Good Times or Bad Times?  
Investors’ Uncertainty and Stock Returns

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This paper investigates empirically the dynamics of investors’ beliefs and Bayesian uncertainty about the state of the economy as state variables that describe the time-variation in investment opportunities. Using measures of uncertainty constructed from the state probabilities estimated from two-state regime-switching models of aggregate market return and of aggregate output, I find a negative relationship between the level of uncertainty and asset valuations. This relationship shows substantial cross-sectional variation across portfolios sorted on size, book-to-market, and past returns, especially conditional on the state of the economy. I show that a conditional model with investors’ beliefs and an uncertainty risk factor is remarkably successful in explaining a large part of the cross-sectional variation in average portfolio returns. The uncertainty risk factor retains its incremental explanatory power when compared to other conditional models such as the conditional CAPM.  
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Uncertainty is a central tenet of finance. Investors often do not have perfect knowledge of the processes associated with macro-level variables or stock dividends but instead must make intelligent estimates on key state variables using whatever information is available to them. As more data gradually become available, existing beliefs are revised into posterior beliefs that take into account the new information. This paper argues that when investors learn in this way, with each piece of new information over time changes in the variance of their own beliefs, or their Bayesian uncertainty, introduces a new source of

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time-variation in the investment opportunity set by changing the volatility of returns.\footnote{I follow Veronesi (2004) in using the term “Bayesian uncertainty” to define the dispersion of agents’ posterior beliefs and to differentiate the term “uncertainty” from other references, such as “uncertainty aversion” (as aversion to “Knightian uncertainty”) or “model uncertainty.”}

This simple observation that economic agents do, in fact, learn over time about the fundamental structure of the economy represents a major departure from the main premises of traditional asset pricing models of perfect knowledge and complete information, and it stands at the heart of a growing theoretical literature on the effect of learning on asset prices. In this paper, I contribute to this literature by investigating empirically the role of investors’ conditional beliefs and uncertainty as state variables that describe the time-variation in investment opportunities. I explore whether fluctuations in investors’ beliefs and uncertainty about the state of the economy can provide a coherent explanation that “(1) relates the cross-sectional properties of expected returns to the variation in expected returns through time, and (2) relates the behavior of expected returns to the real economy in a rather detailed way.”\footnote{Fama (1991, p. 1610).} Specifically, I examine how the level of uncertainty is related to expected returns, and how uncertainty risk is priced in the cross section of stock returns.

This paper offers three main empirical findings. First, using proxies for investors’ conditional beliefs and uncertainty about the state of the economy, I show that investor beliefs and uncertainty are important in describing future investment opportunity sets. Second, investors’ uncertainty about the state of the economy has a negative impact on asset valuations both at the aggregate market level and at the portfolio level, with substantial variation across portfolios sorted by size, book-to-market, and past returns. Finally, in asset pricing tests, an uncertainty risk factor associated with fluctuations in investors’ beliefs and uncertainty about the state of the economy shows remarkable success in explaining the cross-sectional variation of average stock returns. These findings underscore the importance of the consequences of incomplete information and dynamic learning for the risk-return trade-offs investors face.

The theoretical motivation for this study comes from the work of David (1997) and Veronesi (1999). These papers develop a dynamic rational expectations model of the aggregate market in which investors do not observe the current state of the economy, and they examine how changes in investors’ uncertainty affect stock returns and volatility. The key assumptions in Veronesi (1999) are that economic fundamentals, modeled as the drift of a dividend process, may shift from a high-growth state to a low-growth state at random times, and that investors need to form posterior beliefs about the true state based on their observations of past dividends through a Bayesian updating scheme. In this setting, the model shows that investors anticipate that their expectations of future cash flows will react more to new information at times of high
uncertainty, and therefore they expect higher levels of uncertainty to generate greater asset price volatility. This induces risk-averse investors to hedge against unexpected changes in their own uncertainty and to require greater compensation for bearing more risk at times of high uncertainty in anticipation of the greater asset price sensitivity to news.\footnote{David (1997) shows how fluctuations in investors’ own level of uncertainty generate a new class of risk and a hedging demand in an intertemporal portfolio choice setting.}

In this paper, I build on the economic insights of these models in an investigation of the role of investors’ conditional beliefs and their uncertainty about the economy as state variables in describing the time-variation in the investment opportunity set. In the spirit of Merton’s (1973) intertemporal CAPM (ICAPM), I test whether there is a premium associated with uncertainty risk, measured as the covariance between stock returns and unexpected changes in investors’ own uncertainty, as a risk factor. The central hypothesis is that fluctuations in investors’ conditional beliefs and uncertainty due to learning generate priced risk factors that can explain the cross-sectional variation in stock returns.

In a traditional ICAPM framework, if risk-averse investors are willing to hedge against unexpected changes in their own uncertainty, then they should require compensation for holding assets that covary positively with their own confidence. These assets command a higher risk premium because they pay off only at times of low uncertainty, and hence reduce investors’ hedging ability in periods of high uncertainty. Alternatively, investors should desire assets whose payoffs covary positively with their uncertainty, because such assets would enhance their hedging ability against unexpected changes in their own uncertainty. These observations have the following testable cross-sectional implication: when investors have relative risk aversions greater than one, assets that have a lower covariance with unexpected changes in investor uncertainty have greater exposure to uncertainty risk, and should therefore have greater expected returns. This is the main cross-sectional prediction tested in this paper.

Bayesian uncertainty is an elusive concept. Here, I use it to describe the variability of investors’ beliefs upon receiving new information. Since neither beliefs nor their variance is directly observable, however, I construct empirical proxies to measure investors’ conditional beliefs and uncertainty about the aggregate state of the economy. Specifically, I consider a two-state regime-switching model of the aggregate market return that captures the intuition that the economy fluctuates between states in a random and unobservable fashion. Estimating this two-state model provides a time-series of conditional state probabilities; I then use these probabilities to characterize empirically the dynamics of investors’ beliefs over time and to construct a measure of investor uncertainty. In addition, I also estimate a two-state Markov-switching model of the aggregate output and construct output-based measures of beliefs and uncertainty. While both models yield very similar measures of investors’ beliefs
and uncertainty about the economy, this alternative approach also provides an opportunity to test the robustness of the findings.

I begin the empirical analysis by providing preliminary evidence on the ability of investors’ conditional beliefs and uncertainty to predict future investment opportunity sets. Specifically, I estimate predictive regressions of the first and second moments of the aggregate market return on lagged values of investor beliefs and uncertainty measures. The evidence indicates that investors’ beliefs and the level of their uncertainty have some ability to predict future returns and volatility. First, investors’ beliefs about being in the low state are positively, albeit insignificantly, related to both higher expected returns and higher volatility. These findings are consistent with the idea that bad times are associated with a higher risk premium and higher volatility; they also suggest that the probability investors assign to being in bad times (the proxy for investors’ current beliefs in this paper) is an important state variable for describing future investment opportunities. Second, I find evidence that there exists a positive, albeit nonlinear, relationship between the level of uncertainty and expected returns. Specifically, higher uncertainty predicts greater expected returns, especially when investors’ current beliefs are most sensitive to news. These findings are important for relating the behavior of expected returns to the fundamentals of the real economy.

I next study the impact of investor beliefs and uncertainty on asset valuation by examining their relationship to price-dividend ratios. If risk-averse investors expect to be compensated for bearing more risk, they will require a greater discount on asset prices in times of high uncertainty. The results suggest that this prediction is supported empirically: investors’ beliefs and uncertainty both have an impact on asset valuations. First, stronger beliefs about being in bad times have a negative effect on asset prices. Second, in regressions of valuation ratios on uncertainty measures, I find a statistically significant and negative relationship between asset prices and the level of investor uncertainty. This negative relationship is especially pronounced when the output-based uncertainty measure is used in the analysis, and it confirms Veronesi’s (1999) prediction that agents demand higher expected returns when uncertainty is high. Furthermore, the evidence suggests that the negative relationship between the level of uncertainty and asset prices holds not only for the aggregate market, but also at the portfolio level, with substantial cross-sectional variation across portfolios sorted on size, book-to-market, and past returns. While greater uncertainty depresses valuation ratios across all portfolios, it has the greatest unconditional impact on small stocks and past losers. Finally, I find that the impact of uncertainty varies significantly with the current state of the economy. This last finding underscores the need to allow for a time-varying relationship between the level of uncertainty and risk premia.

It is important to note, however, that at times the choice of the uncertainty measure seems to matter to some extent in testing these relationships. In particular, the level of uncertainty seems negatively, though insignificantly, related to
future expected returns when the returns-based measure of uncertainty is used in the analysis. The returns-based measure of uncertainty also fails to show the predicted negative relationship with valuation for all portfolios. It has a positive, but insignificant, impact on asset valuations across some of the portfolios; its effect turns negative only for extreme winners. Given the similarities between the two sets of measures, the disparity in these findings is surprising and puzzling. On the other hand, it underlines the importance of using both sets of measures in the cross-sectional asset pricing tests.

Finally, I address the central question of whether fluctuations in investors’ beliefs and uncertainty can explain the cross section of average stock returns. The main analysis is a test of a conditional three-factor model that includes factors associated with investor beliefs and uncertainty risk, together with the return on the market portfolio. To capture the variation in the conditional moments, I follow Cochrane (1996) and Lettau and Ludvigson (2001a) and model the parameters in the discount factor as dependent on current information. Specifically, I incorporate the conditioning information by interacting each fundamental risk factor with the current probability of being in bad times and express the conditional three-factor model as a scaled multifactor model.

The main finding of the paper is that the uncertainty risk factor is remarkably successful in explaining a large part of the cross-sectional variation in average returns of thirty portfolios sorted by size, book-to-market, and past returns. The addition of investors’ beliefs and uncertainty as state variables significantly improves the performance of the CAPM and the Fama-French (1996) three-factor models in both unconditional and conditional tests. The average pricing errors are considerably smaller for the scaled model, and the null hypothesis that the pricing errors across the thirty portfolios are jointly zero cannot be rejected when the conditional probability of being in the low state and uncertainty risk factors are added to the Fama-French three-factor model. Finally, I note that the findings are generally stronger with the probability and uncertainty measures obtained from a Markov model of aggregate output.

Recently, there have been several other studies that use economically motivated or macroeconomic factors to explain the cross section of average returns. I therefore next compare the performance of the conditional model with conditional beliefs and uncertainty risk to the conditional versions of the CAPM and the consumption CAPM. The results show that accounting for investors’ conditional beliefs and uncertainty risk improves the performance of both the conditional CAPM and the conditional consumption CAPM, even when I use the consumption-to-wealth ratio as the scaling variable, as proposed previously by Lettau and Ludvigson (2001a). This result suggests that the risk that arises from fluctuations in investors’ uncertainty about the economy is distinct from that arising from news associated with the future state of the economy.

In the next section, I review the related literature. In Section 2, I provide a brief theoretical motivation for the empirical research questions addressed in the paper. Section 3 explains the construction of the measures that proxy
for investors’ beliefs and uncertainty and describes their empirical properties. Section 4 presents the portfolio data. In Section 5, I first present predictive regressions to assess the ability of investors’ conditional beliefs and uncertainty as state variables to describe future investment opportunities; I then address the question of how the level of uncertainty affects risk premia by presenting evidence from time-series regressions of asset valuations on the level of uncertainty. In Section 6, I introduce the conditional linear factor model, which includes the probability of being in the low state and uncertainty risk factors, and I express the stochastic discount factor as a scaled multifactor model. I then present the empirical results from the analysis of the cross section of average returns. This section concludes with a comparison of the performance of the conditional three-factor model to other traditional models such as the CAPM, the Fama-French three-factor model, and other conditional models proposed in the literature. Finally, in Section 7, I conclude.4

1. Relation to Existing Literature

Several studies have theoretically examined the effect of incomplete information and learning on asset prices and portfolio choice. Early works by Detemple (1986), Gennette (1986), and Ghysels (1986) analyze the portfolio problem of an investor who does not observe the true state of the economy but who knows the underlying stochastic process. These authors show that in such a setting, the conditional expectation of the unobservable state variable itself acts as the state variable in the investor’s dynamic optimization problem and, as in Merton (1973), hedging needs against unanticipated changes in this state variable become relevant for optimal portfolio choice. More recently, Brennan (1998) shows that the possibility of learning about an unobservable state variable affects optimal portfolio choice. Brennan and Xia (2001) argue that allowing for uncertainty over fundamentals and learning over time can help resolve the equity premium puzzle by increasing the volatility of stock prices.5 Barberis (2000) and Xia (2001) extend earlier work by Kandel and Stambaugh (1996) to incorporate the effects of uncertainty about return predictability and learning on dynamic portfolio choice over long horizons. Finally, Lewellen and Shanken (2002) take a different angle, arguing that the process of learning about an unknown parameter of equilibrium asset prices can itself be a source of predictability.

4 The full details of the analysis of the pricing errors are not reported here for the sake of brevity, but they are available in the working paper version of this paper. The working paper version also includes a comparison of the model with a model that includes a GDP news factor, GMM estimation results, and the analysis of Hansen-Jagannathan distance comparisons across the different models.

5 Timmermann (1993, 1996) explores to what extent agents’ learning and incomplete information about the true underlying model can explain the excess volatility and predictability of returns. He shows that estimation uncertainty about the growth rate of the dividend process and learning over time can increase volatility of stock prices.
Other theoretical models—outside of learning models—have also considered uncertainty as a source of risk premium. Bansal and Yaron (2004), for example, consider the implications of fluctuating economic uncertainty in a model based on Epstein and Zin (1989) preferences. They show that a rise in economic uncertainty, modeled as the time-varying volatility of consumption, lowers asset prices, and fluctuations in economic uncertainty increase the equity risk premium. They provide supporting evidence at the aggregate market level from the United States and other large economies that financial markets dislike uncertainty. Bansal and Yaron show that greater economic uncertainty, measured as the estimated conditional volatility of consumption, lowers valuation ratios such as dividend price ratios.

Recently, a number of studies have empirically investigated the impact that investor uncertainty about factors related to the state of the economy has on returns. Bittlingmayer (1998), for example, suggests that return volatility is related to political uncertainty. David and Veronesi (1999) find that an uncertainty measure (obtained similarly to the one used in this paper) from a regime-switching model of real earnings growth is related to options’ implied volatility. David and Veronesi (2001) also show that uncertainty about future inflation and earnings growth rates helps explain stock and bond monthly volatilities and cross-covariances. Locarno and Massa (2001) focus on the relationship between stock returns and inflation; they show that monetary policy uncertainty affects risk premia and can help explain the observed correlation between inflation and stock returns.

While there is a growing theoretical interest in the way the process of learning generates uncertainty and affects investors’ portfolio choice decisions, there has been surprisingly little empirical work examining how this process affects risk and asset prices. In this respect, one study by Massa and Simonov (2005), which also addresses the effect of investors’ uncertainty associated with learning on asset prices, is of particular interest to the present paper. Massa and Simonov first aim to distinguish learning uncertainty from dispersion of beliefs and then investigate whether their respective proxies are related to the time-variation in risk premia. In particular, they rely on forecast errors estimated from a state-space model of stock returns to construct their uncertainty measure. While their work differs from the present paper in both focus and methodology, they find evidence consistent with the result that learning uncertainty is an important risk factor in explaining cross-sectional returns.

Finally, this study is related to a second strand of literature that addresses the long-time goal of financial economics to understand the empirical linkages between macroeconomic variables and asset prices. Several authors have used macroeconomic variables as factors to explain average returns. Chen, Roll, and Ross (1986), for example, use industrial production and inflation among other variables; Jagannathan and Wang (1996) use labor income, and Cochrane (1996) looks at investment growth. More recently, Flannery and Protopapadakis (2002) document the impact of macroeconomic factors on aggregate stock
returns. Empirical evidence suggests that expected returns are related to the covariances of returns with macroeconomic variables. However, the evidence, as pointed out by Cochrane (2001), only describes the variation in expected returns; it does not explain it. The observation that expected returns vary with the business cycle does not answer the question “What real risks cause the market return to vary?”6 By exploring how uncertainty about the underlying state of the economy affects risk and asset prices, this paper suggests one such source of risk as a rational explanation of predictable variation in returns.

2. Theoretical Motivation

Fama (1970) shows that the one-period CAPM can no longer apply to a multiperiod setting if the investment opportunity or investor preferences change over time. Merton’s (1973) ICAPM shows that when investment opportunities vary over time, there are risk premia associated with the covariance between asset returns and unanticipated changes in state variables that describe the time-variation in the investment opportunity set. Accordingly, in the traditional ICAPM framework with power utility, when the representative agent has a risk aversion coefficient \( \gamma \) greater than one, assets that covary positively with future investment opportunities have higher average returns. These assets command a higher risk premium because they reduce the agent’s hedging ability when investment opportunities deteriorate.

This paper builds on the observation that in a world in which investors learn about economic fundamentals using Bayesian updating, the accompanying change in the variance of their beliefs introduces a new source of time-variation in the investment opportunity set by changing assets’ price sensitivity to news. Specifically, when agents learn about the state of the economy gradually over time, the new information is incorporated into beliefs dynamically with some variable weight. This weight, by the very nature of Bayesian updating, depends on the precision of agents’ prior beliefs. When their prior precision is high, agents give little weight to incoming news and do not revise their beliefs by much. In contrast, the new piece of information gets the largest weight when agents are least certain; in that case, the arrival of a piece of news leads them to revise their beliefs significantly. A key measure of the sensitivity of prior beliefs to news (innovation) in such an economy with learning thus is investors’ changing level of confidence, or the precision of their beliefs.

Building on this intuition, David (1997) shows how fluctuations in investors’ own level of uncertainty can generate a new class of risk and hedging demands in an intertemporal portfolio choice setting. Veronesi (1999) develops a dynamic rational expectations model of the aggregate market with a similar insight. In his model, investors rationally anticipate that their expectations of future cash flows will react more to news in times of high uncertainty, therefore

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generating greater asset price sensitivity and asset price volatility. This, in turn, induces risk-averse investors to hedge against changes in their own uncertainty and to require greater compensation for bearing more risk in periods of high uncertainty.

The state variables on which this paper focuses are investors’ conditional beliefs and their fluctuating uncertainty about the aggregate economy due to learning over time. Motivated by these theoretical models, I address (i) how investors’ conditional beliefs and the level of their uncertainty are related to expected returns over time, and (ii) how exposure to fluctuations in these state variables is priced in the cross section of stock returns.

To fix ideas, consider an economy characterized by a dividend process whose growth rate $\theta(t)$ shifts randomly and in an unobservable fashion between a high-growth state $\theta$ and a low-growth state $\bar{\theta}$. Investors are faced with a signal extraction problem and need to form beliefs on the true $\theta(t)$ conditional on the signal provided by the observed dividend level. Let $\pi(t)$ denote the probability $\Pr(\theta(t) = \theta)$, that is, the conditional probability that the economy is in the good state. In this setting, Veronesi (1999) shows that the equilibrium price function is given by a negative discount over the discounted expected dividends. Moreover, the discount changes with the probability $\pi(t)$. This effect arises because in anticipation of the greater asset price sensitivity to news and, hence, the greater volatility in the market, investors require greater compensation for holding equity in periods of high uncertainty. This predicts a negative relationship between the level of uncertainty and asset valuations, which I test in the time series.

To consider the cross-sectional implications, it is useful to extend the intuition in Veronesi (1999) via a mental exercise. Assume there are two assets in the economy, $a$ and $b$, where asset $a$ covaries positively with the state of the economy and asset $b$ has a negative covariance with the aggregate economy. Suppose now that investors attach a low probability to being in the good state, that is, $\pi(t)$ is close to 0, and that investors receive a piece of good news. This has the effect of increasing the conditional probability $\pi$; however, at the same time it lowers the precision of investors’ beliefs, increasing their uncertainty, because $\pi$ now gets closer to 0.5. Since investors anticipate their expectations of future dividends to vary by a great deal when they are uncertain, they will try to hedge against unexpected changes in their own uncertainty. To achieve this, they will increase their demand for assets that pay off at times of high uncertainty and at the same time will require an additional risk premium to hold the assets that perform poorly when the precision of their beliefs deteriorates. Since an increase in $\pi$ implies that the economy now has a greater chance to revert to the good state, in this example, asset $a$ effectively pays off in states of greater uncertainty (lower confidence) and asset $b$ pays off in states of low uncertainty (higher confidence). Therefore, holdings in asset $a$ would help hedge the risk of fluctuating uncertainty, and holdings in asset $b$ would command an additional premium for their positive covariance with investor confidence.
Conversely, when investors believe that the economy is in a good state, with $\pi(t)$ close to 1, a piece of bad news would have the effect of reducing the conditional probability $\pi$ and therefore investor confidence about the state of the economy. In this case, since a decrease in $\pi$ signals that the economy has a greater likelihood to switch to the bad state, asset $b$ effectively pays off in the state of greater uncertainty (lower confidence) and asset $a$ pays off in states of low uncertainty (higher confidence). Therefore, holdings in asset $b$ would help hedge against the uncertainty risk, and holdings in asset $a$ would require greater compensation due to this asset’s positive covariance with investor confidence.

These examples illustrate the intuition that when the representative agent is more risk-averse than log utility—as in the traditional ICAPM framework—assets that covary positively with changes in investors’ own confidence should have greater returns. These assets command a higher risk premium because they reduce investors’ hedging ability in periods of higher uncertainty. Conversely, in this setting, assets that covary negatively with unexpected changes in uncertainty have greater exposure to uncertainty risk. This leads to the testable prediction that exposure to uncertainty risk should be priced in the cross section of stock returns.

While the economic mechanism on which this paper focuses is investors’ learning and resulting fluctuations in their uncertainty, there are other theoretical models that have similar implications. For example, Chen (2003) extends Campbell’s (1993, 1996) version of the ICAPM to a heteroscedastic environment that allows for time-varying covariances and stochastic market volatility. In his model, Chen also shows that risk-averse investors would like to hedge against changes in future market volatility. The main difference between Chen’s model and the models with learning considered here is that investors’ fluctuating uncertainty provides an economic motivation explaining the reasons that volatility might change over time in the first place. Indeed, Veronesi (1999, p. 995) shows that an economy with a regime-switching dividend process and unobservable state variables is equivalent to a full-information model in which dividends follow an Ito process and the instantaneous dividend growth rate changes according to a Ornstein-Uhlenbeck process with stochastic variance.

Motivated by these theoretical insights and Merton’s (1973) ICAPM, I consider a model with two new state variables, the conditional probability $\pi$ and investors’ uncertainty $UC$. This implies the following conditional multifactor representation of expected returns in the cross section:

$$E_t[R_{i,t+1}] = \beta_{m,t}^i \lambda_{m,t} + \beta_{\Delta \pi}^i \lambda_{\Delta \pi,t} + \beta_{\Delta UC}^i \lambda_{\Delta UC,t},$$

where $\beta_{m,t}^i$ is the loading on the excess market return; $\beta_{\Delta \pi}^i$ is the asset’s sensitivity to changes in $\pi$; $\beta_{\Delta UC}^i$ is the asset’s sensitivity to uncertainty risk, defined as the sensitivity of the return on asset $i$ to an unanticipated change in uncertainty; $\lambda_{m,t}$ is the price of market risk; and finally $\lambda_{\Delta \pi,t}$ and $\lambda_{\Delta UC,t}$ denote the risk
premia associated with changing investor beliefs and uncertainty risk. In the next section, I discuss how I construct proxies to measure investors’ beliefs and their uncertainty over time.

3. Investors’ Beliefs and Uncertainty over Fundamentals

As neither investors’ beliefs nor their uncertainty about the fundamentals is directly observable, a difficulty faced in this study is to find a reasonable proxy for investors’ Bayesian uncertainty. Some common measures of aggregate uncertainty used in the literature include survey-based measures, including dispersion of forecasts on the future of the economy constructed from surveys, such as the Livingston Survey or the ASA-NBER Survey of Professional Forecasters, and market-based measures, such as stock market turnover. Neither of these measures is desirable for the purposes of this study, however, for two reasons. First, since such surveys are conducted at most on a quarterly or biannual basis, survey-based measures lack the high frequency preferred for the nature of the empirical tests in this study. Second, and more importantly, both forecast dispersion and stock market turnover are more appropriately measures of dispersion of opinion, which makes them unsuitable as proxies for investors’ Bayesian uncertainty considered here. In this section, I present the empirical framework used to construct the measures that characterize investors’ beliefs and uncertainty about the state of the economy.

3.1 A Markov-switching model of the stock market return

I model investors’ beliefs and uncertainty about the state of the economy using a Markov-switching regime framework. Hamilton’s (1989, 1994) Markov-switching model of the aggregate fluctuations in the economy has been widely used to formalize occasional, but recurrent, regime shifts associated with business cycles. The Markov model, by design, captures the intuition that the economy may shift from an expansion state to a recession state at random times in a way that is unobservable to investors, who, as a result, must make inferences about the probability of being in a given state at any point in time. In this respect, the Markov-switching regime model provides a convenient framework with which to characterize the evolution of investors’ beliefs and uncertainty about the state of the economy over time.

An important modeling decision in this context is the choice of the proxy for the underlying fundamentals. I begin by presenting a regime-switching model of the stock market return that explicitly accounts for state dependence in risk

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7 See, for example, David and Veronesi (2001) and Connolly, Stivers, and Sun (2005).
8 For example, it is possible to have a significant degree of dispersion across a sample of forecasters, but each of them could be fairly certain about his beliefs. Measuring uncertainty through dispersion, therefore, does not proxy for the degree of sensitivity of beliefs to news or the degree of investors’ Bayesian uncertainty in the sense considered in this paper.
and expected returns. Estimating beliefs and uncertainty by means of a Markov model of returns is a natural choice since it ensures, by construction, that the state variables describing the future investment opportunity sets are obtained directly from forward-looking stock returns data in this setting.

Specifically, I follow Perez-Quiros and Timmermann (2000) and model the excess return \( \tilde{r}_{m,t} \) on the market portfolio in period \( t \) as the following Markov switching specification:

\[
\tilde{r}_{m,t} = \beta_0, s_t + \beta', s_t X_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, h_{s_t}), \tag{2}
\]

where \( X_{t-1} \) is a vector of conditioning information used to predict the excess return. The Markov specification in Equation (2) provides a very general model; it allows the intercept term, the regression coefficients, and the volatility of excess returns to be functions of an unobservable state variable \( S_t = \{1, 2\} \) that follows a two-state Markov chain with a time-varying transition matrix:

\[
Pr(S_t = s_t \mid S_{t-1} = s_{t-1}, z_{t-1}) = \Lambda = \begin{pmatrix}
    p(z_{t-1}) & 1 - q(z_{t-1}) \\
    1 - p(z_{t-1}) & q(z_{t-1})
\end{pmatrix}, \tag{3}
\]

where \( z_{t-1} \) is a vector of variables that are publicly known at time \( t - 1 \).

In his formulation of the Markov-switching model, Hamilton (1989) assumes that the transition probabilities between states are constant. Recent models, however, have introduced additional flexibility into the modeling of the evolution of the state variable by allowing the state transition probabilities to vary on the basis of certain predetermined variables.\(^9\) This additional feature is particularly important for modeling investors’ conditional beliefs, since it is plausible that the information available to investors in making inferences about the current state of the economy is greater than that assumed by a model of constant transition probabilities. I therefore follow Filardo (1994) and Perez-Quiros and Timmermann (2000) and model the state transition probabilities to be a linear function of the composite leading indicator. This choice allows the transition probabilities to capture the currently available information on the future economic conditions. The state transition probabilities are, thus, defined as follows:

\[
p_t = Pr(s_t = 1 \mid s_{t-1} = 1, z_{t-1}) = \phi(\eta_0 + \eta_1 \Delta CLI_{t-1}), \quad q_t = Pr(s_t = 2 \mid s_{t-1} = 2, z_{t-1}) = \phi(\eta_0 + \eta_2 \Delta CLI_{t-1}), \tag{4}
\]

where \( \Delta CLI_{t-1} \) is the one-month lagged value of the change in the log composite leading indicator, and \( \phi(\cdot) \) is the cumulative density function of a standard normal variable.

Equations (2) through (4) together describe a simple model of the evolution of the investment opportunity set in the economy. They state that at any given

\(^{9}\) See Filardo (1994); Diebold, Lee, and Weinbach (1994); Durland and McCurdy (1994); and Gray (1996).
point in time, the expected return and risk both depend on the underlying state of the economy, which, with some probability, changes from one state to another over time. Furthermore, the probability that the economy moves from one state to another also varies according to the present conditions in the economy.

An attractive feature of the regime-switching model above is that the state probabilities implicit in the data can be estimated together with the parameters of the model by maximizing a log-likelihood function for a given sample of return data. Letting \( \theta \) denote the set of all the parameters in the model, \( \theta = \{\beta_0, \beta_2, \beta_1, h_1, h_2, \eta_0, \eta_1, \eta_2\} \), imagine for a moment that the value of \( \theta \) is known with certainty. Then, using this knowledge and the model, an observer could make inferences about the underlying state of the economy at time \( t \). This inference takes the form of a conditional probability \( P\{S_t = s_t | \mathcal{F}_t\} \) assigned to the possibility that the state variable \( S_t \) is equal to \( s_t \), where \( \mathcal{F}_t \) denotes all the information available up to time \( t \). Since, however, \( \theta \) is not known with certainty, estimating the state probabilities jointly with the parameters repeats this procedure iteratively to maximize the log-likelihood function for a given sample.

The Markov model therefore provides a convenient framework with which to construct empirical measures of investors’ beliefs and uncertainty. Using this framework, I first define investors’ beliefs about the state of the economy as the conditional probability \( \pi_t \) that an investor who does not observe the true state would assign to being in a given state at time \( t \):

\[
\pi_t \equiv Pr(S_t = s_t | \mathcal{F}_t),
\]

where \( \mathcal{F}_t \) denotes the investor’s information set at time \( t \), which consists of all the available market return and conditioning information up to time \( t \). I then use the time series of the state probabilities estimated from the Markov model to characterize the evolution of investors’ beliefs empirically.

Next, I turn to the construction of an empirical measure for investors’ uncertainty about the state of the economy. Uncertainty has many different interpretations. In the current study, the term Bayesian uncertainty is used to describe the responsiveness of investors’ beliefs to the arrival of news. When investors are uncertain, the arrival of a new piece of information shifts posterior beliefs by a greater amount than when they are confident. Put differently, greater uncertainty is equivalent to a lower precision on the prior, whereas an increase in confidence is equivalent to an increase in the precision of investors’ beliefs. Using the estimated time series of investors’ beliefs, I construct an index of

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10 To be fair, the expected return in Equation (2) is not an ex ante expected return. This is because while Equation (2) describes the returns as a function of the underlying state variable \( S_t \), agents do not know the true \( S_t \). Conditional on the time \( t - 1 \) information, the expected return will actually be some weighted average of the return computed for \( S_t = 1 \) and \( S_t = 2 \). Equations (2) through (4) more accurately describe, jointly, the process by which the agents update their beliefs over the state of the economy conditional on time \( t - 1 \) information set and, at the same time conditional on these beliefs, what the expected return and risk will be. I thank the editor for pointing out this subtle difference.
uncertainty that captures this intuition:

$$UC_t \equiv \pi_t (1 - \pi_t), \quad (6)$$

where, by construction, $UC_t$ increases as $\pi_t$ nears 0.5 and falls to zero when $\pi_t$ approaches zero or one. The index of uncertainty $UC_t$ given in Equation (6) is similar to the measures of uncertainty used in David and Veronesi (2001) and Locarno and Massa (2001). It captures the spirit of the statistical concept of entropy, which measures the distance between probabilities;\(^\text{11}\) indeed, O’Hara (1995) uses entropy in a similar fashion as an index of uncertainty.

$UC_t$ has a straightforward interpretation as an uncertainty index. Suppose $\pi_t$ is the probability of being in the low state. When investors are confident that the economy is in the low state, $\pi_t$ is close to one and $UC_t$ takes a value close to zero. Conversely, when investors are sure about strong growth prospects in the economy in the near future, both $\pi_t$ and $UC_t$ approach zero. On the other hand, when investors are uncertain about the future direction of the economy, suggesting that their beliefs linger around $\pi_t = 0.5$, $UC_t$ approaches its maximum value of 0.25.

Thus, the time series of the state probability $\pi_t$ and the uncertainty index $UC_t$ can be used together to describe the evolution of investors’ beliefs and uncertainty about the state of the economy over time. The asset pricing tests considered below explore the importance of investor beliefs $\pi_t$ and uncertainty $UC_t$ as state variables that describe the future investment opportunity sets available to investors. To proxy for the change in these state variables, I define the innovation series $S_{\Delta \pi}$ and $S_{\Delta UC}$. The innovation in investor beliefs $S_{\Delta \pi}$ is constructed as the change in investor beliefs from time $t-1$ and $t$.\(^\text{12}\) To construct the innovation in investor uncertainty $S_{\Delta UC}$, I run the following second-order autoregression of $UC_t$:

$$UC_t = a + b UC_{t-1} + c UC_{t-2} + \nu_t. \quad (7)$$

This regression captures the slow-moving, highly persistent nature of uncertainty and produces residuals that appear serially uncorrelated. I define the innovation $S_{\Delta UC,t}$ to denote the surprise change in the level of investors’ uncertainty as proxied by the fitted residuals of the regression equation (7).\(^\text{13}\)

### 3.2 An alternative model with aggregate output

The previous section considered the fitting of a Markov-switching regime model directly to the excess return on the aggregate market. While this approach

\(^{11}\) For a discrete random variable that takes on values $x_i$, $i = 1, \ldots, n$ with probabilities $p_i$, $i = 1, \ldots, n$, the entropy is defined as $-\sum_{i=1}^{n} p_i \log(p_i)$. The entropy for the investor beliefs would then be given by $-[\pi_t \log(\pi_t) + (1 - \pi_t) \log(1 - \pi_t)]$. One can show that this is very similar to $UC_t$ given in Equation (6).

\(^{12}\) Taking the first difference of the conditional probabilities is sufficient to produce innovations that are serially uncorrelated.

\(^{13}\) In the analysis, I scale both $S_{\Delta \pi}$ and $S_{\Delta UC}$ by dividing by 10, which has no economic effect but makes the reporting of the betas more convenient.
has the beneficial property that the state variables are obtained directly from return distributions, an alternative way to obtain the state variables might be to fit a regime-switching model to a macroeconomic indicator such as the aggregate output in the economy. This alternative modeling choice might be interesting for three reasons. First, aggregate output is a direct proxy for the underlying fundamentals of the aggregate economy. Second, to the extent that the aggregate market return may be a noisy summary of the aggregate economy, measures of beliefs and uncertainty estimated from a regime-switching model of aggregate output may differ from those estimated from a model of the market return. Finally, asset pricing tests that consider investor beliefs and uncertainty measures estimated from an alternative model would provide a robustness check to ensure that the results are not driven by the modeling choice.14

To explore this alternative approach, I consider a parsimonious model of the aggregate output in which the growth in aggregate output evolves according to a two-state Markov-switching process:

\[ \Delta I P_t \sim N \left( \mu_{s_t}, \sigma_{s_t}^2 \right), \quad (8) \]

where \( \Delta I P_t \) denotes the monthly growth rate in industrial production and \( S_t \) is an unobservable state variable \( S_t = \{ 1, 2 \} \) that follows a two-state Markov chain with time-varying transition probability matrix, as characterized in Equations (3) and (4). Together with the time-varying probability transition matrix, the Markov model in Equation (8) delivers a description of the economy similar to that from the model of the market return. At any given point in time, both the average growth rate in aggregate output and its standard deviation depend on a latent state variable that, with some probability, changes from one state to another over time. The empirical measures \( \pi_t \) and \( UC_t \) are constructed analogously from the estimated state probabilities from this model, as in the previous section.

### 3.3 Estimation of the Markov-switching model

I estimate both of the Markov-switching regime models presented above and report the estimation results in Table 1. First, Model (1) in Table 1 estimates the Markov model of the aggregate market return described in Equations (2)–(4). In particular, Model (1) specifies the excess return on the stock market to be a function of an intercept term and lagged values of the default premium, the term spread, the one-month T-bill rate, and the dividend yield. All of these variables are common regressors drawn from the return predictability literature.15 The

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14 On the other hand, macroeconomic indicators such as industrial production are likely to suffer from some measurement problems, since these data become available with some lag, and are subject to change.

15 Perez-Quiros and Timmermann (2000) provide a detailed discussion of each regressor and how the role of each predictor varies over the business cycle. Fama and Schwert (1977); Campbell (1987); Glosten, Jagannathan, and Runkle (1993); and Whitelaw (1994) find that the one-month T-bill rate is negatively correlated with future returns. Keim and Stambaugh (1986); Fama and French (1988, 1989); and Kandel and Stambaugh (1996) find that the default premium is positively correlated with future stock returns.
Table 1
Markov-switching models for the state of the economy

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model (1)</th>
<th>Model (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low-state</td>
<td>High-state</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>$-0.025$</td>
<td>$-0.005$</td>
</tr>
<tr>
<td></td>
<td>($-1.19$)</td>
<td>($-0.71$)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$59.420$</td>
<td>$15.683$</td>
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<tr>
<td></td>
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<tr>
<td>$\beta_2$</td>
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<td>$0.177$</td>
</tr>
<tr>
<td></td>
<td>($0.25$)</td>
<td>($1.59$)</td>
</tr>
<tr>
<td>$\beta_3$</td>
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<td>$0.069$</td>
</tr>
<tr>
<td></td>
<td>($-1.18$)</td>
<td>($0.06$)</td>
</tr>
<tr>
<td>$\beta_4$</td>
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<td></td>
</tr>
<tr>
<td></td>
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</tr>
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<td>Variance</td>
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<td></td>
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<tr>
<td>$\lambda_0$</td>
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<td>$0.033$</td>
</tr>
<tr>
<td></td>
<td>($17.80$)</td>
<td>($16.50$)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
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<td>$0.028$</td>
</tr>
<tr>
<td></td>
<td>($1.98$)</td>
<td>($0.37$)</td>
</tr>
<tr>
<td>Transition probabilities</td>
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<tr>
<td></td>
<td>($7.72$)</td>
<td></td>
</tr>
<tr>
<td>$\eta_1$</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>($-3.46$)</td>
<td></td>
</tr>
<tr>
<td>$\eta_2$</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>($1.50$)</td>
<td></td>
</tr>
<tr>
<td>Log-likelihood value</td>
<td>$797.83$</td>
<td></td>
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</table>

This table presents the parameter estimates of the Markov-switching models (1) and (2) for the state of the economy with time-varying state transition probabilities. Model (1) specifies a Markov-switching regime model for the excess return on the market that allows the intercept, the regression coefficients, and the volatility of the excess return to be functions of an unobservable state variable $S_t$. Model (2) specifies a Markov-switching regime model of industrial production growth that allows the average growth rate and its standard deviation to depend on an unobservable state variable $S_t$. Both models employ time-varying state transition probabilities, modeled as a linear function of the composite leading indicator.

Model (1) : $\tilde{r}_{m,t} = \beta_0 S_t + \beta_1 S_t Def_{t-1} + \beta_2 S_t T e r m_{t-1} + \beta_3 S_t Y i e l d_{t-1} + \beta_4 Y i e l d_{t} + \epsilon_t$

$\epsilon_t \sim N(0, h_{S,t})$.

Model (2) : $\Delta IP_t \sim N\left(\mu_{S,t}, \sigma_{S,t}^2\right)$.

The time-varying state transition probabilities are given by

$p_t = Pr(s_t = 1 | s_{t-1} = 1, z_{t-1}) = \phi (\eta_0 + \eta_1 \Delta CLI_{t-1}),$ and

$q_t = Pr(s_t = 2 | s_{t-1} = 2, z_{t-1}) = \phi (\eta_0 + \eta_2 \Delta CLI_{t-1}),$ where $\tilde{r}_{m,t}$ is the value-weighted return on the CRSP index, and $\Delta IP_t$ is the monthly industrial production growth. Default premium, $Def$; dividend yield, $Y i e l d$; term spread, $T e r m$; and the one-month T-bill rate, $I_t$, are conditioning information variables predicting the excess return. $\Delta CLI$ is the growth rate in the log composite leading indicator. The sample period is 1959:12–2001:12. The $t$-statistics are reported in parentheses below each parameter estimate. All parameters are estimated by maximum likelihood.

The short-term rate serves as a proxy for investors’ expectations for future economic activity, as well as a proxy for the credit market conditions. The default premium is defined as the spread between yields on Baa and Aaa corporate bonds. The dividend yield is defined as the dividends on the value-weighted CRSP index over the past twelve months and is commonly used to proxy for the time variation in risk premium in modeling expected returns. In addition, Model (1)
allows the conditional variance of excess returns $h_{St}$ to depend on the state of the economy, as well as on the level of the one-month T-bill rate.\textsuperscript{16} Finally, the aggregate market return is proxied by the return on the value-weighted CRSP index.

Model (2) specifies the Markov-switching regime model of aggregate output, given in Equation (8), using the industrial production index. The industrial production index series has the advantage over other aggregate output series (such as the GDP) of being available at a monthly rather than quarterly frequency. The monthly industrial production index data are obtained from the Federal Reserve Economic Database (FRED) at the Federal Reserve Bank of St. Louis. The growth series is constructed by taking the change in the log of the index. The data on the composite leading indicator (CLI) are available from DRI Basic Economics Database.\textsuperscript{17} Finally, the implicit state probabilities and the parameters of each of the Markov-switching models are estimated over the 1959:12–2001:12 sample period by maximum likelihood estimation.

Table 1 reports all the parameter estimates obtained from the Markov-switching regime models along with the parameters of the transition probabilities. Both models appear successful in distinguishing the high and the low states of the economy. The parameter estimates in the mean equation in Model (1) show that the coefficient $\beta_3$ on the lagged short rate is negative in state one; similarly, the coefficient $\beta_1$ on the default premium is much more important in state one than in state two. Turning to the variance equation, the parameters are all significant, and the estimate values for both the intercept term $\lambda_0$ and the coefficient $\lambda_1$ imply a higher conditional volatility in state one than in state two. These estimated parameter values suggest that state one and state two identified by the model correspond, respectively, to recession and expansion states of the economy.\textsuperscript{18}

Turning next to the parameter estimates of Model (2) with aggregate output, we see in Table 1 that they are all highly significant and the estimates agree

\textsuperscript{16} The conditional variance equation is motivated by Glosten, Jagannathan, and Runkle (1993), who find that lagged interest rates are important in modeling the conditional volatility of monthly stock returns.

\textsuperscript{17} In 1995, the Bureau of Economic Analysis of the Department of Commerce selected the Conference Board as the custodian of the official composite leading, coincident, and lagging indexes, previously published by the Commerce Department. The Board, through its monthly news release of U.S. Leading Economic Indicators and Related Composite Indexes, is the direct source for the composite leading indicator. The series begins in 1959 and consists of ten components: average weekly hours in manufacturing, average weekly initial claims for unemployment insurance, manufacturers’ new orders for consumer goods and materials, the vendors’ performance component of the NAPM index, manufacturers’ new orders for nondefense goods, building permits, the S&P 500 measure of stock prices, the M2 measure of the money supply, the interest rate spread between the 10-year Treasury Bonds and the Federal Funds rate, and the University of Michigan index of consumer expectations. The composite leading indicator is released in the third week of each month for the prior month.

\textsuperscript{18} I also estimated a simplified version of Model (1) without the conditioning information variables in which the average market return and its volatility are allowed to depend on an unobservable state variable $S_t$:

$$r_{mt} \sim N\left(\mu_{S_t}, \sigma_{S_t}^2\right).$$

The parameter estimates are consistent with those of Model (1). The correlation coefficient between the conditional probability series is 0.99.
with the interpretation of the two states. Specifically, the mean growth rate in output is negative in state one, whereas it is positive in state two; furthermore, consistent with the prior evidence in the literature, the estimated volatility of aggregate output is greater in the low-growth state than it is in the high-growth state.

Finally, the behavior of the $\eta_1$ and $\eta_2$ coefficients on the lagged value of the change in the composite leading indicator in the transition probabilities also lends support to this interpretation of the latent states. Table 1 shows that in both models, $\eta_1$ is estimated as negative, but $\eta_2$ is positive. This suggests that while an increase in the leading indicator decreases the probability of staying in the first (low-growth) state, it increases the probability of staying in the second (high-growth) state. This is consistent with the recession and expansion interpretations of states one and two, respectively.\(^{19}\)

I next confirmed the identification of the latent states by examining the time series of the state probabilities estimated from the Markov models. The estimated conditional probabilities of being in the first state (recession) from both models are highly correlated with two widely used business cycle indices, the NBER indicator and the Stock and Watson Experimental Recession Index. The former is an indicator function that takes a value of one during NBER recessions and zero otherwise; the latter is an estimate of the probability, constructed from several variables, that the economy will be in a recession in six months. The correlation coefficients between the conditional probability of being in state one (recession) estimated from the returns-based Markov model and the business cycle indices are 0.57 and 0.40, respectively. The corresponding correlation coefficients are 0.70 and 0.53, respectively, for the conditional probability of low state estimated from the Markov model of aggregate output.\(^{20}\)

Finally, using these time series of the estimated state probabilities, I construct the index of investor uncertainty UC as defined in Equation (6). I refer to the uncertainty measures estimated from the Markov models of returns and of output as the returns-based measure and the output-based measure of uncertainty, respectively.

### 3.4 Empirical properties of belief and uncertainty measures

Figures 1 and 2 plot the investor beliefs $\pi$ and uncertainty UC measures over a four-decade period, respectively. In each figure, the top panel plots the returns-based measure and the bottom panel plots its output-based counterpart.

\(^{19}\) A likelihood ratio test of the restriction that the intercept terms in the transition probability are identical in the two states cannot be rejected. I therefore have chosen to estimate this parsimonious specification with a single common intercept.

\(^{20}\) While these correlation coefficients seem to suggest that a Markov model of aggregate output better captures the underlying states of the economy, the higher correlation coefficients observed with the output-based model are likely to be due to the fact that industrial production growth itself is probably an input for these business cycle indices.
The relationship of the estimated state probability $\pi$ to the business cycles over the sample period is illustrated in Figure 1, which plots the time series of the conditional probability of being in the low state, together with the NBER recessions represented by shaded areas in the figure. Both the top and bottom panels show that latent state one, identified as the low state by the models, indeed coincides with the recessionary periods of the business cycle. The probability of being in a recession shoots up in each of the NBER recessions. This holds true also for the latest recession, beginning in March 2001. Figure 2 plots the time series of the returns-based and output-based measures of UC, together with the NBER recessions.

There are several noteworthy features in these figures. First, it is interesting that while the time-series behavior of the returns-based and the output-based measures of $\pi$ and UC are generally quite similar across the two figures, there are some episodes of differences. It is plausible that events such as the 1987 market crash and more recently the Asian financial crisis and the LTCM crisis would have had very different impacts on market returns and on aggregate output, in a way that could generate important differences between the returns-based and output-based measures of $\pi$ and UC. Second, the returns-based measures appear to fluctuate more over time than do the output-based
Third, both measures in Figure 2 show several spikes in the behavior of investors’ uncertainty over time. Indeed, the returns-based measure of UC displays more frequent spikes. These spikes, however, are not necessarily limited to the turning points of the business cycles. Although recessions are consistently accompanied by a greater level of uncertainty, a sporadic increase in UC is not necessarily followed by a recession. This observation suggests that the fluctuations in investors’ uncertainty about the state of the economy, though correlated with the fundamentals, are not entirely explained by the aggregate fluctuations in the fundamental risk in the economy.

The summary statistics for the $\pi$ and UC measures are reported in Table 2, panel A. Here, $\pi$ is defined as the probability that investors assign to being in the good state of the economy. The table shows that the distribution of $\pi$ is highly skewed to the right; the returns-based $\pi$ and the output-based $\pi$ (denoted as $ret - \pi$ and $IP - \pi$ in the table) have unconditional means of 0.68 and 0.79, respectively, over the period from January 1961 through December 2001. This reflects the fact that the U.S. economy has experienced relatively few recessions in its recent history. Accordingly, the average uncertainty over the same period measures approximately 0.07, compared to its maximum value of 0.25. The autocorrelation coefficients suggest that both series are highly persistent, as also observed in Figures 1 and 2. Finally, it is reassuring that,
Table 2
Summary statistics

Panel A: Beliefs and uncertainty about fundamentals

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>10%</th>
<th>50%</th>
<th>90%</th>
<th>Max</th>
<th>p1</th>
<th>p3</th>
<th>p6</th>
<th>p12</th>
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<tr>
<td>Ret−π</td>
<td>0.679</td>
<td>0.382</td>
<td>0.000</td>
<td>0.006</td>
<td>0.913</td>
<td>0.981</td>
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<td>0.894</td>
<td>0.698</td>
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<tr>
<td>IP−π</td>
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<td>0.321</td>
<td>0.000</td>
<td>0.123</td>
<td>0.958</td>
<td>0.992</td>
<td>0.998</td>
<td>0.890</td>
<td>0.653</td>
<td>0.381</td>
<td>0.029</td>
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<tr>
<td>Ret−UC</td>
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<td>0.071</td>
<td>0.000</td>
<td>0.005</td>
<td>0.047</td>
<td>0.191</td>
<td>0.250</td>
<td>0.562</td>
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<td>0.006</td>
<td>0.030</td>
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<td>0.250</td>
<td>0.698</td>
<td>0.378</td>
<td>0.190</td>
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Panel B: Correlation matrix

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<th>IP − π</th>
<th>Ret</th>
<th>IP</th>
<th>MKT</th>
<th>SMB</th>
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<th>Ret</th>
<th>IP</th>
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<tr>
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</tbody>
</table>

Panel A reports the summary statistics for the measures for investors’ beliefs and uncertainty about the fundamentals, π is the conditional probability of being in a high-growth state and UC is the index of uncertainty constructed from the state probabilities as UC = π(1 − π). IP refers to variables estimated from a Markov-switching regime model of aggregate output and Ret refers to variables that are estimated from a Markov switching regime model of the market return. The sample period is from January 1961 through December 2001. Panel B reports the correlation coefficients among the belief and uncertainty measures and among the innovation measures. SΔπ and SΔUC; their correlation with the value-weighted return on the market MKT; and the book-to-market and size-related Fama-French factors HML and SMB.

despite their differences, the returns-based and the output-based Markov models yield rather similar measures of π and UC.

Panel B of Table 2 reports the correlation coefficients among the belief and uncertainty measures. First, the returns-based and the output-based measures of investor beliefs, ret−π and IP−π, are highly correlated with each other, with a correlation coefficient of 0.57. The two uncertainty measures, ret-UC and IP-UC, are also positively correlated, but they have a smaller correlation coefficient of 0.12. Second, the uncertainty measures are negatively correlated with investors’ conditional beliefs π that the economy is in the high state. This confirms that investors’ uncertainty about the economy is greater in bad times than in good times.

Finally, panel B of Table 2 reports the correlation between the innovation measures SΔπ and SΔUC, and their correlation with commonly used risk factors such as the return on the value-weighted market index and the Fama-French size and book-to-market factors. None of the correlation coefficients for the market return and the Fama-French factors is very large, suggesting that the state variables based on investors’ beliefs and uncertainty about the state of the economy are not highly correlated with other risk factors.
4. Data

In this section, I present a description of the cross-section of assets that are used in the asset pricing tests in the following sections.

The cross-section of test assets consists of portfolios formed on size, book-to-market, and past returns. Sorting on these characteristics has been shown to produce significant cross-sectional dispersion in risk premia and has posed the greatest empirical challenge to traditional asset pricing models. Furthermore, the Fama-French three-factor model relies on book-to-market and size-sorted portfolios to construct factors to price other assets. The economic interpretation of these factors, however, remains controversial. Explaining the risk premia for these portfolios should provide important insights on what these factors actually capture. Finally, the returns to momentum strategies of buying winner stocks and selling loser stocks over intermediate horizons, documented first by Jegadeesh and Titman (1993), have remained to date as one of the greatest anomalies of the asset pricing literature. Recently, Chordia and Shivakumar (2002) have found support for time-varying expected returns as an explanation for momentum payoffs. They show that the profitability of momentum strategies disappears once returns are adjusted for their predictability based on the macroeconomic variables. Motivated by their results, I explore whether exposure to uncertainty risk can explain the profitability of momentum strategies.

The monthly data on the value-weighted portfolio returns are taken from Kenneth French’s website. Size-sorted portfolios are formed from the set of all stocks covered by CRSP by ranking on the basis of their market equity value at the end of June of each year using NYSE breakpoints. Book-to-market portfolios are constructed from all firms in CRSP with Compustat book values by ranking in June of each year on the basis of their book-to-market ratios, using NYSE breakpoints. Finally, momentum portfolios are formed by sorting NYSE and AMEX stocks on CRSP on the basis of their cumulative returns over months \( t - 12 \) through \( t - 1 \).

I proxy for the market portfolio using the return on the value-weighted CRSP index. The risk-free rate is the 30-day T-bill rate from CRSP. I take the Fama-French factors HML and SMB from Kenneth French’s website.

5. Predictive Ability of Investor Beliefs and Uncertainty

If investor beliefs and uncertainty are indeed state variables in an ICAPM sense, they should have some ability to summarize the future investment opportunity sets. In this section, I begin my empirical analysis by exploring whether the probability and uncertainty measures have any ability to predict the first and second moment of the aggregate market return.

21 I thank Kenneth French for making the portfolio data available.
Good Times or Bad Times? Investors’ Uncertainty and Stock Returns

Table 3
Predictive ability of the first-state probability \( \pi_L \) and the uncertainty measure \( \text{UC} \)

Panel A: Predicting the first moment of market return

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.225</td>
<td>0.292</td>
<td>0.295</td>
<td>0.809</td>
<td>0.841</td>
</tr>
<tr>
<td>(0.40)</td>
<td>(0.52)</td>
<td>(0.45)</td>
<td>(1.15)</td>
<td>(1.23)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-3.820</td>
<td>-3.959</td>
<td>0.734</td>
<td>-0.439</td>
<td></td>
</tr>
<tr>
<td>(1.19)</td>
<td>(1.22)</td>
<td>(0.23)</td>
<td>(-0.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{UC}_{\text{High}} )</td>
<td>-4.685</td>
<td>-3.911</td>
<td>6.593</td>
<td>7.418</td>
<td></td>
</tr>
<tr>
<td>(-0.87)</td>
<td>(-0.69)</td>
<td>(1.68)</td>
<td>(1.95)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{UC}_{\text{Low}} )</td>
<td>-3.594</td>
<td>-3.973</td>
<td>-0.969</td>
<td>-3.746</td>
<td></td>
</tr>
<tr>
<td>(-1.08)</td>
<td>(-1.14)</td>
<td>(-0.27)</td>
<td>(-0.99)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Panel B: Predicting the second moment of market return

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.761</td>
<td>0.743</td>
<td>0.743</td>
</tr>
<tr>
<td>(3.43)</td>
<td>(3.25)</td>
<td>(3.25)</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>1.410</td>
<td>1.056</td>
<td>0.222</td>
</tr>
<tr>
<td>(1.34)</td>
<td>(1.08)</td>
<td>(0.28)</td>
<td></td>
</tr>
<tr>
<td>( \text{R}^2 )</td>
<td>-0.002</td>
<td>0.001</td>
<td>-0.002</td>
</tr>
</tbody>
</table>

This table presents the regression results for the ability of the first-state probability \( \pi \) and uncertainty measure \( \text{UC} \) to predict the first and second moments of the aggregate market return, proxied for by the value-weighted CRSP index. Panel A reports the estimated coefficients from the following predictive regression of the aggregate market return:

\[
r_{t,t} = \alpha + \gamma X_{t-1} + \epsilon_t,
\]

where \( r_{t,t} \) is the monthly return on the aggregate market and the independent variable \( X_{t-1} \) is the vector of predictive variables at time \( t-1 \). \( \text{UC}_{\text{Low}} \) and \( \text{UC}_{\text{High}} \) are defined as the contemporaneous level of the uncertainty measure \( \text{UC} \), interacted with the dummy variables \( D_{\text{Low},t} \) and \( D_{\text{High},t} \). The dummy variable \( D_{\text{Low},t} \) \( (D_{\text{High},t}) \) takes a value of one (zero) if the conditional probability of being in the low state is greater than 0.3 and less than 0.7, and zero (one) otherwise. Panel B presents the coefficients from the estimation of a predictive regression of the second moment of the aggregate market return:

\[
\sigma^2_{t,t} = \alpha + \gamma X_{t-1} + \nu_t,
\]

where \( \sigma^2_{t,t} \) is the volatility of the aggregate market return estimated from a GARCH(1,1) model and \( X_{t-1} \) is the vector of predictive variables at time \( t-1 \). The sample period is from January 1961 to December 2001. Each numbered column corresponds to a different specification of the predictive regression. In each panel, the numbered columns on the left-hand side report the coefficients estimated using the probability and uncertainty measures obtained from a Markov-switching regime model of the aggregate market return; the numbered columns on the right-hand side in each panel present the corresponding coefficient estimates obtained with probability and uncertainty measures from a Markov model of the aggregate output. The values in parentheses are \( t \)-statistics corrected for autocorrelation and heteroscedasticity using the Newey-West estimator with five lags. The \( R^2 \) represents the regression \( R^2 \) adjusted for the degrees of freedom.

5.1 Predictive regressions

Table 3 presents the estimation results, with the \( t \)-statistics reported in parentheses. Each column in Table 3 corresponds to a different regression specification. In panels A and B, the columns on the left-hand side labeled
“returns” present the coefficient estimates obtained using the returns-based measure of $\pi$ and UC, whereas those on the right-hand side report their analogues obtained with the output-based probability and uncertainty measures.

In column (1) of panel A in Table 3, I assess the ability of the first-state probability to summarize future investment opportunities by estimating a univariate time-series regression of the monthly return of the value-weighted CRSP index on the lagged value of the probability $\pi$ measure. Using the returns-based $\pi$ measure, the estimated coefficient is 0.225, while the coefficient estimate on the output-based $\pi$ is 0.809. These coefficient estimates suggest that the conditional probability of being in the first state, as proxied for by either the returns-based or the output-based measures of $\pi$, is positively related to the future market return. Although the signs of the coefficients are consistent with the idea that bad times are associated with a higher risk premium, neither of these coefficients on $\pi$ is statistically significant.

I next examine the predictive ability of the uncertainty measure by estimating a univariate time-series regression of the monthly return of the value-weighted CRSP index on the lagged value of the uncertainty UC measure. If higher levels of uncertainty increase the required ex ante expected return to compensate risk-averse investors, we should expect to see a positive relation between the level of uncertainty and future returns. The specification in column (2) of Table 3 tests for this prediction. The coefficient estimates in column (2) on the left-hand and right-hand sides of panel A are, however, very different. The estimated coefficient on the left-hand side, using the returns-based UC, is negative and large in magnitude but statistically insignificant. The sign of the estimated value is not consistent with the prediction that greater uncertainty should predict higher returns. On the other hand, the coefficient on the right-hand side using the output-based measure of UC is positive. While it is consistent with the predicted relationship, the coefficient estimate is only a fraction of the former in magnitude, and it also is statistically insignificant. Column (3) considers the multivariate time-series regression of the monthly return of the value-weighted CRSP index on lagged values of the $\pi$ and UC measures together. The coefficient on $\pi$ remains positive, using either the returns- or the output-based measures; however, the coefficient on the uncertainty measure UC becomes negative but insignificant.

The estimated coefficients in columns (1) through (3) of panel A in Table 3 fail to support the predicted positive relationship between the level of uncertainty and future expected returns. It is plausible, however, that there exists a positive (albeit nonlinear) relationship between the level of uncertainty and expected returns. That is, for $\pi$ close to one or close to zero, there may be little effect of uncertainty on expected returns, but for intermediate values of $\pi$, we may expect to see a positive relationship between the level of uncertainty and future returns.22 The regressions in columns (1) through (3) in

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22 Indeed, Veronesi (1999) suggests that there is an inverse U-shaped relationship between return volatility and expected returns.
Good Times or Bad Times? Investors’ Uncertainty and Stock Returns

Panel A do not address such nonlinearity. To explore this possibility, in column (4) I consider a regression specification regressing the monthly return of the value-weighted CRSP index on the lagged value of UC interacted with dummy variables $D_{\text{low},t}$ and $D_{\text{high},t}$, which are defined to characterize high-uncertainty and low-uncertainty periods. Specifically, $D_{\text{high},t}$ ($D_{\text{low},t}$) takes a value of one (zero) if the conditional probability of being in the low state is greater than 0.3 and less than 0.7, and zero (one) otherwise. The values of the estimated coefficients on UC_{high} and UC_{low} using the output-based measure in column (4) are 6.593 and −0.969, respectively, and the coefficient on UC_{high} is statistically significant at the 10% level. These coefficients suggest that higher uncertainty predicts greater expected returns when the conditional probability is close to 0.5, but has little impact on expected returns when $\pi$ is close to zero or one. Furthermore, the addition of the conditional probability $\pi$ itself to the regression in column (5) strengthens the relation of expected returns with the level of uncertainty, as well as with the conditional probability $\pi$. In contrast, the coefficients on UC_{high} and UC_{low} using returns-based measures in columns (4) and (5) on the left-hand side of the panel are both negative, but insignificant. These estimates suggest that there is evidence of a positive, nonlinear relationship between expected returns and the level of uncertainty, but when the latter is measured from aggregate output.

There may be several explanations for this apparent difference between the results using the returns-based and output-based measures of $\pi$ and UC. First, it has already been noted in Figure 2 that the two measures of uncertainty, though similar in distribution, display at certain times rather different time-series behavior. It is possible that these short episodes make the measures noisier and cause them to have different coefficient estimates in panel A. Furthermore, it is also plausible that when the returns-based measure of UC is compared to its output-based analogue, the former is likely to be more closely related to conditional market variance, by construction. Previous studies have found mixed results on the nature of the relation between market risk premium and conditional market variance. Specifically, Campbell (1987) and Glosten, Jagannathan, and Runkle (1993) also report a significant negative relationship. The negative coefficient on the returns-based measure of UC in panel A may be capturing a similar relationship.

Finally, I examine the ability of the probability and uncertainty measures to predict the second moment of the aggregate market return. For this purpose, I first obtained volatility estimates from a GARCH (1, 1) model of the monthly return of the value-weighted CRSP index over the sample period.23 I then estimated a time-series regression of the monthly volatility on the lagged value of the $\pi$ and UC measures, respectively. Panel B of Table 3 reports these estimation results. First, the coefficient estimate in column (1) shows that the conditional probability of being in the first state is positively and significantly

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23 Bollerslev (1986) was the first to suggest the GARCH model. The most commonly used model in the GARCH class is the simple GARCH(1,1) (Campbell, Lo, and MacKinlay 1997).
related to return volatility. The coefficient estimates obtained with the returns-based and output-based measures of $\pi$ are similar in magnitude, and both are highly significant. This is consistent with the earlier literature that shows that return volatility tends to be higher in recessions. I next turn to the predictive ability of the uncertainty measure UC in column (2). The coefficient estimate on either the returns-based UC or the output-based UC confirms that higher uncertainty predicts greater return volatility, as we should expect, although the relationship is not statistically significant for either of the measures. Finally, in column (3), I consider a multivariate specification in which the market volatility is regressed on the probability $\pi$ and the uncertainty UC measures together. The coefficient estimates suggest that the conditional probability measure $\pi$ is a stronger predictor of future market volatility.

5.2 Investors’ conditional beliefs, uncertainty, and valuation ratios

This section examines the impact of investors’ beliefs and uncertainty about the state of the economy on asset valuation ratios. This relationship is interesting for several reasons. First, Veronesi (1999) shows that uncertainty increases the risk premium; this effect occurs because investors, knowing that their beliefs will be more variable, expect greater compensation in times of uncertainty in anticipation of greater asset price sensitivity to changes in their own beliefs. In Veronesi’s model, this effect of uncertainty appears as an additional discount term in the price function. This implies the testable prediction that there should be a negative relationship between asset valuation ratios such as the price-dividend ratio and investors’ uncertainty about the fundamentals. I therefore first test whether this relationship empirically holds on average in time series. Second, since the level of uncertainty is greater on average in bad times, I also examine whether there is an empirical negative relationship between investors’ conditional beliefs about being in the low state and asset valuation ratios. Finally, by looking across different assets, I show that there is substantial cross-sectional variation in the impact of investor beliefs and uncertainty on valuation ratios. These findings help motivate the cross-sectional analysis in the next section.²⁴

I construct monthly price-dividend ratios on thirty test portfolios from portfolio dividend series formed in the same way as in Campbell (2000), and Bansal, Dittmar, and Lundblad (2005). The level of dividends are constructed as

$$D_{t+1} = y_{t+1} P_t$$

and

$$P_{t+1} = h_{t+1} P_t,$$

²⁴ This approach can also shed some additional light on the ability of uncertainty to predict future returns. In their empirical analysis, Bansal and Yaron (2004) suggest that when the uncertainty level is highly persistent, the effects of uncertainty may be difficult to detect solely in returns, and may be more visible in valuation ratios.
with $P_0 = 100$, where $y_{t+1}$ is the dividend yield computed from the total returns $R_{t+1}$ by subtracting the price gain series $h_{t+1}$: $y_{t+1} = R_{t+1} - h_{t+1}$. I then take twelve-month trailing averages to deseasonalize the monthly dividend series. The price-dividend ratio is constructed by taking the log of the ratio of $P_t$ to $D_t$. The aggregate market price-dividend ratio is constructed in a similar fashion.

I test whether investors’ conditional beliefs about being in the low state and their uncertainty are negatively related to the equity risk premium by regressing the market and portfolio price-dividend ratios on the contemporaneous value of the conditional probability measure $\pi$ and on the contemporaneous level of uncertainty $U_C$, respectively. I present the regression results in Table 4, with the $t$-statistics reported in parentheses. In each panel, the columns labeled “Returns” report the coefficient estimates obtained using the returns-based measures, and the columns labeled “Industrial Production” present the results for the output-based measures. For brevity, I report the results for only the lowest, middle, and top deciles for each group of portfolios.

The coefficient estimates in columns (1) and (2) of panel A in Table 4 indicate that the conditional probability $\pi$ of being in the low state is strongly and negatively related to the market and portfolio price-dividend ratios. First, all of the coefficient estimates are negative, and most are statistically significant. The value of the coefficient estimate on the returns-based measure of $\pi$ is $-0.017$ in the first row, suggesting that valuation ratios are lower in bad times at the aggregate level. This result gets even stronger and becomes statistically significant in column (2) when the output-based measure $\pi$ is used as the conditional probability measure. Second, the individual coefficient estimates suggest that there is substantial variation in the impact of $\pi$ across different portfolios. Moving down the momentum portfolios and book-to-market portfolios, the coefficients estimates $b_{\pi}$ become more negative almost monotonically. In contrast, moving down the size portfolios, the coefficients estimates increase and become less negative nearly monotonically. These coefficients suggest that investors’ conditional beliefs about bad times have the greatest impact on the valuation of winners, small stocks, and value stocks. It is also interesting to note that while the two probability measures yield largely similar patterns in panel A, the average $R^2$ is considerably higher in regressions estimated with the output-based probability measure $\pi$.

Panel B of Table 4 reports the estimation results for the relationship between the level of uncertainty and valuation ratios. The coefficients in columns (3) and (4) show strikingly different patterns. First, all of the coefficient estimates on the output-based uncertainty measure $U_C$ in column (4) are negative, and statistically significant. This confirms that the negative relationship between the level of uncertainty and risk premium, predicted by Veronesi (1999), holds at both the aggregate market and the portfolio level. In contrast, the coefficient

25 The price appreciation series $h_t$ is equivalent to the ret$_t$ series in CRSP. I thank Robert Dittmar for making the portfolio-level data available on his website.
Table 4
Conditional beliefs, uncertainty, and price-dividend ratios

<table>
<thead>
<tr>
<th>Panel A: Beliefs</th>
<th>Panel B: Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t - d_t = b_0 + b_x \pi_t + \epsilon_t$</td>
<td>$p_t - d_t = b_0 + b_{UC} \text{UC}_t + \epsilon_t$</td>
</tr>
<tr>
<td>Returns</td>
<td>Industrial production</td>
</tr>
<tr>
<td>$b_x$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Market</td>
<td>−0.017</td>
</tr>
<tr>
<td>(−0.17)</td>
<td>(−0.280)</td>
</tr>
<tr>
<td>M1</td>
<td>−0.208</td>
</tr>
<tr>
<td>(−2.01)</td>
<td>(−2.280)</td>
</tr>
<tr>
<td>M5</td>
<td>−0.198</td>
</tr>
<tr>
<td>(−3.43)</td>
<td>(−5.03)</td>
</tr>
<tr>
<td>M10</td>
<td>−0.445</td>
</tr>
<tr>
<td>(−3.52)</td>
<td>(−3.17)</td>
</tr>
<tr>
<td>S1</td>
<td>−0.460</td>
</tr>
<tr>
<td>(−4.35)</td>
<td>(−6.84)</td>
</tr>
<tr>
<td>S5</td>
<td>−0.342</td>
</tr>
<tr>
<td>(−3.26)</td>
<td>(−6.18)</td>
</tr>
<tr>
<td>S10</td>
<td>−0.151</td>
</tr>
<tr>
<td>(−1.82)</td>
<td>(−4.76)</td>
</tr>
<tr>
<td>BM1</td>
<td>−0.019</td>
</tr>
<tr>
<td>(−0.21)</td>
<td>(−3.18)</td>
</tr>
<tr>
<td>BM5</td>
<td>−0.221</td>
</tr>
<tr>
<td>(−2.61)</td>
<td>(−5.51)</td>
</tr>
<tr>
<td>BM10</td>
<td>−0.433</td>
</tr>
<tr>
<td>(−6.26)</td>
<td>(−6.83)</td>
</tr>
</tbody>
</table>

This table presents the results of regressing price-dividend ratios on investors’ current beliefs $\pi_t$, about the state of the economy and on the contemporaneous level of uncertainty UC$_t$. Panel A reports the estimation results for investors’ current beliefs $\pi_t$, and panel B reports the estimation results for investors’ uncertainty, UC. The coefficients in columns (1) and (3) are estimated using the probability and uncertainty measures obtained from a Markov-switching regime model of the aggregate market return; columns (2) and (4) show the corresponding coefficients obtained with the probability and uncertainty measures from a Markov model of the aggregate output. In each panel, the top row corresponds to the regression of the market price-dividend ratio. The decile portfolios used are ten momentum (M), ten size (S), and ten book-to-market (BM) portfolios. M1(loser), S1(smallest), and BM1(lowest B/M) correspond to the lowest decile. Results for the lowest, the top, and the middle deciles are included in the table. The sample period for this regression is from January 1961 through December 2001. The t-statistics are corrected for autocorrelation and heteroscedasticity using the Newey-West estimator with five lags. The $\bar{R}^2$ represents the regression $R^2$ adjusted for degrees of freedom.

The estimates on the returns-based uncertainty measure UC in column (3) are mostly positive and insignificant. Second, the individual coefficient estimates in column (4) show substantial variation in the impact of uncertainty across different portfolios. Moving down this column, the estimates of $b_{UC}$ increase and become less negative nearly monotonically across each set of portfolios. This pattern is most pronounced within the momentum portfolios. In contrast, the $b_{UC}$ estimates in column (3) display a much different pattern. Specifically, the estimates of $b_{UC}$ decrease going down column (3) across each set of portfolios. Finally, the coefficients on average are not very precisely estimated using the returns-based measure of uncertainty in panel B; the $\bar{R}^2$ statistics associated with the estimates in column (3) are, on average, much smaller than those reported for the estimates in column (4).
Given the similarities between the results obtained with the returns-based and output-based $\pi$ measures in panel A of Table 4, the differences between the coefficient estimates in panel B are striking. At the same time, they seem to mirror the differences across the two uncertainty measures reported in Table 3. In order to gain additional insight regarding what contributes to these differences, I explored the extent to which occasional differences in the time-series behavior of the returns-based and output-based measures of uncertainty can explain the differences in the findings in Table 4, panel B. Specifically, I estimated the specification in column (3) with the returns-based uncertainty measure over five-year periods. The results show that the sign of the coefficient on the returns-based UC indeed varies considerably across the sub-periods. In periods in which the coefficient estimates are negative, however, they get statistically stronger and the $R^2$ statistics substantially increase. Furthermore, the pattern of the coefficient estimates across portfolios resembles more closely the one observed in column (4) across portfolios. This confirms that the differences between the findings across the two measures in panel B can be at least partially attributed to episodes in which the returns-based UC exhibits more noise than its output-based counterpart.26

Overall, the evidence reported in Tables 3 and 4 supports the conjecture that investors’ conditional beliefs and uncertainty summarize future investment opportunities. The findings indicate that greater uncertainty depresses valuation ratios at the aggregate level. This is consistent with the notion that the greater variability of beliefs in periods of high uncertainty about the state of the economy induces investors to require compensation for risk. More importantly, the data in Table 4 reveal that the impact of investors’ conditional beliefs and uncertainty exhibits substantial variation across the cross-sectional of portfolios. In the next section, I explore whether these differences in exposure to conditional beliefs and uncertainty can explain cross-sectional stock returns.

6. Empirical Methodology and Asset Pricing Tests

In this section, I test the conditional model in Equation (1) in which the conditional expected return is expressed in terms of conditional betas and the risk premia associated with each source of risk. An alternative way of writing the conditional model in Equation (1) is to consider its equivalent conditional linear factor model.27 In the next subsection, I first present this factor model and

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26 In untabulated results, I have also investigated the effect of investor beliefs and uncertainty on asset prices in good times and bad times by regressing the market and portfolio price-dividend ratios on the contemporaneous values of $\pi$ and UC, respectively, interacted with the dummy variables $D_{\text{low},t}$ and $D_{\text{high},t}$. $D_{\text{low},t}$ (or $D_{\text{high},t}$) takes a value of one (zero) if the conditional probability of being in the low state is equal to or greater than 0.5, and zero (one) otherwise. I find that coefficient estimates on both interaction terms are still negative and statistically significant. In addition, the results suggest that an increase in the conditional probability of being in the low state has a greater negative impact on asset prices when the economy is in the good state. On the other hand, greater uncertainty has a more pronounced negative effect on asset prices in bad times than in good times.

27 The discussion in this section draws from Cochrane (2001) and Lettau and Ludvigson (2001a).
then model the variation in conditional moments by allowing the parameters in the stochastic discount factor to depend on current information variables, as in Cochrane (1996) and Ferson and Harvey (1999).

6.1 Scaled multifactor models

It is well known that in the absence of arbitrage opportunities, there exists a stochastic discount factor $M_{t+1}$ such that any traded asset $i$ with a return $R_{i,t+1}$ satisfies

$$1 = E_t[M_{t+1}(1 + R_{i,t+1})],$$

where $E_t$ denotes the expectation conditional on the information available at time $t$. Each asset pricing model has a different specification for $M_{t+1}$. In the conditional CAPM, for example, the implied $M_{t+1}$ is linear in the market portfolio return:

$$M_{t+1} = a_t + b_t R_{MKT,t+1},$$

where $a_t$ and $b_t$ are time-varying coefficients. One can show that the conditional linear factor model given in Equation (10) is equivalent to the conditional beta representation given by

$$E_t(R_{i,t+1}) = R_{0,t} + \beta_{t}^{i} \lambda_t,$$

where $R_{0,t}$ is the return on a zero-beta portfolio uncorrelated with $M_{t+1}$, and

$$\beta_{t}^{i} = \frac{\text{Cov}_t(R_{MKT,t+1}, R_{i,t+1})}{\text{Var}(R_{m,t+1})}$$

and the risk premium

$$\lambda_t = -R_{0,t} \frac{\text{Var}(R_{MKT,t+1})}{b_t}.$$

If fluctuations in investors’ conditional beliefs and uncertainty matter for the pricing of assets, the conditional model in Equation (1) can be expressed as the following conditional linear factor model:

$$M_{t+1} = a_t + f'_{t+1} b_t,$$

where $f_{t+1}$ denotes the vector of fundamental factors $f_{t+1} = (R_{MKT,t+1}, S_{\Delta \pi,t+1}, S_{\Delta UC,t+1})'$, and $a_t$ and $b_t$ are time-varying coefficients.

Following Cochrane (1996) and Lettau and Ludvigson (2001a), I rewrite the conditional linear factor model in (11) as a scaled multifactor model by expressing the time-varying coefficients $a_t$ and $b_t$ as linear functions of instruments $z_t$ containing time $t$ information, $M_{t+1} = a(z_t) + f'_{t+1} b(z_t)$, and then expanding the set of factors with the scaled factors. For example, with one instrument $z_t$.
and one fundamental factor $R_{MKT,t+1}$, the conditional factor model in Equation (10) becomes a linear factor model with constant coefficients:

$$M_{t+1} = (\gamma_0 + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t) R_{MKT,t+1}$$

$$= \gamma_0 + \gamma_1 z_t + \eta_0 R_{MKT,t+1} + \eta_1 (z_t R_{MKT,t+1}).$$

Similarly, the conditional linear factor model in Equation (11) can be expressed as a scaled multifactor model with constant coefficients:

$$M_{t+1} = c' F_{t+1},$$

where $F_{t+1} = (1, \tilde{f}_{t+1}', \tilde{f}_{t+1}' z_{t+1})$, $c$ is the constant vector $c = (\gamma_0, b')$, and $b = (\gamma_1, \eta_01, \eta_11, \eta_02, \eta_12, \eta_03, \eta_13)$. The scaled multifactor representation of $M_{t+1}$ in the form given above implies an unconditional multifactor beta representation for asset $i$ with constant betas given by

$$E(R_{i,t+1}) = E(R_{0,t}) + \beta_{zi} \lambda_z + \beta_{MKTi} \lambda_{mkt} + \beta_{MKTzi} \lambda_{mkt}$$

$$+ \beta_{pi} \lambda_{\Delta pi} + \beta_{pi} \lambda_{\Delta pi} + \beta_{uci} \lambda_{\Delta UC} + \beta_{uczi} \lambda_{\Delta ucez},$$

(12)

where $E(R_{0,t})$ is the average return on a zero-beta portfolio that is uncorrelated with the stochastic discount factor and $\beta_{zi}, \beta_{MKTi}, \beta_{MKTzi}, \beta_{pi}, \beta_{uci}, \beta_{uczi}$ are regression coefficients of returns on the scaled factors $\tilde{f}_{t+1} = (z_t, \tilde{f}_{t+1}', \tilde{f}_{t+1}' z_{t+1}).$

An important consideration for the scaled multifactor model approach is the choice of the scaling variable $z_t$. I use the conditional probability $\pi_t$ as the scaling variable $z_t$ for two reasons. First, the analysis in the previous section suggests that the effect on risk premia of a change in investor beliefs and uncertainty critically depends on investors’ current beliefs about the state of the economy. Second, the conditional probability $\pi_t$, by definition, is a summary measure of investors’ beliefs and expectations and should proxy for investors’ unobservable information sets. Finally, as suggested by Ferson, Sarkissian, and Simin (2003), I use the de-meaned value of the scaling variable in all of the empirical analysis.

It is important to note that the individual coefficients $\lambda$ from Equation (12) do not have the same interpretation as the risk prices in Equation (1). When the representative agent is more risk-averse than log utility, as in the traditional CAPM, in this model with uncertainty risk we would expect assets that covary positively with changes in investors’ own confidence to have greater returns. These assets would command a higher risk premium because they reduce investors’ hedging ability in periods of higher uncertainty. Put differently, when investors have relative risk aversion greater than one, the conditional three-factor model in Equation (1) implies that assets with a lower covariance with the uncertainty innovation should have higher expected returns. Let $\tilde{\lambda}$ denote the risk premia on the fundamental factors in Equation (1); this relation has implications for the sign of the risk prices $\tilde{\lambda}$ and for the sign of the $b$ coefficient.
on $S_{\DeltaUC,t+1}$ in the equivalent conditional factor model in (11). Specifically, it implies that the risk price $\tilde{\lambda}_{\DeltaUC,t} < 0$ and that $b_{\DeltaUC,t+1} > 0$. These provide testable restrictions that can be used to evaluate the performance of the specific models considered in the empirical analysis.

These conditions do not, however, imply that the $\lambda$ coefficients from the unconditional beta representation in Equation (12) should be negative. Specifically, it can be shown that

$$\lambda_t = -E(R_{0,t})\text{Cov} (\bar{f}_{t+1}, \bar{f}'_{t+1}) b_t,$$

where $\bar{f}$ denotes the vector of scaled factors. Similarly, the vector of risk prices $\tilde{\lambda}$ on the fundamental factors in Equation (1) also depend on $b_t$:

$$\tilde{\lambda}_t = -E(R_{0,t})\text{Cov} (f_{t+1}, f'_{t+1}) b_t.$$

Clearly, the coefficients $\lambda$ are very different from the risk prices $\tilde{\lambda}$ in Equation (1). They can be related, however, if the unconditional mean of the scaling variable is zero, as is the case here, and if we assume that the average zero-beta rate and the conditional covariance matrix for the factors are constant. Under these assumptions, the $\lambda$ coefficients from the unconditional beta representation in Equation (12) are equal to the average $\tilde{\lambda}$ coefficients from the conditional model in Equation (1). For each model investigated below, it is possible then to check and verify that, conditional on these assumptions, the estimated $\lambda$ values from the unconditional beta representation are consistent with the implications of the model; that is, $E(\tilde{\lambda}) < 0$.

Below, I use the unconditional beta representation given in Equation (12) as the basis for assessing the ability of the conditional three-factor model in Equation (1) to explain the cross-section of average returns. The scaled multifactor model has the advantage that for each asset pricing model under consideration it nests the corresponding unconditional models in which the betas on the scaling variable and the scaled factors are zero. This facilitates the comparison of the results to both conditional and unconditional asset pricing models that have been proposed in the literature.

6.2 Cross-sectional estimation and results

In this subsection, I estimate the unconditional model in Equation (12) using the cross-sectional methodology of Fama and MacBeth (1973). For comparison, I also estimate the cross-sectional specification implied by standard unconditional models, such as the CAPM and the Fama-French three-factor model. The estimation results are presented in Table 5. Rows (1) through (8) in panels A and B report the estimated $\lambda$ coefficients for the given model specifications and the autocorrelation-corrected (and uncorrected) $t$-statistics obtained from Fama-MacBeth standard errors. To compare the relative performance of the different specifications, I report in the last column of each panel the cross-sectional regression $R^2$ adjusted for degrees of freedom, which shows the fraction of the cross-sectional variation in average returns that can be explained by the model.
Finally, panel A presents the estimated $\lambda$ coefficients using the $S_{\Delta \pi}$ and $S_{\Delta \text{UC}}$ factors obtained from the returns-based $\pi$ and uncertainty measures, and panel B analogously reports the coefficients obtained with the output-based measures.

I begin by presenting first the results from the static CAPM and the Fama-French three-factor models in rows (1) and (2) of Table 5. The static CAPM implies the following cross-sectional specification:

$$E(R_{i,t}) = E(R_{0,t}) + \beta_{MKT,i} \lambda_{MKT}.$$  \hspace{1cm} (13)

To test the cross-sectional specification in Equation (13), I employ the two-pass cross-sectional methodology of Fama and MacBeth (1973). Specifically, I first obtain the factor loadings $\beta_{MKT,i}$ for each portfolio $i$ in a first-pass, single time-series regression of the monthly return of portfolio $i$ on the market portfolio return:

$$R_{i,t+1} = \alpha_i + \beta_{MKT,i} \lambda_{MKT} + \epsilon_{i,t+1},$$

where the return of the market portfolio is proxied for by the monthly value-weighted return on the CRSP index. I then estimate a second-pass cross-sectional regression in which returns across portfolios are regressed for each month on their first-pass factor loadings to estimate the risk premium. The risk premium $\lambda_{MKT}$ in (13) is estimated as the average of the cross-sectional estimates:

$$\lambda_{MKT} = \frac{1}{T} \sum_{t=1}^{T} \lambda_{MKT,t},$$

where the standard deviation of the cross-sectional regression estimates is used to generate the sampling error of the estimate of the risk premium.

The results are presented in row (1) in Table 5, panel A. Consistent with the evidence in the literature, the static CAPM does not explain portfolio returns very well. First, the coefficient $\lambda_{MKT}$ is estimated to be negative. Second, the $t$-statistic shows that the market beta is not a statistically significant determinant of the cross-section of average returns on the thirty benchmark portfolios. Finally, the poor performance of the model is also evidenced by the low $R^2$ reported in the last column.

The Fama-French three-factor model implies the following cross-sectional specification:

$$E(R_{i,t}) = E(R_{0,t}) + \beta_{MKT,i} \lambda_{MKT} + \beta_{SMB,i} \lambda_{SMB} + \beta_{HML,i} \lambda_{HML},$$  \hspace{1cm} (14)

where SMB and HML are the size and book-to-market factors constructed by Fama and French (1993).\hspace{1cm}28 The factor loadings are obtained similarly, by

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28 Recall that the SMB is the difference in returns between small and large firms, while HML is the return difference between the portfolios of stocks with high book-to-market ratios and stocks with low book-to-market ratios.
Table 5
Cross-sectional regressions

Panel A: Using returns-based probability and uncertainty measures

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Panel B: Using output-based probability and uncertainty measures

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| (Continued overleaf)
This table presents estimates from the cross-sectional Fama-MacBeth regressions of portfolio returns. The regressors are the slope coefficients estimated in a single time-series multivariate regression of portfolio returns on the relevant factors. The individual \( \lambda_j \) estimates are the average cross-sectional regression estimates, expressed in percentages. \( S_{\Delta \pi} \) is the innovation in investors’ beliefs about the state of the economy, measured as the change in the probability of being in the low state \( \pi_t \). \( S_{\Delta \text{UC}} \) is the innovation in uncertainty, measured by the unanticipated change in \( \text{UC}_t \). \( \text{MKT} \) denotes the value-weighted return on the CRSP index, and SMB and HML are the Fama-French factor portfolios related to size and book-to-market ratios. The scaling variable, \( \pi_t \), is the conditional probability of being in the low state. The uncertainty and probability variables \( S_{\Delta \text{UC}} \) and \( S_{\Delta \pi} \) in panel A are estimated from a Markov-switching regime model of the market return; those in panel B are obtained from a Markov-switching regime model of aggregate output. Two sets of \( t \)-statistics are reported below each coefficient estimate. The values in parentheses use the uncorrected Fama-MacBeth standard errors; \( t \)-values in brackets have been corrected for potential autocorrelation in measurement errors in betas. The \( R^2 \) is the cross-sectional \( R^2 \), adjusted for degrees of freedom. The thirty portfolios used are ten momentum (M), ten size (S), and ten book-to-market (BM) portfolios. M1 (loser), S1 (smallest), and BM1 (growth) correspond to the lowest deciles. The sample period is from January 1961 through December 2001.

Estimating a single multivariate time-series regression of the monthly return of portfolio \( i \) on the return of the market portfolio and on the Fama-French size and book-to-market factors. The results of the second-pass cross-sectional estimation of Equation (14) are presented in row (2) of panel A in Table 5.

The three-factor model performs better than the static CAPM in explaining the cross-sectional variation in returns: the cross-sectional \( R^2 \) rises to 15%. This is low in comparison to the findings of Fama and French (1996), who show that together with the market factor, the size and book-to-market factors can explain most of the cross-sectional variation in average returns. The apparent weakness of the three-factor model in Table 5 may be due to the choice of the test portfolios used in this study. Indeed, the thirty portfolios used as test assets differ from the typically used twenty-five size and book-to-market portfolios constructed from the intersection of size and book-to-market quintiles. When the three-factor model in Equation (14) is tested with the usual twenty-five portfolios, the cross-sectional \( R^2 \) jumps to 79%. However, these are the very same portfolios that are used in the construction of the SMB and HML factors. In addition, the test assets in this study also include momentum portfolios, and it is well known that the three-factor model has difficulty explaining momentum returns.

Before turning to the estimation of the cross-sectional specification in (12), to facilitate its comparison with the unconditional models of the static CAPM and the Fama-French three-factor model, I also estimate each model with the \( S_{\Delta \text{UC}} \) and \( S_{\Delta \pi} \) factors. For the static CAPM, this corresponds to the unscaled model nested in Equation (12) in which the betas on the scaling variable and
the scaled factors are zero. For the three-factor model, this becomes:

\[ E(R_{i,t}) = E(R_{0,t}) + \beta_{\pi} \lambda_{\Delta \pi} + \beta_{\text{UC}} \lambda_{\Delta \text{UC}} + \beta_{\text{MKT}} \lambda_{\text{MKT}} + \beta_{\text{SMB}} \lambda_{\text{SMB}} + \beta_{\text{HML}} \lambda_{\text{HML}}. \] (15)

I report the results of the specification in Equation (15) and its CAPM counterpart in rows (3) and (4) of Table 5, panel A. The risk loadings are estimated similarly in a single multivariate time-series regression of the portfolio returns on the uncertainty and \( \pi \) risk factors, \( S_{\Delta \text{UC}} \) and \( S_{\Delta \pi} \), the return on the market portfolio, and the Fama-French size and book-to-market factors. The estimated \( \lambda \) coefficients are reported in rows (3) and (4). First, the estimated \( \lambda \) coefficients associated with \( S_{\Delta \text{UC}} \) and \( S_{\Delta \pi} \) both appear significant when added either to the CAPM or to the Fama-French three-factor model. Furthermore, accounting for uncertainty and \( \pi \) risk factors unconditionally improves significantly the ability of the CAPM and the Fama-French three-factor model to explain the cross-sectional variation in returns. The cross-sectional \( R^2 \) rises to 57% in row (3), and it is approximately 70% in row (4) in panel A.

I next test the scaled conditional three-factor model considered in Equation (12) with

\[ f_{t+1} = (\text{MKT}_{t+1}, S_{\Delta \pi, t+1} S_{\Delta \text{UC}, t+1}) \] and \( z_t = \pi_t \). To obtain the risk loadings, I first estimate a single multivariate time-series regression for each portfolio \( i \) over the sample period:

\[ R_{i,t+1} = \alpha_i + \beta_{\pi} \lambda_{\Delta \pi} S_{\Delta \pi, t+1} + \beta_{\text{MKT}} \lambda_{\text{MKT}} (\text{MKT}_{t+1} \times z_t) + \beta_{\text{UC}} \lambda_{\text{UC}} (S_{\Delta \text{UC}, t+1} \times z_t) + \epsilon_{i,t+1}. \]

I then estimate the cross-sectional specification in Equation (12) by regressing for each month the portfolio returns on the estimated risk loadings. Row (5) of Table 5 presents the results of the cross-sectional estimation.

First, scaling helps improve the overall fit of the model. The cross-sectional \( R^2 \) statistic in row (5) of Table 5 is higher than that reported in row (3) for the unscaled version of the model: the addition of the scaling variable increases the \( R^2 \) to 79%. Second, the estimated coefficients on the uncertainty factor loadings \( \lambda_{\Delta \text{UCz}} \) and \( \lambda_{\Delta \text{UC}} \), scaled and unscaled, are both strongly significant. Furthermore, in contrast to the static CAPM, the coefficient \( \lambda_{\text{MKTz}} \) on the scaled market factor is now also statistically significant, albeit still negative. The coefficient estimates on the first-state probability loadings \( \lambda_{\Delta \pi z} \) and \( \lambda_{\Delta \pi} \) in row (5) seem to suggest that the first-state probability \( \pi \) risk factor does not help the pricing model; however, when the time-varying component of the intercept \( \beta_{zi} \) is eliminated from the specification in row (6), the coefficient \( \lambda_{\Delta \pi z} \) also becomes statistically significant. Dropping the scaling variable in row (6) does not appear to affect the magnitude and the significance of the other estimated coefficients, although it slightly reduces the fit of the model,
Good Times or Bad Times? Investors’ Uncertainty and Stock Returns

as indicated by the $R^2$ statistic. These estimates support the scaled conditional three-factor model considered in Equation (12).

I next examine whether the uncertainty risk and first-state probability factors remain important after the Fama-French factors are controlled for. This is important, since one of the major criticisms of the Fama-French factors has been their “empirical” nature, which leaves them open to various economic interpretations, and hence, to a great deal of controversy. Therefore, I also estimate the scaled multifactor model with $f_{t+1} = (S_{\Delta \pi, t+1}, S_{\Delta UC, t+1}, \text{MKT}_{t+1}, \text{HML}_{t+1}, \text{SMB}_{t+1})$ and scaling variable $z_t = \pi_t$. The estimation results are reported in rows (7) and (8) of Table 5, panel A.

The addition of the size and book-to-market factors improves the overall fit of the regression to 89%, as indicated by the $R^2$ in rows (7) and (8) of Table 5. While the significance of the time-varying component of the uncertainty risk premium $\lambda_{\Delta ucz}$ drops considerably in the presence of SMB and HML, the estimated coefficient $\lambda_{\Delta UC}$ remains statistically significant but is lower in magnitude. Furthermore, the first-state probability $\pi$ does not seem to matter for the pricing of assets; the coefficients on both the scaled and unscaled first-state probability loadings $\beta_{\Delta \pi}$ and $\beta_{\Delta \pi z}$ are insignificant. It is plausible that the Fama-French factors HML and SMB proxy for news related to GDP future growth, as suggested by Vassalou (2003). Since the Fama-French factors are constructed from stock returns themselves, it is likely that HML and SMB capture the same information as the news related to a returns-based measure of $\pi$. Indeed, this would explain why the coefficients $\lambda_{\Delta ucz}$ and $\lambda_{\Delta \pi z}$ appear significant in panel B, where the estimation is carried out using output-based uncertainty and $\pi$ measures. I turn to this next.

For robustness, I also estimated all of the cross-sectional specifications considered above with the innovation factors $S_{\Delta \pi}$ and $S_{\Delta UC}$ obtained from the output-based $\pi$ and uncertainty measures. The cross-sectional estimation results are reported in rows (1) through (8) in Table 5, panel B. To facilitate comparison, the estimation results for the CAPM and the Fama-French three-factor models are presented again in rows (1) and (2). The results in panel B are, to a large degree, similar to those in panel A. For this reason, here I only highlight the differences between the two panels across the difference specifications in rows (3) through (8).

The first thing to note is the significant improvement in the overall fit of the models in panel B of Table 5. The cross-sectional $R^2$ ranges from a minimum of 78% in row (3), where $S_{\Delta \pi}$ and $S_{\Delta UC}$ are added to the static CAPM, to 93% in row (8), which considers the scaled multifactor specification with uncertainty risk and first-state probability factors together with the Fama-French three-factors. Second, the coefficients $\lambda_{\Delta UC}$ and $\lambda_{\Delta ucz}$ are both statistically significant across all specifications. The addition of the size and book-to-market factors lowers their magnitude and statistical significance only slightly in both the scaled and the unscaled models. Third, the time-varying component of the risk
premium associated with the first-state probability $\pi$ is statistically important across all of the scaled models in rows (5) through (8). Finally, it is interesting to note that the coefficient $\lambda_{MKT}$ on the scaled market factor is now significant in all the scaled models in panel B; in contrast, the coefficients on the size and book-to-market factors are not significant in any of the specifications.

There are several other noteworthy features of the results presented in Table 5. First, I check whether the estimated $\lambda$ coefficients indeed imply negative average risk prices for the uncertainty and $\pi$ risk factors in the conditional linear factor model in Equation (1). Assuming that the covariance matrix for the factors, the scaled factors, and the scaling variable is constant, the coefficient estimates in the unscaled model in row (3) in panels A and B imply positive average risk prices for $\lambda_{\Delta \pi}$ and $\lambda_{\Delta U}$\. Although the models explain a large part of the cross-sectional variation in portfolio returns, as indicated by the cross-sectional $R^2$ statistics in Table 5, positive average risk prices are not consistent with the implications of the model. I next check the average risk prices implied by the coefficients in the scaled version of the model in row (5). The average risk prices $\lambda_{\Delta \pi}$ and $\lambda_{\Delta U}$ will now be a weighted average of $\lambda_{\Delta \pi}$ and $\lambda_{\Delta U}$, and $\lambda_{\Delta U}$ and $\lambda_{\Delta U}$, respectively. The estimated values indeed yield a negative average risk price on the first-state probability beta in row (5), but they still imply a positive average risk price of investor uncertainty.

It is possible that these findings are driven by the correlations between the factors, the scaled factors, and the conditioning variable $z_t$. To check for this possibility, I estimated the model specifications in rows (3) and (5) in Table 5 using betas obtained from univariate regressions in which I regressed the monthly returns of each portfolio $i$ on the market return, on $S_{\pi, t+1}$, and on $S_{U, t+1}$ separately. While there is indeed greater evidence of negative average risk prices in the scaled models when betas are estimated in this way, the estimation results indicate that the findings in Table 5 cannot be entirely attributed to the correlation between the factors, the scaled factors, and the conditioning variable $z_t$.

Overall, Table 5 presents strong evidence that a conditional model with uncertainty risk and first-state probability factors explains a very substantial fraction of the variation in average returns across portfolios sorted by size, book-to-market, and past returns. The results suggest that the scaling variable $\pi_t$ is important in capturing the time variation in the estimated risk premium coefficients. A concern remains, however, that the signs of the average risk prices implied by the estimated coefficients in Table 5 are not entirely consistent

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29 I also estimated the cross-sectional regressions in Table 5 with risk loadings obtained from 60-month rolling-window multivariate time-series regressions. While the cross-sectional results are largely qualitatively similar and explain a comparable magnitude of the variation in cross-sectional returns (as in Table 5), the significance of the uncertainty premiums drops slightly in some of the specifications. Interestingly, uncertainty risk becomes more important in the presence of the Fama-French three-factors. Finally, it is important to note that the scaled cross-sectional regressions yield largely negative average risk prices for uncertainty, consistent with the predictions of the model.
6.3 Comparison with conditional (C)CAPM

Recently, there have been several other studies that use economically motivated or macroeconomic factors to explain the cross-section of average returns.\textsuperscript{30} The conditional versions of the CAPM and the consumption CAPM—jointly, the (C)CAPM—have been found to do much better than the unconditional versions. Several papers find the (C)CAPM to explain the average returns on the Fama-French portfolios when used with a conditioning variable, such as the consumption-to-wealth ratio, as in Lettau and Ludvigson (2001a). In this section, I compare the performance of the conditional model with uncertainty risk and $\pi$ factors to the conditional versions of the (C)CAPM.

Table 6 presents the cross-sectional empirical analysis of the conditional (C)CAPM. I follow Lettau and Ludvigson (2001a) and use the log consumption-to-wealth ratio, $cay_t$, as the scaling variable.\textsuperscript{31} Panel A presents the estimated $\lambda$ coefficients using $S_{\Delta \pi}$ and $S_{\Delta UC}$ factors obtained from the returns-based $\pi$ and uncertainty measures; in panel B, I report the analogous coefficients for the output-based measures.

The first row of Table 6, panel A, shows the results for the scaled conditional CAPM with one fundamental factor, $f_{t+1} = MKT_{t+1}$, and one scaling variable, $z_t = cay_t$. The $\lambda$ coefficients are obtained similarly from the two-pass cross-sectional methodology. I first estimated a single time-series multivariable regression for each portfolio $i$ over the sample period. I then estimated the second-pass cross-sectional specification by regressing for each month the portfolio returns on the estimated first-pass risk loadings. Neither of the estimated coefficients on $\beta_{mkti}$ and $\beta_{mktzi}$ in row (1) is statistically different from zero; however, the $R^2$ statistic for the conditional CAPM reported in the last column is higher than the one reported in Table 5 for the static CAPM, consistent with previous studies.

Next, I add to the conditional CAPM the returns-based first-state probability $\pi$ and uncertainty risk factors, $S_{\Delta \pi}$ and $S_{\Delta UC}$, and consider the following cross-sectional specification:

$$
E(R_{i,t+1}) = E(R_{0,t}) + \beta_{zi} \lambda_z + \beta_{MKTi} \lambda_{MKT} + \beta_{MKTzi} \lambda_{MKTz} + \beta_{\pi i} \lambda_{\Delta \pi} + \beta_{\pi zi} \lambda_{\Delta \pi z} + \beta_{UCi} \lambda_{UC} + \beta_{UCzi} \lambda_{UCz},
$$

\textbf{Equation 16}

\textsuperscript{30} See also Lustig and Van Nieuwerburgh (2005) and Santos and Veronesi (2006). Bansal, Dittmar, and Lundblad (2005) and Parker and Julliard (2005) use the long-run growth rate in nondurable consumption to explain average returns. Piazzesi, Schneider, and Tuzel (2006) augment the (C)CAPM with the growth rate in housing services; Yogo (2006) augments the (C)CAPM with the growth rate in durable consumption.

\textsuperscript{31} I thank Martin Lettau for providing the $cay_t$ series on his website. The reader is referred to Lettau and Ludvigson (2001b), where $cay_t$ is shown to be a strong predictor of excess stock returns on aggregate stock market indexes, for details on data construction and a description of the estimation procedure.
Table 6
Comparison with the conditional (C)CAPM

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Panel B: Using output-based probability and uncertainty measures

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This table presents $\lambda$ coefficients estimates from the cross-sectional Fama-MacBeth regressions of portfolio returns. The regressors are the slope coefficients estimated in a single time-series multivariate regression of portfolio returns on the relevant factors. The individual $\lambda_f$ estimates are the average cross-sectional regression estimates, expressed in percentages. $S_{\Delta\pi}$ is the innovation in investors’ beliefs about the state of the economy, $S_{\Delta\pi}$ is the log change in the probability of being in the low state $\pi_i$. $S_{\Delta\pi}$ is the innovation in uncertainty, measured by the unanticipated change in UC. MKT denotes the value-weighted return on the CRSP index, and $\Delta c$ is the log difference in consumption. The scaling variable is $cay_t$, the log consumption-to-wealth ratio, as defined by Lettau and Ludvigson (2005). The uncertainty and probability variables $S_{\Delta\pi}$ and $S_{\Delta\pi}$ in panel A are estimated using a Markov-switching regime model of the market return; those in panel B are estimated using a Markov-switching regime model of aggregate output. Two sets of $t$-statistics are reported below each coefficient estimate. The values in parentheses use the uncorrected Fama-MacBeth standard errors; $t$-values in brackets have been corrected for potential autocorrelation in measurement errors in betas. The $\bar{R}^2$ is the cross-sectional $R^2$, adjusted for degrees of freedom. The thirty portfolios used are ten momentum (M), ten size (S), and ten book-to-market (BM) portfolios. M1(loser), S1(smallest), and BM1(growth) correspond to the lowest deciles. The sample period for this regression is from January 1961 through December 2001.

in which the scaling variable is $z_t = cay_t$. The estimation results are reported in Table 6, row (2). First, the coefficient estimates $\lambda_{\Delta\pi}$ and $\lambda_{\Delta\pi}$ associated with uncertainty risk are both statistically different from zero. Furthermore, when log consumption-to-wealth is used as the scaling variable in row (2), the estimated $\lambda$ coefficients imply average risk prices for $\pi$ and uncertainty factors that are negative, which make the findings more consistent with the model. Finally, although the coefficient estimates on first-state probability risk loadings $\beta_{\pi_i}$ and $\beta_{\pi_2}$ are not statistically significant, the addition of $\pi$ and uncertainty risk factors to the multifactor model implied by the scaled conditional CAPM
improves its performance somewhat. The magnitude of the $\bar{R}^2$ is, however, low relative to the other model specifications considered in Table 5.

In panel B of Table 6, row (2) reports the corresponding $\lambda$ coefficients estimates obtained with the output-based $\pi$ and uncertainty measures. There are several noteworthy differences compared to those in panel A. First, in addition to $\lambda_{\Delta UC}$ and $\lambda_{\Delta ucz}$ coefficients associated with uncertainty risk, the coefficients on the first-state probability factor loadings $\beta_{\Delta \pi}$ and $\beta_{\Delta \pi z}$ now are also significant. Second, the magnitudes of the coefficients $\lambda_{\Delta UC}$ and $\lambda_{\Delta ucz}$ are similar to their estimated values reported in Table 5. Third, the $\lambda$ coefficients associated with the scaled and unscaled MKT factors are both statistically significant. Finally, the overall fit of model improves significantly: the scaled three-factor CAPM with $\pi$ and uncertainty risk factors explains more than 80% of the cross-sectional variation in average returns. On the other hand, using output-based measures of $\pi$ and UC yields positive average risk prices that are not consistent with the implications of the model.

I next turn to the (C)CAPM. For comparison, I first estimate the scaled multifactor consumption CAPM with $z_t = cay_t$ as the conditioning variable and report the $\lambda$ coefficients in row (3) in panels A and B of Table 6. Consistent with previous studies, the (C)CAPM performs better than the conditional CAPM, explaining close to 50% of the cross-sectional variation in average returns. I then test the explanatory power of the $\pi$ and uncertainty risk factors when they are added to the consumption CAPM. The scaled multifactor consumption CAPM with $\pi$ and uncertainty risk factors takes the following form:

$$E(R_{i,t+1}) = E(R_{0,t}) + \beta_{zi} \lambda_z + \beta_{zi} \lambda_{z} + \beta_{\Delta czi} \lambda_{\Delta c} + \beta_{\Delta czi} \lambda_{\Delta cz}$$

$$+ \beta_{\pi i} \lambda_{\Delta \pi} + \beta_{\pi zi} \lambda_{\Delta \pi z} + \beta_{uci} \lambda_{\Delta UC} + \beta_{uczi} \lambda_{\Delta ucz},$$

(17)

where $\Delta c$ denotes the log consumption growth, and the scaling variable is $z_t = cay_t$.

In row (4) of Table 6, panel A, the estimated $\lambda$ coefficients are reported. First, the $\lambda_{\Delta \pi}$ and $\lambda_{\Delta \pi z}$ associated with the first-state probability $\pi$ are statistically significant. The estimated coefficients $\lambda_{\Delta UC}$ and $\lambda_{\Delta ucz}$ drop slightly in magnitude compared to row (2) but also remain statistically significant. These estimates suggest that investors’ beliefs and uncertainty retain their explanatory power for cross-sectional returns, even after accounting for other macroeconomic variables, such as consumption. I next check whether the estimated coefficients are consistent with the implications of the model in the presence of consumption risk. The average risk price for the first-state probability from the associated conditional linear factor model will now be a weighted average of $\lambda_{\Delta \pi}$ and $\lambda_{\Delta \pi z}$, the latter multiplied by $z_t$. Given these estimates, and the assumption that the covariance matrix for the factors, the scaled factors, and the scaling variable is constant, the estimated coefficients $\lambda_{\Delta \pi}$ and $\lambda_{\Delta \pi z}$ imply a positive average risk price for the first-state probability $\pi$, contrary to the model. Under similar assumptions, however, the same calculation with
\( \lambda_{\Delta UC} \) and \( \lambda_{\Delta uz} \) yields a negative average risk price for the uncertainty risk, as predicted by the model, after accounting for consumption risk. Finally, the \( R^2 \) statistic is considerably higher: it increases to 75% in panel A.

The estimation results for the output-based measures of \( \pi \) and UC reported in row (4) of panel B are largely similar. The model has a greater overall fit, as indicated by a \( R^2 \) of over 90%, but the problem with this model is that the estimated coefficients suggest that there is a positive average risk price on the beta for both the first-state probability \( \pi \) and the uncertainty risk.

The findings in panels A and B of Table 6 suggest that investors’ uncertainty risk is independent of other risk factors associated with macroeconomic fundamentals, and taking this source of risk into account improves the ability of the conditional consumption CAPM to explain the cross-section of average returns by a great deal.

7. Conclusion

Economic agents rarely have the perfect knowledge that traditional asset pricing models ascribe to them. Instead, they learn little by little, incorporating new information into their existing beliefs with some variable weight through a Bayesian updating scheme. This paper has examined the role of investors’ conditional beliefs and their own uncertainty as state variables describing the time variation in the risk-return trade-offs investors face.

By the nature of Bayesian updating, beliefs are more sensitive to news in periods of high uncertainty. When beliefs are about future cash flow expectations, greater uncertainty implies greater price sensitivity to news, and hence greater asset price volatility. This induces risk-averse investors to hedge against changes in their own uncertainty and to require greater compensation for bearing more risk, as they anticipate greater asset price sensitivity to news in periods of high uncertainty. In the spirit of Merton’s intertemporal CAPM, I have examined whether the covariance between asset returns and innovations in investors’ conditional beliefs and uncertainty about the state of the economy gives rise to risk sources important for explaining cross-sectional returns.

One of the difficulties in this study has been the empirical measurement of investors’ beliefs and Bayesian uncertainty about the state of the economy. For this purpose, I constructed proxies for investors’ beliefs and uncertainty from time series of state probabilities estimated using a regime-switching model of the aggregate market return. The time series of the conditional state probability provides a convenient instrument that summarizes investors’ future expectations and their unobservable information set. I also considered, alternately, beliefs and uncertainty measures obtained similarly from a Markov model of the aggregate output. It is reassuring that the two sets of measures are positively correlated and yield largely similar empirical results, even though they may at times be capturing slightly different dimensions of uncertainty. While both measures of uncertainty increase around the business cycle turning points,
investors’ uncertainty about the economy shows substantial variation outside the business cycles as well, suggesting that fluctuations in investors’ uncertainty are not entirely explained by aggregate fluctuations in the fundamental risk in the economy.

This paper suggests that investors’ conditional beliefs and uncertainty about the state of the economy are fundamental determinants of the relationship between expected returns over time and their cross-sectional properties. First, I have presented evidence that investors’ conditional beliefs regarding being in bad times have a negative impact on asset prices. This result holds for both returns-based and output-based measures of conditional beliefs, and it is consistent with the idea that bad times are associated with higher risk premiums. Second, I have found that there is a negative and possibly nonlinear relationship between the level of uncertainty and risk premiums. The evidence indicates that uncertainty has a negative impact on asset prices, both at the aggregate and at the portfolio level. This is consistent with Veronesi’s (1999) prediction that agents demand higher returns when there is greater uncertainty. Moreover, I have documented that this relationship shows substantial cross-sectional variation across portfolios sorted by size, book-to-market, and past returns, especially conditional on the state of the economy. This finding highlights the time-varying nature of the relationship between the level of uncertainty and risk premiums.

Next, I explored whether changes in investors’ conditional beliefs and uncertainty about the state of the economy can help explain the cross-sectional properties of expected returns. Following previous work that explicitly models the parameters of the discount factor as dependent on current-period information, I scaled the fundamental factors by a proxy for investors’ current beliefs about the state of the economy. A scaled model that accounts for investors’ conditional beliefs and uncertainty risk factors is remarkably successful in explaining the cross-sectional variation across thirty portfolios sorted on size, book-to-market, and past returns. The addition of these two state variables to account for investors’ beliefs and uncertainty improves the performance of the CAPM and the Fama-French three-factor model in both unscaled and scaled versions. Furthermore, the uncertainty risk factor retains its incremental explanatory power when contrasted with other economically motivated conditional models, such as (C)CAPM, even with an alternative scaling variable, such as Lettau and Ludvigson’s consumption-to-wealth ratio, \( cay \).

The main contribution of this study to the literature is its identification and empirical refinement of the interaction between investors’ beliefs and uncertainty and economic risk premia. This is important for several reasons. First, the findings suggest that in a world in which investors do not have perfect knowledge, investors’ beliefs and the evolution of their uncertainty in response to news play a critical role in the determination of the risk premium in the economy. Second, the results in this paper respond to Fama’s challenge of providing a coherent story that relates the time variation in expected returns to cross-sectional returns and that, at the same time, is fundamentally related
to the aggregate economy. Finally, the cross-sectional results lend significant support to the recent rational theories that attempt to explain anomalies by risk premia that arise from rational uncertainty.32

References


32 See Brav and Heaton (2002).


