Financial Contracts as Lasting Commitments: 
The Case of a Leveraged Oligopoly*

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Received May 9, 1991

The commitment value of financial contracts is limited by the ability of contracting parties to renegotiate them away, if it becomes mutually beneficial to do so. When debt contracts are used by oligopolistic firms to commit to aggressive output strategies as in Brander-Lewis, we show that renegotiation may undermine commitment under symmetric information, but not generally under asymmetric information. Lasting contracts that survive renegotiation are proposed. It is shown that there exist lasting debt contracts which preserve the commitment value and in which not all debt is renegotiated away. Journal of Economic Literature Classification Numbers: G32, L12, and D82. © 1992 Academic Press, Inc.

INTRODUCTION

A growing literature views debt and agency contracts as strategic commitments to take actions that are optimal *ex ante* but not *ex post*. This idea is traceable to Schelling (1960). For instance, Dybvig and Zender

* We thank Thomas Chemmanur, Masako Darrough, Raghuram Rajan, Paolo Siconolfi, Matt Spiegel, Avanidhar Subrahmanyam, Josef Zechner, and seminar participants at the Columbia Business School, Rutgers University, the WFA meetings in Jackson Hole, and especially two anonymous referees and the editor for very valuable comments. We remain responsible for any residual errors. Paolo Fulghieri gratefully acknowledges the support of the Faculty Research Fund, Columbia Business School.
CONTRACTS AS LASTING COMMITMENTS

(1991) show that optimal managerial contracts may be used to address a class of agency problems arising in capital markets. In Dewatripont (1988), an incumbent firm uses a labor contract to commit to predatory pricing. Brander and Lewis (1986) argue that debt contracts could be used to commit oligopolistic firms to aggressive output decisions, and similar results were obtained by Maksimovic (1987, 1988). Fershtman and Judd (1987) and Sklivas (1987) study the use of managerial contracts as commitment devices in oligopolies. In Nagarajan (1988), stockholders use a managerial contract to commit to value-increasing takeover resistance. As pointed out by Holmström and Myerson (1983) and Hart and Tirole (1988), however, contracts differ from other commitment devices such as capital investment (Dixit, 1980) in the sense that firms can easily undo the commitment in the short run by renegotiating the contract away if it becomes mutually optimal for the parties to do so. With the exception of Dewatripont (1988), the above works ignore the implications of such renegotiation possibilities.

This paper shows that the requirement of renegotiation proofness may undermine the commitment value of contracts under symmetric information, but not generally under asymmetric information. For concreteness, we employ the leveraged oligopoly model of Brander and Lewis (1986) (henceforth, BL) and show that even if firms have the option to renegotiate a financial commitment, informational asymmetries may impair the renegotiation process enough to allow some debt contracts to function as credible commitments.

In the BL model, two firms play a two-stage oligopoly game under complete information. Both firms have the option to issue debt in the first stage and to play a Cournot output game in the second stage. BL show that under standard assumptions, debt can act as a strategic commitment to overproduce, forcing the competitor to cut its output. In a symmetric equilibrium, both firms have the incentive to issue ex ante a positive amount of debt, leading them to greater outputs and lower overall profits. In contrast, if only one firm is allowed to issue debt, this firm will gain an advantage by forcing the competitor to cut output, thus increasing its own output and profits.

We first formalize the claim that in the symmetric information BL model if the two firms could privately buy back their debt just prior to

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1 For a similar critique, but in the context of financial markets, see Ravid and Spiegel (1989).

2 We have borrowed the BL framework, rather than introducing a fresh one, for two reasons. First, its imperfect market of oligopoly offers a rich and well-understood setting in which to examine the commitment value of debt contracts. Second, and perhaps more important from a technical standpoint, the model remains tractable when renegotiation under asymmetric information is introduced.
choosing their outputs, they would repurchase all of it—thus completely undoing the ex ante commitment. This result obtains for two reasons. First, any leverage-driven overproduction in the output market lowers profits and is dominated by unlevered production. Second, in the absence of either a credible commitment not to renegotiate or any significant costs of renegotiation, the symmetric information structure in the BL model allows the firms to repurchase all the debt, internalizing all the output gains.

We then modify the BL model by introducing private information. One of the firms, say firm 1, after issuing its debt, receives some private information—either high (H) or low (L)—about the quality of its future performance. In this paper, debt renegotiation under asymmetric information is modeled in two steps in Sections 2 and 3. In Section 2, firms are given the option to privately repurchase some or all of their debt only in the final stage, just prior to choosing their outputs. In this case, the amount and the price of the debt repurchased by firm 1 may reveal to the creditors its private information in the ensuing (sub)game of asymmetric information, thus impairing the renegotiation process. By applying the intuitive criterion of Cho and Kreps (1987), we show that there exists a unique debt contract surviving the debt repurchase process. This contract is referred to as a lasting contract in this paper. Roughly speaking, a lasting debt contract is immune to renegotiation when the uniformed creditors of firm 1 form sophisticated beliefs about the out-of-equilibrium behavior of the informed firm. Such beliefs help rule out a potential pooling equilibrium in Section 2. We then relate the lasting contracts in this paper to the notions of durability (Holmstrom and Myerson, 1983) and renegotiation proofness (Maskin and Tirole, 1992).

In the unique separating equilibrium of Section 2, both firm 2 and the high-quality type of firm 1 buy back all their debt, whereas the low-quality type repurchases only a part of it—leaving a residual debt that acts as a credible commitment. The intuition behind this result is as follows. As indicated earlier, both types of firm 1 will have the ex post incentive to buy back all their debt, given the option to do so. If both types pool together by repurchasing all their debt, however, the low-quality firm will find that its debt is overvalued by the debtholders, thus increasing the cost of the repurchase. Hence, the low-quality firm may prefer to signal its type to its creditors by not repurchasing the entire debt, thus inducing them to tender at a lower price. While the competitor does not observe this repurchase immediately, it rationally expects that firm 1 will not have repurchased all its debt if it turned out to be of low-quality and keeps

3 For expositional clarity, the case of both firms being privately informed is not modeled here, but is briefly discussed in Section 4.
its output low, while both types of the informed firm produce more. Thus, the residual debt gives both types of firm 1 a commitment value in equilibrium.

In Section 3, both firms are allowed to renegotiate their debt in the interim as well, before the last stage of the game. Since debt repurchases executed at this interim stage are observed by the competitor, they may undermine the original commitment value. Hence, in the game of full information, no debt will be repurchased until the very last stage, just prior to choosing the outputs. In contrast, in the game of asymmetric information, any debt repurchase by the informed firm in the interim (i.e., after it learns its type) may reveal its private information to the competitor as well. The informed firm thus faces a problem of signaling to two audiences, its creditors and the competitor, and faces a richer set of tradeoffs. On one hand, both types of the informed firm can pool in the interim and stay anonymous until the final stage, when they separate as in Section 2, obtaining a commitment value from debt. On the other hand, they can try to convince their creditors and the competitor that they are of either high or low quality. Claiming a low quality may reduce the repurchase price demanded by the creditors but may induce the competitor to be aggressive in the product market. In contrast, claiming a high quality may intimidate the competitor while increasing the repurchase price. Corresponding to these two cases, we show that there may exist two equilibria in which the informed firm prefers to forego the renegotiation option in the interim, thereby preserving the ex ante commitment value of debt. Roughly speaking, these equilibria obtain when the commitment value of debt dominates the incentives to be identified as either type of the informed firm.

The role of asymmetric information in the renegotiation process is also relevant to the literature on reorganization under bankruptcy. In particular, our model implies that bargaining during Chapter 11 reorganizations may not always eliminate deadweight losses in the presence of asymmetric information, in contrast to Haugen and Senbet (1978). In this regard, our results are consistent with Giammarino (1989).

Our paper is related to that of Dewatripont (1988), who shows that an incumbent firm can commit to deter entry by entering into a labor contract that is renegotiation proof in equilibrium. This paper differs from Dewatripont...
Fulghieri and Nagarajan (1988) in that we model the renegotiation game as a \textit{sequential signaling} game rather than as a screening game and analyze the two-audience signaling issues. The latter have also been studied by Gertner \textit{et al.} (1988). The main differences between the games in the two papers are that we focus on the commitment value of debt (in addition to its signaling value), and we examine an \textit{additional} single-audience signaling problem at the last stage.


The paper is organized as follows. The next section introduces renegotiation in the BL model. In Section 2, asymmetric information is introduced, and the ex ante incentives to issue debt are examined. In Section 3, we examine the implications of allowing renegotiation in the interim as well. Section 4 presents some thoughts on possible extensions of the paper. We close with a brief discussion of the empirical implications of the model. All proofs can be found in the Appendix.

1. \textbf{RENEGOTIATION IN THE BL GAME}

In this paper, we study a modification of the duopoly game analyzed in BL that allows for renegotiation. As in BL, we assume that two firms play a two-stage game in a duopolistic market. In the first stage, they have the option to simultaneously issue debt with face values $D_i$, $i = 1, 2$. We assume that the debt is privately placed\textsuperscript{6} and that its level is publicly

\textsuperscript{6} We model privately placed debt because of its increasing importance, especially since the liberalization of SEC Rule 144A. Indeed, by the late 1980's, privately placed debt accounted for nearly 75\% by volume of the total issues and almost 60\% by dollar value (Investment Dealers Digest Information Services, 1991).
known after a brief (grace) period. In the second stage, after observing each other's level of debt, the firms simultaneously choose their output in a Cournot game. As in BL, these output choices are not observed by either the creditors or the competitor.

In this section, we model the renegotiation process as follows. At any time prior to choosing its output level $q_i$, each firm has the option to repurchase some or all of the private debt $D_i$ it had issued earlier. Again, the repurchase terms will become common knowledge only after the grace period. Given this structure, it can be shown that firms have no incentive to repurchase their debt until the very last moment. This is because repurchases made earlier would be observed by the competitor, and any reduction in debt would undermine the original commitment. Consequently, the repurchase option will be valuable only if exercised at the very last moment—that is, just prior to the output choice of both the firms. More importantly, each firm has an incentive to time its repurchases so that the competitor would learn its repurchase terms only when the latter's output is already chosen. Accordingly, we structure the modified game in two stages as follows.

Stage 1. The stockholders of the two firms simultaneously make take-it-or-leave-it offers $\{D_i, v_i\}$ to their respective creditors, where $v_i$ is the issue price of private debt $D_i$. Creditors can either accept or reject these offers. The debt levels and issue prices $\{D_i, v_i\}$ are then observed by the respective competitors.

Stage 2. The stockholders of both firms simultaneously make repurchase offers to their creditors. Each firm $i$ makes a prorated, all-or-nothing, take-it-or-leave-it offer to repurchase an amount $B_i \in [0, D_i]$ of its debt at a total price $r_i$. Again, the creditors can either accept or reject the repurchase offer. Immediately afterward, before observing the competitor's repurchase terms, both firms choose their output levels $q_i$ in a traditional Cournot-Nash game.

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7 This is consistent with the prevailing disclosure rules for publicly traded firms. For instance, the SEC reporting requirement involves filing Form 10Q every quarter to report, among other things, any debt issues or repurchases—thus allowing a maximum of 3 month's delay for the terms of a privately placed debt issue or repurchase to become public knowledge.

8 On the other hand, if one of the firms receives private information before the repurchase stage, the timing of the repurchase decision is much more complicated and will be explicitly analyzed in Section 3. This is due to the possibility of signaling to two audiences—namely, the competitor and the creditors (see Gertner et al., 1988).

9 Although we model privately placed debt in this paper, one can equivalently consider public debt as well. All the results in the paper go through, in the case of public debt, as long as the competitor is able to observe the terms of the public debt repurchase after an arbitrarily small time lag.
We also assume that each firm begins the second stage with initial cash of \( C_i \), which is sufficient for repurchase purposes. Any remaining cash is assumed to be paid out to the original stockholders in the form of a dividend or a stock repurchase. Again, these transactions will become common knowledge only after the outputs have been chosen. For notational ease, \( v_i \) will be subsumed in \( C_i \). For consistency, we shall use BL's notation as much as possible, except when noted. Specifically, the operating profit for firm \( i \) is \( R'(q_i, q_j, z_i) \), where \( z_i \) is a random variable distributed as \( F(\cdot) \) over the interval \([z, \bar{z}]\). The random variable \( z_i \) is not observed before the production levels are determined. The operating profit will be assumed to have all the convenient properties: \( R_{ii} < 0, R_{ij} < 0, R_{ij} < 0, \) and \( R_{iz} > 0 \). In addition, we assume throughout that the marginal profits increase with the random variable, i.e., \( R_{iz} > 0 \), in order to help focus on the commitment value of debt.\(^{10}\) We assume no discounting.

The two-stage game described above can be solved backward. In the last stage, the stockholders of each firm will choose the level of output that maximizes the expected payoff after repurchase, which is

\[
q_i^*(q_j, N_i) \in \arg \max_{q_i} V_i[q_i, q_j, N_i] = C_i - r_i
\]

\[
+ \int_{\hat{z}(N_i)}^{\bar{z}} \{R_i(q_i, q_j, z_i) - N_i\} f(z_i) dz_i, \tag{1.1}
\]

where \( N_i = D_i - B_i \) is the residual debt (after repurchase) and \( \hat{z}(N_i) \) is defined by

\[
R_i(q_i, q_j, \hat{z}) - N_i = 0. \tag{1.2}
\]

Following BL, we assume an interior solution for the output choice, given by the first-order condition\(^{11}\)

\[
V_i = \int_{\hat{z}(N_i)}^{\bar{z}} R_i(q_i, q_j, z_i) f(z_i) dz_i = 0. \tag{1.3}
\]

\(^{10}\) The case of \( R_{iz} < 0 \) is uninteresting since, as BL have shown, the debt does not give any incentive to overproduce and hence will not have any strategic commitment value.

\(^{11}\) The standard Cournot–Nash assumptions guaranteeing the reaction function stability and equilibrium uniqueness will be assumed here: \( V_{ii} < 0 \) and \( V_{ii} V_{ii} - V_{ij} V_{ji} > 0 \). It is well known that these conditions can be violated by quite reasonable specifications of \( R^i \) and \( f(z) \) (see Roberts and Sonnenschein, 1976). They are, however, satisfied for the standard case involving linear demand functions, constant marginal costs, and uniform distribution of \( z \). As in much of the literature and in the interest of focus, we leave open the more general issue.
Note that for any given repurchase decision $B_i = D_i - N_i$, the output strategy must satisfy (1.3). Thus, the impact of the repurchase decision $B_i$ on the output strategy $q_i^*(q_j, N_i)$ can be seen from the partial derivative $\frac{\partial q_i^*}{\partial B_i}$:

$$\frac{\partial q_i^*}{\partial B_i} = - \frac{V_{ii}}{V_{ii}} = \frac{R_i(z_i) f(z_i) dz_i}{V_{ii}}. \quad (1.4)$$

Since by assumption $R_i(z_i) > 0$, it follows from (1.3) that $R_i(z_i) < 0$. Also, from (1.2), $d\hat{z}_i / dB_i = -1/R_i(z_i) > 0$. Since $V_{ii} < 0$, it follows that $\partial q_i^* / \partial B_i < 0$. That is, repurchasing more debt decreases the output strategy of the firm. Thus, debt buyback works in the opposite direction of the initial debt, undermining its original commitment value.

Debt repurchases by the two firms are modeled as simultaneous, pro-rated, all-or-nothing, take-it-or-leave-it offers $\{B_i, r_i\}$ to their creditors. The creditors will accept the offer if and only if the repurchase price $r_i$ satisfies an appropriate individual rationality constraint. The assumption of prorated, all-or-nothing, take-it-or-leave-it offers serves two purposes. First, it simplifies the bargaining problem between the stockholders and debtholders in the issuing and repurchasing stages, by effectively giving all the bargaining power and hence all the surplus to the stockholders. Second, it solves a potential free-rider problem: Since the repurchase reduces the amount of outstanding debt and generates output gains, if the entire debt is not repurchased those creditors who do not accept the offer stand to benefit from the reduction in bankruptcy risk and to share these output gains with the stockholders. This free-rider problem would be similar to the one in Grossman and Hart (1980) in the takeover context.

Given the all-or-nothing, prorated, take-it-or-leave-it offer, the stockholders will proceed with the repurchase if and only if all the creditors tender their quota of debt. This makes every creditor pivotal, and thus eliminates the free-rider problem. Furthermore, since the offer is prorated, all creditors will tender the same fraction of debt. This allows the stockholders to extract all the surplus, avoiding financial expropriation by the creditors.

The creditors' individual rationality constraint is satisfied if they are offered the status quo value of their holdings. First, define

$$W_i[q_i, q_j, D_i] = \int_{\hat{z}_i}^{\hat{z}_j} R_i(q_i^*(D_i), q_j, z_i)f(z_i) dz_i + D_i[1 - F(\hat{z}_j(D_i))] \quad (1.5)$$

12 Other schemes that allow for different divisions of the surplus are more complex, but would not qualitatively alter the results.

as the status quo value of the existing debt $D_i$. Then the stockholders’ problem at the repurchase stage is to

$$\max_{[b_i, r_i]} V_i[q_i, q_j, N_i] = C_i - r_i + \int_{\Re} \{R_i(q_i^*(N_i), q_j, z_i) - N_i\} f(z_i) dz_i$$

s.t. \(r_i + W_i[q_i^*(q_j, N_j), q_j, N_j] \geq W_i[q_i^*(q_j, D_i), q_j, D_i], \) \quad (1.6)

where $N_i = D_i - B_i$. The creditors’ individual rationality constraint in program (1.6) requires that the cash offered plus the value of their residual debt after the repurchase be at least equal to the value of their status quo debt. Note that the repurchase can be equivalently interpreted as a cash plus debt exchange offer for the original debt. The following proposition characterizes the equilibrium of the BL game with the repurchase option.

**Proposition 1.1.** For any arbitrary initial debt $D_i$,

$$v_i^* = r_i^*$$

$$B_i^* = D_i$$

$$r_i^* = \int_{\Re} R_i(q_i^*, q_j^*, z_i) f(z_i) dz_i + D_i[1 - F(\hat{z}(D_i))]$$

$$q_i^* = q_c^*,$$

where $q_c^*$ is the symmetrical equilibrium output in the standard unlevered Cournot game.

**Proof.** See Appendix.

That is, in equilibrium, both firms will repurchase all their debt and play the unlevered Cournot–Nash output game. Since the debtholders rationally anticipate the end game, the ex-ante issue price $v_i^*$ will be equal to the repurchase price $r_i^*$.

Proposition 1.1 formalizes the criticism of Holmström and Myerson (1983) and Hart and Tirole (1988) as applied to the BL model: Firms have an incentive to renegotiate contractual commitments that involve taking suboptimal actions. This result obtains for two reasons. First, as BL acknowledge, any leverage-driven overproduction by both firms will be unprofitable and is dominated by unlevered production. Second, in the absence of either a credible commitment not to renegotiate or significant costs of renegotiation, the symmetric information structure in the BL model allows the firms to renegotiate the debt with their creditors. Note that the ex ante debt $D_i$ is rendered irrelevant and its choice, indeterminate. This observation leads to the following result.
COROLLARY 1.1. In the presence of arbitrarily small issue costs and no other benefits, neither firm will issue any ex ante debt in equilibrium.

We now introduce a simple asymmetric information structure to the BL model.

2. THE ASYMMETRIC INFORMATION GAME

The game in this section is the same as the game with full information, save for the following modification: after stage 1, and before stage 2, stockholders of firm 1 privately observe a signal \( s \in \{L, H\} \) about the realization of \( z_1 \), where \( H \) is a "better" signal than \( L \), in the sense of first-order stochastic dominance (Milgrom, 1981): \( F(z_1|H) \leq F(z_1|L) \) and \( \alpha = P\{s = H\} \). Later, however, in order to prove the single crossing property in the general case, we will need the condition

\[
F(z_1^H|H) \leq F(z_1^L|L),
\]

for any given level of residual debt and output strategies of the two firms, where \( z_1^H \) and \( z_1^L \) are the bankruptcy states corresponding to the two signals. Condition (2.1) implies that the better signal indicates a lower probability of bankruptcy in equilibrium. In the standard case where the demand is linear, the marginal cost is constant, and \( z_i \) is distributed uniformly, it is straightforward to show that first-order stochastic dominance implies condition (2.1). In general, however, condition (2.1) is somewhat stronger than first-order stochastic dominance. This is because the signal has two opposing effects: if it improves the likelihood of profits, it may also give the firm an incentive to produce more in stage 2, thus lowering profits in bad states and increasing the bankruptcy state \( z_1 \). Therefore, the natural definition of a good signal in this context is one that improves the overall profitability and lowers the bankruptcy probability, accounting for this overproduction incentive. Given the generality of \( R^i \) and \( f(z_i) \) in our model, however, there exist no simple restrictions on the parameter space that yield (2.1). Hence, in the interest of exposition, we shall use condition (2.1) as an assumption, keeping in mind that first-order stochastic dominance will suffice in many cases, including the standard one.

Firm 2 observes no signal and has no private information. Note that the game with asymmetric information has three (virtual) players—firm H, firm L, and firm 2. Accordingly, we generalize the assumptions on the stability of reaction functions and the uniqueness of the equilibrium in the Cournot game as follows: let \( V_i^i < 0 \), for \( i \in \{H, L, 2\} \) and the determinant

\[14\] The alternative formulation in which firm 1 observes its type before stage 1 is discussed in Section 4.
of the Jacobian $|J|$ of the first-order conditions be negative (Takayama, 1985).\footnote{These are more conveniently displayed as (A2) in the Appendix.}

Before studying the equilibrium for the entire asymmetric information game, it is useful to consider the subgame starting in stage 2. In this subgame, firm 1, having privately observed the signal $s \in \{L, H\}$, determines, simultaneously with firm 2, the amount of debt to be repurchased $B_i$, the repurchase price $r_i$, and subsequently, the amount of output $q_i$. Again, due to the reporting lag, the repurchase terms of the informed firm become known to its competitor only after the output of the uninformed firm is chosen. This also implies that the debtholders of firm 2 observe only $B_2$ and not $B_1$, at the time they decide to tender their debt for repurchase.

2.1. The Uninformed Firm

We first characterize the (uninformed) firm 2’s optimal strategies at the repurchasing and production stages. Note that the output strategy of the uninformed firm will depend on its (in equilibrium, correct) conjectures about the output decisions of both types of the informed firm 1, $\{q_1^L, q_1^H\}$. The output strategy of the uninformed firm is given by

$$q_2^*(q_1^H, q_1^L, N_2) \in \arg \max_{q_2} \alpha \int_{\hat{z}_2(N_2)} \{R^2(q_1^H, q_2, z_2) - N_2\} f(z_2) dz_2$$

$$+ (1 - \alpha) \int_{\hat{z}_2(N_2)} \{R^2(q_1^L, q_2, z_2) - N_2\} f(z_2) dz_2,$$

where $\hat{z}_2^H(N_2)$ and $\hat{z}_2^L(N_2)$ are defined by $R^2(q_1^H, q_2, \hat{z}_2^H) = N_2$, $R^2(q_1^L, q_2, \hat{z}_2^L) = N_2$.

The status quo value of existing debt $D_2$ at the repurchasing stage depends on the output of both types of firm 1 and can be defined as

$$W^2[q_1^H, q_1^L, q_2, D_2] = \alpha W^2[q_1^H, q_2, D_2] + (1 - \alpha) W^2[q_1^L, q_2, D_2],$$

where $W^2[q_1^L, q_2, D_2]$ is as defined before in (1.5). Firm 2 chooses the repurchase amount and its price by solving the program:

$$\max_{(B_2, r_2)} C_2 - r_2 + \alpha \int_{\hat{z}_2(N_2)} \{R^2(q_1^H, q_2(N_2), z_2) - N_2\} f(z_2) dz_2$$

$$+ (1 - \alpha) \int_{\hat{z}_2(N_2)} \{R^2(q_1^L, q_2(N_2), z_2) - N_2\} f(z_2) dz_2$$

s.t. $r_2 + W^2[q_1^H, q_1^L, q_2^*(N_2), N_2] \geq W^2[q_1^H, q_1^L, q_2^*(D_2), D_2].$ (2.2)
As before, the constraint in (2.2) guarantees the tendering debtholders at least the expected value of their status quo holdings. The following lemma characterizes the uninformed firm's optimal repurchase strategy.

**Lemma 2.1.** Given any initial level of debt $D_2$, and for any output choices $\{q_2^L, q_2^H\}$ of the competitor, firm 2 will optimally repurchase all its debt, i.e., $B_2^* = D_2$ at a price equal to $W^2[q_2^H, q_2^L, q_2^*(D_2), D_2]$.

**Proof.** See Appendix.

The intuition for the uninformed firm's optimal strategy is similar to that in the BL game: absent any private information, debt has no commitment value at this stage. If the debt has a buyback option, the uninformed firm will fully repurchase all its debt, irrespective of its competitor's strategy in the output game. This renders the initial choice of $D_2$ irrelevant, as we see later.

### 2.2. The Informed Firm

Consider now the informed firm 1 at stage 2, after its debt $D_1$ has been announced and after it has privately observed the signal $s \in \{L, H\}$. For any given output of the uniformed firm $q_2$, firm 1 makes a take-it-or-leave-it offer $\{B_1, r_1\}$ and its debtholders accept or reject the offer. Given that the firm has observed $s$ and has repurchased $B_1$ of the outstanding debt $D_1$, its output choice $q_1^*$ is given by

$$q_1^*[q_2, N_1|s] \in \arg\max_{q_1} \int_{\tilde{z}(N_1)} \{R^1(q_1, q_2, z_1) - N_1\} f(z_1|s)dz_1,$$

where $\tilde{z}(N_1)$ is defined as $R^1(q_1, q_2, \tilde{z}(N_1)) = N_1$, $\forall s \in \{L, H\}$.

At the output-repurchasing stage, firm 1 faces a signaling problem, since it moves first: its offer of $B_1$ may reveal its private information $s$ to its debtholders. Therefore, its repurchase offer must satisfy the appropriate incentive compatibility constraint. Define the status quo value of the existing debt $D_1$ for the $s$ type firm as

$$W^1[q_1, q_2, D_1|s] = \int_{\tilde{z}(D_1)} R^1[q_1, q_2, z_1] f(z_1|s)dz_1 + D_1[1 - F(\tilde{z}(D_1))].$$

As before, the debtholders will accept the offer if the sum of the proposed repurchase price $r_1$ and the status quo value of the residual debt $N_1$ is at least equal to the expected status quo value of the existing debt $D_1$,

$$r_1 + \mu W^1[q_1^*(N_1|H), q_2, N_1, H] + (1 - \mu)W^1[q_1^*(N_1|L), q_2, N_1|L] \geq \mu W^1[q_1^*(D_1), q_2, D_1|H] + (1 - \mu)W^1[q_1^*(D_1), q_2, D_1|L],$$
where \( \mu \) represents the debtholders’ subjective (conditional) belief at the accept/reject stage that the stockholders have observed the signal \( s = H \). Moreover, since the debtholders can condition their belief on the entire history of the game thus far, the belief \( \mu \) will depend on the amount of debt repurchased: \( \mu(B_1): [0, D_1] \to [0, 1] \). Following the game theoretic literature, beliefs of the form \( \mu(B_1) = \alpha, \forall B_1 \in [0, D_1] \) will be referred to as passive conjectures.

As with many signaling games, there may exist both a pooling equilibrium and a continuum of separating equilibria. The following proposition characterizes the pooling equilibrium for this subgame.

**Proposition 2.1.** For any initial debt \( D_1 \) and for any output level \( q_2 \) chosen by firm 2, there exists a pooling equilibrium \( \{B^*(s) = D_1, r^* | \forall s\} \) in the repurchase subgame between firm 1 and its debtholders that is supported by passive conjectures, if

\[
- r^* + \int_{z_1} R^1[q^*|q_2, 0|L, q_2, z_1]f(z_1|L)dz_1 \\
\geq \int_{z_1(0)} R^1[q^*|q_2, D_1|L, q_2, z_1] - D_1]f(z_1|L)dz_1, \tag{2.3}
\]

where

\[
\begin{align*}
 r^* &= \alpha W^1[q^*|q_2, D_1|H, q_2, D_1, H] \\
 &\quad + (1 - \alpha)W^1[q^*|q_2, D_1|L, q_2, D_1, L].
\end{align*}
\]

**Proof.** The proof closely follows that of Lemma 2.1 and is omitted.

In the pooling equilibrium, both types of firm 1 recall all their debt at the same (average) price \( r^* \). Clearly, the high-quality firm is willing to repurchase is entire debt in the pooling equilibrium because its debt is now worth more than the average price. In contrast, the low-quality firm faces the following tradeoff: On one hand, it can benefit from repurchasing its debt, since levered production is unprofitable; on the other hand, it is forced to repurchase the debt at a loss, since the pooling price is greater than its true value. Condition (2.3) guarantees that the first effect dominates the second and that the low-quality firm will also prefer to repurchase all its debt, as compared to the status quo.

Note that condition (2.3) represents a significant restriction on the parameter space. If it fails to hold, then the pooling equilibrium cannot exist. Moreover, pooling equilibria are not very interesting if a single-crossing property is satisfied and thus separating equilibria exist. This is because the pooling equilibrium will not satisfy the intuitive criterion of Cho and Kreps (1987), which is, in fact, satisfied by the least-cost separating equilibrium. The following definition will be useful later.
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DEFINITION. A contract \( \{D_1, N_1(s), r_1(s)\mid N_1(s) = D_1 - B_1(s), s = L, H\} \) is a lasting contract if it survives the intuitive criterion.

The following lemma will be useful in showing the existence of a separating equilibrium.

**Lemma 2.2.** (Single-crossing property:) \[ \frac{\partial r_1(H)}{\partial B_1(H)} > \frac{\partial r_1(L)}{\partial B_1(L)}. \] (2.4)

**Proof.** See the Appendix.

The single-crossing property implies that the maximum price at which the stockholders of firm 1 are willing to repurchase their debt is different for the two types. By condition (2.1), type \( H \) firm has a lower probability of bankruptcy and, therefore, a debt with a higher value. Ceteris parabus, type \( H \) firm will be willing to repurchase its debt at a higher price than type \( L \) firm. This difference in their marginal willingness to pay opens up the possibility of a separating equilibrium. In particular, the low-type firm may find it optimal to reveal its true type to the creditors, if, in doing so, it can repurchase its debt at a lower price. In what follows, we show that the amount of debt repurchased can indeed convey such information giving rise to a separating equilibrium. First, define \( \hat{B}_1 \) as

\[
-W_1[q_1^t(q_2, D_1|H), q_2, D_1|H] + \int_{z_1}^{\xi} R_1[q_1^t(q_2, 0|H), q_2, z_1]f(z_1|H)dz_1 = -W_1[q_1^t(q_2, D_1, D_1|L), q_2, D_1] + W_1[q_1^t(q_2, D_1 - \hat{B}_1|L), q_2, D_1 - \hat{B}_1|L] + \int_{z_1}^{\xi} \{R_1[q_1^t(q_2, D_1 - \hat{B}_1|H), q_2, z_1] - (D_1 - \hat{B}_1)f(z_1|H)\}dz_1.
\]

(2.5)

That is, \( \hat{B}_1 \) is such that type \( H \) is indifferent between repurchasing all its debt, i.e., \( B_1^H = D_1 \) while being perceived as \( H \) type, and repurchasing only \( \hat{B}_1 < D_1 \) at a lower price, conveying the (false) impression that it is type \( L \). Note that the single crossing property (2.4) implies that \( \hat{B}_1 \) is uniquely defined by Eq. (2.5).16

As indicated earlier, a common property of signaling games is that there may be a continuum of separating equilibria. These equilibria are characterized by repurchase levels \( B_1 \leq \hat{B}_1 \). They can be ranked from the point of view of the informed firm, however, and it can be shown (see Cho and Kreps, 1987) that the dominant (also known as the Riley or the least-cost-

16 See also the proof of Proposition 2.2.
separating) equilibrium alone will survive the intuitive criterion. Hence, in what follows, we focus on the dominant separating equilibrium.

**Proposition 2.2.** For any initial debt $D_1$ and for any output $q_2$ chosen by firm 2, the following is an equilibrium of the repurchase game:

- Type $H$ offers $\{B_H^1, r_H^1\}$ such that
  
  $B_H^1 = D_1$
  
  $r_H^1 = W'[q_H^1(q_2), D_1 | H], q_2, D_1, H].$

- Type $L$ offers $\{B_L^1, r_L^1\}$ such that
  
  $B_L^1 = \hat{B}_1$
  
  $r_L^1 = W'[q_L^1(q_2), D_1 | L], q_2, D_1, L]
  
  $- W'[q_L^1(q_2), D_1 - \hat{B}_1 | L], q_2, D_1 - \hat{B}_1, L].$

This equilibrium is supported by the following beliefs held by the debtholders:

$$
\mu(B_1) = \begin{cases} 
0 & \text{if } B_1 \leq \hat{B}_1 \\
1 & \text{if } B_1 > \hat{B}_1.
\end{cases}
$$

**Proof.** See Appendix.

That is, in the dominant separating equilibrium, the high type repurchases all its debt at the price $r_H^1$ equal to the full-information value of its existing debt $D_1$. The low type undertakes only a partial repurchase, where the quantity and price of its debt repurchase $\{\hat{B}_1, r_L^1\}$ are chosen according to (2.5) in such a way that the high type will not attempt to mimic the low type—given the above equilibrium beliefs.

Thus, the only equilibrium to survive the intuitive criterion is the dominant separating equilibrium. Interestingly, this result can be generalized to cases with more than two types, in contrast to the standard signaling models. This is because, in our model, the informed party proposes both the repurchase amount $\hat{B}_1$ as well as the offer price $r_1$. Finally, note that given our two-type information structure, the dominant equilibrium is also *divine* in the sense of Banks and Sobel (1987).

Recalling the earlier definition, a lasting contract can now be written as $\{D_1, (N_T^1, r_T), (0, r_H^1)\}$, where $N_T^1 = D_1 - \hat{B}_1$. It is easy to show that it is interim incentive-efficient and durable, in the sense of Holmström and

Myerson (1983). It is also *ex-post* efficient, and strongly Cho-Kreps *renegotiation proof* in the sense of Maskin and Tirole (1992). We omit these proofs.

Thus far, we have characterized the optimal unilateral behavior of each firm in the output-repurchase subgame for given strategies of the opponent. The following proposition summarizes the *equilibrium* of the output-repurchase subgame in stage 2.

**Proposition 2.3.** For any given levels of initial debt \( \{D_1, D_2\} \), the unique equilibrium that satisfies the intuitive criterion in the output-repurchase game is a strategy combination, \( \{(B_1^{*H}, r_1^{*H}, q_1^{*H}), (B_1^{*L}, r_1^{*L}, q_1^{*L})\}, \{B_2^{*}, r_2^{*}, q_2^{*}\} \) and a belief function \( \mu^* (B_1) \) such that

\[
\begin{align*}
B_1^{*H} &= D_1, \\
r_1^{*H} &= W_1^1[q_1^{*H}(D_1), q_2^{*}, D_1|H] \\
B_1^{*L} &= \hat{B}_1, \\
r_1^{*L} &= W_1^1[q_1^{*L}(D_1), q_2^{*}, D_1|L] - W_1^1[q_1^{*L}(D_1 - B_1^{*L}), q_2^{*}, D_1 - B_1^{*L}|L] \\
q_1^{*s} &= q_1^{*[s]}(q_1^{*s}, D_1 - B_1^{*s}|s) \quad \forall \ s = L, H \\
B_2^{*} &= D_2, \\
r_2^{*} &= W_2^2[q_1^{*H}, q_1^{*L}, q_2^{*}(D_2), D_2] \\
q_2^{*} &= q_2^{*[0]}(q_1^{*H}, q_1^{*L}, 0)
\end{align*}
\]

and

\[
\mu(B_1) = \begin{cases} 
0 & \text{if } B_1 \leq \hat{B}_1 \\
1 & \text{if } B_1 > \hat{B}_1.
\end{cases}
\]

**Proof.** Follows immediately from Lemma 2.1 and Proposition 2.2 and is omitted.

Proposition 2.3 says that, in the equilibrium of the output-repurchase subgame, the high type of firm 1 as well as firm 2 repurchase all their debt, while the low type of firm 1 undertakes a partial repurchase as in Proposition 2.2. The repurchase terms for the low type \( \{B_1^{*L}, r_1^{*L}\} \) are chosen such that the high type will not attempt to mimic the low type in equilibrium. The repurchase price of the uninformed firm \( r_2^{*} \) is chosen according to Lemma 2.1. The output of firm 1, \( q_1^{*s} \), will depend on its type \( s \) and the residual debt \( D_1 - B_1^{*s} \), whereas the output choice of firm 2, \( q_2^{*} \), will account for the possibility that it may be playing against an unlevered high type or a levered low type of firm 1.
From Propositions 2.2 and 2.3 it is clear that while the high type repurchases all its debt \( N_{TH} = 0 \), the equilibrium repurchase by the low type \( B_{TL} \) depends on \( D_1 \). Moreover, the residual debt of the low type \( N_{TL} = D_1 - B_{TL} > 0 \), and it will influence the choice of outputs by both firms in the output game. We can write the equilibrium outputs as \( \{ q_{TH}(N_{TL}), q_{TL}(N_{TL}), q_{TL}(N_{TL}) \} \). To demonstrate the commitment value of this residual debt, it is necessary to determine the impact of \( N_{TL} \) on the output of the uninformed firm.

**Lemma 2.3.** The output of the uninformed firm is decreasing in the residual debt of the type L firm, i.e., \( dq_{f}/dN_{TL} < 0 \).

**Proof.** See Appendix.

Lemma 2.3 illustrates the commitment value of the residual debt carried by the low-type firm. As in BL, this residual debt commitment forces the rival firm to reduce its output. More importantly, here is also lasting—as Propositions 2.2 and 2.3 indicate—since the debt contract now is immune to renegotiation. It is interesting to note that the high-quality firm, despite not carrying any debt ex post, benefits from this residual debt of the low-quality firm. This is because the uninformed firm, uncertain about the type of the informed firm, rationally anticipates the possibility of its competitor being a low-quality one and cuts its output. Thus, both types of the informed firm benefit from the residual debt commitment of the low-type firm.

### 2.3 The ex ante Commitment Value of Debt

Next, we turn to the analysis of the firms' ex ante incentive to issue debt. Since the uninformed firm repurchases all the debt it had issued earlier, in the presence of even small but positive transaction costs, it has no incentive to issue any in stage 1. Hence, we need to consider only the ex ante incentives of the informed firm. Given the equilibrium output levels \( \{ q_{TH}(N_{TL}), q_{TL}(N_{TL}), q_{TL}(N_{TL}) \} \) and the fact that the debt is fairly priced, the ex ante expected value of the firm from an ex post residual debt \( N_{TL} \) is given by

\[
EV = \alpha \int_{\xi}^{\zeta} R^{T\lambda}[q_{TH}(N_{TL}), q_{TL}(N_{TL}), z_1]f(z_1|H)dz_1
+ (1 - \alpha) \int_{\xi}^{\zeta} R^{T\lambda}[q_{TH}(N_{TL}), q_{TL}(N_{TL}), z_1]f(z_1|L)dz_1. \tag{2.6}
\]

Computing the derivative of this ex ante expected value at \( N_{TL} = 0 \), we get
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\[ \frac{\partial EV}{\partial N^*_1 \ell} \bigg|_{N^*_1 = 0} = \alpha \int_{\tilde{z}}^{\hat{z}} R_{1,2}^1[q_1^H, q_2, z_1]f(z_1|H)dz_1 \frac{dq_2}{dN^*_1 \ell} + (1 - \alpha) \int_{\tilde{z}}^{\hat{z}} R_{1,2}^2[q_1^L, q_2, z_1]f(z_1|L)dz_1 \frac{dq_2}{dN^*_1 \ell} > 0. \] (2.7)

Thus, residual debt is desirable ex ante. We have proved the following.

**Proposition 2.4.** In equilibrium, firm 1 issues a positive amount of debt ex ante.

Let \( D_1^* \) be the equilibrium ex ante debt issued by the informed firm.\(^ {18} \) Thus, in the game of asymmetric information, only the informed firm gains a commitment value from debt. By committing itself to higher output, the informed firm obtains greater profits at the expense of the competitor. This result is similar to the one we would obtain in the basic BL model, if only one firm is allowed to issue debt.

An empirical implication of (2.7) is that debt issues have a positive announcement effect on the stock price of the informed firm, but a negative impact on that of the competing firm. The former is consistent with the findings in Myers and Majluf (1986). Their paper differs from ours, however, since the positive announcement effects of debt issues in their paper derive from adverse selection alone, whereas the announcement effects in our paper arise from the commitment value of debt. To distinguish the predictions of our model from that of Myers and Majluf (1986), an event study could be conducted for the announcement effect on the competing firm's stock returns, not just on the firm that issues debt.

3. **Timing of the Repurchase Decisions**

In this section, we relax the assumption in Section 2 that the firms can repurchase their debt just prior to their output choice and unobserved by their competitors. We now assume that, in addition to the repurchase option at the last stage examined earlier, both firms can repurchase all or part of their debt before the last stage of the game as well. This process of interim renegotiation is modeled as follows. Both firms, after firm 1 learns its type, and before choosing their outputs, can again make simultaneous, all-or-nothing, prorated, take-it-or-leave-it repurchase offers to their creditors.\(^ {19} \) The terms of these interim offers become common knowledge to

\(^{18}\) The interior solution can be obtained by setting the derivative of (2.6) equal to zero.

\(^{19}\) There is a trivial possibility where firms can repurchase their debt before the private information is received. Since debt repurchases executed at this stage are observable by the competitor, the firms face the same problem as they did at the ex ante stage, when they were
both firms and their respective creditors, well before the outputs are chosen. Any debt outstanding after the interim repurchase will be subject to possibly another round of renegotiation at the last stage, which will proceed exactly as in Section 2.

Note that the new assumption of interim renegotiation does not change either the equilibrium of the full information game in Section 1 or the optimal strategies of the uninformed firm in the game of asymmetric information of Section 2.1. This follows from the fact that in either case debt has no commitment value and it will be entirely repurchased at the last stage, as in Sections 1 and 2.2, leaving the actual timing of the repurchase in the expanded game indeterminate. We, therefore, focus only on the equilibrium strategies of the informed firm.

The informed firm has two nontrivial repurchasing decisions to make, one at the interim stage and the other at the last stage. Note that the difference in the timing of these two decisions lies in the fact that the terms of the repurchase undertaken at the interim stage are known to the competitor, whereas those done at the last stage are not. At the interim stage, the informed firm faces a problem of signaling to two audiences—namely, its creditors and the uninformed competitor—similar to the one studied by Gertner et al. (1988).* The results in Section 2 suggest that if both types of the informed firm pool in the interim, however, the informed firm will still gain a commitment value from having some debt in its capital structure at the last stage. We focus on such equilibria in this section.*

The existence of equilibria in which both types of the informed firm pool in the interim will depend on the relative gains from signaling in the interim (the signaling value), rather than remaining anonymous (pooled) in the interim until the last stage (the commitment value). There are two possible benefits from signaling in the interim: First, by convincing its creditors that it is of lower quality, firm 1 may be able to buy back its debt. It both firms issued a positive amount of debt that is ex ante optimal, they will have no incentive to repurchase any debt at this stage, before the private information is received.

The main differences between the games in the two papers are that we focus on the commitment value of debt (in addition to its signaling value), and we examine an additional single audience signaling problem at the last stage.

Equilibria in which the two types of the informed firm separate in the interim by repurchasing all their debt at their (different) full information prices may also exist under more restrictive conditions. An interesting question then is which equilibria will survive a relevant refinement criterion. Our task is complicated by the possibility of sequential signaling by the informed firm, which requires more sophisticated refinement criteria, perhaps as in Cho (1987). This issue is beyond the scope of the current paper. Nevertheless, given the insight in Gertner et al. (1988), it seems likely that equilibria with interim pooling will survive the criterion of perfect sequentiality, eliminating equilibria involving separation in the interim.
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at a lower price, and the second incentive is to convince the competitor that it is of high quality, so as to induce the competitor to cut its output. Correspondingly, two types of equilibria that involve pooling in the interim may result, depending on whether the commitment value exceeds the signaling value in each of these two cases—which in turn, depend on the parameters of the model. Of the two equilibria, the one where the alternative to pooling strategy for firm 1 is to convince both its creditors and the competitor that it is of low quality is simpler, and hence will be taken up first.

To this end, note that by influencing the competitor’s beliefs, the informed firm can affect the competitor’s choice of the equilibrium output in the final Cournot game. Let $q_{2L}^s$ be the equilibrium output choice of firm 2, if it believes that firm 1 is of type L, and $q^s = q^s(N_{1L}^L)$ if it believes that it is of type H with probability $\alpha$ and that type L will have a residual debt of $N_{1L}^L$. It is easy to check that $q_{2L}^s > q_{2H}^s$ and that $q_{2L}^s > q^s$. Define $\Delta V_1(L)$ as the incremental expected profit for an informed firm of type s from being perceived as type H with probability $\alpha$, rather than being perceived as type L with probability 1 and having no residual debt. That is,

$$\Delta V_1(L) = \int_{z}^{\hat{z}} R_1[q^s(q_{2L}^s, N_{1L}^L | s), q_{2L}^s, z_1] f(z_1 | s) dz_1$$

$$- \int_{z}^{\hat{z}} R_1[q^s(q_{2H}^s, 0 | s), q_{2H}^s, z_1] f(z_1 | s) dz_1,$$  \hspace{1cm} (3.1)

where $N_{1H}^L = 0$ and $N_{1L}^L > 0$. Similarly, let $\Delta W_1(I)$ be the incremental status quo value of the debt of a firm of type s, if it is perceived by its competitor and creditors as type H with probability $\alpha$, rather than as type L with probability 1:

$$\Delta W_1(L) = W_1[q^s(q_{2L}^s, D_1 | s), q_{2L}^s, D_1 | s]$$

$$- W_1[q^s(q_{2H}^s, D_1 | s), D_1 | L], q_{2L}^s, D_1 | L].$$  \hspace{1cm} (3.2)

Denote the interim-stage strategies and beliefs with primes. We are now ready to establish the existence of the following equilibrium outcome in the subgame starting at the interim stage.22

22 Since the equilibrium will be determined with respect to the beliefs of the two uninformed parties—namely, the creditors and the competitor—there may exist other less interesting equilibria as well. This is due to the possibility that their beliefs, if uncoordinated, could be different. As in Gertner et al. (1988), we consider only coordinated beliefs that can be achieved through, for instance, simple “cheap-talk” communication. For more on this, see Farrell (1986).
PROPOSITION 3.1. For a given level of ex ante debt $D_1$, in the subgame starting at the interim stage, there exists an equilibrium with pooling in the interim where no debt is repurchased, followed by separation at the last stage as in Proposition 2.3, if

$$\Delta V_j(L) > \Delta W_j(L) \quad \forall \ s = H, L. \tag{3.3}$$

This equilibrium is supported by the interim beliefs

$$\mu'(B'_1) = \begin{cases} \alpha & \text{if } B'_1 = 0 \\ 0 & \text{else} \end{cases}. \tag{3.4}$$

Proof. See Appendix.

This result can be understood as follows. The terms of debt repurchases executed in the interim are observed by both the creditors and the competitor, determining their beliefs about firm 1's true type. Optimal strategy of the informed firm will then depend on the interactions of the effects induced on the creditors and the competitor in the interim as well as in the last stage of the game. As discussed in Section 2, debt has a commitment value for the informed firm only if the two types are still pooled together before entering the last stage. The informed firm then has the option to forgo debt repurchases in the interim and postpone them to the last stage, where it will behave exactly as in Proposition 2.3. Furthermore, given the belief structure (3.4), type $H$ of the informed firm has the option to repurchase all its debt in the interim, at a (lower) price reflecting the creditors' beliefs that it is of the low type. These savings, however, obtain at a cost, since the competitor will now infer that firm 1 is of low type and hence will increase its output. A type $L$ firm, in contrast, has the option to repurchase all its debt at its (low) full information price. Since if type $L$ pools in the interim, it will eventually repurchase only $B_1$, full repurchase in the interim allows this type to avoid the overproduction losses resulting from a partial repurchase in the last stage (the cost of the later separation). These savings again obtain at the cost of inducing the competitor to increase its output. The incentive compatibility constraint (3.3) guarantees that the benefits in the interim from the commitment value of debt outweigh the net gains from being identified as type $L$.

The second equilibrium, where the informed firm's alternative to pooling in the interim is to convince its creditors and the competitor that it is of higher quality, is more complicated for the following reason. Since the competitor does not observe the final stage repurchase terms, the low type can signal in the interim that it is of higher quality by repurchasing an arbitrarily small amount of debt at a higher price, thus inducing its competitor to cut its output. Later on, in the final stage, it can repurchase up
to $\hat{B}_1$ at the lower price, thus revealing its true quality, but only to its creditors. The incentive compatibility constraint for the second equilibrium, therefore, must account for this possibility. Accordingly, define $\Delta V^*_i(H)$ as the incremental expected profit for an informed firm of type $s$ from being perceived as type $H$ with probability $\alpha$, rather than being perceived as type $H$ with probability 1. That is,

$$\Delta V^*_i(H) = \int_{\mathbb{R}} R^1[q^*_i(q^t, N^t|s), q^t, z_i]f(z_i|s)dz_i$$

$$- \int_{\mathbb{R}} R^1[q^*_i(q^{*H}, N^*|s), q^{*H}, z_i]f(z_i|s)dz_i.$$  

Similarly, let $\Delta W^*_i(H)$ be the incremental status quo value of the debt of a firm of type $s$, if it is perceived by its competitor and creditors as type $H$ with probability $\alpha$, rather than as type $H$ with probability 1:

$$\Delta W^*_i(H) = W^1[q^*_i(q^t, D_1|s), q^t, D_1|s] - W^1[q^*_i(q^{*H}, D_1|s), q^{*H}, D_1|s].$$

We have the following equilibrium.

**Proposition 3.2.** For any initial debt $D_1$, a second equilibrium with pooling in the interim exists in the subgame starting at the interim stage where no debt is repurchased, followed by separation in the last stage, as in Proposition 2.3, if

$$\Delta V^*_i(H) > \Delta W^*_i(H) \quad \forall \quad s = H, L.$$  

This equilibrium is supported by the interim beliefs

$$\mu'(B'_i) = \begin{cases} \alpha & \text{if } B'_i = 0 \\ 1 & \text{else.} \end{cases}$$

**Proof.** See Appendix.

In the second equilibrium involving pooling in the interim, a firm of type $H$ faces the following incentives. If it does not repurchase any debt at this stage, it will remain pooled with type $L$. Separation will then occur in the

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23 Such a strategy is typically not credible in many sequential signaling games. It is a viable strategy in our model because one of the audiences in the interim, namely the competitor, does not observe the final stage signals. This feature is also different from that of the sequential renegotiation of Beaudry and Poitevin (1990), in which all the contracting agents observe all the signals at every stage.
last stage, when all its debt will be repurchased as in Section 2, at a price reflecting an uninformed competitor and informed creditors. Since, as in Proposition 2.3, type L will have residual debt and the competitor will rationally anticipate this possibility, type H gains the commitment value of debt. On the other hand, given the interim beliefs (3.8), by repurchasing some or all its debt in the interim, the firm can signal a higher quality to its competitor as well, and may possibly induce it to cut its output even further. As a result, the cost of repurchase may be different for these two strategies. A firm of type L, in contrast, faces the following incentives. If it does not repurchase any debt in the interim, it will remain pooled with type H until the last stage, when it repurchases \( \hat{B}_1 \) at a lower price. On the other hand, a firm of type L may repurchase an arbitrarily small amount of debt in the interim, leading the competitor to believe that it is of type H, forcing the competitor to reduce output. Subsequently, in the last stage, the firm may still execute a partial debt repurchase, which will enable it to reveal its true type only to its creditors, thus lowering the repurchasing price. Condition (3.7) ensures that the benefits for both types from the commitment value of debt dominates the overall gains from being perceived as type H, supporting pooling in the interim.

In summary, the informed firm may find in the interim conflicting incentives in the execution of debt repurchases. These derive from the fact that, at this stage, the firm signals its private information to both its creditors and the competitor. However, if the commitment value of debt is sufficiently large and either condition (3.3) or (3.7) is satisfied, the informed firm will not repurchase any debt in the interim, even if it has the option to do so. In this case, the commitment value of ex ante debt established in Proposition 2.4 will be preserved.

4. EXTENSIONS AND FUTURE RESEARCH

Extending the model to include both firms being privately informed is straightforward, but messy. The results in this case will be similar to the results in the present paper: Under conditions analogous to (3.3) and (3.7), there will exist equilibria where both types of each firm will pool in the interim, followed by separation in the last stage where high-quality types of both firms will repurchase all their debt, while the low-quality types of both firms will undertake partial repurchases. Since both types of the two firms would have credibly committed to overproduce, neither firm will gain any advantage in the output game, as in BL.24

The above discussion implies that the leverage option, if available to

24 There may be other reasons, such as predatory pricing, why firms may resort to aggressive output strategies with little debt. For instance, the Levitz Furniture Corporation had almost no debt prior to its output expansion in the late 1960's and early 1970's and financed most of its expansion with equity (Forbes, 1972).
both firms, is suboptimal, and if there is some credible means for both firms to commit themselves not to be levered, they may agree to do so ex ante. One such case is a repeated game, which may involve a richer strategy space, including tit-for-tat strategies that can eventually lead to cooperative outcomes, similar to those in Abreu (1983) and Maksimovic (1988). Such cooperative equilibria will be sustained by punishment strategies, or penal codes, that would severely punish a firm deviating from unlevered Cournot–Nash strategies in any period.

Tax considerations normally associated with debt issues can be easily incorporated in our model. The value of tax shields serves to reinforce the debt commitment, further impairing the debt repurchase process. Debt contracts will be even more robust to renegotiation in this case. As a result, both the uninformed firm as well as the high-quality type of the informed firm would retain residual debt in their capital structure, in addition to the low-quality type. Both types of the informed firm and the uninformed firm will overproduce, resulting in a loss of producers’ surplus as in BL. Whether or not the overall welfare is improved depends on whether the value of the tax shields offsets the reduction in the producers’ surplus. Note, however, that while overproduction decreases the producer surplus, it increases the consumer surplus. Thus, from a public policy perspective, tax shields on debt have a competitive side effect: They may increase competition in the output markets, thus increasing consumer welfare.

It is interesting to ask how our results would be affected if Firm 1 learns its type ex ante, before any debt is issued, while maintaining the repurchase options later on. In this case, the choice of the debt level itself may convey information to the competitor and its creditors, similar to the two-audience signaling problem analyzed in Section 3. The results from that section suggest that under conditions analogous to (3.3) and (3.7), equilibria may exist in this alternative formulation that involve pooling at the ex ante stage—with both types of firm 1 issuing the same level of debt at the same (average) price—followed by separation at the output-repurchase stage, along the lines of Section 3. Thus, there will still remain a useful role for residual debt as a lasting commitment device. More importantly, pooling at the debt-issuing stage implies that this reformulation would be observationally equivalent to the model in this paper.

5. CONCLUSION

We have shown that debt contractual commitments can be made lasting in the presence of asymmetric information. In the leveraged oligopoly

25 We are grateful to Anjan Thakor for raising this issue.

26 Note that there is only one nontrivial option to repurchase, i.e., at the final stage, since the interim stage collapses into the ex ante stage.
framework of BL, there exist equilibria that involve a nonzero residual debt for the low-quality firm, which can be exploited by both types of the informed firm, by forcing the competitor to cut its output. The insight is that the low-quality firm may find its debt overvalued and may be willing to separate itself from the high-quality type by undertaking a partial repurchase.

There are many examples in which debt commitments induce aggressive behavior in output markets. In the U.S. airline industry, the half-dozen highly leveraged airlines that operate under bankruptcy protection are among the most competitive price setters in the industry. Consumer-product firms such as RJR Nabisco, recently taken private using high leverage (LBOs), are also aggressive in their brand name markets.

A potential test of our model will involve searching for announcement effects of debt issue and repurchase by firms, not on their own stock returns, but on their competitors' stock returns. Such a test will be able to isolate the impact of commitment from other traditional effects due to debt, such as tax shields or other confounding events. The model in this paper predicts that a debt issue will have a negative effect on the competitors' stock returns.

Our results also have implications for prepackaged bankruptcy plans that are increasingly being used to reduce the high cost of lengthy bankruptcy reorganizations. While there exists a risk of these plans unraveling once the firm files for bankruptcy, they may well survive the bankruptcy proceedings nearly intact if there is sufficient asymmetric information.

The fundamental insight used in this paper is that informational asymmetries may prevent contracting parties from making Pareto improvements ex post (which can otherwise be made under symmetric information), helping to sustain threats made ex ante to a third party, such as a competitor. While the paper deals only with the case of debt contracts in an oligopolistic market, the same insight can be extended to other types of financial contracts (e.g., agency contracts) and other markets (e.g., capital markets) as well. We conjecture that the main insights from the literature involving agency contracts used as commitments (e.g., Fershtman and Judd, 1987; Sklivas, 1987; Nagarajan, 1988; Dybvig and Zender, 1991; and others) can be preserved using this approach.

**APPENDIX**

*Proof of Proposition 1.1.* Note that the constraint in (1.6) will be binding at the optimum. The objective function becomes, after making the substitutions,
\[ \max_{B_i} \mathcal{L}(B_i) = C_i + \int_{z} \mathcal{R}^{i}(q_i^{*}(q_j, D_i - B_i), q_j, z_i)f(z_i)dz_i \\
- \mathcal{W}^{i}[q_i^{*}(q_j, D_i), q_j, D_i]. \]

Differentiating and using the first-order condition (1.3), we obtain

\[ \mathcal{L}'(B_i) = \frac{\partial q_i^{*}}{\partial B_i} \int_{z} \mathcal{R}^{i}(q_i^{*}(D_i - B_i), q_j, z_i)f(z_i)dz_i \]

\[ = \frac{\partial q_i^{*}}{\partial B_i} \int_{z}^{s(D_i - B_i)} \mathcal{R}^{i}(q_i^{*}(D_i - B_i), q_j, z_i)f(z_i)dz_i > 0 \]

since \( \mathcal{R}^{i}(z_i) < 0 \) for all \( z_i < z_i(D_i - B_i) \) and since we have already shown that \( \frac{\partial q_i^{*}}{\partial B_i} < 0 \). The optimum is thus achieved at \( B_i^{*} = D_i \). The repurchase price is given by, \( r_i = \mathcal{W}^{i}[q_i^{*}(q_j, D_i), q_j, D_i] \). The rest of the results follow easily.

**Proof of Lemma 2.1.** This proof closely follows that of Proposition 1.1, and hence only an outline is given here. As in Proposition 1.1, the constraint will be binding, and the program reduces to

\[ \max_{B_2} \mathcal{L}(B_2) = C_i - W^2[q_i^{ll}, q_i^{l}, q_2, D_2] \]

\[ + \alpha \int_{z}^{\hat{z}} \mathcal{R}^{2}[q_i^{ll}, q_i^{l}, q_2, D_2 - B_2, z_2]f(z_2)dz_2 \]

\[ + (1 - \alpha) \int_{z}^{\hat{z}} \mathcal{R}^{2}[q_i^{ll}, q_i^{l}, q_2, (D_2 - B_2), z_2]f(z_2)dz_2. \]

Since \( q_i^{ll}(q_i^{ll}, q_i^{l}, D_2 - B_2) \) maximizes only the equity value, the maximum here is obtained at \( B_2^{*} = D_2 \).

**Proof of Lemma 2.2.** At stage 2, for a given residual debt \( N_1 \), the informed firm’s objective function is given by

\[ V_1[q_1, q_2, N_1|s] \]

\[ = C_1 - r_1 + \int_{\xi(N_1)}^{\hat{z}} \{ \mathcal{R}^{1}[q_1^{ll}(q_2, N_1|s), q_2, z_1] - N_1 \} f(z_1|s)dz_1. \]

\[ \frac{\partial r_1}{\partial B_1} \bigg|_{V_i} = - \frac{V_{B_1}}{V_{r_1}} \]

\[ = 1 - F(\hat{z}_1|s), \]

and the result follows easily from (2.1).
Proof of Proposition 2.2. Let $W^{1L}(B_1) = W^I[q^*_1(q_2, D_1|s), q_2, D_1|s] - W^I[q^*_2(q_2, D_1 - B_1|s), q_2, D_1 - B_1|s]$. Given the specified belief function, the individual rationality constraint for the debtholders is satisfied if

$$r_1 \geq \begin{cases} W^{1L}(B_1) & \text{if } B_1 \leq \hat{B}_1 \\ W^{1H}(B_1) & \text{else.} \end{cases}$$  \hspace{1cm} (A1)

Note that the above constraint must be binding at the optimum. We must show that, given the above constraints, firm 1 chooses $B_1 = D_1$ if it is of type $H$, and $B_1 = \hat{B}_1$ if it is of type $L$.

Consider first type $H$. From Proposition 1.1, we know that $(0, \nu, \tau) \in \Gamma$ is a global optimum when type $H$ optimizes over the full information constraint $r_1 \geq W^{1H}(B_1)$ for all $B_1$. We now need to show that this optimum is unchanged when the constraint becomes (A1).

Claim. $\hat{B}_1$ is the unique solution to (2.5).

Suppose that this claim is not true and let there be another solution $\hat{B}_1$ such that $\hat{B}_1 < \hat{B}_1$ (WLOG). By the single crossing property of Lemma 2.2, $\hat{B}_1$ and $\hat{B}_1$ will not be on the same indifference curve of type $L$, and furthermore, $\hat{B}_1$ will dominate $\hat{B}_1$ for type $L$. But this contradicts Proposition 1.1.

Since $\hat{B}_1$ is unique, all the offers $r_1 = W^{1L}(B_1)$, where $B_1 < \hat{B}_1$ must be dominated by those on the indifference curve of type $H$ through $\{B_1 = D_1, r_1^H\}$.

Consider now the optimization problem of type $L$. Proposition 1.1 again implies that this type prefers $\hat{B}_1$ to any other $B_1 < \hat{B}_1$. Furthermore, the single crossing property implies that $\{r_1^L, \hat{B}_1\}$ is at least as good as $\{r_1^H, D_1\}$ and strictly dominates any choice $\{B_1, W^{1H}(B_1)\}$ for $\hat{B}_1 < \hat{B}_1 < D_1$. \hfill \Box

Proof of Lemma 2.3. For any level of residual debt $N^{\pi L}$, the equilibrium outputs must satisfy the following first-order conditions:

$$V_1^H = \int_{q_1^H}^{\hat{q}_1^H} R[q_1^H, q_2, z_1] f(z_1|H) dz_1 = 0$$

$$V_1^L = \int_{q_1^L}^{\hat{q}_1^H} R[q_1^L, q_2, z_1] f(z_1|L) dz_1 = 0$$  \hspace{1cm} (A2)

$$V_2^L = \alpha \int_{\hat{q}_2^L}^{\hat{q}_2^F} R_2^L[q_1^H, q_2, z_2] f(z_2) dz_2 + (1 - \alpha) \int_{\hat{q}_2^L}^{\hat{q}_2^F} R_2^F[q_1^L, q_2, z_2] f(z_2) dz_2 = 0.$$

Totally differentiating, by Cramer's rule,
where \(|J|\) is the Jacobian determinant of the system (A2) and is assumed to be negative. The numerator \(|J_2|\) is given by

\[
|J_2| = \begin{vmatrix} V_{11}^H & 0 & V_{11}^H \\ 0 & V_{11}^L & V_{11}^L \\ V_{2H}^2 & V_{2L}^2 & 0 \end{vmatrix} = -V_{11}^H V_{11}^L V_{2H}^2 - V_{11}^H V_{11}^L V_{2L}^2 < 0,
\]

where \(V_{11}^H\) and \(V_{11}^L\) are negative from second-order conditions and

\[
\begin{align*}
V_{2H}^2 &= \frac{\partial^2 V^2}{\partial q_2 \partial q_1^H} = \alpha \int R_{21}^2[q_1^H, q_2, z_2] f(z_2) dz_2 < 0 \\
V_{2L}^2 &= \frac{\partial^2 V^2}{\partial q_2 \partial q_1^L} = (1 - \alpha) \int R_{21}^2[q_1^L, q_2, z_2] f(z_2) dz_2 < 0 \\
V_{11}^H &= \frac{\partial^2 V^H}{\partial q_1^H \partial N_{1L}^*} = -R_{11}[q_1^H, q_2, z_1^H] f(z_1^H) \frac{\partial z_1^H}{\partial N_{1L}^*} > 0 \\
V_{11}^L &= \frac{\partial^2 V^L}{\partial q_1^L \partial N_{1L}^*} = -R_{11}[q_1^L, q_2, z_1^L] f(z_1^L) \frac{\partial z_1^L}{\partial N_{1L}^*} > 0.
\end{align*}
\]

Thus, \(dq_2/dN_{1L}^* < 0\).

**Proof of Proposition 3.1.** If types \(H\) and \(L\) pool in the interim, their true type will not be known to either their creditors or firm 2 at this stage. Both types will enter the final stage with all the debt they had issued ex ante, and the equilibrium of the final stage will be as in Proposition 2.3: \(B_{1L}^* = \hat{B}_1, B_{1H}^* = D_1\) and \(r_{1H}^* > r_{1L}^*\). The equilibrium output choices are: \([q_{1L}^* f(N_{1L}^*), q_{1L}^* f(N_{1L}^*), q_{1L}^* f(N_{1L}^*)]\). To show that this equilibrium obtains under condition (3.3), rewrite it using (3.1) and (3.2) as

\[
\begin{align*}
\int R_{11}[q_1^H(q_2^H, N_{1L}^*|s), q_2^H, z_1|s] f(z_1|s) dz_1 &= - W^I[q_1^H(q_2^H, D_1|s), q_2^H, D_1|s] \\
&\geq \int R_{11}[q_1^L(q_2^L, 0|s), q_2^L, z_1|s] f(z_1|s) dz_1 \\
&\quad - W^I[q_1^L(q_2^L, D_1|L), q_2^L, D_1|L], (A3)
\end{align*}
\]

where \(N_{1H}^*=0\) and \(N_{1L}^*= D_1 - \hat{B}_1 > 0\). The left-hand side of (A3) represents the payoff to type \(s\) from not repurchasing any debt and pool-
ing in the interim and separating in the last stage. The right-hand side is the payoff from undertaking a complete repurchase in the interim and being perceived as type \(L\), as per the beliefs (3.4). The result follows from the fact that type \(H\) strictly prefers to repurchase all its debt in the interim at the lower price, while type \(L\) is indifferent between repurchasing at the interim or later.

Proof of Proposition 3.2. The equilibrium strategies with interim pooling followed by final stage separation will be exactly as in the Proof of Proposition 3.1. To show that it is obtained under condition (3.7), rewrite it using (3.5) and (3.6) as

\[
\int_{z} R^1[q^t(q^*_H, N^*_H|s), q^*_H, z_1]f(z_1|s)dz_1 - W^1[q^t(q^*_H, D_1|s), q^*_H, D_1|s]
\]

\[
\geq \int_{z} R^1[q^t(q^*_H, N^*_L|s), q^*_H, z_1]f(z_1|s)dz_1 - W^1[q^t(q^*_H, D_1|s), q^*_H, D_1|s], \quad (A4)
\]

where \(N^*_H = 0\) and \(N^*_L = D_1 - \hat{B}_1 > 0\). As before, the left-hand side represents the payoff to type \(s\) from not repurchasing any debt and pooling in the interim. For type \(H\), the right-hand side represents the payoff from full repurchase at the high price (in the interim or in the last stage) and, for type \(L\), the payoff from repurchasing \(\$e\) worth of debt in the interim at the higher price and the rest of \(\hat{B}_1\) at the lower price in the final stage, according to the beliefs (3.8).

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