Online Appendix for "Are Strategic Customers Bad for a Supply Chain?"

Analysis of Equilibrium in Section 5.3

We derive sub-game perfect equilibrium in this section in three steps following backward induction: (1) the retailer's optimal pricing decisions in $t=2$; (2) the retailer's and customers' RE equilibrium decisions in $t=1$; and (3) the manufacturer's wholesale price decision. We analyze each step in the following.

1. Retailer's optimal price in $t=2$

In this stage-game we characterize the retailer's optimal choice of p_2 for any given θ and Q. For any given p_2, the retailer faces $k(\delta-p_2)^+$ and $(\theta-\frac{p_2}{\delta})^+$ units of demand from the new (if the product turns out popular) and remaining strategic customers, respectively. Let D_2^H and D_2^L denote the retailer's demand when the market turns out to be high and low, respectively. Then we have

$$D_2^H = \begin{cases} k(\delta-p_2)+\theta-\frac{p_2}{\delta} & \text{if } p_2 \le \delta\theta \\ k(\delta-p_2)^+ & \text{if } p_2 > \delta\theta \end{cases}, \tag{1}$$

$$D_2^L = (\theta-\frac{p_2}{\delta})^+. \tag{2}$$

Let $Q_R = Q-(1-\theta)$ denote the remaining inventory in $t=2$. Then the retailer's profit in $t=2$ is

$$\pi_R^{t_2}(\theta,Q_R) = p_2(h(D_2^H \wedge Q_R)+(1-h)(D_2^L \wedge Q_R)), \tag{3}$$

where $(x\wedge y)=min(x,y)$ and $(x\vee y)=max(x,y)$. Then the retailer's problem in $t=2$ is formulated as follows

$$\text{(Problem 1)} \ \max_{p_2} \pi_R^{t_2}(\theta,Q_R).$$

Problem 1 does not have a concave objective function because the slope of D_2^H is piecewise linear and hence $\pi_R^{t_2}$ is not concave in p_2. In light of this observation, we solve Problem 1 by solving the following two sub-problems, each restricting to $p_2\le\delta\theta$ and $p_2\ge\delta\theta$, respectively:

$$\text{(Sub-Problem 1.a)} \max_{p_2 \le \delta\theta} \pi_R^{t_2}(\theta, Q_R)$$

$$\text{(Sub-Problem 1.b)} \max_{p_2 \ge \delta\theta} \pi_R^{t_2}(\theta, Q_R)$$

Then the solution of Problem 1 is given by the sub-problem that yields the highest profit. In the following, Lemmas 1 and 2 characterize the solution to sub-problems 1.a and 1.b respectively, and then Lemma 3 provides the optimal solution to Problem 1 by comparing results of Lemmas 1 and 2.

LEMMA 1. *The optimal solution of sub-problem 1.a is given in Table 1.*

Table 1 Optimal Solution of Sub-Problem 1.a

Case	Conditions	p_2^*	$Q_R \lesseqqgtr D_2^H$	$Q_R \lesseqqgtr D_2^L$
(i)	$\theta \ge \frac{\Omega}{1+2\Delta}$, $Q_R \ge \eta_1$	$\frac{\delta(\Omega+\theta)}{2(1+\Omega)}$	$\ge$	$\ge$
(ii)	$\theta \le \frac{\Omega}{1+2\Delta}$, $Q_R \ge \eta_2$	$\delta\theta$	$\ge$	$\ge$
(iii)	$(\eta_2 \vee \eta_3) \le Q_R \le \eta_1$	$\frac{\delta(k\delta+\theta-Q_R)}{1+k\delta}$	$=$	$\ge$
(iv)	$\eta_5 \le Q_R \le (\eta_3 \wedge \eta_4)$	$\frac{\delta(h(Q_R-\theta)+\theta)}{2(1-h)}$	$\le$	$\ge$
(v)	$\eta_4 \le Q_R \le \eta_2$	$\delta\theta$	$\le$	$\ge$
(vi)	$Q_R \le \eta_5$	$\delta(\theta - Q_R)$	$\le$	$=$

$\Omega \doteq hk\delta$, $\eta_1 \doteq \frac{k\delta(\Omega+2-\theta-h(1-2\theta))+\theta}{2(1+\Omega)}$, $\eta_2 \doteq k\delta(1-\theta)$, $\eta_3 \doteq \frac{(1-h)(k\delta(2-\theta)+\theta)}{2-h(1-k\delta)}$, $\eta_4 \doteq \theta(\frac{1}{h}-1)$, $\eta_5 \doteq \theta(\frac{1-h}{2-h})$

Proof: First consider case (i) where $Q_R \ge D_2^H$ and $Q_R \ge D_2^L$. In that case the retailer's profit is $\pi_R = p_2(h(k(\delta - p_2) + \theta - \frac{p_2}{\delta}) + (1-h)(\theta - \frac{p_2}{\delta}))$, and we drop the superscript t_2 for the retailer's profit for brevity. It is straightforward to show that π_R is concave in p_2 and the first-order condition yields $p_2^* = \frac{\delta(\Omega+\theta)}{2(1+\Omega)}$. Given this p_2^*, $Q_R \ge D_2^H$ implies $Q_R \ge \eta_1$ and $p_2^* \le \delta\theta$ implies $\theta \ge \frac{\Omega}{1+2\Delta}$. In case (ii), we consider $\theta \le \frac{\Omega}{1+2\Delta}$. The retailer's profit is the same as case (i) but $p_2^* = \delta\theta$. In this case, $Q_R \ge D_2^H$ implies $Q_R \ge \eta_2$. When $Q_R \le \eta_1$, p_2^* is given by the solution to $Q_R = D_2^H$ and that leads to case (iii). In that case, $p_2^* \le \delta\theta$ implies $Q_R \ge \eta_2$. Moreover, we need to ensure that the retailer does not get better off by setting $p_2 < \frac{\delta(k\delta+\theta-Q_R)}{1+k\delta}$ so that $Q_R < D_2^H$. This condition requires that $\frac{\partial p_2(hQ_R+(1-h)(\theta-\frac{p_2}{\delta}))}{\partial p_2}\Big|_{p_2=\frac{\delta(k\delta+\theta-Q_R)}{1+k\delta}} \ge 0$, which implies $Q_R \ge \eta_3$. In contrast, when $Q_R < \eta_3$ the retailer sets $p_2 < \frac{\delta(k\delta+\theta-Q_R)}{1+k\delta}$ and earns the profit $\pi_R = p_2(hQ_R + (1-h)(\theta - \frac{p_2}{\delta}))$ which corresponds to case (iv). In that case, π_R is concave in p_2 so the first-order condition yields $p_2^* = \frac{\delta(h(Q_R-\theta)+\theta)}{2(1-h)}$. In this case, $p_2 \le \delta\theta$ implies $Q_R \le \eta_4$ and $Q_R \ge D_2^L$ implies $Q_R \ge \eta_5$. Moreover, $Q_R \le \eta_2$ in case (iii) and

$Q_R \geq \eta_4$ in case (iv) imply that $p_2^* = \delta\theta$ when $\eta_4 \leq Q_R \leq \eta_2$, and $\pi_R = p_2 h Q_R$ in that case. Finally, case (vi) implies that when $Q_R \leq \eta_5$, the condition $Q_R \geq D_2^L$ is violated, and hence p_2^* is given by the solution to $D_2^L = Q_R$ which yields case (vi) where $p_2^* = \delta(\theta - Q_R)$. □

LEMMA 2. *The optimal solution of sub-problem 1.b is given in Table 2.*

Table 2 Optimal Solution for Sub-Problem 1.b

Case	Conditions	p_2^*	$Q_R \lesseqqgtr D_2^H$	$Q_R \lesseqqgtr D_2^L$
(i)	$\theta \leq \frac{1}{2}$, $Q_R \geq \frac{k\delta}{2}$	$\frac{\delta}{2}$	$\geq$	$\geq$
(ii)	$\theta \geq \frac{1}{2}$, $Q_R \geq \eta_2$	$\delta\theta$	$\geq$	$\geq$
(iii)	$(\frac{k\delta}{2} \wedge \eta_2) \geq Q_R$	$\delta - \frac{Q_R}{k}$	$=$	$\geq$

Proof: First suppose $Q_R \geq k(\delta - p_2)$ so $\pi_R = p_2 h k(\delta - p_2)$. Since π_R is concave in p_2, the first-order condition yields $p_2^* = \frac{\delta}{2}$ as given in case (i). In this case, $Q_R \geq k(\delta - p_2)$ implies $Q_R \geq \frac{k\delta}{2}$ and $p_2^* \geq \delta\theta$ implies $\theta \leq \frac{1}{2}$. When $\theta \geq \frac{1}{2}$, $p_2^* \geq \delta\theta$ is violated in case (i) and hence $p_2^* = \delta\theta$, leading to case (ii). In that case, $Q_R \geq k(\delta - p_2)$ implies $Q_R \geq \eta_2$. Finally, combining cases (i) and (ii), we conclude that when $Q_R \leq (\frac{k\delta}{2} \wedge \eta_2)$, the condition $Q_R \geq k(\delta - p_2)$ is violated in both of those cases, and hence p_2^* is given by the solution to $Q_R = k(\delta - p_2)$ and $\pi_R = p_2 h Q_R$. □

Next we compare the retailer's profit between sub-problems 1.a and 1.b. The solution to Problem 1 will be given by the sub-problem that yields the highest profit. This comparison leads to Lemma 3:

LEMMA 3. *The solution to Problem 1 is given in Table 3.*

2. Retailer's and Customers' RE Equilibrium in $t = 1$

Next, we consider customers' choices and the retailer's problem in $t = 1$. Given the possibility of new customer arrivals in $t = 2$, the demand in $t = 2$ may be higher than the remaining inventory. In that case we assume that all of the customers have equal chance of obtaining a product in $t = 2$. Let α be the chance that a customer obtains a product if she waits for $t = 2$:

$$\alpha(h, k, \theta, \hat{Q}_R) = h\left(\frac{\hat{Q}_R}{D_2^H(p_2^*(\hat{Q}_R, \theta), \theta)} \wedge 1\right) + (1 - h)\left(\frac{\hat{Q}_R}{D_2^L(p_2^*(\hat{Q}_R, \theta), \theta)} \wedge 1\right) \tag{4}$$

where D_2^H and D_2^L, respectively, are given in equations (1) and (2), $\hat{Q}_R = [\hat{Q} - (1 - \theta)]^+$ and p_2^* is given in Lemma 3. Also, $(\frac{\hat{Q}_R}{D_2^H(p_2(\hat{Q}_R, \theta), \theta)} \wedge 1)$ and $(\frac{\hat{Q}_R}{D_2^L(p_2(\hat{Q}_R, \theta), \theta)} \wedge 1)$ are the rationing probabilities if the market turns out to be high and low, respectively. Therefore, the marginal customer θ satisfies

$$\theta - p_1 = \alpha(h, k, \theta, \hat{Q}_R)(\delta\theta - p_2^*(\hat{Q}_R, \theta)). \tag{5}$$

Table 3 Optimal Solution for Problem 1

Case	Conditions	p_2^*	$Q_R \lesseqqgtr D_2^H$	$Q_R \lesseqqgtr D_2^L$
(i)	$(t_1 \vee t_6) \le \theta \le t_2$	$\delta(1-Q)$	$\le$	$=$
(ii)	$t_1 \le \theta \le (t_6 \wedge t_3)$	$\delta - \frac{Q-(1-\theta)}{k}$	$=$	$\ge$
(iii)	$(t_2 \vee t_6) \le \theta \le t_4$	$\frac{\delta(h(Q-1)+\theta)}{2(1-h)}$	$\le$	$\ge$
(iv)	$(t_4 \vee t_6) \le \theta \le t_5$	$\frac{1-Q+k\delta}{1+k\delta}$	$=$	$\ge$
(v)	$t_3 \le \theta \le t_6$	$\frac{\delta}{2}$	$\ge$	$\ge$
(vi)	$(t_6 \vee t_5) \le \theta$	$\frac{\delta(\Omega+\theta)}{2(1+\Omega)}$	$\ge$	$\ge$

$\Omega \doteq hk\delta, t_1 \doteq 1-Q,\ t_2 \doteq (2-h)(1-Q),\ t_3 \doteq 1-Q+\frac{k\delta}{2}$

$t_4 \doteq \frac{2\delta k-2Q+2-h(\delta k+Q(\delta k-1)+1)}{\delta k+1}$

$t_5 \doteq \frac{(\delta k+1)(\Omega+2)-2Q(\Omega+1)}{\delta k+1}$

$t_7 \doteq \sqrt{\Omega(1+\Omega)}-\Omega,$

$t_8 \doteq \frac{(\delta k+1)^2(\Omega+4)+4Q^2(\Omega+1)-4Q(\delta k+1)(\Omega+2)}{4(\delta k+1)(\delta k-Q+1)}$

$t_9 \doteq \sqrt{(1-h)\Omega}+h(1-Q)$

$t_{10} \doteq \frac{h^2(2\delta k-4Q+4)-2\sqrt{\delta(h-1)^2hk\left(h\left(\delta k+Q^2-2Q+1\right)-(Q-1)(-\delta k+Q-1)\right)}+h(-3\delta k+Q(\delta k+4)-4)}{4h^2-4h-\delta k}$

$t_{11} \doteq \frac{h(1-Q+k\delta)-k(1-Q)\delta}{h},\ \bar{Q}_1 \doteq \frac{(\delta k+1)\left(2\Delta-\sqrt{(\delta hk+1)}+2\right)}{2\Delta+2},\ \bar{Q}_2 \doteq \frac{(\delta k+1)\left(\sqrt{\delta(-(h-1))hk}+2h-2\right)}{2(h-1)}$

$\bar{Q}_3 \doteq \frac{2\sqrt{\Omega(1-h)}+2h-\delta k-2}{2(h-1)},\ \bar{Q}_4 \doteq \frac{(h-1)(h+\delta k)}{h^2-h-\delta k}$

$$t_6 \doteq \begin{cases} t_7 & \text{if } Q \ge \bar{Q}_1 \\ t_8 & \text{if } \bar{Q}_2 \le Q \le \bar{Q}_1 \\ t_9 & \text{if } \bar{Q}_3 \le Q \le \bar{Q}_2 \\ t_{10} & \text{if } \bar{Q}_4 \le Q \le \bar{Q}_3 \\ t_{11} & \text{if } 1-h \le Q \le \bar{Q}_4 \end{cases}$$

The retailer earns the following profit

$$\pi_R = p_1(1-\theta) + \pi_R^{t_2}(\theta, Q_R) - wQ, \tag{6}$$

where $\pi_R^{t_2}$ is given by equation (3). Then we seek for RE equilibrium (Q^*, p_1^*) that satisfies the following conditions

$$(p_1^*, Q^*) = \arg\max_{p_1, Q} \pi_R \tag{7}$$

$$\hat{Q} = Q^* \tag{8}$$

In the following lemma, we identify all of the equilibrium candidates. We will next analyze when those candidates are indeed RE equilibrium.

Lemma 4.

There are only two candidates for the RE equilibrium:

(1) In $t=1$, for a given h, the retailer orders Q and sets p_1 given in Table 4, and in $t=2$ it sets $p_2 = \delta(1-Q)$ listed in Table 3 case (i). In this case, all of the inventory is always sold out in $t=2$ in both high and low market scenarios. Rationing occurs only in the high market scenario.

(2) In $t=1$, for a given h, the retailer orders Q and sets p_1 given in Table 5, and in $t=2$ it sets $p_2=\delta-\frac{Q-(1-\theta)}{k}$ listed in Table 3 case (ii). In this case, all of the inventory is sold out in $t=2$ in a high market. However, in $t=2$ no product is sold at all in a low market. Rationing does not happen in either high or low market scenarios.

Table 4 **Equilibrium Decisions when Inventory Is always Sold Out in $t=2$**

Case	Condition	Q	p_1
(i)	$h_1 \le h \le \bar{h}$	$\frac{\delta(\delta(k(\delta(2h-3)+3)+7)-7)+2w(3-\delta(\delta(h-1)k+k+3+\sqrt{\epsilon_1})}{2\delta(\delta(k(\delta(3h-4)+4)+4)-4)}$	$\frac{1}{2}(1-\delta((1+\epsilon_3)Q-1))$
(ii)	$\bar{h} \le h \le h_2$	$\frac{(\delta-1)(h-3)h+2\delta(h-1)k(\delta(h-2)(h+1)+h+2)-h\sqrt{\epsilon_2}}{2\delta h^3-4h^2\left(\delta+\delta^2k-1\right)+4(\delta-1)h(\delta k-1)+4(\delta-1)\delta k}$	$\frac{1}{2}(1-\delta((1+\epsilon_4)Q-1))$
(iii)	$h \le (h_1 \wedge h_3)$	$\frac{1-w}{2-\delta(1-h)}$	$\frac{1+w-(1-h)\delta}{2-(1-h)\delta}$
(iv)	$h \ge (h_2 \vee h_3)$	$1-h$	h

$h_1 \doteq \frac{(\delta-1)(2w-\delta)}{\delta(w-\delta)}$, $h_2 \doteq \frac{3\delta-2+\sqrt{4-8\delta+5\delta^2}}{2\delta}$, $h_3 \doteq \frac{\delta-1+\sqrt{1-(1-w)\delta}}{\delta}$, $\bar{h}$ is the relevant root of $2w\left(\delta^3(h-1)k^2+\delta^2k(h(3-2h)+k+3)-\delta(3(h+1)k+h)+h\right)-h\sqrt{\epsilon_5+\epsilon_6}-k\delta\sqrt{\epsilon_7+\epsilon_8-\epsilon_9}+\delta\epsilon_{10}=0$, where ϵ_1 to ϵ_{10} are defined in Table 6.

Table 5 **Equilibrium Decisions when Inventory Is Sold Out only in High Market in $t=2$**

Case	Condition	Q	p_1
(i)	$h \ge (h_4 \vee h_5)$	$\frac{\delta hk+h(1-w)+h-kw}{2h}$	$\frac{w+1}{2}$
(ii)	$h_6 \le h \le h_5$	$\frac{1-w}{2}$	$\frac{1+w}{2}$
(iii)	$h \le (h_4 \wedge h_7)$	$\frac{2\delta(h(1-k)+k)-h(1+\delta h)}{2((\delta-1)h+\delta k)}$	$\frac{\delta h^2+h(\delta^2k-1)+\delta k}{2((\delta-1)h+\delta k)}$
(iv)	$h_7 \le h \le h_6$	$1-h$	h

Proof: First we derive the equilibrium candidate that leads to decisions given in case (i) of Table 3. In this case, the retailer sells out all the remaining inventory in $t=2$ in both low and high market outcomes, and rationing only occurs if the market turns out to be high. Hence, the rationing risk is

$$\alpha=\frac{h(\hat{Q}+\theta-1)}{(\theta-p_2/\delta)+k(\delta-p_2)}\Big|_{p_2=\delta(1-\hat{Q})}+(1-h)=\frac{h(\hat{Q}+\theta-1)}{\hat{Q}(1+k\delta)+\theta-1}+(1-h). \tag{9}$$

The the marginal customer satisfies $\theta-p_1=\alpha(\delta\theta-\delta(1-\hat{Q}))$, leading to

$$\theta=\frac{p_1+\alpha\delta(\hat{Q}-1)}{1-\alpha\delta}. \tag{10}$$

In that case, the retailer's profit is $\pi_{t_1} = p_1(1-\theta) + \delta(1-Q)(Q-(1-\theta)) - wQ$, where θ is given in equation (10). Hence we can obtain an equilibrium candidate $(\tilde{Q}, \tilde{p_1})$ leading to decisions given in case (i) of Table 3 by solving the following problem

$$\begin{aligned}(\tilde{Q}, \tilde{p_1}) = \arg\max_{p_1, Q} \pi_{t_1}(\alpha, \theta, p_1, Q, \hat{Q})\\ s.t. \quad Q = \hat{Q},\ (t_1 \vee t_6) \leq \theta \leq t_2,\end{aligned}$$

Solving this problem leads to the decisions given in Table 4. We drop the accent $\sim$ on $\tilde{Q}$ and $\tilde{p_1}$ in Table 4 for brevity. Note that we cannot claim that $(\tilde{Q}, \tilde{p_1})$ is RE equilibrium yet because so far we only show that they are RE equilibrium only subject to $(t_1 \vee t_6) \leq \theta \leq t_2$. We need to ensure that the retailer does not deviate to any other quantity $(Q \neq \tilde{Q})$ that violates $(t_1 \vee t_6) \leq \theta \leq t_2$ before claiming that $(\tilde{Q}, \tilde{p_1})$ is RE equilibrium.

Now we consider case (ii) of Table 3. In this case, the retailer sells out all the remaining inventory in $t = 2$ to new customers in the high market and remaining strategic customers cannot purchase at all in $t = 2$. Therefore $\alpha = 0$ and $\theta = p_1$ in this case, and the retailer's profit is $\pi_{t_1} = p_1(1-\theta) + h(\delta - \frac{Q-(1-\theta)}{k})(Q-(1-\theta)) - wQ$. Hence we can obtain an equilibrium candidate $(\tilde{Q}, \tilde{p_1})$ leading to decisions given in case (ii) of Table 3 by solving the following problem

$$\begin{aligned}(\tilde{Q}, \tilde{p_1}) = \arg\max_{p_1, Q} \pi_{t_1}(\alpha, \theta, p_1, Q, \hat{Q})\\ s.t. \quad Q = \hat{Q},\ t_1 \leq \theta \leq (t_6 \wedge t_3),\end{aligned}$$

Solving this problem leads to the decisions given in Table 5. Likewise, we drop the accent $\sim$ in Table 5 for brevity. We apply the same analysis to derive equilibrium candidates for cases (iii) to (vi) of Table 3 but find that RE equilibrium does not exist for those cases and hence they do not lead to RE equilibrium. $\square$

Essentially, Lemma 4 states that Tables 4 and 5 present all possible RE equilibrium candidates for the retailer's order quantity. The decisions in Tables 4 and 5 would result in a RE equilibrium if the consistency condition $\hat{Q} = Q^*$ holds (so far the consistency condition holds only under certain ranges of Q as shown in proof of Lemma 4). So next we need to check whether the retailer benefits from deviating to order quantities that are different from those in Tables 4 and 5; the consistency condition holds if the retailer never finds such deviation attractive. Unfortunately, this analysis cannot be done analytically because the threshold $\bar{h}$ in Table 4 does not have a closed form expression, and hence the exact conditions that would ensure the retailer does not deviate cannot be characterized analytically. In particular, as given in Table 4, $\bar{h}$ is the relevant root to the following equation:

$$2w\left(\delta^3(h-1)k^2 + \delta^2 k(h(3-2h)+k+3) - \delta(3(h+1)k+h) + h\right) - h\sqrt{\epsilon_5+\epsilon_6} - k\delta\sqrt{\epsilon_7+\epsilon_8-\epsilon_9} + \delta\epsilon_{10} = 0$$

where ϵ_5 to ϵ_{10} are defined in Table 6. In light of this observation, we need to resort to numerical studies for the remaining part of the analysis.

Specifically, for any given w, h, k and δ, we first calculate Q and p_1 from Tables 4 and 5. Then for each (Q, p_1) pair we numerically check if the retailer would benefit from deviating to any other quantity. We conclude that (Q, p_1) is a RE equilibrium if no attractive deviation exists. In the case that multiple RE equilibria exist, as in the base model, we refer to the one with the largest (smallest) order quantity as the optimistic (pessimistic) case.

3. Manufacturer's Optimal Price

Finally, we move to the first stage-game where the manufacturer chooses w. There is only one decision variable in this stage game so we perform a simple grid search to obtain the optimal wholesale price w^*. Specifically, we consider $w \in [0, 0.001, 0.002, ..., 1]$ and calculate the manufacturer's profit for each w. Then w^* is given by w that yields the highest profit.

Table 6 Parameter Values in Table 4

Parameter	Value
ϵ_1	$4(w-\delta)\delta(1+2k\delta)\left(\delta+(1-h)k(3-\delta)\delta^2+w((1-2(1-h)k)\delta-2)\right)+\left(\delta-2w(1+k\delta(1+(1-h)\delta))+\delta^2(1+k(3+\delta-2h\delta))\right)^2$
ϵ_2	$4(1-h)\delta(2k\delta-1)(1-(3-h)h+(2k+(2-h)h(1-2k))\delta)+(1-3h+(1+h+2k-2hk)\delta+2(1-h)^2k\delta^2)$
ϵ_3	$\frac{\delta(\delta^2(2h-1)k-\delta(3k+1)-1)+w(2\delta k(\delta-\delta h+1)+2)+\sqrt{\epsilon_1}}{2\delta(2\delta k+1)(w-\delta)}$
ϵ_4	$\frac{1-3h+\delta+\delta h+2\delta k-2\delta hk+2\delta^2k-4\delta^2hk+2\delta^2h^2k-\sqrt{\epsilon_2}}{2\delta(h-1)(2\delta k+1)}$
ϵ_5	$(\delta+\delta^2(k(\delta-2\delta h+3)+1)+2w(\delta k(\delta(h-1)-1)-1))^2$
ϵ_6	$4\delta(2\delta k+1)(w-\delta)\left(\delta+(\delta-3)\delta^2(h-1)k+w(\delta(2(h-1)k+1)-2)\right)$
ϵ_7	$\delta^2\left(1+(-2+6k)\delta+\left(1-12k+8hk+9k^2\right)\delta^2+2k(3-4h-9k+6hk)\delta^3+(3-2h)^2k^2\delta^4\right)$
ϵ_8	$4w^2\left(1+2(-1+k)\delta+\left(1-4k+k^2\right)\delta^2+2k(1+(-1+h)k)\delta^3+(-1+h)^2k^2\delta^4\right)$
ϵ_9	$4w\delta\left(1+(-2+4k)\delta+\left(1+2(-4+h)k+3k^2\right)\delta^2+k(4-6k+h(-2+5k))\delta^3+\left(3-5h+2h^2\right)k^2\delta^4\right)$
ϵ_{10}	$\delta^3(6(2-h)h-5)k^2+\delta^2k(h(4h-8k-13)+5k+1)+\delta(13hk+h-k)-h$