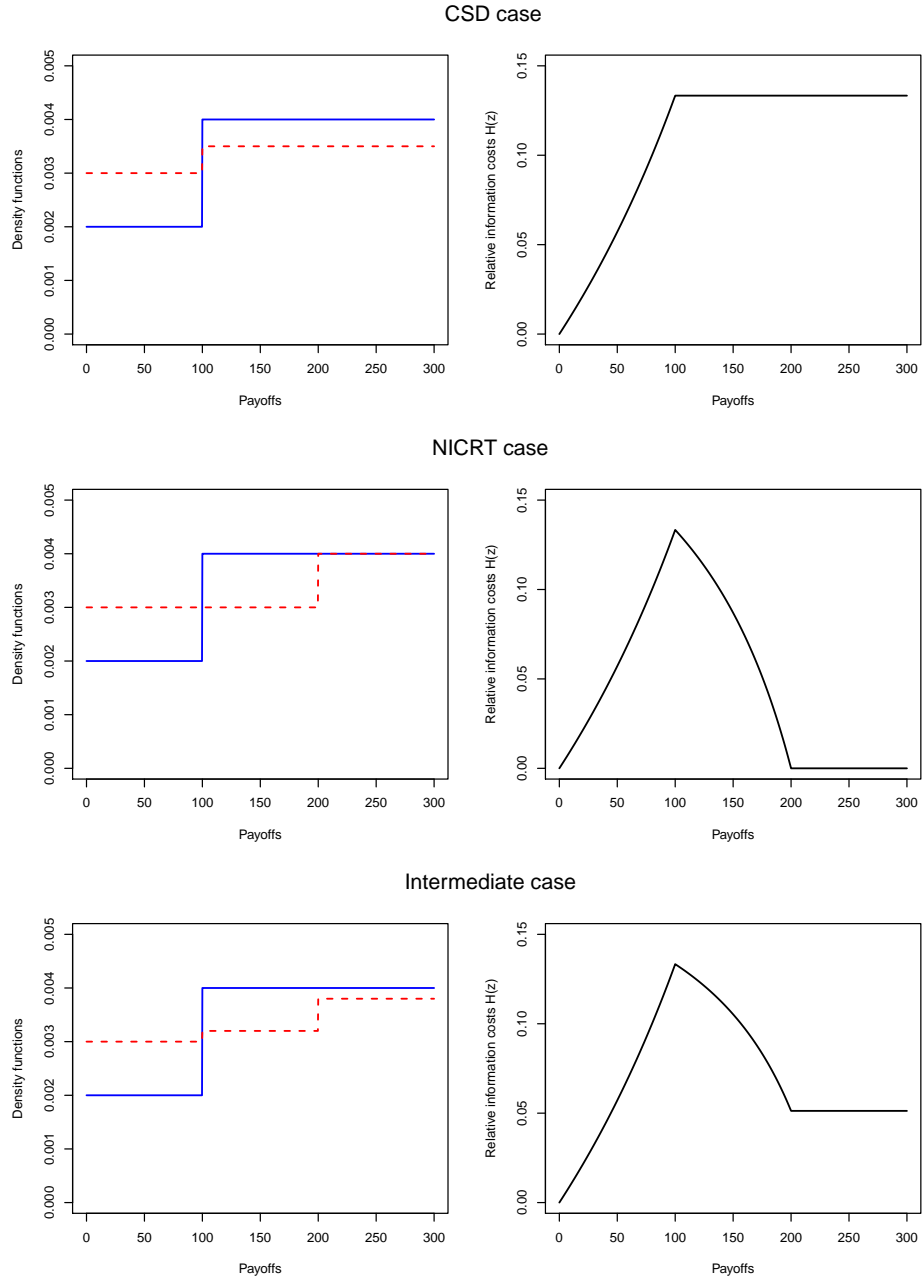


Figure 1: The left panels plot the densities for the high-type (in blue) and the bad-type (in red), whereas the right panels plot the relative information costs, the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$. The parameter values correspond to the example discussed in section 3.3, with $x = 0.05$ for the top two graphs (“CSD case”), with $x = 0.10$ for the middle graphs (“NICRT case”), and $x = 0.09$ for the bottom graphs (“Intermediate case”).

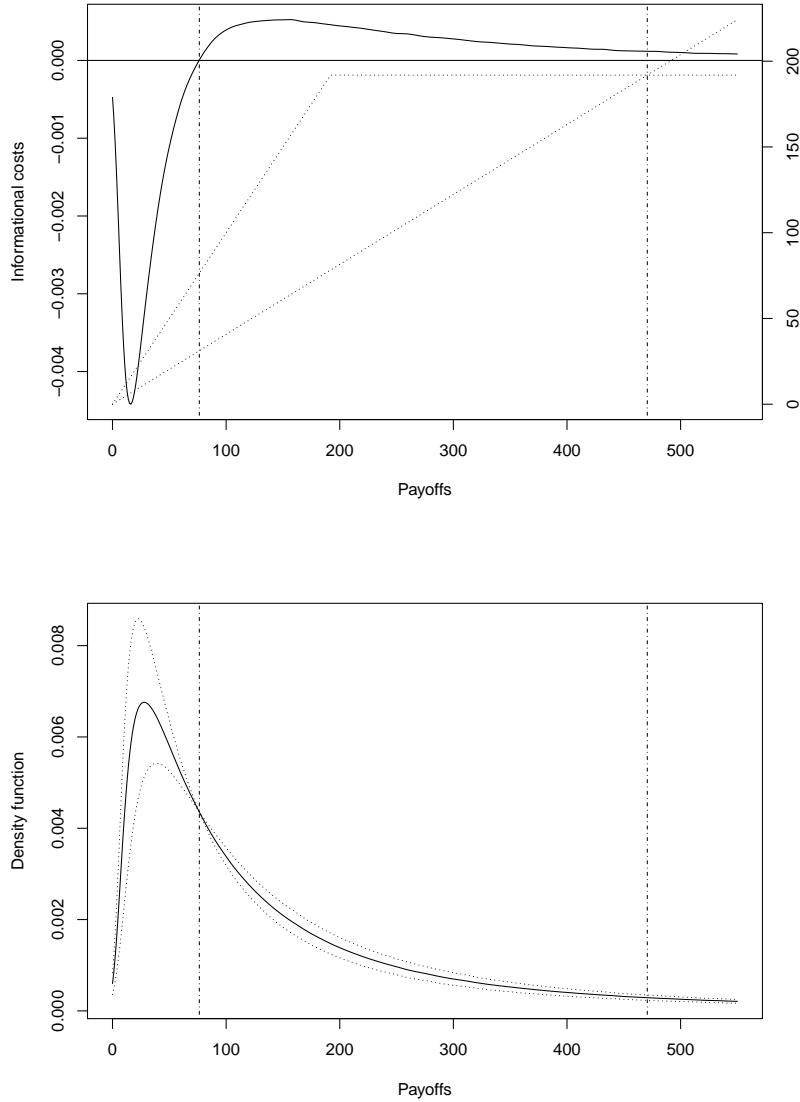


Figure 2: The top graph plots on the x -axis the payoffs from the firm at maturity, and on the y -axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ (y -axis labels on the left), and as dotted lines the payoffs from debt and equity (y -axis labels on the right). The left-most vertical dashed line is the point \hat{z} for which $f_G(\hat{z}) = f_B(\hat{z})$, so points to the right of that line have positive information costs. The right-most vertical dashed line is the point \bar{z} for which $K = \lambda\bar{z}$, so for payoffs to the right of that line equityholders receive more than debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal. The parameter values used in the figures are $\bar{X}_G = 125$, $\bar{X}_B = 75$, $\bar{Y}_G = 205$, $\bar{Y}_B = 195$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\rho = 0$, $T = 15$, $p = 0$, $I = 100$, $I_T = 50$. The dilution costs of debt for these parameters are $\mathcal{D}_D = 23.7$, whereas those of equity are $\mathcal{D}_E = 21.5$.

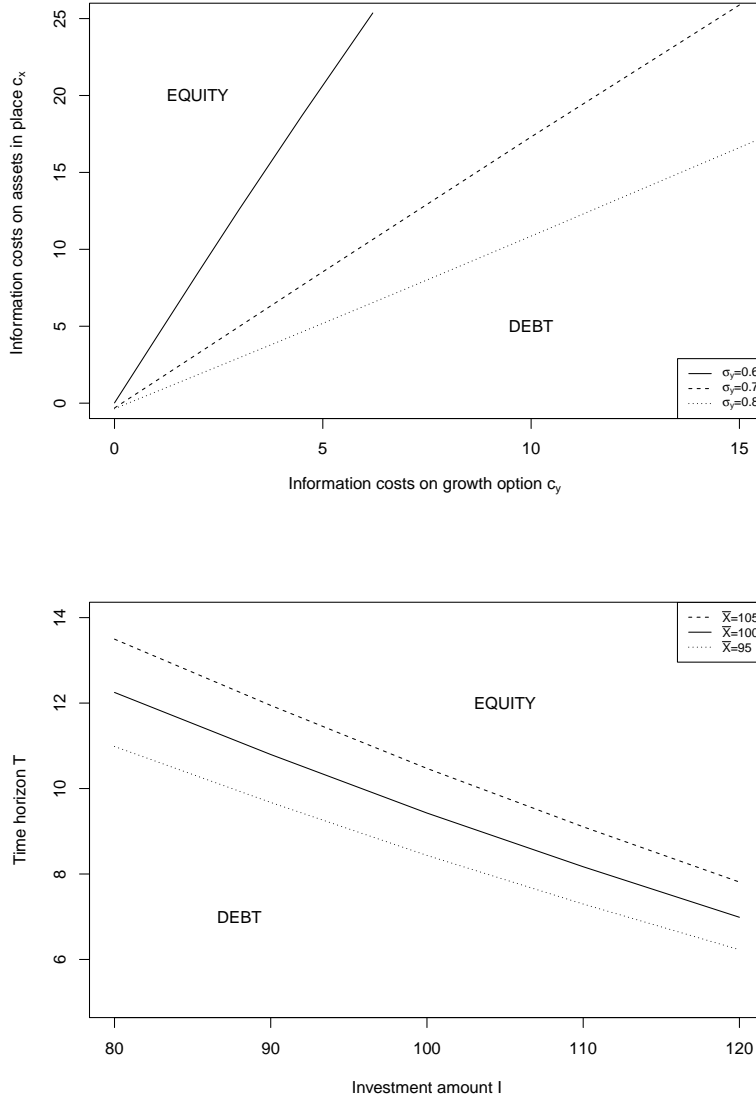


Figure 3: The top graph plots the set of points (c_y, c_x) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $\bar{X} = 100$, $\bar{Y} = 200$, $\sigma_x = 0.3$, $I = 120$, $I_T = 50$, $T = 10$, $\rho = 0$ and $p = 0.5$. Recall we set $\bar{X}_G = \bar{X} + c_x$ and $\bar{X}_B = \bar{X} - c_x$, and similarly $\bar{Y}_G = \bar{Y} + c_y$ and $\bar{Y}_B = \bar{Y} - c_y$. The solid line corresponds to the case where $\sigma_y = 0.6$, whereas the other two lines correspond to $\sigma_y = 0.7$ and $\sigma_y = 0.8$. Debt is optimal for pairs of (c_y, c_x) below the lines, whereas equity is optimal above the lines. The bottom graph plots the set of points (I, T) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $\bar{Y} = 200$, $\sigma_x = 0.3$, $I_T = 50$, $T = 10$, $c_x = 25$, $c_y = 0$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case where $\bar{X} = 100$, whereas the other two lines correspond to $\bar{X} = 105$ and $\bar{X} = 95$. Debt is optimal for pairs of (I, T) below the lines, whereas equity is optimal above the lines.

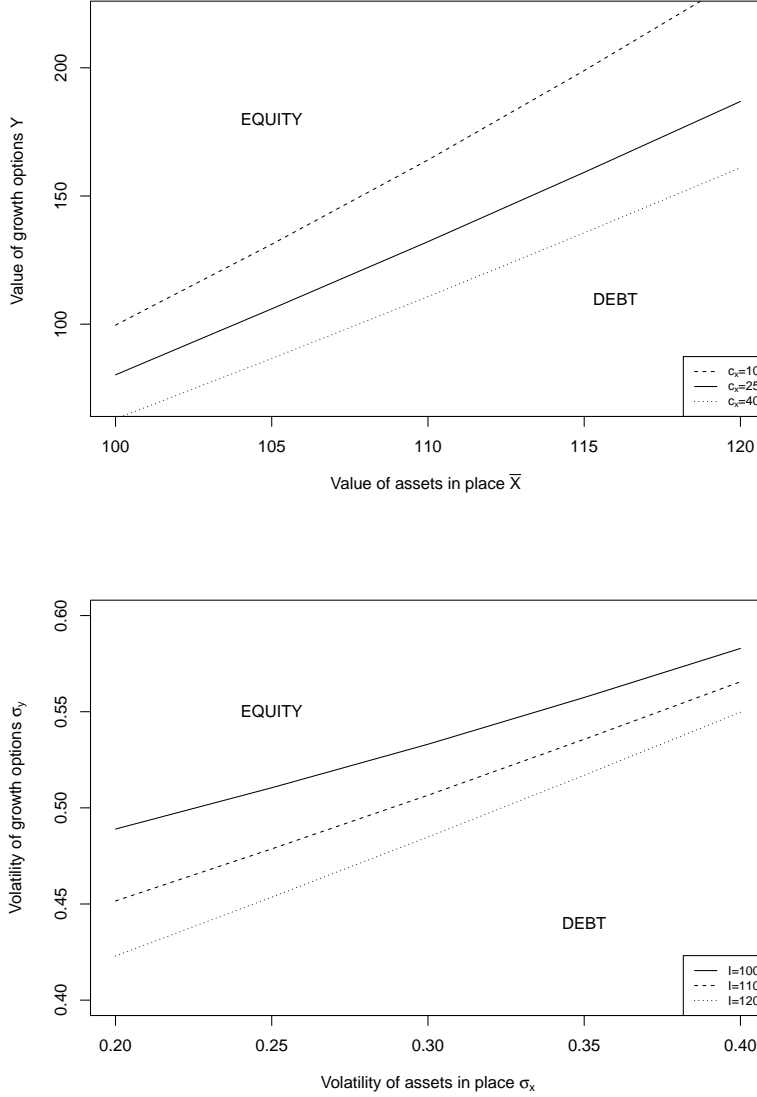


Figure 4: The top graph plots the set of points (\bar{X}, \bar{Y}) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $I = 110$, $T = 15$, $I_T = 50$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $c_x = 25$, whereas the other two lines correspond to $c_x = 10$ and $c_x = 40$. Debt is optimal for pairs of (\bar{X}, \bar{Y}) below the lines, whereas equity is optimal above the lines. The bottom graph plots the set of points (σ_x, σ_y) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\bar{X} = 100$, $\bar{Y} = 150$, $T = 15$, $I_T = 50$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $I = 110$, whereas the other two lines correspond to $I = 100$ and $I = 120$. Debt is optimal for pairs of (σ_x, σ_y) below the lines, whereas equity is optimal above the lines.

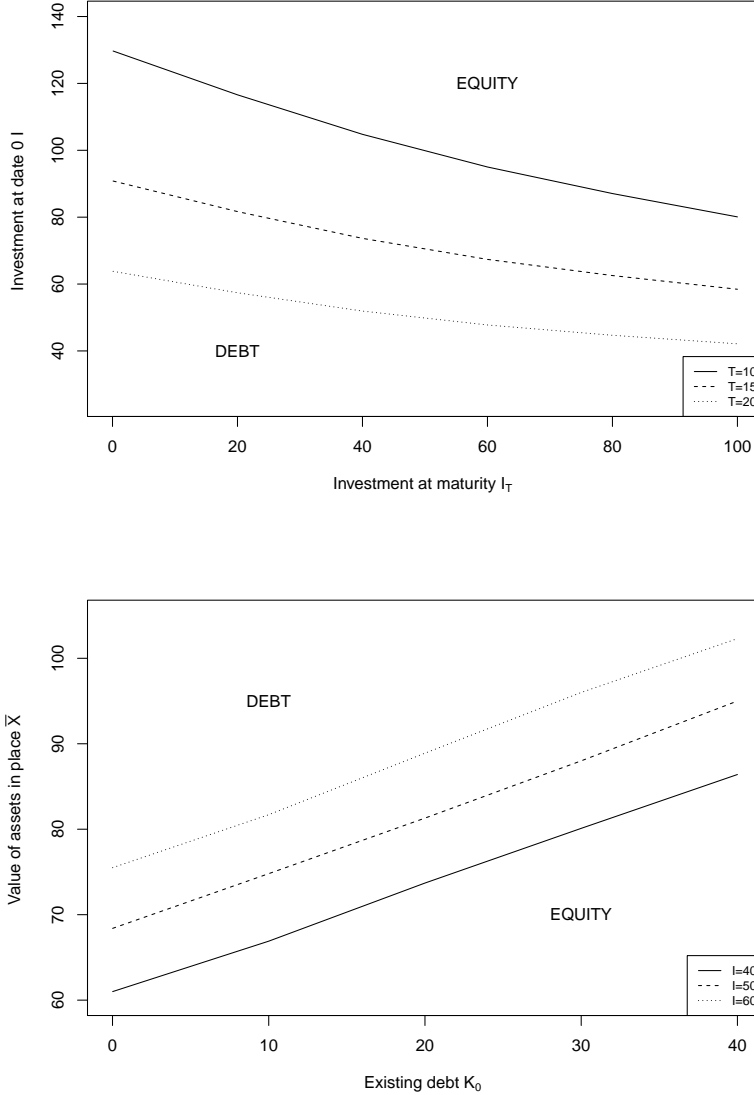


Figure 5: The top graph plots the set of points (I_T, I) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $Y_G = Y_B = 175$, $\bar{X} = 100$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $T = 10$, whereas the other two lines correspond to $T = 15$ and $T = 20$. Debt is optimal for pairs of (I_T, I) below the lines, whereas equity is optimal above the lines. The bottom graph plots the set of points (K_0, \bar{X}) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\bar{Y} = 175$, $I_T = 0$, $T = 10$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $I = 40$, whereas the other two lines correspond to $I = 50$ and $I = 60$. Equity is optimal for pairs of (K_0, \bar{X}) below the lines, whereas debt is optimal above the lines.

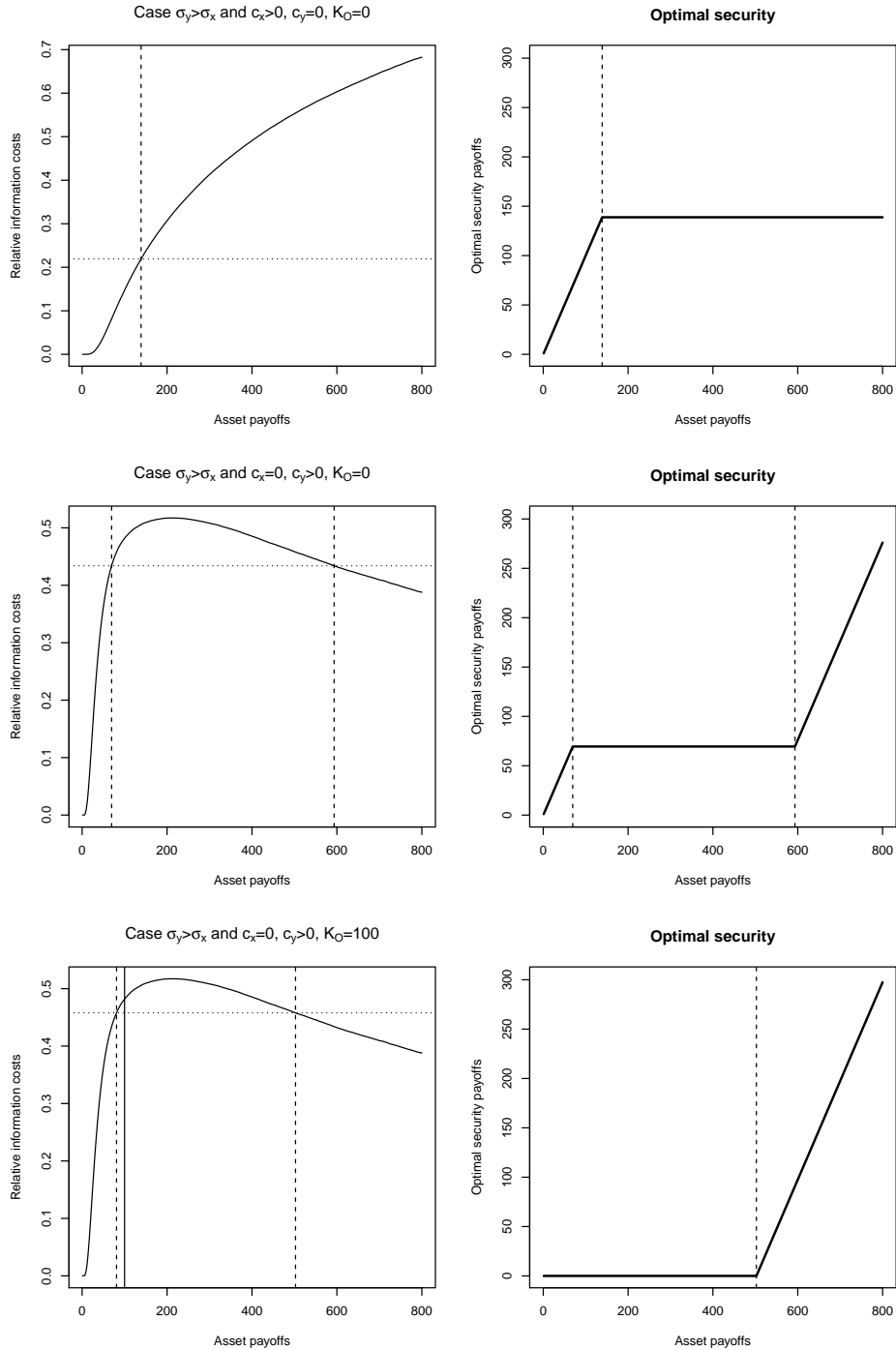


Figure 6: The left panels plot the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$, whereas the right panels plot the optimal securities. The parameter values correspond to the cases listed in Table 3: Case A is depicted in the top two graphs, Case B corresponds to the middle figure, and Case C to the bottom plots. The vertical dashed lines mark the points z for which $H(z) = \gamma$, where γ is given by the dotted horizontal line in the left panels. The vertical solid line in the bottom left graph shows the value of existing debt in Case C, namely $K_0 = 100$.