A Theory of the Distribution of Underpriced Initial Public Offers by Investment Banks

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It is well documented that a firm may choose to offer underpriced securities in an initial public offer. An open question is why investment banks do not retain underpriced offers in their portfolio. We argue that the distribution of underpriced securities allows banks of high quality to signal their value to their customers, promoting in this way their other product lines. We show that the total dollar value of underpriced securities distributed (rather than the percentage value) acts as the signal. We also find that, all else equal, larger customers and those with more elastic demand functions receive a larger total dollar value of underpricing.

1. Introduction

In 1975 Ibbotson discovered that in the first few trading days following an initial public offering (IPO), a stock’s price increases by an average of 16% (see Ibbotson, 1975). Systematic underpricing in IPOs has since been documented in Ritter (1984, 1985), Chalk and Peavy (1987), and Tinic (1988). In reaction to these empirical findings, several papers, such as those by Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989, 1992), have shown that the sale of securities at less than market prices in an IPO may be part of an optimal strategy for the issuer. In this paper we take as given the possible motivations of the issuer to sell underpriced securities, and we focus instead on two related questions. First, what is the incentive for an investment

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bank engaged in a firm commitment offer to distribute underpriced securities to investors? A profitable alternative is to retain as much of the issue as possible and then resell it at the higher secondary market price. This possibility is even more attractive if the success of the IPO depends on the marketing efforts exerted by the underwriter. Second, if the investment bank decides not to retain a portion of the issue for its own portfolio, how are shares allocated among its clients; that is, which investors will become the final beneficiaries of the underpriced IPO?

In this paper we provide a theory of the distribution of underpriced IPOs by investment banks. Our model is based on the complementarity between marketing new equity issues by a bank and supplying other investment services. An investment bank that has a large customer base purchasing a variety of general services other than IPOs (such as risk management, investment advice, foreign currency transactions, and general brokerage), automatically has a group of potential clients for new issues. Conversely, a bank engaged only in marketing IPOs must constantly solicit customers with whom it does not otherwise transact and, thus, operates at a competitive disadvantage.

While banks try to sell potential clients a variety of services, customers must decide which bank to use. From a client’s point of view, all investment banks are not the same: The desirability of services depends on the bank’s quality, which is not directly observable. Service “units” obtained from a high-quality bank are more valuable than those from a low-quality one. Thus, investment banks that are perceived to be of high quality can sell more of their services at a higher price. At the same time, investment banks of higher quality are also likely to be more efficient in the production of services other than IPOs, giving them a cost advantage over their lower-quality counterparts. Because high-quality banks can increase output more cheaply than low-quality ones, burning money, such as selling underpriced IPOs instead of retaining them, can act as a credible signal for quality. This leads to the implication that the total dollar amount of underpriced IPOs distributed is a signal for underwriter quality. Furthermore, because in our model, each signal (the allocation of underpriced IPOs) is privately observed by every recipient, we are also able to obtain restrictions on the distribution of IPOs across customers.1

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1. A useful way to view our model is in the context of the advertising literature. Here underpriced IPOs act as a form of advertising in the same sense as Nelson (1974) and Milgrom and Roberts (1986). In those papers, customers observe the total amount spent by a firm on advertising and attribute expensive promotional campaigns to high-quality producers. By contrast, issuing stocks is a form of “mass marketing” to them. In a sense, as opposed to the models above, the impact of mass marketing is the product of the status of mass marketing
By explicitly modeling investment banks as multiproduct firms, this paper provides a theory of why banks do not retain underpriced issues for their own portfolios and how IPOs are distributed among their clients. We show that, all else equal, larger customers receive a greater dollar value of underpricing from banks of higher quality. A related finding is that customers with more elastic demand functions receive larger dollar values of underpricing. Intuitively, these effects arise from the investment bank’s desire to promote future sales of its other services by giving undervalued IPOs to its best and least secured customers. In equilibrium, the allocation of underpriced IPOs to a client will be greater the more the bank can expect to gain from future transactions.

The importance of reputation in the investment banking community has been recognized in the literature. For instance, Carter and Manaster (1990) suggest that the marketplace has access to information that allows the classification of investment banks into “prestige tiers.” However, within these tiers, banks actively compete for customers. Thus, our model may be interpreted as showing how investment banks may use the distribution of underpriced IPOs to keep old customers and lure new ones from other competitors with similar public reputations.

As noted earlier in our model, a signal consists of the total dollar value of underpricing allocated to a customer. This is given by the difference between the cost of the IPO lot and its true market value. Because the allocation of IPOs to each customer is fixed by the investment bank, the percentage of underpricing on the allocated IPOs (i.e., the rate of return) may not properly reflect the actual profits accruing to a customer. This feature differentiates our work from existing research relating percentage of underpricing and underwriter reputation. For instance, Carter and Manaster (1990) find that high-quality firms underprice on a percentage basis by less than low-quality ones. However, Ritter (1984) shows that high-prestige banks market larger issues. These findings are consistent with the model discussed here, because by marketing larger issues (and perhaps a higher volume of issues), high-quality banks can use a smaller percentage return to distribute to their customers a larger dollar value of underpricing. Thus, the net result can be a negative statistical relationship between quality producers. By contrast, in our model, customers only see their own allocation of IPOs and therefore only know how much the investment firm spends on “advertising” to them. In a sense, our model can be viewed as one of “direct mail advertising,” as opposed to the models of Nelson (1974) and Milgrom and Roberts (1986), who examine the impact of “mass media advertising.”
the quality of the investment firm marketing the issue and the percentage of underpricing.\footnote{This issue will be discussed in more detail in Section 3.}

Another empirical prediction of our model is that scope economies may result in the cross-subsidization of an investment firm’s clients through the allocation of underpriced IPOs. We show that the value of the underpriced IPOs allocated to any customer depends on the properties of his demand function as well as on that of all other customers’. Furthermore, we show how the presence of scope economies in a multiproduct investment bank may allow signaling and separation in markets in which no signaling will occur if each service is produced by a single product firm. Our analysis of how scope economies can induce these spillover effects across markets is, to our knowledge, a novel feature in the literature.

Our model examines explicitly the problem of who is the final beneficiary of underpricing in IPOs, an issue that has received relatively little attention in the literature. While several models, such as those of Allen and Faulhaber (1989) and Welch (1989), have shown that it may be in the interest of the issuer to sell its securities at less than their expected market value, these models do not determine who will in fact be the final recipient of these underpriced securities. One potential equilibrium has the underwriter purchase the issue at the underpriced value and then resell it at market value. In this case, the underwriter’s commission serves as a signal for the issuing company’s quality, and underpricing at the retail level becomes unnecessary. Note that competition among underwriters will not eliminate these profits, because commissions must act as a signal of the issuer’s quality. Alternatively, an investment bank may still underprice an issue but retain a large fraction of it for its own portfolio. Such a strategy leaves the issuing firm’s signal undiluted and increases the investment bank’s profits. In contrast, our model provides an explanation for why investment banks choose to pass on the benefits of underpricing to their customers.

The paper closest in scope to ours is that of Benveniste and Spindt (1989). In that paper, purchasers of IPOs have more information than either the investment bank or the issuer. IPO allocations then act as a way to elicit this private information from the bank’s “regular” customers, leading to a theory of how IPOs are allocated within a heterogeneous customer base. Another difference is that in Benveniste and Spindt, IPO allocations are used to benefit the issuer, while in our paper allocations work to benefit the investment banks.

The question of who is the final beneficiary of underpricing in

IPOs is also addressed in the literature. In the single message to every bank (the sender) to promote (the recipient), we still yield us among customers.

Finally, our paper closely relates the empirical results to additional tests that can be run.

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IPOs is also addressed in Tinic (1988) and Hughes and Thakor (1991). These works argue that investment firms insure themselves against law suits for "overpricing" by selling only underpriced issues to the public. This approach is compatible with our model, and one can view our analysis as providing an additional motivation for why the value from underpricing accrues to the final purchasers of the securities.

Finally, our paper introduces a feature that is new in the signaling literature. In the standard signaling models, a sender transmits a single message to every recipient. In contrast, our model allows the bank (the sender) to potentially issue a different signal to each customer (the recipient), via the individual IPO allocations (the message). This results in a system of incentive compatibility conditions. Our analysis shows that, under conditions that will be made precise later, this system may lead to a fundamental multiplicity of equilibria that is not found in standard signaling models. These multiple equilibria, however, still yield useful restrictions on the final distribution of IPOs among customers.

The paper is organized as follows: The next section presents the model and describes the main results of the paper. In Section 3 we closely relate the empirical literature to our findings and explain what further tests can be used to verify our results. Finally, the conclusion is contained in Section 4. All the proofs are in the Appendix.

2. THE MODEL

2.1 THE BASIC GAME

We assume that investment banks market IPOs and sell other services to their customers in a two-stage game. (A time line is provided in Figure 1.) There are investment banks of high quality (denoted by \( T = H \)) and low quality (\( T = L \)), in proportions \( \theta_0 \) and \( 1 - \theta_0 \), respectively.\(^3\) The type \( T \) is selected at the beginning of the game and is private information to the investment banks. In the first stage of the game, banks market the IPOs they have available at that time. Some of these issues may be underpriced by the issuer, while the remainder are priced at their conditional expected value. Because we are interested in modeling explicitly how IPOs are distributed across clients, we assume that an investment bank retains or allocates securities among its clientele network in order to maximize overall profits.\(^4\) For

\(^3\) As we have already argued, the probability \( \theta_0 \) may depend on other available noisy information that may allow to respect investment houses in "prestige tiers."

\(^4\) This is in contrast to the traditional models involving IPO sales, which instead assume that issues are distributed pro rata among customers; see, for example, Rock (1986).
Stage 1:

i. Nature selects investment banks types $T$: Type $H$ is chosen with probability $\theta_0$ and type $L$ with probability $1 - \theta_0$;

ii. The investment banks allocate to customer $i = 1, 2$ a lot of IPOs with a total dollar value of underpricing equal to $v_i$;

Stage 2:

i. Nature reveals a production cost parameter $z$ to the investment bank. Lower values of $z$ characterize more efficient production. Parameter $z$ is distributed according to $F(z | T)$, with $F(z | L)$ dominating $F(z | H)$ by first order stochastic dominance;

ii. The investment bank sells a quantity $q_i$ of non-IPO services to customer $i$. The investment bank’s total production cost equals $c(q_1, q_2, z)$.

iii. Given beliefs $\theta_i$ on bank’s quality, customers pay $p_i(\theta_i, q_i)$ per unit of services.

Game ends.

**FIGURE 1. TIME LINE FOR THE MODEL**

simplification of exposition, we assume here that a bank has only two customers, indexed by $i = 1, 2$. In the first stage of the game, the investment bank selects a vector $\mathbf{v} = (v_1, v_2)$ representing the total dollar value of underpricing (in present value terms) allocated to its customers. That is, $v_i$ is equal to the difference between the present value of the allocated lot of IPOs minus its cost (i.e., the issue price). From the point of view of the investment bank, this also constitutes the implicit net wealth transfer that occurs when the bank decides to allocate that portion of under-priced IPOs to customer $i$ rather than retaining it in its own portfolio. Let $\mathbf{v}^T$ represent the vector of allocations across customers chosen by a bank of type $T$. Customers observe only their own allocation $v_i$ but cannot see (or equivalently verify) the allocation received by the other customer. Thus, each $v_i$ acts as a private signal to each client. This opens up the possibility that different customers will see different signals and come to different conclusions about the investment bank’s quality.

After the IPO’s distribution, the game enters stage two. At this point: in time the bank clients. Let $q = (q_1, q_2) = (p_1, p_2)$ the vector of services supplied by banks of customers than those $i$’s willingness to pay bank in quantity $q_i$ decreasing in $q_i$. For services from a high-$c$ $\phi > 1$. Finally, assume $p_i = p_i(L, q_i)(\theta_i \phi +$ the investment bank.

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point in time the banks provide services other than IPOs to their clients. Let $q = (q_1, q_2)$ represent the quantity vector of services and $p = (p_1, p_2)$ the vector of unit prices charged to each customer. Services supplied by banks of type $H$ are assumed to be more valuable to their customers than those supplied by type $L$. Denote $p_i(L, q_i)$ as customer $i$'s willingness to pay for one unit of services if provided by a type $L$ bank in quantity $q_i$. Let $p_i$ be twice continuously differentiable and decreasing in $q_i$. For simplicity, let customers' willingness to pay for services from a high-quality firm be equal to $p_i(H, q_i) = \phi p_i(L, q_i)$ with $\phi > 1$. Finally, assume that customer $i$ is willing to pay up to $p_i(\theta_i, q_i) = p_i(L, q_i) (\theta_i \phi + (1 - \theta_i))$ for one unit of service if he believes the investment bank is of type $H$ with probability $\theta_i$.

As in any signaling game, separation can occur only if type $H$ firms receive a larger benefit from identification than type $L$ ones. In a model involving production, such as the one examined here, this asymmetry naturally arises through heterogenous levels of efficiency. Intuitively, better firms produce their output more efficiently. Thus, a high-quality firm will earn a larger marginal profit from any outward shift in its customer's demand curve, making signaling possible. To capture the efficiency differentials, we assume that the investment banks have access to a technology that produces services other than IPOs at a cost $c(q, z)$. Here, $z \in [0, \infty)$ represents an efficiency parameter. Higher values of $z$ identify less efficient producers, that is, $\partial c / \partial z > 0$ and $\partial^2 c / \partial z^2 > 0$. Production also exhibits increasing marginal costs and scope economies: $\partial^2 c / \partial q_1 \partial q_2 > 0$ for $i = 1, 2$, and $\partial^2 c / \partial q_1 \partial q_2 < 0$.

In our model the distribution of underpriced IPOs allows banks of good quality to signal their type to potential customers and in this way attract future business. At the time IPOs are marketed, however, banks may not have complete information about their future cost structure. In this spirit, we assume that the parameter $z$ is observed only at the second stage, after the distribution of available IPOs. This

6. For expositional clarity, we will use the term services only to refer to bundles $q_i$ of services other than IPOs. Note also that an investment bank may supply these services in a different proportion to each customer. Then, $q_i$ may be thought as representing the particular bundle of services purchased by customer $i$. Furthermore, each customer may be thought of either as a single unit or as a group of homogeneous agents forming a market. Hence, in what follows, we will speak interchangeably of customers as well as markets.

7. More general functions yield qualitatively identical results.

8. The variable $z$ may then be thought of as representing all the residual uncertainty over future costs that still exists in the first IPO stage. This residual uncertainty may depend on several factors, such as tax increases, wage hikes, regulatory changes, and others. A similar approach is adopted in Maksimovic and Zechner (1991), where it is assumed that firms are initially uncertain on their future cost structure.
formulation is very useful because it allows us to simplify the model considerably. Suppose, for instance, that investment banks could instead observe the variable \( z \) prior to the IPO stage. In this case it would be possible for a bank to use the IPO allocation \( (v_i) \) and the choice of output \( (q_i) \) together to signal the combination of the variables \( z \) and type \( T \). This alternative specification of the model would significantly complicate the analysis, without changing the nature of our results.

While banks may not know the realization of \( z \) for sure when they allocate the IPOs, they do have some prior information on their future cost structure. This information is embodied in a probability distribution function \( F(T, z) \), with density \( f(T, z) \neq 0 \) for any \( z \geq 0 \) and \( T = L, H \). Because we wish to capture the notion that better banks tend also to be more efficient, we assume that \( F(L, z) \) first order stochastically dominates \( F(H, z) \). This implies that high-quality banks are more likely to have lower costs. Finally, the realization of \( z \) is privately observed by the investment bank, and it is not known to its clients. Clients can infer the value of the parameters \( T \) and \( z \) only from an investment bank’s observable actions.

Given an investment bank’s realized cost structure \( c(q, z) \) and the demand functions it faces, the bank chooses the quantity of services supplied to each customer.\(^9\) Banks are assumed to operate under monopolistic competition: They do not behave as price takers but rather face individual demand functions for services. Let \( q_i^T(z) \) be the level of service supplied to client \( i \) by an investment bank of type \( T \) with costs \( z \). Let \( q = (q_1^T, q_2^T) \) represent the combination of services supplied to all clients.

After observing the output \( q_i \) supplied by the bank, each customer uses Bayes’ rules to update his beliefs about the investment bank’s type, given all the information he has available at that point. Customer \( i \)’s posterior beliefs \( \theta_i \) are then formed solely on the basis of his allocation of underpriced IPOs and, possibly, on the quantity \( q_i \) of services supplied, that is, \( \theta_i(v_i, q_i) \). Given the posterior \( \theta_i \) and the total services offered by the investment bank, client \( i \) then pays up to a total of \( p_i(\theta_i, q_i)q_i \), concluding the game.

2.2 A Numerical Illustration

Before proceeding to the derivation of the equilibrium for the game, we present here a numerical example showing its main features. Table

9. The assumption that \( f(T, z) \neq 0 \) ensures that customer beliefs are always restricted by Bayes’ rule, given any \((v_i, q_i)\) combination.

10. An equivalent formulation would be to assume that investment banks set prices for the bundle of services other than IPOs purchased by each customer. Customers then set their demands accordingly. The price and quantity specifications would produce identical results as long as demand functions are invertible.
us to simplify the model. Instead, banks could invest. In this case it would be useful to consider the choice of one of the variables \( z \) and model would significantly improve the nature of our results. The situation of \( z \) for sure when we have information on their embodied in a probability \( F(L, z) \neq 0 \) for any \( z \geq 0 \) would not fit the notion that better order statistic \( z \) first order is that high-quality banks have the realization of \( z \) and it is not known to its parameters \( T \) and \( z \) only is.

Consider the structure \( \theta(q, z) \) and the quantity of services used to operate under the notion of price takers but services. Let \( q^T(z) \) be the investment bank of type \( T \) combination of services by the bank, each customer is aware of the investment available at that point. The idea is to offer services to customers who receive the posterior \( \theta_i \) and bank, client \( i \) then pays me.

I shows the expected profits for high- and low-quality banks as a function of customers’ beliefs. Because in what follows we will be interested only in equilibria in which banks of high-quality separate, we focus here only on beliefs that characterize a bank as being of low quality, \( \theta_i = 0 \), or high quality, \( \theta_i = 1 \). For example, if only customer 2 believes that the bank is of high quality (so that \( \theta_1 = 0 \) and \( \theta_2 = 1 \)) the profit levels are $5.5 for a type \( H \) bank and $3 for a type \( L \) one. Customers differ in importance to a bank: Here customer 1 represents a smaller client than customer 2. Also note that the value to a type \( H \) bank of a change of its reputation from \( \{0,0\} \) to \( \{1,0\} \) is $2, and of a change from \( \{0,1\} \) to \( \{1,1\} \) is $6. In both cases only customer 1 changes his beliefs, but the presence of production scope economies makes the latter change more valuable. A similar pattern holds also for the low-quality investment bank, although the effect is reduced by its inferior production technology.

Given the profit opportunities displayed in Table 1, the investment bank can try to influence the posterior probabilities \( \theta_i \) by selling underpriced IPOs to its customers. In standard signaling models, high-type firms must send a signal large enough to deter low-type ones from mimicking. The analog in this case would be to consider the incremental profits for the two types of banks when customer beliefs change from \( \{0,0\} \) to \( \{1,1\} \). The profit differential for high-type banks is $10, while for the low-type ones it is only $5. Thus, if the high-type banks send their customers IPOs with a total value of \( v_1 + v_2 = \$$5, the low-type ones will not have an incentive to copy them with all customers.

The procedure described earlier determines the total amount of underpricing necessary to prevent mimicking. Additional restrictions, however, are necessary to insure incentive compatibility. This is due to the fact that while a customer observes his own IPO allocation, he cannot see the signal received by the other customer. This results in additional restrictions on the distribution of signals across customers. For example, suppose that a bank of type \( H \) sets \( v_1 = $4 \) and \( v_2 = $1 \). In this case, a type \( L \) bank can profitably copy the signal to customer 2.

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<tr>
<th>Bank Type</th>
<th>Customer Beliefs: ( \theta_1, \theta_2 )</th>
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Table 1.
Payoff Matrix
only. This changes the investors’ beliefs from \{0, 0\} to \{0, 1\}, and the type \(L\) bank can then increase its profits by $2 at a cost of only $1. Thus, to signal their quality the high types must now only spend a total of $5, but they must also allocate this value appropriately to each customer so as to deter mimicking to any customer at a time. In the next section, we show that the following procedure will give the desired incentive compatible allocation. First, send a signal to customer 1 equal in value to the incremental profits of a low type when beliefs change from \{0, 1\} to \{1, 0\}; that is, set \(v_1 = $1\) in the example earlier. Customer 2 then receives a signal whose value equals the incremental profits to a low type when beliefs change from \{1, 0\} to \{1, 1\}, which is equal to \(v_2 = $4\) in the example. Hence, a separating allocation is \(v^H = (1, 4)\). This procedure would also work when customers are considered in the reverse order, implying that there are multiple incentive compatible allocations.\(^{11}\) Our model, however, places restrictions on the minimum allocation that must be guaranteed to each client. We examine these restrictions in Section 2.4. There, we show that the value of each signal to any individual customer must be at least equal to the low type’s profit from convincing that customer only that the bank is of high quality. In our example, the minimum allocations to customers 1 and 2 are $1 and $2, respectively.

### 2.3 Equilibrium for the Full Game

In this section of the paper we derive explicitly the separating equilibrium for the full game. Those readers who are primarily interested in the main implications of our model may wish to skip directly to Section 2.4.

In the second stage of the game, customers form posterior beliefs on each bank’s quality, \(\theta_i\), based only on the observation of the equilibrium selections of \(v_i\) (their allocation of underpriced IPOs) and \(q_i\) (the quantity of services they are supplied). Separation occurs in markets for all customers if equilibrium strategies imply that \(\theta_i(v_i^H, q_i^H) = 1\) if \(T = H\), and \(\theta_i(v_i^L, q_i^L) = 0\) if \(T = L\), for \(i = 1, 2\). Given the belief functions \(\theta_i\), separation may be induced by the level of underpricing \(v_i\) and/or the choice of output levels \(q_i\). Separation is achieved by the choice of output if \(q_i^H \neq q_i^L\); \(\theta_i(v_i, q_i) = 1\) for all \(i\), and in equilibrium both types choose the same vector \(v\) of underpricing \(v^H = v^L\). Conversely, separation with all customers is obtained at the IPO stage, by the allocation of underpricing, if in equilibrium \(v_i^H \neq v_i^L\) for all \(i\).

To reduce the set of candidate equilibrium strategies and belief functions, it is useful may not occur in the initial IPO stage and mand functions for tli with common z have incentives to choose probability conditions req the same profits as t separation. Therefor posterior beliefs will giving \(\theta_i = \theta_i(v_i)\).

Consider then the quantity of servic functions \(p_i(q_i, \theta_i)\) ti of the game. Profits then given by

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R(\theta(v), z) = \max_{\theta} f
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where \(\theta = (\theta_1, \theta_2)\) eq. (1) determines the bank with cost struct a (posterior) probat optimal amount of \(v_i\) of its customer’s be

**Lemma 1**: The so posterior probabilities

An implication of L effects across custo any customer dep beliefs held by the is increasing in \(\theta_i\), generating an ince higher quality. Fun increases with

From eq. (1), IPOs, overall profi \(r(v | T)\) and \(\pi(v | T; z)\), given that the underpriced secur order stochastic d
functions, it is useful to begin by showing that separation on quality may not occur in the production stage of the game. If separation occurs in the production stage, \( v^H = v^L \). In this case, both types pool at the initial IPO stage and face in the second stage an identical set of demand functions for their products. After the realization of \( z \), all banks with common \( z \) have the same cost structure \( c(q, z) \) and face identical incentives to choose any particular output. Hence, incentive compatibility conditions require that, conditional on \( z \), banks of type \( L \) earn the same profits as type \( H \) ones, preventing any effective gain from separation. Therefore, separation must occur at the IPO stage, and posterior beliefs will depend only on the amount of underpricing, giving \( \theta_i = \theta_i(v_i) \).

Consider then an investment bank’s final problem of choosing the quantity of services supplied to each customer. Banks face demand functions \( p_i(q_i, \theta_i) \) that depend upon customer beliefs \( \theta_i \) at this stage of the game. Profits \( R \), from sales of services other than IPOs, are then given by

\[
R(\theta(v), z) = \max_{q_1, q_2} \sum_{i} p_i(q_i, \theta_i(v_i))q_i + \sum_{i} p_i(q_i, \theta_i(v_i))q_i - c(q, z),
\]

where \( \theta = (\theta_1, \theta_2) \) is the vector of posterior beliefs. The solution to eq. (1) determines the output vector \( q^*(\theta, z) \) optimal for an investment bank with cost structure \( z \), which is perceived as being of type \( H \) with a (posterior) probability \( \theta \). The following lemma establishes that the optimal amount of services sold by any bank is an increasing function of its customer’s beliefs.

**Lemma 1:** The solutions \( q_i \) to problem (1) are increasing functions of the posterior probabilities \( \theta_i \): \( \partial q_i / \partial \theta_i > 0 \), for all \( i, j \).

An implication of Lemma 1 is that scope economies introduce spillover effects across customers: The optimal amount of services supplied to any customer depends positively on his belief \( \theta_i \), as well as on the beliefs held by the other customer. Because the willingness to pay \( p_i \) is increasing in \( \theta_i \), profits \( R(\theta, z) \) are also increasing in the vector \( \theta \), generating an incentive for investment banks to be perceived as being of high quality. Furthermore, because of scope economies, this incentive increases with the number of customers the bank is signaling to.

From eq. (1), and accounting for the distribution of underpriced IPOs, overall profits are equal to \( \Pi(v, z) = R(\theta(v), z) - v_1 - v_2 \). Let \( r(v | T) \) and \( \pi(v | T) \) represent the expected values of \( R(v, z) \) and \( \Pi(v, z) \), given that the bank is of type \( T \) and it distributes the vector \( v \) of underpriced securities. Because \( F(z, L) \) dominates \( F(z, H) \) by first-order stochastic dominance, expected benefits from separation will
be greater for a type \( H \) bank than a type \( L \) one. This will make separation of the two types of banks possible.

We now proceed to the characterization of the incentive compatibility conditions that ensure separation. Because each individual customer observes only his own allocation \( \nu_i \), we must account for the possibility that the two customers may receive different signals: Investment banks must truthfully reveal their type to every customer. In particular, investment banks of type \( L \) must be deterred from attempting to imitate type \( H \) ones to one customer at a time as well as to both customers simultaneously. Because type \( L \) banks in equilibrium have nothing to gain from distributing underpriced IPOs, they will optimally set \( \nu^i_L = 0 \), for \( i = 1, 2 \). Given any updating rule such that \( \theta_i(0) = 0 \) and \( \theta_i(1) = 1 \), an allocation \( \nu^H \) is incentive compatible if the following conditions are satisfied:

\[
\begin{align*}
\pi(\nu^H, 0 | L) & \leq \pi(0, 0 | L), \\
\pi(0, \nu^H_L | L) & \leq \pi(0, 0 | L), \\
\pi(\nu^H_l, \nu^H_r | L) & \leq \pi(0, 0 | L).
\end{align*}
\]

Conditions (2) and (3) deter a type \( L \) from mimicking with one customer only, and condition (4) deters from mimicking with both customers simultaneously. These equations place restrictions on the distribution, customer by customer, of underpriced IPOs that ensures truthful revelation and allow the characterization of signaling costs for type \( H \) banks. In particular, we will show that they imply that there is a minimum amount \( \nu^H_t \) of underpriced IPOs that type \( H \) banks must supply to deter imitation in any single market, and a total dollar value \( \nu^H_t + \nu^H_r \) of underpricing needed to deter imitation on all markets simultaneously.

The next lemma provides a basic result leading to the characterization of separating equilibria. Intuitively, it states that the additional profits from deviating in two markets simultaneously exceeds the sum of the incremental profits from deviating in markets 1 and 2 separately.

**Lemma 2:** For posterior beliefs such that \( \theta_i(0) = 0 \) and \( \theta_i(1) = 1 \), we have:

\[
\pi(\nu^H, \nu^H | L) - \pi(0, 0 | L) > \{ \pi(\nu^H, 0 | L) - \pi(0, 0 | L) \}
+ \{ \pi(0, \nu^H | L) - \pi(0, 0 | L) \}.
\]

An implication of Lemma 2 is that production scope economies introduce a form of signaling diseconomies. These derive from the property that investment banks...
that investment banks of type $L$ have an incentive to imitate the behavior of type $H$ banks with the greatest number of customers. For example, suppose a bank of type $H$ chooses levels of underpricing $v_i$ satisfying conditions (2) and (3) with equality. In the example from Section 2.2, this amounts to setting $v_1 = $1, $v_2 = $2. While this allocation dissuades type $L$ banks from imitating type $H$ ones with every customer, one at a time, it is not overall incentive compatible. Due to production scope economies, a type $L$ can profitably imitate a type $H$, simultaneously to both customers. In our example, type $L$ may profitably distribute the proposed allocation to customers 1 and 2 at a total cost of $3$, which produces a gain of $5$. As a result, the total signaling costs of a type $H$ bank must be greater than the sum of the minimum costs necessary to prevent imitation to any single customer alone. This also implies that there exist a multiplicity of incentive compatible vectors $v^M$, generating an indeterminacy in the equilibrium allocation across customers. The equilibrium restrictions can only characterize a minimum total allocation of underpriced securities for the entire group of customers, and a minimum amount that must be allocated to each customer.

Using Lemma 2, we can now show that there are equilibria in which banks separate at the IPO stage through their choice of the allocation of underpriced IPOs. Furthermore, we show that signaling costs are minimized when condition (4) binds, setting a lower bound on total signaling costs.

**Proposition 1:** There are separating equilibria in which $0 < v^L < v^H$. Furthermore, the equilibrium with lowest total signaling cost $v^L + v^H$ occurs when the incentive compatibility condition (4) holds with equality.

Proposition 1 establishes a lower bound on the total cost a type $H$ investment bank must incur to induce separation. In addition, because investment banks only care about total signaling costs, it establishes that the tightest incentive compatibility constraint is the one preventing imitation simultaneously on all markets. This condition differs from the usual restriction holding for a single-product bank. It characterizes the minimum signaling cost for the group of all customers in its entirety, but it does not determine its distribution across product markets. This indeterminacy does not generally arise in the usual single-market signaling models, and it depends on the fact that each signal is privately received by its recipient.

To obtain restrictions on the distribution of IPOs across customers, we must examine the remaining incentive compatibility conditions. The next proposition shows that there is a minimum amount $v^H$ of underpricing that banks of type $H$ must guarantee to each cus-
Proposition 2: In any separating equilibrium, investment banks of type H must distribute a minimum dollar value \( v_i^H \) of underpriced securities to purchasers of service \( i \), whose value is obtained by setting (2) and (3) to equality.

The results given above readily generalize to the case in which there are more than two customers per bank (see Fulghieri and Spiegel, 1993). In this case, a bank of type H must distribute to its \( n \) customers a vector \( v = (v_1, v_2, \ldots, v_n) \) of underpriced IPOs that must satisfy a set of incentive compatibility conditions equivalent to (2)-(4). An incentive compatible allocation \( v^H \) may then be found by following a procedure similar to the one described here. In particular, find first an allocation \( v_1 \) such that the incentive compatibility condition for signaling only to customer 1 is met with equality. Then, given this value of \( v_1 \), find a value \( v_2 \) such that the incentive compatibility condition preventing mimicking simultaneously to customers 1 and 2 together hold with equality. Again, given the economies of scope, this will require an allocation of underpricing to the second customer that is greater than the one needed to deter imitation with that customer alone. This process continues until a value for \( v_n \) is established. Intuitively, a type H investment bank must first meet the weakest constraint and then set each subsequent \( v_i \) to satisfy an ever tighter constraint. The resulting vector \( v \) may be shown to satisfy all the necessary incentive compatibility conditions. Furthermore, because the ordering of customers is arbitrary, there is again a multiplicity of allocations satisfying all the desired conditions. Finally, we can obtain in a similar way the minimum amount \( V_K \) of underpricing that must be guaranteed to deter imitation simultaneously in a given subset \( K \) of customers. This can be computed by requiring that the corresponding incentive compatibility constraint holds with an equality sign, and it will be equal to the incremental profit that a type L bank may earn by signaling to be a type H one to each member of the group \( K \).

2.4 The Distribution of Underpriced IPOs

Propositions 1 and 2 show that scope economies lead to an indeterminacy in the actual distribution of underpriced IPOs among an investment bank's clientele. However, the characterization of each customer's minimum incentive compatible allocation enables us to formulate some empirical predictions for particular classes of clients. These predictions may then be tested against aggregate data for an investment bank, as \( w \) by client.

The correlation size of the recipient customer interest in the literature that, all else equal, of underpricing. How controlling for both

Proposition 3: All of customer i increases the signaling costs for type

Proposition 3 verifies that multiple IPOs with high investment bank must because there is more bank is of high quality.

The comparative aimed at analyzing how the prices of underpricing now show that the more elastic the demand of incentives from a positive is the incentive type H ones. This for

Proposition 4: The demand function such that of underpricing \( v^H \) re (demand elasticity).

The potential impact of underpriced IPOs on demand elasticities propositions 3 and 4 show that one can expect to receive else equal, large client bank, may not receive.

Elastic demand functions clients of underpriced demand elasticities.

The presence of...
investment bank, as well as against observations disaggregated client by client.

The correlation between the amount of underpricing and the size of the recipient customer is a variable that has attracted the greatest interest in the literature. The next proposition confirms the intuition that, all else equal, larger customers do receive a greater allocation of underpricing. However, an empirical test of this prediction requires controlling for both production costs and demand elasticities.

**Proposition 3:** All else equal, a multiplicative increase in the demand of customer $i$ increases the minimum amount of underpricing $v_i^H$ and the total signaling costs for type $H$ banks.

Proposition 3 verifies the intuition that larger customers receive packages of IPOs with higher dollar values of underpricing. A high-quality investment bank must send a stronger signal to its larger customers, because there is more to be gained from convincing them that the bank is of high quality.

The comparative statics property examined in Proposition 3 is aimed at analyzing how size influences the distribution of IPOs. We now show that the minimum amount of underpricing allocated to any customer also increases with his demand elasticity. Intuitively, the more elastic the demand function, the greater is the increase in revenues from a positive change in customers’ beliefs and, therefore, the greater is the incentive type $L$ banks have to imitate the behavior of type $H$ ones. This forces type $H$ banks to divert more valuable lots of underpriced IPOs toward this class of customers.

**Proposition 4:** Let $\beta$ represent an elasticity parameter within the demand function such that $\partial^2 p/\partial q \partial \beta > 0$. All else equal, the minimum amount of underpricing $v_i^H$ received by a customer is an increasing function of $\beta$ (demand elasticity).

The potential impact of a customer’s business volume on its allocation of underpriced IPOs has been suggested in the literature. The effect of demand elasticities has not received analogous attention. Propositions 3 and 4 show that the value of underpriced securities a customer can expect to receive is affected by both factors. This implies that all else equal, large clients, for some reason locked into the investment bank, may not receive as many IPOs as other clients having more elastic demand functions. Empirical tests of the distribution across clients of underpriced IPOs must then account for both size and demand elasticities.

The presence of economies of scope in a multiproduct investment bank also generates interdependencies across product markets.
and spillover effects. Parameters characterizing one customer class may affect the dollar value of underpriced IPOs distributed to another class of customers. First, Propositions 3 and 4 can be extended to examine these cross-market effects. The minimum amount of underpricing that must be guaranteed to any customer class is an increasing function of the size and demand elasticities in any other class, resulting in cross-subsidization of customers. Second, economies of scope may also allow signaling and separation in a product line if supplied by a multiproduct bank, even when no separation is possible with a single-product bank. To see this, consider an investment bank with a cost function $c_1(q_1) + c_2(q_2, z) + c_3(q_1, q_2)$. Note that the production cost for service 1 does not depend directly on the parameter $z$. Hence, if the bank does not produce service two, both types of banks produce service 1 at the same cost, and no separation in that market is possible. On the other hand, production of the second service does allow separation and induces signaling to customers of both products.

**Proposition 5**: Consider a multiproduct bank whose cost function can be written as $c_1(q_1) + c_2(q_2, z) + c_3(q_1, q_2)$. Then there are separating equilibria in which customers for both services are signaled to: $v_t^i > 0$, for $i = 1, 2$.

Proposition 5 shows that signaling in a given product may occur even if all banks are equally efficient at producing that particular product. With scope economies, efficiency gains in a product line may be enough to sustain signaling even in other lines that otherwise will not allow it.

3. The Empirical Evidence and Predictions

In contrast to the existing literature, the predictions made by this paper concern the total dollar value of underpricing of IPOs distributed by a particular investment bank. Therefore, empirical tests regarding the percentage of underpricing on particular IPOs cannot be directly applied to our model. However, if carefully analyzed, some of the existing evidence can be used to draw some preliminary conclusions.

Beatty and Ritter (1986) find that investment banks failing to underprice issues are driven out of the market. This phenomenon can be interpreted in the context of our model as being determined by a loss of overall business. Investment banks failing to signal that they are of high quality over time will find customers across their service lines leaving them. These results are also consistent with the claim that IPOs are one of the factors responsible for the reputation on its quality, and that those banks that have also suffered reductions in the distribution of IPOs.

Propositions 3 and 5 allow us to examine the extent to which larger amounts of underpricing are required. The evidence examined in this section also helps to explain the distribution of IPOs across banks. As noted, the distribution of IPOs is significantly different from what one would expect if on this, he concludes that these banks are not particularly favored.

Tinic’s (1988) analysis is further refined by examining the dollar value of the underpricing. As the underpricing is reflected in the distribution of IPOs, the high underpricing is not only a function of the demand for underpriced IPOs. With this, as well as estimates of the underpricing, a direct test of the predictions of our model can be made.

In a related paper, Beatty and Ritter (1986) find that investment banks failing to underprice issues are driven out of the market. This phenomenon can be interpreted in the context of our model as being determined by a loss of overall business. Investment banks failing to signal that they are of high quality over time will find customers leaving them. These results are also consistent with the claim that IPOs are one of the factors responsible for the reputation on its quality, and that those banks that have also suffered reductions in the distribution of IPOs.

12. Tinic (1988) disputes this finding, leading to future losses because both Tinic and I the results in our paper. The evidence is that IPOs have been underpriced across broad movements in market
that IPOs are one of the tools an investment bank uses to enhance the reputation on its entire product line. Thus, our model predicts that those banks that left, in Beatty and Ritter (1986), the IPO market also suffered reductions across their other products.\textsuperscript{12}

Propositions 3 and 4 also predict that, all else equal, larger customers, and customers with more elastic demand functions receive larger amounts of underpriced securities. To some extent, Tinic (1988) examines the distribution of IPOs among clients and finds that institutional investors purchased about 31\% of the shares issued. He also notes that the distribution of returns received by institutions does not significantly differ from that received by the general population. Based on this, he concludes that the best customers of the investment banks are not particularly favored.

Tinic's (1988) analysis, however, is based on percentage returns. As noted earlier, our model predicts that the relevant variable is the dollar value of the underpricing received by customers. On this basis, institutional investors appear to receive almost a third of all the value investment banks were handing out through the distribution of underpriced IPOs. Without data on volume for each individual client, as well as estimates of demand elasticities, it is not possible to have a direct test of the predictions of our model. At first reading, however, our prediction that large customers are likely to receive a larger dollar volume of underpriced IPOs seems to conform with the available evidence.

In a related paper, Carter and Manaster (1990) find that high-quality banks underprice on a percentage basis by less than low-quality banks.\textsuperscript{13} To conduct this test, they use a bank's general placement in "tombstone" announcements as a measure of quality under the assumption that high-quality banks are able to force their way to the top of the announcement.

Under close scrutiny, this evidence may be consistent with the prediction of our Proposition 1 that investment banks of higher quality engage in a larger dollar value of underpricing. There are three aspects to this issue. The first aspect is whether the existence of the tombstone eliminates the need for signaling. The answer to this question seems

\textsuperscript{12} Tinic (1988) disputes Beatty and Ritter's (1986) interpretation that low underpricing leads to future losses in IPO market shares. However, he does agree that there are broad movements in market shares from low- to high-prestige investment houses. Because both Tinic and Beatty and Ritter examine returns rather than dollar values, the results in our paper indicate that a good deal of power may be lost in the statistical tests. As a result, when Tinic subdivides the data for his tests, the differences in returns that survive across broad groups of firms may be lost in narrower classes.

\textsuperscript{13} The relationship between underpricing and reputation of the investment bank marketing an issue is discussed in Chemmanur and Fulghieri (1994).
to be negative. To be placed at the top of a tombstone, a bank must first prove it is of sufficient stature. Such proof must require costly reputational development. Furthermore, if investment bank quality evolves through time, banks must continuously signal to the marketplace over time. A second, related aspect is the fact that the marketplace may have access to other information allowing the classification of investment banks in broad "prestige tiers." However, if quality differences persist among investment houses in the same class, the distribution of underpriced IPOs may be used to build up (or maintain) reputation within a tier. IPO allocations then become an instrument for similarly situated investment banks to compete for customers.

Finally, Ritter (1984) shows that high-prestige banks are able to market larger issues. Thus, high-quality banks can use a smaller percentage return to provide their customers with a stronger signal. This would be consistent with a negative relationship between the quality of the investment bank marketing an issue and the percentage of underpricing. An interesting test in this direction would be to check whether higher-quality investment banks do in fact engage in a greater dollar volume of underpricing.

Furthermore, even if investment banks wish to deliver to their clients IPOs with the greatest value at minimum cost, they are restricted by the available supply from issuers. This raises the issue of why securities are underpriced and how issuers choose their leading underwriters. Several papers, such as those by Grinblatt and Hwang (1989), Welch (1989), and Allen and Faulhaber (1989) derive underpricing as a signal from the issuer. These models are not only fully compatible with the one presented here but form a necessary underpinning. The aim of our paper is to explain how a given dollar amount of underpricing is allocated in the investment community and not where these underpriced IPOs come from. To explain the source of underpricing, one needs alternative theories, such as those mentioned earlier.

Finally, as an example of how issuer-based models may interact with the theory presented here, consider the case where issuers underprice to signal their quality. The question now is the determination of which underwriter will market the issue. It seems plausible to assume that investment banks with the strongest incentives to handle the issue will in fact be more likely to win the bidding to obtain the right to its underwriting and distribution. In the context of our model, this implies that high-quality investment banks are more likely to handle larger issues and issues with a greater total dollar value of underpricing. While the a partial signal of investing. For example, a bank because it is unusual wants the issue to enc as there remains some motivations, the proc additional information

This paper has presented an investment bank as a theory of how IPOs tomers. The model pr banks signal their ow ing across all the IPO each individual IPO. IPOs for $18 million $ bank that markets $5 this is true despite th is engaged in "more" vestedment banks distri bities to their customers. Second, we find demand elasticities rece priced IPOs. Third, subsidization of cust This leads to an anon that only deals in a sin mies, the bank will n through the distribu tiproduct environm the same product lin standing among its c

Proof of Lemma 1. First Order Condition
Distribution of Underpriced Initial Public Offers

4. Conclusions

This paper has presented an explicit model of the interaction between an investment bank and its client base. This has enabled us to develop a theory of how IPOs are allocated among an investment bank's customers. The model provides three primary findings. First, investment banks signal their own quality through the dollar value of underpricing across all the IPOs they handle, rather than the rate of return on each individual IPO. Thus, a bank marketing $20 million worth of IPOs for $18 million sends a stronger signal to its clients than another bank that markets $5 million worth of IPOs for $4 million. Note that this is true despite the fact that, in percentage terms, the second bank is engaged in "more" underpricing. Furthermore, higher-quality investment banks distribute a larger dollar volume of underpriced securities to their customers.

Second, we find that larger customers and those with high demand elasticities receive, all else equal, a more valuable lot of underpriced IPOs. Third, we find that signaling can result in the cross-subsidization of customers within a multiproduct investment bank. This leads to an anomalous possibility. Consider an investment bank that only deals in a single non-IPO product line. Without scope economies, the bank will not engage in reputation building to its customers through the distribution of underpriced IPOs. However, in a multiproduct environment with scope economies, a division marketing the same product line will allocate underpriced IPOs to improve its standing among its client base.

Appendix

Proof of Lemma 1. Follows immediately by differentiation of the First Order Conditions of problem (1) and Cramer's rule. \(\square\)
Proof of Lemma 2. Represent \( \pi(v^H_1, v^H_2 | L) \) as follows

\[
\pi(v^H_1, v^H_2 | L) = \pi(0, 0 | L) + \int_0^{v^H_2} \frac{\partial \pi(w, 0 | L)}{\partial v_1} \, dw \\
+ \int_0^{v^H_1} \frac{\partial \pi(v^H_1, w | L)}{\partial v_2} \, dw.
\]

Similarly, express \( \pi(v^H_1, 0 | L) \) and \( \pi(0, v^H_2 | L) \) as increments from \( \pi(0, 0 | L) \). Substituting these expressions for the profit functions, after simplifying, eq. (5) holds if

\[
\int_0^{v^H_2} \frac{\partial \pi(v^H_1, w | L)}{\partial v_2} \, dw > \int_0^{v^H_1} \frac{\partial \pi(0, w | L)}{\partial v_2} \, dw.
\]

To verify this inequality, for any \( z \) consider the artificial price function \( P_i^*(q_i, v_i) = p_i(q_i, \theta_i/v^H_1) \), in which \( q_i \) solves (1) for \( \theta_i = v_i/v^H_1 \). Notice if \( v_i \) equals zero then \( v_i/v^H_1 \) equals zero, and if \( v_i \) equals \( v^H_1 \) then \( v_i/v^H_1 \) equals 1. Therefore, the price schedule \( P_i^* \) takes on the same value as the equilibrium price schedule whenever \( v_i \) equals zero or one. Next let \( \Pi^o \) represent \( \Pi \) under the price function \( P^o \). Now substitute \( R(\theta, z) \) into \( \Pi \). Then given \( P^o \), the definition of \( p_i(q_i, \theta) \), and the envelope theorem, one has

\[
\frac{\partial \Pi^o(v, z)}{\partial v_i} = \frac{\partial P_i^o/\partial q_i}{\partial v_i} q_i - 1 = q_i(\phi - 1)p_i/v^H_1 - 1,
\]

where \( p_i \) is evaluated at \( q_i \) and \( L \). Because \( \partial P_i^o/\partial v_i > 0 \), scope economies also imply \( \partial q_i/\partial v_i > 0 \) for all \( i, j = 1, 2 \). Hence,

\[
\frac{\partial^2 \Pi^o(v, z)}{\partial v_i \partial v_j} = q_i \frac{\partial p_i \partial q_j}{\partial q_i \partial v_j} (\phi - 1) + q_i \frac{\partial q_i (\phi - 1)p_i}{\partial v_i} > 0,
\]

for all \( z \), where Lemma 1 and our previous definitions imply the inequality holds for the price function \( P^o \). However, \( P_i = p_i \) for all \( i \) whenever \( v_i = v^H_1 \) or 0. Because \( \Pi^o = \Pi \) under either \( p \) or \( P \) whenever \( v_i = v^H_1 \) or 0 for all \( i \), the inequality holds under \( p \) as well. Finally, integrating over \( z \) we obtain the desired result. \( \square \)

Proof of Proposition 1. The proof provides a constructive method to obtain incentive compatible vectors \( v^H \) for which constraint (4) holds, and hence minimizes the total signaling costs. Construct vector \( v^H \) satisfying the following constraints:

\[
\begin{align*}
\pi(v^H_1, 0 | L) &= \pi(0, 0 | L) \\
\pi(v^H_1, v^H_2 | L) &= \pi(0, 0 | L) \tag{6}
\end{align*}
\]

To prove the proposition, vector prevents limits show that

\[
\pi(0, v^H_2 | L) \leq \pi(0, 0 | L)
\]

holds. From Lemma \( \pi(v^H_1, v^H_2 | L) \). Using

Finally, FOSD impli

Proof of Proposition incentive compatibil

Proof of Proposition

From eq. (1), and us because

\[
p_i(q_i, 1)q_i(0, 0, z) > 0
\]

for all \( z \). Similarly for \( (4) \) verifies that agg.

Proof of Proposition 3, set \( i = 1 \). Different

\[
\frac{\partial \pi(v^H_1, 0 | T)\beta > 0}
\]

Again, by the envelope in equilibrium

\[
q_i(1, 0, z)\partial p_i(q_i, 1).
\]

for all \( z \), which followings or \( \beta \). A simil

Proof of Proposition of product 1 deeper. We can then find \( v \) the incentive comp
ics & Management Strategy

\( \pi(0, v^H \mid L) \leq \pi(0, 0 \mid L) \) holds. From Lemma 2, \( \pi(v^H, 0 \mid L) + \pi(0, v^H \mid L) - \pi(0, 0 \mid L) \leq \pi(v^H, v^H \mid L) \). Using eq. (6) in this inequality proves that eq. (7) holds. Finally, FOSD implies that \( \pi(v^H \mid H) - \pi(0 \mid H) > \pi(v^H \mid L) - \pi(0 \mid L) = 0 \), and type \( H \) is willing to separate.

Proof of Proposition 2. If eq. (2) does not hold for security \( i \), the incentive compatibility condition is violated. Furthermore, because \( v^i \) enters linearly in the other conditions, the constraint can always be made binding, establishing the lower bound.

Proof of Proposition 3. Set, say, \( i = 1 \). Let \( \alpha \) be a multiplicative parameter such that \( ap_1 \). Then differentiating eq. (2) with respect to \( \alpha \), we have that \( \partial \pi^i / \partial \alpha > 0 \) if

\[ \partial \pi(v^i, 0 \mid T) / \partial \alpha > \partial \pi(0, 0 \mid T) / \partial \alpha. \]

From eq. (1), and using the envelope theorem, the inequality follows because

\[ p_1(q^i, 1)q^i(1, 0, z) > p_1(q^i, 0)q^i(0, 0, z) \]

for all \( z \). Similarly for \( i = 2 \). A similar argument applied to condition (4) verifies that aggregate signaling costs increase in \( \alpha \) as well.

Proof of Proposition 4. In a similar spirit to the proof of Proposition 3, set \( i = 1 \). Differentiating condition (2) yields that \( \partial v^i / \partial \beta > 0 \) if

\[ \partial \pi(v^i, 0 \mid T) / \partial \beta > \partial \pi(0, 0 \mid T) / \partial \beta. \]

Again, by the envelope theorem, the above inequality is satisfied if in equilibrium

\[ q^i(1, 0, z) \partial p_1(q^i, 1) / \partial \beta > q^i(0, 0, z) \partial p_1(q^i, 0) / \partial \beta \]

for all \( z \), which follows immediately from Lemma 1, and the assumptions on \( \beta \). A similar result holds for \( i = 2 \).

Proof of Proposition 5. Due to economies of scope, the marginal cost of product 1 depends, through the choice of \( q_2 \), on the parameter \( z \). We can then find values \( v_1 \) and \( v_2 \) strictly greater than zero satisfying the incentive compatibility conditions (2), (3), and (4).
REFERENCES