Online Supplement to The Value of Commitments When Selling to Strategic Consumers: A Supply Chain Perspective

In this online supplement, we provide explicit expressions of period 2 equilibrium values in Appendix B. Our proofs of main model including coordination results are provided in Appendix C. These proofs are presented in the order their corresponding results appear in the paper. We present the myopic consumers benchmark in Appendix D. We discuss various extensions in Appendices E-J. Specifically, we consider an alternative quantity commitment model in Appendix E, an alternative utility discounting assumption in Appendix G, no-inventory benchmark in Appendix H, heterogeneity in Customer Patience Level in Appendix I and Simultaneous Price and Quantity commitments in Appendix J. Proofs of these extensions are in Appendix K. Finally, we illustrate how our equilibrium results would change when we allow $\delta > 4/5$ in Appendix L.

Appendix B: Explicit Statement of Period 2 Equilibrium Outcomes

In the main body of the paper, equilibrium 2 values are stated in terms of period 1 quantities. This is because those expressions are obtained by backward induction. By plugging in equilibrium values of period 1 variables into those expressions, we can derive explicit expressions for period 2 equilibrium outcomes. These are stated in the following table.

Table 2 Summary of the period 2 equilibrium values

	Region	$\bar{\theta}$	c_2	p_2	Q_2
NC	$\delta \leq \frac{16}{21}$	$\frac{3}{4}$	$\frac{3\delta}{8}$	$\frac{9\delta}{16}$	$\frac{3}{16}$
	$\frac{16}{21} < \delta$	$\frac{2(4-3\delta)}{16-15\delta}$	$\frac{\delta(4-3\delta)}{16-15\delta}$	$\frac{3\delta(4-3\delta)}{2(16-15\delta)}$	$\frac{4-3\delta}{2(16-15\delta)}$
RQC	$\delta \leq 0.249$	$\frac{3}{4}$			0
	$\delta > 0.249$	$\frac{8}{5\sqrt{16-5\delta}} + \frac{2}{5}$	$\frac{\delta(4(\sqrt{16-5\delta}+4)-5\delta)}{5(16-5\delta)}$	$\frac{3\delta(4(\sqrt{16-5\delta}+4)-5\delta)}{10(16-5\delta)}$	$\frac{2}{5\sqrt{16-5\delta}} + \frac{1}{10}$
RPC	$\delta \leq \frac{4}{5}$	$\frac{3}{4}$		$\frac{3\delta}{4}$	0
SQC	$\delta < \frac{1}{2}$	$1 - \frac{1}{2(2-\delta)}$			0
	$1/2 \leq \delta < \frac{199-3\sqrt{137}}{218}$	$\frac{5-4\delta}{7-5\delta}$			0
	$\frac{199-3\sqrt{137}}{218} \leq \delta < 0.754$	$\frac{3}{4}$	$\frac{3\delta}{8}$	$\frac{9\delta}{16}$	$\frac{3}{16}$
	$0.754 \leq \delta$	$\frac{4}{3}(1-K_2)$	$\frac{2}{3}\delta(1-K_2)$	$\delta(1-K_2)$	$\frac{1-K_2}{3}$
SPC	$\delta \leq 1 - \frac{1}{\sqrt{17}}$	$\frac{2(6-5\delta)}{16-13\delta}$	$\frac{2\delta(5-4\delta)}{16-13\delta}$	$\frac{\delta(11-9\delta)}{16-13\delta}$	$\frac{1-\delta}{16-13\delta}$
	$\delta > 1 - \frac{1}{\sqrt{17}}$	$\frac{2-\delta}{4-3\delta}$	$\frac{\delta(6-5\delta)}{2(4-3\delta)}$	$\frac{\delta(10-7\delta)}{4(4-3\delta)}$	$\frac{2-3\delta}{4(4-3\delta)}$

Appendix C: Proofs

We present the proofs of main model results including coordination results in the order they appear in the paper.

Lemma 1 Since $c_2 > 0$, the constraint in (9) binds. So we can write p_2 in terms of Q_2. Plugging this expression into the optimization problem in (2) results in a concave function of Q_2. First-order condition (FOC) yields optimal (p_2, Q_2) in part (i). Plugging them into (3) yields a concave function of c_2 and its solution leads to part (ii).

Proposition 1 The retailer's total profit in equation (12) is jointly concave in $(Q_1, \bar{\theta})$ since the eigenvalues of the hessian are non-positive. Hence, KKT conditions and equation (14) fully characterize the solution, which yield the equilibrium regions and outcomes.

Proposition 2 To find the supplier's best response to the retailer's optimal strategies we need to analyze all three segments in Proposition 1. Π_S is continuous in c_1 and concave in c_1 within each segment. The sign of $\frac{d\Pi_S}{dc_1}$ in these segmentd show that the solution can be only in segment (ii). Furthermore, it is at the boundary when $\delta > \frac{16}{21}$ and in the interior otherwise.

Lemma 2 Centralized system's profit in periods 1 and 2 are concave in p_1 and p_2 respectively. The FOCs give the optimal prices.

Lemma 3 We prove the result for quantity commitment, proof of price commitment is similar. When the seller's quantity commitment binds $p_2^* = \delta(1-Q_{max})$ following (9). Furthermore, Q_1 and Q_2 can be stated in terms of Q_{max} and $\bar{\theta}$ following (8) and $Q_1+Q_2=Q_{max}$. Likewise, p_1 can be stated in terms of $\bar{\theta}$ similar to (11). Eventually, Π_C^{QC} becomes jointly concave in $(Q_{max}, \bar{\theta})$. The FOC gives the solution.

Theorem 1 $\Pi_C^{NC} = \frac{(2-\delta)^2}{4(4-3\delta)}$ and $\Pi_C^{QC} = \Pi_C^{PC} = \frac{1}{4}$. Thus, the value of commitment is $\frac{\delta(1-\delta)}{4(4-3\delta)} > 0$.

Lemma 4 The proof follows from the discussion following Lemma 4 in the text.

Lemma 5 Similar to the proof of Lemma 1 p_2 can be expressed in terms of Q_2 in $\Pi_{R,2}$. A binding commitment should satisfy the conditions in (17). c_2^* is the maximum c_2 that satisfy $\frac{dPi_{R,2}}{dQ_2}\Big|_{Q_2=Q_{max}-Q_1} = \delta(2(1-Q_{max})-\bar{\theta}))-c_2 \geq 0$. Similarly, the last condition in the lemma follows from the second condition in (17). Furthermore, Q_2^* and p_2^* directly follow from the definition of a binding quantity commitment.

Proposition 3 The retailer's objective function in (19) is a quadratic concave function of Q_{max}. So Q_{max}^* must be at one of two end points of the feasible region: Case.A $Q_{max} = Q_1$ and Case.B $Q_{max} = \frac{Q_1+1}{2} - \frac{\bar{\theta}}{4}$. The global optimal solution can be found by comparing the solutions to these two cases of the problem. In Case A, $\Pi_{R,A}$ is jointly concave in $(Q_1, \bar{\theta})$ since the eigenvalues of the hessian are nonpositive. The KKT conditions lead to $Q_1^* = \frac{1-c_1}{2}$,$\bar{\theta}^* = \frac{1+c_1}{2}$ for all c_1 and δ. In case B, $\Pi_{R,B}$ is also jointly concave in $(Q_1, \bar{\theta})$. The KKT conditions yield three cases, B1: $Q_1^* = \frac{1}{60}\left(36 - \frac{40c_1}{\delta} - \frac{6+5c_1}{6-5\delta}\right)$, $\bar{\theta}^* = \frac{c_1-4\delta+6}{2(6-5\delta)}$ for $c_1 < \frac{2\delta}{16-15\delta}$; B2: $Q_1^* = \frac{8(1-c_1)-3\delta}{16-5\delta}$, $\bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta}$ for $\frac{2\delta}{16-15\delta} \leq c_1 < \frac{8-3\delta}{8}$; and B3: $Q_1^* = 0$,$\bar{\theta}^* = 1$ for $c_1 \geq \frac{8-3\delta}{8}$.

We show that $\Pi_{R,A}^* - \Pi_{R,B1}^* > 0$ because the difference is concave in c_1 and is positive at the end points of B1. Likewise, $\Pi_{R,A}^* - \Pi_{R,B3}^* < 0$ because the difference is convex in c_1 and is negative at the end points of B3. Finally, $\Pi_{R,A}^* - \Pi_{R,B2}^* > 0$ within B2 if and only if $c_1 < \frac{\sqrt{16-5\delta}-1}{5}$. Here, we exploit the fact that this difference is quadratic and concave in c_1. Therefore, the global optimal follows from case A when $c_1 < \frac{\sqrt{16-5\delta}-1}{5}$; case B2 when $\frac{\sqrt{16-5\delta}-1}{5} \leq c_1 < \frac{8-3\delta}{8}$; and case B3 otherwise.

Proposition 4 To find the best c_1 of supplier to we have to consider each case of Proposition 3 that accounts for different ranges of c_1. It is straightforward to show that case (iii) is dominated by case (ii) as Π_S is constant in c_1 in case (iii) and is continuous at the boundary. Therefore we need to compare the optimal solutions in cases (i) and (ii) to find the overall optimal solution. Π_S is concave in c_1 within each case. In case (i), $c_1^* = \frac{1}{2}$ when $\delta < \frac{3}{4}$, otherwise c_1^* is at the boundary. In case (ii), $c_1^* = \frac{\sqrt{16-5\delta}-1}{5}$. The difference between Π_S of these two cases at their c_1^* is decreasing in δ. The difference is zero at $\bar{\delta}_1$ which solves $\delta(693 - 4(5\delta + 22\sqrt{16-5\delta})) = 16(113 - 28\sqrt{16-5\delta})$. Thus, when $\delta \leq \bar{\delta}_1$ global optimal follows from case (i) and for $\delta > \bar{\delta}_1$ global optimal follows from case (ii).

Theorem 2 Following Propositions 2 and 4, we have two cutoff points for δ hence, there are three cases to consider. Below we present the proof for each case separately. *(i)* When $\delta \leq \bar{\delta}_1$, $\Pi_S^{NC} - \Pi_S^{RQC} = 3\delta/64$ and $\Pi_R^{NC} - \Pi_R^{RQC} = 3\delta/256$. *(ii)* When $\bar{\delta}_1 < \delta \leq \frac{16}{21}$, we show that the denominators of $\Pi_S^{NC} - \Pi_S^{RQC}$ and $\Pi_R^{NC} - \Pi_R^{RQC}$ are positive, furthermore their numerators are concave in δ and they take positive values at the boundary points. *(iii)* When $\frac{16}{21} < \delta \leq \frac{4}{5}$, the denominator of $\Pi_S^{NC} - \Pi_S^{RQC}$ is positive and its numerator is concave in δ and takes positive values at the boundary points. In this case, $\Pi_R^{NC} - \Pi_R^{RQC}$ is decreasing in δ and it is equal to zero at $\bar{\delta}_2$ which is given by solution of $5\delta(824 + 135(2-3\delta)\delta) = 6\left(576 - (16-15\delta)^2\sqrt{16-5\delta}\right)$. Furthermore, the difference of supply chain profits is concave in δ in this region, and it is positive at the boundary points.

Lemma 6 Supplier sets $c_2^* = p_2$ as explained following the Lemma. We can express p_2 in terms of Q_2 as in the proof of Lemma 1. However, in this case (2) decreases in Q_2. Hence, $Q_2^* = 0$.

Proposition 5 Following Lemma 6 and the fact that $c_1 > 0$, the constraint in (9) binds. Thus, $Q_1^* = 1 - \frac{p_2}{\delta}$. Likewise, (8) leads to $\delta p_1 \geq p_2$. When Q_1^* and Q_2^* are plugged into retailer' problem in (5), it becomes concave in p_2, furthermore, it is increasing at its upperbound, hence $p_2^* = \delta p_1$. Consequently, (5) becomes concave in p_1 and the FOC leads to p_1^*. The supplier problem is concave in c_1 and the FOC gives c_1^*.

Theorem 3 Following Propositions 2 and 5, we need to consider two δ intervals. *(i)* When $\delta \leq \frac{16}{21}$, $\Pi_S^{NC} - \Pi_S^{RPC} = 3\delta/64$ and $\Pi_R^{NC} - \Pi_R^{RPC} = 3\delta/16$, hence the result follows. *(ii)* When $\delta > \frac{16}{21}$, it is straightforward to show that the denominators of $\Pi_S^{NC} - \Pi_S^{RPC}$ and $\Pi_R^{NC} - \Pi_R^{RPC}$ are positive and their numerators are concave in δ and positive at the boundary points, hence the result follows.

Theorem 4 The result for supplier profit follows from the fact that supplier can always match the no-commitment scenario profit by committing to an arbitrary large non-binding quantity. We will prove the results for retailer and supply chain profits. Following Propositions 2 and 10, there are five δ regions. *(i)*

When $\delta < 1/2$, the result follows from simple algebra and the fact that $0 \le \delta < 4/5$. *(ii)* When $1/2 \le \delta < \frac{199-3\sqrt{137}}{218}$, the denominator of $\Pi_R^{SQC} - \Pi_R^{NC}$ is positive and its numerator is increasing in δ, furthermore $\bar{\delta}_3$ sets the numerator equal to zero and it is given by the solution of $\delta(6-5\delta(3-\delta))-14=0$. Similarly, the denominator of the supply chain profit difference, $(\Pi_S^{SQC}+\Pi_R^{SQC})-(\Pi_S^{NC}+\Pi_R^{NC})$ is also positive. In this case, the numerator is convex in δ. Furthermore, its first derivative is negative and it is positive at the upper-bound of the δ interval, hence the result follows. *(iii)* When $\frac{199-3\sqrt{137}}{218} \le \delta < \bar{\delta}_7$, $\Pi_R^{SQC} = \Pi_R^{NC}$, hence the results immediately follow. *(iv)* Consider $\bar{\delta}_7 \le \delta < \frac{16}{21}$. Let $\Delta_1\Pi_R(Q_{max}, c_1) = \Pi_R^{SQC}(Q_{max}, c_1) - \Pi_R^{NC}(Q_{max}, c_1)$. Following Proposition 9(iii).b.2, $K_c < c_1^* \le \frac{3\delta(4-3\delta)}{2(16-15\delta)}$ in this case. $\Delta\Pi_R(K_2, c_1)$ in convex and quadratic in c_1. Furthermore, the smaller c_1 root of $\Delta\Pi_R = 0$ is greater than $\frac{3\delta(4-3\delta)}{2(16-15\delta)}$, thus $\Delta\Pi_R(Q_{max}, c_1^*) > 0$. Because $\Delta\Pi_S > 0$, the supply chain result also follows. *(v)* The proof for $\delta \ge \frac{16}{21}$ is similar to the previous case.

Lemma 7 This proof is similar to the proof of Lemma 1, therefore it is omitted.

Proposition 6 We need to consider two regions for Π_R given in (25) depending on whether $Q_2^* > 0$ or $Q_2^* = 0$. Note that Q_2^* is a function of Q_1 and $\bar{\theta}$ as shown in Lemma 7. In both of these regions Π_R is jointly concave in $(\bar{\theta}, Q_1)$ and Π_R is quasiconcave across these two regions. Hence, KKT conditions are sufficient to characterize the optimal solution which yields the results of this Proposition.

Proposition 7 We analyze the supplier's profits in each case of Proposition 6 and find that the global optimal solution follows from case (ii). Specifically, in cases (i) and (iii), Π_S is monotone in c_2 and their optimal solutions appear at their boundary with case (ii). In case (iv), Π_S is monotone increasing in c_1 and its optimal solution is at the boundary with case (iii). Furthermore, Π_S is continuous at these boundary points. In case (ii), Π_S is jointly concave in (c_1, c_2). Thus, KKT conditions characterize the global optimal solution, which leads to the result of this Proposition.

Theorem 5 Following Propositions 2 and 7, there are three δ regions to consider. *(i):* When $\delta \le 1 - \frac{1}{\sqrt{17}}$, $\Pi_S^{NC} - \Pi_S^{SPC} = \frac{\delta(8-7\delta)}{64(16-13\delta)} > 0$. Furthermore, the denominator of $\Pi_R^{NC} - \Pi_R^{SPC}$ is positive and its numerator is decreasing in δ and positive at δ upper-bound of this region. *(ii):* When $1 - \frac{1}{\sqrt{17}} < \delta \le \frac{16}{21}$, the denominator of $\Pi_S^{NC} - \Pi_S^{SPC}$ is positive and its numerator is decreasing in δ and is positive at δ upper-bound of this region. Similarly, the denominator of $\Pi_R^{NC} - \Pi_R^{SPC}$ is positive and its numerator is concave and positive at the two boundary points. *(iii):* When $\delta > \frac{16}{21}$, denominators of both $\Pi_S^{NC} - \Pi_S^{SPC}$ and $\Pi_R^{NC} - \Pi_R^{SPC}$ are positive and their numerators decrease in δ and positive at $\delta = 4/5$ (upper-bound). The result for the supply chain follows because we show both the retailer and the supplier get worse off.

Proposition 8 Let $\mathbb{1}_{\mathbb{X}}$ be an indicator function which takes the value one if the variable X is strictly positive; otherwise it takes the value zero. The SPNE definition in equations (2) - (9) is modified by replacing (2),(3),(5) and (7) with the following equations:

$$\begin{aligned}
(p_2^*, Q_2^*) &= \arg\max_{(p_2,Q_2)\in S_2} [p_2(\bar{\theta} - p_2/\delta) - c_2 Q_2 - F_2 \mathbb{1}_{\mathbb{Q}_2}],\\
(c_2^*, F_2^*) &= \arg\max_{c_2,F_2} c_2 Q_2^* + F_2 \mathbb{1}_{\mathbb{Q}_2^*},\\
(p_1^*, Q_1^*) &= \arg\max_{(p_1,Q_1)\in S_1} [p_1(1-\bar{\theta}) - c_1 Q_1 - F_1 \mathbb{1}_{\mathbb{Q}_1} + p_2^*(\bar{\theta}^* - p_2^*/\delta) - c_2^* Q_2^* - F_2^* \mathbb{1}_{\mathbb{Q}_2^*}],\\
(c_1^*, F_1^*) &= \arg\max_{c_1,F_1} [c_1 Q_1^* + F_1 \mathbb{1}_{\mathbb{Q}_1^*} + c_2^* Q_2^* + F_2^* \mathbb{1}_{\mathbb{Q}_2^*}].
\end{aligned}$$

We now follow backward induction. The retailer's actions in period 2 are the same as in Lemma 1 because the fixed payment F_2 does not affect them. The supplier sets the maximum F_2 that leaves the retailer indifferent between paying F_2 and ordering $Q_2 > 0$ or selling just its left-over inventory from Q_1 (if available) that is given by $Q_1 - (1-\bar{\theta})$. Therefore, the retailer sets $F_2 = \frac{\delta[1-Q_1-(Q_1-(1-\bar{\theta}))]^2}{4}$ and $c_2 = 0$.

In period 1, retailer's objective function is jointly concave in $\bar{\theta}$ and Q_1 and the KKT conditions yield the following solutions.

(i) When $c_1 < \frac{\delta(2-\delta)}{4(1-\delta)}$, the retailer sells the product in both periods and carries inventory between periods, where $Q_1^* = \frac{1}{2} - \frac{\delta(1-\delta)-c_1(2-\delta)}{\delta(4-3\delta)}$, $p_1^* = \frac{(2-\delta)(2+c_1-\delta)}{2(4-3\delta)}$ and $\bar{\theta}^* = \frac{1}{2} + \frac{2c_1+\delta}{8-6\delta}$.

(ii) When $\frac{2-\delta}{2} > c_1 \ge \frac{\delta(2-\delta)}{4(1-\delta)}$, the retailer sells the product in both periods but it does not carry inventory between periods, where $Q_1^* = 1/2 - \frac{c1}{2-\delta}$, $p^*1 = \frac{1}{4}(2+2c_1-\delta)$ and $\bar{\theta}^* = 1/2 + \frac{c1}{2-\delta}$.

(iii) When $\frac{2-\delta}{2} \le c_1$, the retailer sells the product only in period 2, and it does not carry inventory between periods, where $Q_1^* = 0$ and $\bar{\theta}^* = 1$.

Supplier's profit decreases in c_1 in all regions, therefore $c_1^* = 0$ and the solution is in region (i). The supplier sets $F_1 = \frac{(2-\delta)^2}{4(4-3\delta)}$ to extract the retailer's entire profit.

Theorem 6 Below we present the proof for each case.

When retailer makes a quantity commitment, Lemmas 4 and 5 continue to hold because the fixed payment does not change the second period dynamics. The supplier extracts all of retailer's period 2 profit by setting $F_2^* = \delta\left(Q_{max}^2 + \bar{\theta} + Q_1(2(1-Q_{max}) - \bar{\theta}) - 1\right)$. Therefore, the retailer restricts all sales to period 1 by setting $Q_1^* = \frac{1-c_1}{2}$, $Q_{max}^* = \frac{1-c_1}{2}$, $p_1^* = \frac{1+c_1}{2}$, $\bar{\theta}^* = \frac{1+c_1}{2}$. Consequently, the supplier sets $c_1^* = 0$, $F_1^* = \frac{1}{4}$ and the result in part (i) follows.

When retailer makes a price commitment, similar to Section 6.2 it restricts sales to only first period. In this case, the supplier sets $c_1^* = 0$, $F_1^* = \frac{1}{4}$ and the result in part (ii) follows.

When supplier makes a quantity commitment, it attains the profit of the centralized system by setting $c_1^* = 0$, $F_1^* = \frac{1}{4}$ and $Q_{max}^* = \frac{1}{2}$. The retailer buys all of the committed quantity and sells only in period 1, which yields the result in part (iii).

When supplier makes price commitments, similar to its quantity commitment it sets $c_1^* = 0$, $F_1^* = \frac{1}{4}$ and retailer sells only in period 1, which gives the result in part (iv).

Theorems 7-10 The proofs for these theorems are provided in Appendix K of online supplement along with proofs of other extension results.

Lemma 8 We first show that optimal Q_{max} is never less than 1/4. When $Q_{max} < \frac{1}{4}$, the commitment always binds strictly because (27) always holds. The retailer's profit is given by (19) as a function on Q_1 and $\bar{\theta}$. Observe that (19) becomes with respect to Q_1, so Q_1^* must be at one of the two boundary points: $1-\bar{\theta}$ or Q_{max}, which are given by (22) and (21). We find the solution for each case and compare them to find the global solution. The following lists these solutions.

(i) When $1-(2-\delta)Q_{max} < c_1 < 1-3\delta Q_{max}$, the retailer's optimal solution is same as in Proposition 9.i.

(ii) When $c_1 \le \min(1-(2-\delta)Q_{max}, 1-3\delta Q_{max})$, the retailer's optimal solution is same as in Proposition 9. ii.

(iii) When $c_1 \ge \max(1-(2-\delta)Q_{max}, 1-3\delta Q_{max})$, the retailer sells the product only in period 2 and does not carry inventory between periods, where $Q_1^* = 0$, $p_1^* = 1-\delta Q_{max}$ and $\bar{\theta}^* = 1$.

The corresponding supplier's profit in all of these three cases is jointly concave in (c_1, Q_{max}) and the FOC shows that $Q_{max} \ge 1/4$ in all cases.

The second part of Lemma follows from the fact that equilibrium total quantity in a no-commitment model never exceeds 1/2 (commitment never binds when $Q_{max} > 1/2$) as seen in Propositions 1 and 2.

Proposition 9 When commitment binds, i.e. (27) holds, the retailer's profit is given by (19) as a function on Q_1 and $\bar{\theta}$ and it is linear in Q_1. The feasible region is defined by (22), (21) and (27). Therefore, optimal Q_1 must be at one of the boundary points: $1-\bar{\theta}$, $2Q_{max} - 1 + \frac{\bar{\theta}}{2}$ or Q_{max}. We find the solution for each case and compare them to find the global solution.

(A.i) The quantity commitment binds strictly, the retailer sells the product in both periods and it carries inventory between periods with $Q_1^* = \frac{1-c_1-3\delta Q_{max}}{2(1-2\delta)}$, $p_1^* = \frac{1+\delta(1-c_1)+\delta(Q_{max}(1+\delta)-3)}{2(1-2\delta)}$ and $\bar{\theta}^* = \frac{1+c_1-\delta(4-3Q_{max})}{2(1-2\delta)}$, when $\delta < 1/2$, $\frac{1-c_1}{2-\delta} < Q_{max} < \frac{5-3c_1-4\delta}{8-7\delta}$.

(A.ii) The quantity commitment binds strictly, the retailer sells the product only in period 1 with $Q_1^* = Q_{max}$ and $p_1^* = \bar{\theta}^* = 1 - Q_{max}$, when

a) $\delta < 1/2$, $Q_{max} \le \frac{1-c_1}{2-\delta}$,

b.1) $\delta \ge 1/2$, $c_1 \le \frac{2(1-\delta)(5\delta-1)}{23-25\delta}$, $Q_{max} < K_1$,

b.2) $\delta \ge 1/2$, $c_1 > \frac{2(1-\delta)(5\delta-1)}{23-25\delta}$, $Q_{max} < \frac{4-3c_1-2\delta}{7-5\delta}$.

(A.iii) The quantity commitment binds weakly, the retailer sells the product in both periods and it does not carry inventory between periods with $Q_1^* = \frac{1}{3}(4Q_{max} - 1)$, $p_1^* = \frac{1}{3}(4-\delta-Q_{max})$, and $\bar{\theta}^* = \frac{4}{3}(1-Q_{max})$, when $\max(\frac{4-3c_1-2\delta}{7-5\delta}, \frac{5-3c_1-4\delta}{8-7\delta}) \le Q_{max} \le \frac{10+3c_1-14\delta}{4(4-5\delta)}$.

(A.iv) The quantity commitment binds weakly, the retailer sells the product in both periods and it carries inventory between periods with $Q_1^* = \frac{4Q_{max}(4-3\delta)-6(1-\delta)}{4(2-\delta)}$, $p_1* = \frac{2+(1-\delta)(4\delta-c_1)-2\delta(4-3\delta)Q_{max}}{2(2-\delta)}$ and $\bar{\theta}^* = \frac{2-c_1+2\delta(1-2Q_{max})}{2(2-\delta)}$, when $\max(K_1, \frac{10+3c_1-14\delta}{4(4-5\delta)}) < Q_{max}$.

When the commitment does not bind, Proposition 1 describes the retailer's optimal policy. We use (27) to specify the condition on Q_{max} in these cases as listed below.

(B.i) Retailer sells the product in both periods and it carries inventory between periods with $Q_1^* = \frac{11}{14} - \frac{c_1(112-91\delta)+6\delta}{21\delta(8-7\delta)}$, $p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-7\delta)}$, and $\bar{\theta}^* = \frac{3(4-3\delta)+2c_1}{3(8-7\delta)}$, when $c_1 < \frac{3\delta(4-3\delta)}{2(16-15\delta)}$, $Q_{max} > \frac{1}{42}\left(33 - \frac{14c_1}{\delta} - \frac{2(6+7c_1)}{8-7\delta}\right)$.

(B.ii) Retailer sells the product in both periods and it does not carry inventory between periods with $Q_1^* = \frac{8(1-c_1)-3\delta}{16-9\delta}$, $p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}$, and $\bar{\theta}^* = \frac{8(1+c_1)-6\delta}{16-9\delta}$, when $\frac{3\delta(4-3\delta)}{2(16-15\delta)} \le c_1 < \frac{8-3\delta}{8}$, $Q_{max} > \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$.

(B.iii) Retailer sells the product only in period 2 and it does not carry inventory between periods, where $Q_1^* = 0$, and $\bar{\theta}^* = 1$, when $c_1 \ge \frac{8-3\delta}{8}$.

To find the global optimal we compare the solutions to the subproblems presented above. This comparison leads to the equilibrium regions in Proposition 9. Note that A.i, A.ii B.ii, and B.iii cases remain the same in the global solution. We compare A.iii and A.iv with B.i where we find the thresholds K_2 and K_3 respectively.

Proposition 10 We analyze Π_S in each region of Proposition 9 to find the optimal (c_1, Q_{max}).

In case (i), supplier profit is jointly concave in (c_1, Q_{max}). But the FOCs do not result in an interior solution in this region, furthermore, Π_S is continuous on the boundary points, hence the optimal solution cannot be in this region.

In case (ii), Π_S is linear increasing in Q_{max}, thus we need to consider Q_{max} upper bounds. When $\delta \le 1/2$, $Q_{max} = \frac{1-c_1}{2-\delta}$ and it yields $c_1 = 1/2$,. When $\delta > 1/2$, $Q_{max} = \frac{4-3c_1-2\delta}{7-5\delta}$ and it yields $c_1 = \frac{2-\delta}{3}$. Note $Q_{max} = K_1$ does not yield a solution because $\Pi_S(Q_{max} = K_1)$ is a concave function of c_1 and $\frac{d\Pi_S}{dc_1}$ is positive at c_1 upper-bound of this region.

In case (iii), Π_S is a convex function of Q_{max}, hence we need to consider the boundary points for Q_{max}. Note we have already considered the lower-bound in case (ii). So we only consider the upper-bounds here. *(b.3)*: When $Q_{max} = \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$, $c_1 = \frac{16-3\delta}{32}$ for $\delta \le \frac{16}{21}$. *(b.2)*: When $Q_{max} = K_2$, $c_1 = \arg\max_{c_1} \Pi_S(K_2, c_1)$ for $\delta > \frac{71-\sqrt{241}}{75}$. *(b.1)*: When $Q_{max} = \frac{10+3c_1-14\delta}{4(4-5\delta)}$, Π_S is a quadratic convex function of c_1, therefore, the solution can be only in *(b.2)* and *(b.3)*. However, the two δ regions in these cases intersect. Hence, we compare the solutions in these two cases. Define $\bar{c}_1 = \arg\max_{c_1} \Pi_S(K_2, c_1)$. Then, $\bar{\delta}_7$ solves $\Pi_S(K_2, \bar{c}_1) = \frac{8+3\delta}{64}$ where the right hand side is the value of Π_S in case *(b.3)*.

In case (iv), Π_S is a convex function of Q_{max}, hence optimal Q_{max} must be at one of the boundary points. The only boundary left for analysis is $Q_{max} = K_3$. However, $\Pi_S(Q_{max} = K_3)$ is a concave function of c_1.

In case (iv), Π_S is a convex function of Q_{max}, hence optimal Q_{max} must be at the boundary. The only boundary point left for analysis is $Q_{max} = K_3$. However, $\Pi_S(Q_{max} = K_3)$ is concave in c_1 and $\frac{d\Pi_S}{dc_1}$ is positive at the upper-bound, thus, a solution cannot follow from this case.

Solutions in cases (v) and (vi) are same as in Proposition 2, which are superseded by the solution from case (iii). Thus, to find the optimal (c_1, Q_{max}) one needs to compare the solutions given in cases (ii) and (iii) above, which leads to the equilibrium regions in the Proposition.

Appendix D: Myopic Consumers Benchmark

When customers are myopic they buy the product whenever their value exceeds its retail price. Specifically, they do not consider second period price when they make their purchasing decision in period 1. The SPNE is defined by the following equations.

$$\begin{aligned}
(p_2^*, Q_2^*) &= \arg \max_{(p_2,Q_2)\in S_2} [p_2(p_1 - p_2/\delta) - c_2 Q_2], \qquad (32)\\
c_2^* &= \arg\max_{c_2} c_2 Q_2^* \\
(p_1^*, Q_1^*) &= \arg \max_{(p_1,Q_1)\in S_1^M} [p_1(1-p_1) - c_1 Q_1 + p_2^*(p_1^* - p_2^*/\delta) - c_2^* Q_2^*], \qquad (33)\\
c_1^* &= \arg\max_{c_1} [c_1 Q_1^* + c_2^* Q_2^*].
\end{aligned}$$

where the retailer's feasible strategy sets S_i in each period are given by

$$\begin{aligned}
S_1^M &= \{(p_1, Q_1) : Q_1 \ge 1 - p_1^* \ge 0\}, \qquad (34)\\
S_2 &= \{(p_2, Q_2) : Q_1 + Q_2 \ge 1 - p_2/\delta\}.
\end{aligned}$$

The following lemma then describes the supplier's and retailer's second period optimal policy.

Lemma 9. *Suppose the retailer orders Q_1 units and and charges p_1 in period 1.*

(i) For any given wholesale price c_2, the retailer orders $Q_2^* = \max(\frac{\delta(2(1-Q_1)-p_1)-c_2}{2\delta}, 0)$ units and sets $p_2^* = \delta(1 - Q_1 - Q_2^*)$ in period 2.

(ii) The supplier sets $c_2^* = \frac{\delta}{2}(2(1-Q_1) - p_1)$ in period 2.

This yields profits $\Pi_{R,2} = \frac{\delta}{16}\left(p_1^2 + 12p_1(1-Q_1) - 12(1-Q_1)^2\right)$ and $\Pi_{S,2} = \frac{\delta}{8}(2(1-Q_1) - p_1)^2$ for the retailer and supplier respectively.

Since customer's are myopic the consumer segment in $[1-p_1]$ stays in the market and we can solve for retailer's optimization problem by plugging the values in Lemma 9 into equation (33). The following proposition describes retailer's optimal policy in period 1.

PROPOSITION 11. *The SPNE order quantity Q_1^* and retail price p_1^* in period 1 are as follows.*
(i) When $c_1 < \frac{6\delta}{16-7\delta}$ the retailer sells the product in both periods, and in carries inventory between periods, where $Q_1^* = 1 - \frac{2+c_1}{2(4-\delta)}$ and $p_1^* = \frac{2+c_1}{4-\delta}$.
(ii) When $\frac{6\delta}{16-7\delta} \le c_1 \le 1 - \frac{\delta}{8}$ the retailer sells the product in both periods and it does not carry inventory between periods where $Q_1^* = 1 - \frac{8(1+c_1)}{16-\delta}$, $p_1^* = \frac{8(1+c_1)}{16-\delta}$.
When $c_1 > 1 - \frac{\delta}{8}$ retailer does not sell the product in period 1.

To fully characterize the equilibrium we solve the supplier's optimization problem in period 1, which is stated by the following proposition.

PROPOSITION 12. *The SPNE wholesale price c_1 is $c_1^* = \frac{4}{8-\delta} - \frac{\delta}{32}$.*

Price in Proposition 12 corresponds to part (ii) of Proposition 11. Hence in equilibrium the retailer serves the market in both periods and does not carry inventory.

It is a well established result that strategic customers cause firms to loose profit. The following corollary formally states this result. Let NC^M denote the no commitment scenario when customers are myopic.

COROLLARY 1. *(i) $\Pi_S^{NC^M} > \Pi_S^{NC}$.*
(ii) $\Pi_S^{NC^M} + \Pi_R^{NC^M} > \Pi_S^{NC} + \Pi_R^{NC}$.
(iii) $\Pi_R^{NC^M} > \Pi_R^{NC}$.

When δ decreases waiting for mark-down becomes less attractive as the product's value in period 2 decreases. Thus, the gap between strategic and myopic customer models gets smaller as δ decreases. Recall that we assume $\delta > 0$. However, at the extreme, when $\delta = 0$, strategic and myopic models become identical as customers do not benefit from waiting at all.

Appendix E: Analysis of Supplier's Period-specific Quantity Commitment

In this section we analyze the scenario when supplier makes period-specific quantity commitments.

LEMMA 10. *Suppose the supplier can make period-specific quantity commitments Q_{1max} and Q_{2max}. It is never optimal for the supplier to make a strictly binding commitment in period 1.*

Following Lemma 10 supplier makes a binding quantity commitment only in period 2. Similar to (17) let us define the critical period 2 whole sale price $\bar{c}_2$ below which retailer finds it attractive to procure all of this quantity. Thus, $\bar{c}_2$ solves $Q_2^*(c_2) = Q_{2max}$. In order for supplier's second period quantity commitment to bind, the following conditions need to be satisfied.

$$\left.\frac{d\Pi_{R,2}}{dQ_2}\right|_{Q_2=Q_{2max}} \ge 0, \text{ and } \left.\frac{d\Pi_{S,2}}{dc_2}\right|_{c_2=\bar{c}_2} \le 0. \tag{35}$$

Basically, (35) ensures that retailer procures all of the commitment quantity given supplier's second period wholesale price and it is not optimal for supplier to increase this price. The solution of (35) leads to the following Lemma, which characterizes the firms' equilibrium strategies in period 2.

LEMMA 11. *When supplier's period 2 quantity commitment binds, $Q_2^* = Q_{2max}$, $p_2^* = \delta(1-Q_1-Q_{2max})$ and $c_2^* = \delta(2(1-Q_1-Q_{2max})-\bar{\theta})$ in a SPNE in period 2. This yields $\Pi_{R,2} = p_2^*(Q_1+Q_{2max}-(1-\bar{\theta})) - c_2^*Q_2^*$ and $\Pi_{S,2} = c_2^*Q_2^*$. Furthermore, the suppliers's second period quantity commitment binds if and only if $Q_{2max} \le \frac{1}{4}(2(1-Q_1)-\bar{\theta})$.*

Following lemma 11 retailer buys all of supplier's commitment quantity in period 2, if and only if

$$Q_{2max} \le \frac{1}{4}(2(1-Q_1)-\bar{\theta}). \tag{36}$$

Thus, when (36) holds, period 2 equilibrium is given by lemma 11. On the other hand, when (36) does not hold the commitment is ineffective, and period 2 equilibrium is the same as the no-commitment scenario, which is given by lemma 1. Retailer's period 1 choices determine whether the commitment in period 2 will bind. When the commitment binds i.e. (36) holds the retailer's objective function is given by the equation below.

$$[1-\bar{\theta}]p_1(\bar{\theta},\hat{Q}_1,Q_{2max})-Q_1c_1+\Pi_{R,2}(\bar{\theta},Q_1,Q_{2max}) \tag{37}$$

On the other hand, when the commitment does not bind, i.e., (36) does not hold, the retailer's profit is given by (12). We determine the retailer's optimal policy in period 1 by solving for both of the cases, which is presented in the following proposition.

PROPOSITION 13. *Suppose the supplier commits to sell not more than Q_{2max} units in period 2. The retailer's SPNE policy is characterized as follows.*

(i) The quantity commitment binds, the retailer sells the product in both periods and it carries inventory in between periods with $Q_1^*=\frac{1}{4}\left(3-\frac{2c_1}{\delta}-\frac{\delta Q_{2max}}{1-\delta}\right)$, $p_1^*=\frac{1}{4}\left(2+2c_1-\delta-\frac{(2-3\delta)\delta Q_{2max}}{1-\delta}\right)$ *and* $\bar{\theta}^*=\frac{1-\delta(1-Q_{2max})}{2(1-\delta)}$ *when* $\frac{(2c_1-\delta)(1-\delta)}{\delta^2}<Q_{2max}<\frac{c_1}{4\delta}$.

(ii) The quantity commitment binds, the retailer sells the product in both periods and it does not carry inventory in between periods with $Q_1^*=\frac{1-c_1-\delta Q_{2max}}{2-\delta}$, $p_1^*=\frac{c_1+(1-\delta)(1-\delta Q_{2max})}{2-\delta}$ *and* $\bar{\theta}^*=\frac{1+c_1-\delta(1-Q_{2max})}{2-\delta}$ *when* $Q_{2max}\le\min\left(\frac{(2c_1-\delta)(1-\delta)}{\delta^2},\frac{1+c_1-\delta}{8-5\delta},\frac{1-c_1}{\delta}\right)$.

(iii) The quantity commitment binds, the retailer sells the product only in period 2 and it does not procure any units in period 1 with $Q_1^*=0$ *and* $\bar{\theta}^*=1$ *when* $\frac{1-c_1}{\delta}<Q_{2max}$.

(iv) The quantity commitment does not bind, the retailer sells the product in both periods and it carries inventory in between periods with $Q_1^*=\frac{6-c_1-\delta(6-18Q_{2max})-16Q_{2max}}{8(1-\delta)}$, $p_1^*=\frac{(2+c_1-2\delta)(2-\delta)+2\delta(2-3\delta)Q_{2max}}{8(1-\delta)}$ *and* $\bar{\theta}^*=\frac{2(1-\delta(1+Q_{2max}))+c_1}{4(1-\delta)}$ *when* $\frac{c_1}{4\delta}\le Q_{2max}<\min\left(\frac{c_1}{3\delta},\frac{2(1-\delta)+c_1}{2(8-7\delta)}\right)$.

(v) The quantity commitment does not bind, the retailer sells the product in both periods and it does not carry inventory in between periods with $Q_1^*=1-4Q_{2max}$, $p_1^*=(4-\delta)Q_{2max}$ *and* $\bar{\theta}^*=4Q_{2max}$ *when* $\max\left(\frac{2(1-\delta)+c_1}{2(8-7\delta)},\frac{1+c_1-\delta}{8-5\delta}\right)\le Q_{2max}<\min\left(\frac{c_1}{3\delta},\frac{4(1+c_1)-3\delta}{2(16-9\delta)}\right)$.

(vi) The quantity commitment does not bind, the retailer sells the product in both periods and it carries inventory between periods with $Q_1^*=\frac{11}{14}-\frac{c_1(112-91\delta)+6\delta}{21\delta(8-7\delta)}$, $p_1^*=\frac{1}{2}-\frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-7\delta)}$, *and* $\bar{\theta}^*=\frac{3(4-3\delta)+2c_1}{3(8-7\delta)}$, *when* $c_1<\frac{3\delta(4-3\delta)}{2(16-15\delta)}$ *and* $Q_{2max}\ge\frac{c_1}{3\delta}$.

(vii) The quantity commitment does not bind, the retailer sells the product in both periods and it does not carry inventory between periods with $Q_1^*=\frac{8(1-c_1)-3\delta}{16-9\delta}$, $p_1^*=\frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}$, *and* $\bar{\theta}^*=\frac{8(1+c_1)-6\delta}{16-9\delta}$, *when* $\frac{3\delta(4-3\delta)}{2(16-15\delta)}\le c_1$ *and* $Q_{2max}\ge\frac{4(1+c_1)-3\delta}{2(16-9\delta)}$

Next we solve the supplier's optimization problem to completely characterize the equilibrium. The supplier maximizes $c_1Q_1^*+c_2^*Q_{2max}$, where Q_1^* and c_2^* are given by Proposition 13 and Lemma 11. The following proposition gives the solution to this problem.

PROPOSITION 14. *Suppose the supplier commits to sell not more than Q_{2max} units in period 2. The equilibrium whole sale price* $c_1^*=\frac{1}{2}$ *and the commitment quantity* $Q_{2max}^*=\frac{1-\delta}{2(4-3\delta)}$, *which yields* $c_2^*=\frac{\delta}{2}$, $p_2^*=\frac{\delta(5-4\delta)}{2(4-3\delta)}$ *and* $Q_2^*=\frac{1-\delta}{2(4-3\delta)}$.

The equilibrium solution corresponds to part (*ii*) of Proposition 13, hence in equilibrium supplier sets a first period wholesale price and commits to a second period quantity such that retailer buys and sells in both periods but does not carry inventory.

Interestingly, we find that the supplier achieves a higher profit by committing only to a period 2 quantity instead of making an aggregate quantity commitment. This result is formally stated in the following Corollary.

COROLLARY 2. $\Pi_S^{SQC2}>\Pi_S^{SQC}$.

Appendix F: Concurrent Commitments

We first discuss quantity commitments and then price commitments. Suppose both the supplier and the retailer can make independent quantity commitments following the same sequence of events as in Sections 6.1 and 7.1. Let Π_i^{sQC}, $i:S,R$ show the supplier's and retailer's profit under this scenario. In equilibrium, either the supplier's or the retailer's quantity commitment would dominate. For example, if the supplier sets a more stringent quantity commitment, the retailer's quantity commitment becomes irrelevant and vice versa. The following theorem shows the impact of these commitments on their profitability.

THEOREM 11. *There exists* $\bar{\delta}_1 \approx 0.249$ *and* $\bar{\delta}_3 \approx 0.674$ *such that*
(i) $\Pi_S^{NC} > \Pi_S^{sQC}$ *when* $\delta \le \bar{\delta}_1$.
(ii) $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{sQC} + \Pi_R^{sQC}$ *when* $\delta \le \bar{\delta}_1$.
(iii) $\Pi_R^{NC} \ge \Pi_R^{sQC}$ *when* $\delta < \bar{\delta}_3$.

Note $\bar{\delta}_1$ and $\bar{\delta}_3$ are explicitly characterized in the proofs of Proposition 4 and Theorem 4 in Appendix C.

The Theorem illustrates that the supplier and the retailer as well as the entire supply chain can get worse off due to concurrent quantity commitments. Recall that the supplier always benefits from its own unilateral quantity commitments. However, simultaneous quantity commitment can hurt its profitability when $\delta < \bar{\delta}_1$, because the retailer's quantity commitment dominates in this case.

Now, suppose both the supplier and retailer can make price commitments following the same sequence of events as in Sections 6.2 and 7.2. The following proposition shows that the retailer's price commitment always dominates in this case.

PROPOSITION 15. *When both the retailer and the supplier can make price commitments, the resulting equilibrium is same as that of the retailer's unilateral price commitment.*

Essentially, the retailer restricts its sales to only period 1 in equilibrium, therefore, the supplier's price commitment (to period 2 wholesale price) becomes irrelevant. Because the resulting equilibrium is identical to the retailer's unilateral price commitment, both the supplier and the retailer get worse off compared to no commitment scenario as shown in Theorem 3.

Appendix G: Utility Discounting

In this section we analyze an extension where customer utility is given by (26). Note that with this assumption total demand increases because the set S_2 in equation (9) is replaced with

$$\{(p_2, Q_2) : Q_1 + Q_2 \ge 1 - p_2\}. \tag{38}$$

In addition customers' willingness to pay in period 2 increases, which is reflected in the marginal customer's decision given by the following equation.

$$\bar{\theta}^* = \inf\{\theta : \bar{\theta} - p_1 \ge \delta(\bar{\theta} - p_2^*(\hat{Q}_1))\}. \tag{39}$$

The following Lemma describes the supplier's and retailer's optimal policy in period 2.

LEMMA 12. *Suppose the retailer orders* Q_1 *units in period 1, and consumer segment* $[0, \bar{\theta})$ *remains in the market in period 2.*
(i) For any given wholesale price c_2, *the retailer orders* $Q_2^* = \max(\frac{1}{2}(2 - 2Q_1 - \bar{\theta} - c_2), 0)$ *units and sets* $p_2^* = 1 - Q_1 - Q_2^*$ *in period 2.*
(ii) The supplier sets $c_2^* = 1 - Q_1 - \frac{\bar{\theta}}{2}$ *in period 2.*
This yields profits $\Pi_{R,2} = \frac{1}{16}(\bar{\theta}^2 + 12(1 - Q_1)(Q_1 - 1 + \bar{\theta}))$ *and* $\Pi_{S,2} = \frac{(2-2Q_1-\bar{\theta})^2}{8}$ *for the retailer and supplier respectively.*

Customers conjecture the second period price and they expect $p_2^*(\hat{Q}_1) = \frac{2-2\hat{Q}_1+\bar{\theta}}{4}$. Note that both Q_2^*, c_2^* and p_2^* increase in comparison to the base model. The marginal customer $\bar{\theta}$ in equation (39) leads to $\bar{\theta} = \frac{4p_1 - 2\delta(1-\hat{Q}_1)}{4-3\delta}$, which is the same as equation (10) in the base model.

After reformulating the retailer's total profit in terms of the order quantity Q_1 and target consumer segment $[\bar{\theta}, 1]$ in period 1 we can solve for the retailer's optimal policy.

PROPOSITION 16. *The SPNE order quantity* Q_1^* *and retail price* p_1^* *and the corresponding marginal customer* $\bar{\theta}^*$ *in period 1 are as follows.*
(i) When $c_1 < \min(\frac{3(4-3\delta)}{2(9-8\delta)}, \frac{3(1-\delta)}{3-2\delta})$ *the retailer sells the product in both periods, and it carries inventory between periods, where* $Q_1^* = \frac{8-7\delta-2c_1(5-4\delta)}{2(6-5\delta)}$, $p_1^* = \frac{3(2-\delta)(4-3\delta)-2c_1(3\delta^2+\delta-6)}{6(6-5\delta)}$, *and* $\bar{\theta}^* = \frac{3(4-3\delta)+2c_1(3-2\delta)}{3(6-5\delta)}$.
(ii) When $\frac{3(1-\delta)}{3-2\delta} \le c_1 \le \frac{3}{4}$ *the retailer sells the product only second period and carries inventory, where* $Q_1^* = \frac{1}{6}(3 - 4c_1)$, $p_1^* = 1 - \frac{\delta}{6}(3 - 2c_1)$, *and* $\bar{\theta}^* = 1$.
(iii) When $\frac{3(4-3\delta)}{2(9-8\delta)} \le c_1 \le \frac{1}{8}(7 - 2\delta)$ *the retailer sells the product in both periods, and it does not carry inventory between periods, where* $Q_1^* = \frac{7-8c_1-2\delta}{15-8\delta}$, $p_1^* = \frac{(4-\delta)(4(1+c_1)-3\delta)}{2(15-8\delta)}$, *and* $\bar{\theta}^* = \frac{2(4+4c_1-3\delta)}{15-8\delta}$.
When $c_1 > \max(\frac{1}{8}(7 - 2\delta), \frac{3}{4})$ *the retailer does not buy the product in first period.*

Note that the region in which the retailer carries inventory increases in comparison to the base model, which is due to the fact that the second period demand increases when customers discount not only their valuations but also the retail price.

To find the equilibrium whole sale price we need to solve the supplier's optimization problem. The following Proposition describes the solution.

PROPOSITION 17. *The SPNE whole sale price c_1 in period 1 is as follows.*
(i) When $\delta \leq \frac{1}{132}(127-\sqrt{817})$, $c_1^* = \frac{9(8-7\delta)}{4(33-26\delta)}$.
(ii) When $\delta > \frac{1}{132}(127-\sqrt{817})$, $c_1^* = \frac{9}{16}$.

Parts $(i),(ii)$ of Proposition 17 map to the parts $(i),(ii)$ of Proposition 16. Hence, the supplier allows the retailer to carry inventory in between periods. In addition, c_1^* also increases compared to the base model.

G.1 Retailer's Quantity Commitment when Customers Discount Future Net Utility

In this section we analyze the retailer's quantity commitment when customer utility function is given by (26). Analysis is similar to that of Section 6.1, specifically Lemma 4 continues to hold. Following equation (17), we obtain the following Lemma that describes the optimal policies in the second period.

LEMMA 13. *When the retailer's quantity commitment binds, $Q_2^* = Q_{max} - Q_1$, $p_2^* = 1 - Q_{max}$ and $c_2^* = 2(1-Q_{max}) - \bar{\theta}$ in a SPNE in period 2. This yields $\Pi_{R,2} = p_2^*(Q_{max} - (1-\bar{\theta})) - c_2^* Q_2^*$ and $\Pi_{S,2} = c_2^* Q_2^*$. Furthermore, the retailer quantity commitment binds if and only if $Q_{max} \leq \frac{Q_1+1}{2} - \frac{\bar{\theta}}{4}$.*

The marginal customer $\bar{\theta}$, who is indifferent between buying in period 1 or waiting for period 2 then solves $\bar{\theta} - p_1 = \delta(\bar{\theta} - p_2^*)$ which gives $p_1(\theta, \bar{Q}_{max}) = \bar{\theta} - \delta(Q_{max} - (1-\bar{\theta}))$. Following Lemma 13 and the expression for marginal customer we can state retailer's period 1 problem similar to equations (19 - 22), which leads to the following proposition.

PROPOSITION 18. *Suppose the retailer can make a quantity commitment. Its SPNE order quantity Q_1^*, quantity commitment Q_{max}^*, retail price p_1^* and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.*

(i) When $c_1 < K_{c_2}$, the retailer sells the product in both periods, it carries inventory in between periods and the quantity commitment binds strictly, where $Q_1^* = \frac{2(1-c_1)}{3+d}$, $Q_{max}^* = \frac{2(1-c_1)}{3+\delta}$, $p_1^* = \frac{2+c_1(1+\delta)}{3+\delta}$ and $\bar{\theta}^* = \frac{2+c_1+\delta}{3+\delta}$.

(ii) When $K_{c_2} \leq c_1 < \min(\frac{(3-\delta)(2-\delta)}{9-8\delta}, \frac{3(1-\delta)}{3-2\delta})$, the retailer sells the product in both periods, it carries inventory in between periods and the quantity commitment binds weakly, where $Q_1^* = \frac{12-(11-\delta)\delta-3c_1(5-4\delta)}{2(9-\delta(6+\delta))}$, $Q_{max}^* = \frac{3(4-3c_1)-10\delta+7c_1\delta}{2(9-\delta(6+\delta))}$, $p_1^* = \frac{2(6-(6-\delta)\delta)-c_1(3\delta^2+\delta-6)}{2(9-\delta(6+\delta))}$ and $\bar{\theta}^* = \frac{6-\delta(3+\delta)+c_1(3-2\delta)}{9-\delta(6+\delta)}$.

(iii) When $\frac{3(1-\delta)}{3-2\delta}) \leq c_1 < \frac{3}{4}$, the retailer sells the product only in period 2, it carries inventory between periods and the quantity commitment binds weakly, where $Q_1^* = \frac{1}{6}(3-4c_1)$, $Q_{max}^* = \frac{1}{6}(3-2c_1)$, and $\bar{\theta}^* = 1$.

(iv) When $\frac{(3-\delta)(2-\delta)}{9-8\delta} \leq c_1 \leq \frac{1}{8}(7-2\delta)$, the retailer sells the product in both periods, it does not carry inventory between periods and the quantity commitment binds weakly, where $Q_1^* = \frac{7-8c_1-2\delta}{15-4\delta}$, $Q_{max}^* = \frac{6(3-2c_1)-5\delta}{2(15-4\delta)}$, $p_1^* = \frac{(4-\delta)(4(1+c_1)-\delta)}{2(15-4\delta)}$, and $\bar{\theta}^* = \frac{8(1+c_1)-2\delta}{15-4\delta}$.

(v) When $c_1 \geq \max(\frac{1}{8}(7-2\delta), \frac{3}{4})$, the retailer does not procure the product in first period commitment binds weakly where, $Q_1^* = 0$, $Q_{max}^* = \frac{1}{4}$, and $\bar{\theta}^* = 1$.

The threshold K_{c_2} is defined as follows.

$$K_{c_2} = \frac{(3-\delta)(1-\delta)\delta + \sqrt{\delta^2(27-36\delta+8\delta^3+\delta^4)}}{9+(3-8\delta)\delta} \tag{40}$$

Recall that weak and strict commitments are defined in section 6.1. Finally following Lemma 13 and Proposition 18, we can characterize the equilibrium.

PROPOSITION 19. *Suppose the retailer can make a quantity commitment, the supplier's SPNE wholesale price c_1 in period 1 is as follows. There exists $\bar{\delta}_8 \approx 0.735$ such that*
(i) When $\delta \leq \bar{\delta}_8$, $c_1^* = \frac{108-\delta(159-\delta(41+(15-\delta)\delta))}{198-4\delta(69-2\delta(8+3\delta))}$.
(ii) When $\delta > \bar{\delta}_8$, $c_1^* = \frac{9}{16}$.

Here $\bar{\delta}_8$ is explicitly characterized in the proof of this proposition. Parts (i) and (ii) of Proposition 19 map to parts (ii) and (iii) of Proposition 18. Specifically the retailer's quantity commitment always binds weakly and the retailer always carries inventory.

G.2 Retailer's Price Commitment when Customers Discount Future Net Utility

Suppose the retailer can credibly commit to future prices then similar to the reasoning in Section 6.2 retailer procures all of its needed quantity in period 1. Hence, although customers discount the second period price as well as their valuation of the product, the equilibrium does not change since retailer does not sell in period 2. Therefore, Lemma 6 and Proposition 5 continue to hold.

G.3 Supplier's Quantity Commitment when Customers Discount Future Net Utility

Our results for this section are based on numerical examples and they are stated in Section 9.3.

G.4 Supplier's Price Commitment when Customers Discount Future Net Utility

Similar to Section 7.2, the following Lemma describes the equilibrium in period 2.

LEMMA 14. *Suppose that the supplier commits to wholesale prices c_1 and c_2, the retailer orders Q_1 in period 1 and consumer segment $[0,\bar{\theta})$ remain in the market in period 2. The retailer then orders $Q_2^* = \max(\frac{1}{2}(2-2Q_1-\bar{\theta}-c_2),0)$ units and sets $p_2^* = 1-Q_1-Q_2^*$ in period 2.*

The marginal customer $\bar{\theta}$ solves the following equation $\bar{\theta}-p_1=\delta(\bar{\theta}-p_2^*)$, which leads to $\bar{\theta}=\frac{2p_1-\delta c_2}{2-\delta}$. Note that similar to the base model the marginal customer in this case is also independent from Q_1. Then we can formulate the retailer's optimization problem in period 1 using equation (25), which leads to the following Proposition.

PROPOSITION 20. *Suppose the supplier commits to wholesale prices c_1 and c_2. The retailer's SPNE order quantity Q_1^* and retail price p_1^* in period 1 are as follows.*
(i) When $c_2 \le \frac{2c_1+\delta-1}{1+\delta}$, the retailer sells the product only in period 2, where $Q_1^*=0$.
(ii) When $\frac{2c_1+\delta-1}{1+\delta} < c_2$, the retailer sells the product in both periods, where $Q_1^* = \frac{1-2c_1+c_2-(1-c_2)\delta}{3-2\delta}$ and $p_1^* = \frac{2c_1(2-\delta)+(2-\delta)^2-c_2(2-(2-\delta)\delta)}{2(3-2\delta)}$.
The retailer does not carry inventory between periods, thus the marginal customer is given by $\bar{\theta}^* = 1-Q_1^*$, in addition without loss of generality $c_2 \le c_1$ in both of the cases.

To fully characterize the equilibrium, we solve the supplier's problem.

PROPOSITION 21. *Suppose the supplier can commit to future wholesale prices. The equilibrium wholesale prices c_1 and c_2 are as follows. $c_1^* = c_2^* = \frac{1}{2}$.*

Proposition 21 corresponds to part (*ii*) of Proposition 20, hence in equilibrium the supplier commits to wholesale prices such that the retailer buys and sells in both periods.

Appendix H: No-Inventory Carryover

In this extension we analyze the model where retailer is not allowed to carry any inventory into period 2. Since retailer cannot carry inventory, it sells all the quantity that it buys in the first period, hence Q_1 is always $1-\bar{\theta}$.

The following Lemma describes the supplier's and retailer's optimal policy in period 2.

LEMMA 15. *Suppose consumer segment $[0,\bar{\theta})$ remains in the market in period 2.*
(i) For any given wholesale price c_2, the retailer orders $Q_2^* = \max(\frac{1}{2}(\bar{\theta}-\frac{c_2}{\delta}),0)$ units and sets $p_2^* = \delta\left(\bar{\theta}-Q_2^*\right)$ in period 2.
(ii) The supplier sets $c_2^* = \frac{\delta\bar{\theta}}{2}$ in period 2.
This yields profits $\Pi_{R,2} = \frac{\delta\bar{\theta}^2}{16}$ and $\Pi_{S,2} = \frac{\delta\bar{\theta}^2}{8}$ for the retailer and supplier respectively.

Following Lemma 15, the marginal customer is given by $\bar{\theta} = \frac{4p_1}{4-\delta}$. Rearranging this equality, we get $p_1(\bar{\theta}) = (1-\frac{\delta}{4})\bar{\theta}$. We can now solve for retailer's optimization problem in period 1, which is given in the following Proposition.

PROPOSITION 22. *The SPNE retail price p_1^* and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.*
(i) When $c_1 < \frac{8-3\delta}{8}$ the retailer sells the product in both periods, where $p_1^* = \frac{(4-\delta)(4(1+c_1)-\delta)}{2(16-5\delta)}$, and $\bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta}$.
(ii) When $c_1 \ge \frac{8-3\delta}{8}$ the retailer does not buy the product in first period.

Observe that parts (i) and (ii) of above proposition is the same as the parts (*ii*) and (*iii*) of Proposition 3. Hence by not being able to carry inventory retailer is actually making a weak quantity commitment. The equilibrium is described by the following Proposition.

PROPOSITION 23. *Supplier sets the SPNE wholesale price as* $c_1^* = \frac{4(3-\delta)}{24-9\delta} - \frac{11\delta}{96}$.

In equilibrium supplier sets the price so that retailer always sells the product in period 1.

Appendix I: Heterogeneity in Customer Patience Level with a Fixed θ

I.1 Demand Characterization

Assume that customers are homogeneous in terms of how they value the product, which is given by $\theta > 0$. All customers with $\theta - p_1 \geq \delta\theta - p_2$ buy in period 1 . There exists a critical $\bar{\delta}$ such that $\theta - p_1 = \bar{\delta}\theta - p_2$. Call the utility of this marginal customer in second period $U_{\bar{\delta}}$. All customers with $\hat{\delta} < \bar{\delta}$ buy in period 1 and all customers with $\hat{\delta} \geq \bar{\delta}$ wait for second period. Assume a customer with discount factor $\delta_\epsilon = \bar{\delta} + \epsilon$, where $\epsilon > 0$. Then, utility of this customer in second period $U_{\delta_\epsilon} = \delta_\epsilon\theta - p_2 = \bar{\delta}\theta - p_2 + \epsilon\theta = U_{\bar{\delta}} + \epsilon\theta$. Hence this customer also buys in second period. Therefore, all customers with $\hat{\delta} > \bar{\delta}$ buy in second period and all of the market is served. Total demand is 1.

I.2 Equilibrium

LEMMA 16. *When customers are heterogeneous with respect to discount factor then* $Q_2^* = 1 - Q_1$, $p_2^* = \delta\theta$, $c_2^* = p_2$.

PROPOSITION 24. *When customers are heterogeneous with respect to discount factor then* $c_1^* = p_1^* = \theta$, *marginal customer becomes* $\bar{\delta}^* = 1$ *and* $\Pi_{S,1} = \theta$, $\Pi_{R,1} = 0$.

Appendix J: Simultaneous Price and Quantity Commitments

Recall that retailer always gets worse off from its price commitment. However, it may benefit from its quantity commitment when consumers are extremely patient ($\delta > 0.79$). The following theorem shows that simultaneously committing to both price and quantity does not help the retailer.

THEOREM 12. *When retailer benefits from committing to a maximum quantity* Q_{max} *to sell over two periods, committing additionally to retail prices* p_1 *and* p_2 *never increases its profit. In addition, simultaneous price and quantity commitment never results in a higher profit than no commitment for the retailer.*

When the retailer commits to both price and quantity, the price commitment limits the sales to period 1 only, and therefore the outcome is identical to the retailers price only commitment. We know that retailer's price commitment always hurts its profitability, therefore when the retailer is benefiting from its quantity commitment having an additional price commitment is never profitable.

THEOREM 13. *When supplier commits to a maximum quantity* Q_{max} *to sell over two periods, it does not benefit from committing to wholesale prices* c_1 *and* c_2.

When the supplier benefits from quantity commitment, additional price commitment is never helpful. This is due to the fact that price commitment eliminates the retailer incentive to buy more in period 1 to pay less in period 2.

Appendix K: Additional Proofs

We present the proofs of extension results.

Theorem 7 The result about the supplier profit follows from the Corollary 2. We only need to prove the results about the retailer and supply chain profits.

Following Propositions 2 and 14 there are two cutoff points for δ below we present the proof for each case separately.

- When $\delta \leq \frac{16}{21}$, $\Pi_R^{SQC2} - \Pi_R^{NC} = -\frac{(28-9\delta)\delta}{256(4-3\delta)}$, which is clearly negative. $(\Pi_S^{SQC2} + \Pi_R^{SQC2}) - (\Pi_S^{NC} + \Pi_R^{NC}) = \frac{\delta(20-19\delta)}{256(4-3\delta)}$, which is clearly positive.
- When $\delta > \frac{16}{21}$, $\frac{d(\Pi_R^{SQC2} - \Pi_R^{NC})}{d\delta} = \frac{1}{100}\left(23 + \frac{176}{(16-15\delta)^2} - \frac{25}{(4-3\delta)^2} - \frac{1024}{(16-15\delta)^3}\right)$, which is greater than 0, hence the difference function is increasing. However, the difference is negative at both end points of the feasible region hence the result follows. The proof for the supply chain result is similar.

Theorem 8 The proof of this thoerem is very similar to the proof of Theorem 2. Specifically we need to compare two piecewise functions of δ and determine whether the difference is positive or negative.

First case of the Theorem 8 follows from Propositions 17 and 19. We will show the proof of the result for the supplier profit, the proof for the results for retailer profit and the supply chain profits are similar. Following Propositions 17 and 19 we have two cutoff points for δ so three regions to consider.

When $\delta < \bar{\delta}_8$ the difference $\Pi_S^{NC} - \Pi_S^{RQC}$ can be written as the following.

$$\Pi_S^{NC} - \Pi_S^{RQC} = \frac{24\delta(1680-6283\delta)+32\delta^3(6578-4157\delta)+16\delta^5(2187-130\delta)}{128(6-5\delta)(33-26\delta)\left(3(33-46\delta)+4\delta^2(8+3\delta)\right)} \tag{41}$$

Observe that all parts of the denominator of (41) are positive except $33-46\delta$ when $\delta > \frac{33}{46}$. However, $4\delta^2(8+3\delta) > 3(46\delta-33)$ in that range, hence the denominator is positive. Similarly, all parts of the numerator of (41) are positive except $1680-6283\delta$ when $\delta > \frac{1680}{6283}$. However, in that region the numerator is decreasing and positive at both end points of the region. Therefore, $\Pi_S^{NC} > \Pi_S^{RQC}$ in this region.

When $\bar{\delta}_8 \le \delta < \frac{1}{132}(127-\sqrt{817})$ the difference function can be written as the following.

$$\Pi_S^{NC} - \Pi_S^{RQC} = \frac{9}{64}\left(\frac{58-127\delta+66\delta^2}{(6-5\delta)(33-26\delta)}\right) \tag{42}$$

The denominator of (42) is clearly positive and the numerator is decreasing and has one root which is the boundary point $\frac{1}{132}(127-\sqrt{817})$. Therefore, $\Pi_S^{NC} > \Pi_S^{RQC}$ in this region.

When $\delta \ge \frac{1}{132}(127-\sqrt{817})$ the two profit functions are equal. Therefore, $\Pi_S^{NC} = \Pi_S^{RQC}$ in this region.

Second case of the Theorem 8 follows from Propositions 17 and 5, which is very similar to above second δ region hence they are omitted.

Theorem 9 Proof of this theorem is similar to the proof of Theorem 5. Following Propostion 17 and 21 we have two cutoff points for δ. Below we present the proof for the result for supplier profit. Proofs for retailer and the supply chain profit are similar and they are omitted.

When $\delta < \frac{1}{132}(127-\sqrt{817})$ the following is the difference between the supplier profits.

$$\Pi_S^{NC} - \Pi_S^{SPC} = \frac{96(12-35\delta)+8\delta^2(373-102\delta)}{128(6-5\delta)(33-26\delta)(3-2\delta)} \tag{43}$$

Clearly the denominator of (43) is positive. The numerator of (43) is positive when $\delta < \bar{\delta}_4 \approx 0.659$ which is given by the root of the following equation. Hence equation (44) explicitly characterizes $\bar{\delta}_4$.

$$\delta^2 = \frac{12(35\delta-12)}{373-102\delta} \tag{44}$$

When $\delta \ge \frac{1}{132}(127-\sqrt{817})$, $\Pi_S^{NC} = \frac{9}{64} < \Pi_S^{SPC} = \frac{4-3\delta}{8(3-2\delta)}$, since $\delta < \frac{4}{5}$.

Theorem 10 Below we present the proof for each case separately.

When retailer makes a quantity commitment under the condition that it cannot carry inventory the Lemma 4 continues to hold. In addition Lemma 5 also continues to hold since it is for every Q_1, but now $Q_1 = 1-\bar{\theta}$. Moreover, note that the optimal strategies and the prices for retailer's first period optimization problem given in Proposition 3 do not involve carrying inventory. Hence, when retailer is not allowed to carry inventory it can replicate the best response given in Proposition 3. Therefore Propositions 3 and 4 also continue to hold. To complete the proof for this case we need to compare the profits. Following Propositions 4 and 23 we have two cutoff points for δ. When $\delta \le \bar{\delta}_1$ the difference $\Pi_S^{NC} - \Pi_S^{RQC} = \frac{\delta(8+\delta)}{64(8-3\delta)}$, which is clearly positive. The difference $\Pi_R^{NC} - \Pi_R^{RQC} = \frac{27\delta(192-\delta(112-19\delta))}{6912(8-3\delta)^2}$ is also positive since $192 > \delta(112-19\delta)$ $\forall\delta$. When $\delta > \bar{\delta}_1$ the difference $\Pi_S^{NC} - \Pi_S^{RQC} = \frac{\left(896-256\sqrt{16-5\delta}-\delta\left(456-96\sqrt{16-5\delta}-55\delta\right)\right)^2}{1600(16-5\delta)^2(8-3\delta)}$, which is also clearly positive. Also, the difference $\Pi_R^{NC} - \Pi_R^{RQC} = \frac{3}{25}\sqrt{16-5\delta} + \frac{8}{27(8-3\delta)^2} + \frac{671\delta}{11520} - \frac{4991}{10800}$, where the second derivative is $\frac{16}{(8-3\delta)^4} - \frac{3}{4(16-5\delta)^{3/2}} < 0$. So the difference function is concave but positive at both end points therefore the difference is positive, which completes the proof for case (i) of the Theorem 10.

When retailer commits to future prices it commits not to sell in period 2, which does not change whether it can carry inventory or not. Hence, the analysis in section 6.2 and the Proposition 5 continue to hold. Then following Propositions 5 and 23 we have the differences $\Pi_S^{NC} - \Pi_S^{RPC}$ and $\Pi_S^{NC} - \Pi_S^{RQC}$, the same as the first part of the above proof hence positive and completes the proof for case (ii) of the Theorem 10.

When supplier makes a quantity commitment under the condition that retailer cannot carry inventory, Lemma 8 in Appendix A continues to hold. In addition, following the discussion above in the proof of case (i), the commitment condition that follows from the Lemma 5 also continues to hold. Similar to the analysis for supplier's quantity commitment in Appendix A, the retailer's profit function exibits a piecewise structure where it is given by (19) when the commitment binds; on the other hand, when the commitment does not bind, it is given by the no commitment objective that is studied in Appendix H. Since overall retailer objective function is not quasiconcave we solve for each part and compare in order to find the optimal retail price, order quantity and the marginal customer in period 1, which is given by the following.

(i) The quantity commitment binds strictly and the retailer sells the product in both periods with $p_1^* = \frac{1+(1-\delta)c_1-\delta(3-(1+\delta)Q_{max})}{2(1-2\delta)}$ and $\bar{\theta}^* = \frac{1+c_1-\delta(4-3Q_{max})}{2(1-2\delta)}$, when $\delta < 1/2$, $\frac{1-c_1}{2-\delta} < Q_{max} < \frac{5-3c_1-4\delta}{8-7\delta}$.

(ii) The quantity commitment binds strictly, the retailer sells the product only in period 1 with $p_1^* = \bar{\theta}^* = 1 - Q_{max}$, when

a) $\delta < 1/2$, $Q_{max} \leq \frac{1-c_1}{2-\delta}$,

b.1) $\delta \geq 1/2$, $c_1 \leq \frac{5(3-\delta)\delta-4}{2(2+5\delta)}$, $Q_{max} \leq K_4$,

b.2) $\delta \geq 1/2$, $c_1 > \frac{5(3-\delta)\delta-4}{2(2+5\delta)}$, $Q_{max} < \frac{4-3c_1-2\delta}{7-5\delta}$.

(iii) The quantity commitment binds weakly and the retailer sells the product in both periods with $p_1^* = \frac{1}{3}(4-\delta)(1-Q_{max})$ and $\bar{\theta}^* = \frac{4}{3}(1-Q_{max})$ when $\max\left(\frac{5-3c_1-4\delta}{8-7\delta}, \frac{4-3c_1-2\delta}{7-5\delta}\right) \leq Q_{max} \leq \frac{4(5-3c_1)-7\delta}{2(16-5\delta)}$.

(iv) The quantity commitment does not bind and the retailer sells the product in both periods with $p_1^* = \frac{(4-\delta)(4(1+c_1)-\delta)}{2(16-5\delta)}$, and $\bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta}$, when

a) $\delta < 1/2$, $c_1 < 1 - \frac{3\delta}{8}$, $Q_{max} > \frac{4(5-3c_1)-7\delta}{2(16-5\delta)}$

b.1) $\delta \geq 1/2$, $c_1 < \frac{5(3-\delta)\delta-4}{2(2+5\delta)}$, $K_4 < Q_{max}$

b.2) $\delta \geq 1/2$, $\frac{5(3-\delta)\delta-4}{2(2+5\delta)} \leq c_1 < 1 - \frac{3\delta}{8}$, $Q_{max} > \frac{4(5-3c_1)-7\delta}{2(16-5\delta)}$

When $c_1 \geq 1 - \frac{3\delta}{8}$, the retailer does not procure any units in period 1. Note that in all the cases above optimal quantity $Q_1^* = 1 - \bar{\theta}^*$. The threshold is given by $K_4 = \frac{1}{2}\left(1 - c_1 + \sqrt{\frac{\delta(3-c_1(2+5c_1)-\delta)}{16-5\delta}}\right)$ and is the point when the profits from the cases (ii) and (iv) are equal.

Given above optimal retailer strategies, supplier maximizes its first period profit. However, again the overall supplier objective function is not quasiconcave and we find the optimal strategies by considering each of the above cases separately, which results in the following optimal values.

(i) When $\delta < 1/2$, $c_1^* = 1/2$, $Q^*_{max} = \frac{1}{2(2-\delta)}$.

(ii) When $\delta \geq 1/2$, $c_1^* = \frac{2-\delta}{3}$, $Q_{max} = \frac{2-\delta}{7-5\delta}$.

Both of above optimal supplier strategies correspond to the case (ii) of optimal retailer strategies. Now to complete the proof we need to compare the profits. Following Proposition 23 and the above optimal values for the supplier; when $\delta < 1/2$ the difference for supplier profits $\Pi_S^{NC} - \Pi_S^{SQC} = -\frac{\delta(48-(18-\delta)\delta)}{64(2-\delta)(8-3\delta)} < 0$, because denominator is clearly positive and the numerator is also positive since $\delta \leq 4/5$. For retailer profits $\frac{d(\Pi_R^{NC} - \Pi_R^{SQC})}{d\delta} = \frac{19}{2304} + \frac{1}{2(2-\delta)^3} + \frac{16}{9(8-3\delta)^3} - \frac{1}{4(2-\delta)^2} > 0$, so the difference is increasing but $\left(\Pi_R^{NC} - \Pi_R^{SQC}\right)\big|_{\delta=0} = 0$, hence $\Pi_R^{NC} > \Pi_R^{SQC}$. The result for the supply chain profit when $\delta < 1/2$ follows from these. When $\delta \geq 1/2$, for the supplier profits $\frac{d\left(\Pi_S^{NC} - \Pi_S^{SQC}\right)}{d\delta} = \frac{59}{960} + \frac{4}{3(8-3\delta)^2} - \frac{3}{5(7-5\delta)^2} > 0$, so the difference is increasing, however $\left(\Pi_S^{NC} - \Pi_S^{SQC}\right)\big|_{\delta=1/2} = -\frac{157}{4992}$ and $\left(\Pi_S^{NC} - \Pi_S^{SQC}\right)\big|_{\delta=4/5} = -\frac{43}{2800}$, so $\Pi_S^{NC} < \Pi_S^{SQC}$. For the retailer profits, $\frac{d\left(\Pi_R^{NC} - \Pi_R^{SQC}\right)}{d\delta} = -\frac{673}{11520} - \frac{6}{5(7-5\delta)^2} + \frac{16}{9(8-3\delta)^3} + \frac{18}{5(7-5\delta)^3} < 0$, so the difference is decreasing. $\left(\Pi_R^{NC} - \Pi_R^{SQC}\right)\big|_{\delta=1/2} = \frac{10475}{778752}$ and $\left(\Pi_R^{NC} - \Pi_R^{SQC}\right)\big|_{\delta=4/5} = -\frac{477}{78400}$, so the difference becomes zero once in this delta range. The delta value when the difference function is zero is given by the solution to the following equation, which explicitly characterizes $\bar{\delta}_5$.

$$\frac{3}{256}\left(\frac{7-5\delta}{8-3\delta}\right)^2 = \frac{(2-\delta)(1+5\delta(1-\delta))}{1024-\delta(576-\delta(32+19\delta))} \tag{45}$$

Therefore, $\Pi_R^{NC} \geq \Pi_R^{SQC}$ when $\delta \geq \bar{\delta}_5$. For the supply chain profit, $\frac{d\left((\Pi_S^{NC}+\Pi_R^{NC})-(\Pi_S^{SQC}+\Pi_R^{RQC})\right)}{d\delta} = -\frac{9}{5(7-5\delta)^2} + \frac{7}{2304} + \frac{4}{3(8-3\delta)^2} + \frac{16}{9(8-3\delta)^3} + \frac{18}{5(7-5\delta)^3} < 0$, so the difference function is decreasing. However, $\left((\Pi_S^{NC} + \Pi_R^{NC}) - (\Pi_S^{SQC} + \Pi_R^{RQC})\right)\big|_{\delta=1/2} = -\frac{14017}{778752}$ therefore $\Pi_S^{NC} + \Pi_R^{NC} < \Pi_R^{SQC} + \Pi_R^{RQC}$, which completes the proof for case (iii) of the Theorem 10.

When supplier commits to wholesale prices retailer has no incentive to carry inventory since carrying inventory does not change c_2. Therefore when inventory carrying is not allowed the retailer's actions does not change and the analysis in section 7.2 holds. Specifically the Propositions 6 and 7 continue to hold. To complete the proof we need to compare the profits. Following Propositions 6 and 23 when $\delta \le 1 - \frac{1}{\sqrt{17}}$ then difference $\Pi_S^{NC} - \Pi_S^{SPC} = \frac{\delta(\delta(288-109\delta)-192)}{64(8-3\delta)(16-13\delta)} < 0$, because the denominator is clearly positive and the term in paranthesis in the numerator is a concave increasing function with derivative $288 - 218\delta$ but the value at upperbound is $-\frac{10}{17}\left(33+7\sqrt{17}\right)$. For the retailer profit in this delta range $\frac{d\left(\Pi_R^{NC} - \Pi_R^{SPC}\right)}{d\delta} = \frac{4363}{389376} + \frac{45}{169(16-13\delta)^2} - \frac{16}{9(-8+3\delta)^3} + \frac{64}{169(-16+13\delta)^3} > 0$, so the difference function is increasing but $(\Pi_R^{NC} - \Pi_R^{SPC})\big|_{\delta=0} = 0$, thus $\Pi_R^{NC} > \Pi_R^{SPC}$. For the supply chain profit in the same delta range, $\frac{d^2\left((\Pi_S^{NC}+\Pi_R^{NC})-(\Pi_R^{SQC}+\Pi_R^{RQC})\right)}{d\delta^2} = \frac{2}{13(16-13\delta)^3}\left(19 - \frac{96}{16-13\delta}\right) + \frac{8}{(8-3\delta)^3}\left(1+\frac{2}{8-3\delta}\right) > 0$, so the difference function is convex. $\left((\Pi_S^{NC}+\Pi_R^{NC}) - (\Pi_R^{SQC}+\Pi_R^{RQC})\right)\big|_{\left\{\delta=0,\delta=\frac{1}{3},\delta=1-\frac{1}{\sqrt{17}}\right\}} = \left\{0, -\frac{11153}{8467200}, \frac{59061\sqrt{17}-241757}{735488}\right\} \approx \{0, -0.0013, 0.0024\}$. So the difference function becomes 0 once. The delta value when the difference function is 0 is given by the solution to the following equation, which explicitly characterizes $\bar{\delta}_6$.

$$64\delta(512 - 3\delta(624 - 775\delta)) = (74912 - 12641\delta)\delta^4 \tag{46}$$

Therefore, $\Pi_S^{NC} + \Pi_R^{NC} \ge \Pi_S^{SPC} + \Pi_R^{SPC}$, when $\delta \le \bar{\delta}_6$. The proof for the results when $\delta > 1 - \frac{1}{\sqrt{17}}$ is similar and omitted, which completes the proof for the case (iv) of the Theorem 10.

Theorem 11 To find the equilibrium in simultaneous quantity commitment scenario, it is sufficient to compare the equilibriums of individual quantity commitment scenarios and identify the case when retailer would deviate from supplier's quantity commitment equilibrium. In other words retailer would deviate from supplier's quantity commitment equilibrium when restricting the equilibrium commitment quantity further and when this action increases its profit. This happens when $\delta \le \bar{\delta}_1$ where equilibrium commitment quantity in retailer's quantity commitment is $\frac{1}{4}$ which is less than corresponding equilibrium commitment quantity in supplier's quantity commitment, which is $\frac{1}{2(2-\delta)}$. In addition, in this range $\Pi_R^{RQC} > \Pi_R^{SQC}$ because $\frac{1}{16} < \frac{1-\delta}{4(2-\delta)^2}$. Therefore, the Theorem 11 follows from Propositions 4 and 10.

Proposition 15 Since, supplier commits to c_2 before retailer orders the first period quantity retailer has no incentive to carry inventory. Therefore, $Q_1^* = 1 - \bar{\theta}^*$. Also, since retailer commits to p_1 and p_2 at the beginning of period 1 the marginal customer, $\bar{\theta}$ becomes $\frac{p_1-p_2}{1-\delta}$. Plugging these into retailer's objective function, it becomes $\frac{\delta p_1(1-c_2-\delta-p_1)-c_1\delta(1-\delta-p_1+p_2)+(c_2+2\delta p_1)p_2-p_2^2}{(1-\delta)\delta}$, which is jointly concave in p_1 and p_2. KKT conditions yield the following strategies for the retailer.

(i) When $c_2 \le c_1 + \delta - 1$, the retailer sells only in period 2 where $p_1^* = \frac{2+c_2-\delta}{2}$ and $p_2^* = \frac{c_2+\delta}{2}$.

(ii) When $c_1 + \delta - 1 < c_2 < \delta c_1$, the retailer sells in both periods where $p_1^* = \frac{1+c_1}{2}$ and $p_2^* = \frac{c_2+\delta}{2}$.

To characterize the equilibrium we need to solve the supplier's problem. Supplier maximizes $c_1(1-\bar{\theta}^*) + c_2(\bar{\theta}^* - \frac{p_2^*}{\delta})$, where p_1^*, p_2^* and $\bar{\theta}^*$ are given above. This gives the optimal solution $c_1^* = \frac{1}{2}$ and $c_2^* = \frac{\delta}{2}$, which forces retailer to sell only in period 1 by making second period price too high. Therefore the equilibrium becomes the same as the one in Section 6.2.

Lemma 10 There may be two scenarios where supplier makes a commitment in period 1. The first scenario is when it makes a commitment only in period 1 and the second is when it makes separate quantity commitments in both periods. Proving that supplier does not benefit from the former scenario is easy, we skip that and present the proof for the latter.

The analysis for the scenario when supplier makes a commitment only in period 2 is presented in Appendix E. In this case since supplier makes a commitment in both of the periods Lemma 11 holds. In addition since we are only interested in the commitment binding solutions it is sufficient to consider only the cases where (36) is satisfied. Moreover, customers' belief about the first period quantity i.e. $\hat{Q}_1$ is Q_{1max} and in order there would be an equilibrium this should also be the optimal first period quantity chosen by the retailer. Hence the following is the reatailer's first period optimization problem.

$$\begin{aligned}
\max_{Q_1,\bar{\theta}} \quad & [1-\bar{\theta}]p_1(\bar{\theta}, Q_{1max}, Q_{2max}) - Q_1 c_1 + \Pi_{R,2}(\bar{\theta}, Q_1, Q_{2max}) \\
\text{s.t.} \quad Q_{2max} & < \frac{1}{4}(2(1-Q_1) - \bar{\theta}), \\
Q_1 & \le Q_{1max} \\
(Q_1, \bar{\theta}) & \in S_1'.
\end{aligned}$$

Note that retailer's objective function is jointly concave in Q_1 and $\bar{\theta}$, so KKT conditons are necessary and sufficient. The following are retailer's equilibrium strategies.

(i) When $c_1 \le \frac{\delta(3-4Q_{1max}-\delta(3-4Q_{1max}+Q_{2max}))}{2(1-\delta)}$ and $\frac{(1-\delta)(1-2Q_{1max})}{\delta} \le Q_{2max} \le \frac{(1-\delta)(3-4Q_{1max})}{8-7\delta}$, the retailer sells the product in both periods and it carries inventory between periods where $Q_1^* = Q_{1max}$, $p_1^* = \frac{1}{2}(1+\delta(1-2Q_{1max}-Q_{2max}))$ and $\bar{\theta}^* = \frac{1-\delta(1-Q_{2max})}{2(1-\delta)}$.

(ii) When $c_1 \le 1-(2-\delta)Q_{1max} - \delta Q_{2max}$ and $Q_{2max} \le \max\left(\frac{1-Q_{1max}}{4}, \frac{(1-\delta)(1-2Q_{1max})}{\delta}\right)$ the retailer sells the product in both periods and it does not carry inventory between periods where $Q_1^* = Q_{1max}$, $p_1^* = 1-Q_{1max}-\delta Q_{2max}$ and $\bar{\theta}^* = 1-Q_{1max}$.

Following Lemma 11 and the above regions and optimal values for the retailer we can solve for supplier's optimization problem in terms of c_1, Q_{1max} and Q_{2max}. Note that supplier's objective function is linear and increasing in c_1, hence the optimal c_1 is at the boundary. This simplifies the solution and the optimal comes from the second region given above where $c_1^* = \frac{1}{2}$, $Q_{1max}^* = \frac{2-\delta}{2(4-3\delta)}$ and $Q_{2max}^* = \frac{1-\delta}{2(4-3\delta)}$, which is the same optimal solution when supplier makes a quantity commitment only in period 2, which is presented in Appendix E. Therefore, supplier does not benefit from making a quantity commitment in first period when it makes a quantity commitment in second period.

Lemma 11 This proof is similar to proof of Lemma 5. First part of Equation (35) leads to $\delta\left(2(1-Q_1-Q_{2max})-\bar{\theta}\right)-c_2 \ge 0$. Since the commitment binds this inequality binds. Hence, $\bar{c_2} = c_2^* = \delta\left(2(1-Q_1-Q_{2max})-\bar{\theta}\right)$. Plugging this expression into the second part of Equation (35) yields $Q_{2max} \le \frac{1}{4}(2(1-Q_1)-\bar{\theta})$.

Proposition 13 When $Q_{2max} = \frac{1}{4}(2(1-Q_1)-\bar{\theta})$ the objective function given in (37) is the same as the no commitment objective given by (12). Therefore, when finding the solution of retailer's objective in the region when commitment binds we can restrict the feasible space to where the inequality in (36) is strictly satisfied.

Following Lemma 11 we can formulate retailer's period 1 problem in terms of order quantity Q_1, commitment Q_{2max} and target consumer segment $[\bar{\theta}, 1]$ in period 1 as in the following:

$$\begin{aligned} \max_{Q_1,\bar{\theta}} \quad & p_1(\bar{\theta}, \hat{Q}_1, Q_{2max})[1-\bar{\theta}] - Q_1 c_1 + \Pi_{R,2}(\bar{\theta}, Q_1, Q_{2max}) \\ \text{s.t.} \quad Q_{2max} & < \frac{1}{4}(2(1-Q_1)-\bar{\theta}), \\ \hat{Q}_1 = Q_1^* & \\ (Q_1,\bar{\theta}) & \in S_1^{'}. \end{aligned}$$

Note that $S_1^{'}$ is defined in (15). The retailer's total profit in equation (37) can be written as the following.

$$\Pi_R(Q_1,\bar{\theta},\hat{Q}_1) = \delta\left(Q_{2max}^2 + \bar{\theta} - Q_1(Q_1+\bar{\theta}-2)-1\right) - (1-\bar{\theta})(\delta(\hat{Q}_1+Q_{2max}+\bar{\theta}-1)-\bar{\theta}) - c_1 Q_1. \tag{47}$$

Hessian of (47) is $\begin{pmatrix} -2\delta & -\delta \\ -\delta & -2(1-\delta) \end{pmatrix}$, where the eigenvalues of the hessian are $-1-\sqrt{1-\delta(4-5\delta)}$ and $-1+\sqrt{1-\delta(4-5\delta)}$. Both of the eigenvalues are non-positive for $\delta \le 4/5$, hence (47) is jointly concave in $(Q_1,\bar{\theta})$.

We showed that retailer's objective function in the no-commitment scenario is jointly concave in the Proof of Proposition 1. Given the two functions are continuous at the boundary and both jointly concave, and the overall problem is quasiconcave; KKT conditions together with the belief consistency equation fully characterizes the solution, which yield the regions and the optimal values in Proposition 13.

Proposition 14 Π_S defined by all the parts of the proposition 13 is not quasiconcave. Therefore, to find the global optimal we need to analyze each part separately.

For the first part of the proposition 13, $\Pi_S = \frac{1}{4}\left(c_1\left(3-\frac{2c_1}{\delta}\right) + \frac{c_1(4-5\delta)Q_{2max}}{1-\delta} - 8\delta Q_{2max}^2\right)$, which is a jointly concave function of (c_1, Q_{2max}). First order condition gives $\frac{48(1-\delta)^2\delta}{48-\delta(88-39\delta)}$, $\frac{3(1-\delta)(4-5\delta)}{48-\delta(88-39\delta)}$ for c_1 and Q_{2max} respectively. However, these values do not satisfy the feasible region defined in first part of the proposition 13. In addition, Π_S is continuous in both of the boundaries of the feasible region therefore global optimal solution does not come from this part.

For the second part of proposition 13, $\Pi_S = \frac{(1-c_1)c_1+\delta Q_{2max}(1-\delta-(4-3\delta)Q_{2max})}{2-\delta}$, which is also a jointly concave function of (c_1, Q_{2max}). First order condition gives $c_1^* = \frac{1}{2}$ and $Q_{2max}^* = \frac{1-\delta}{2(4-3\delta)}$. These values satisfy the feasible region defined in second part of the proposition 13. Hence, unless any other solution from the remaining parts gives a better objective value this is the global optimal point.

For the third part of the proposition 13, $\Pi_S = \delta(1-2Q_{2max})Q_{2max}$, which is a function of Q_{2max}. Hence the optimal solution has to be at the boundary but Π_S is continuous at this boundary between parts (ii) and (iii) of proposition 13 and part (ii) has an interior solution. Therefore, the global solution cannot come from part three.

For the fourth part of the proposition 13, $\Pi_S = \frac{c_1(6(1-\delta)-c_1)-2c_1(8-9\delta)Q_{2max}+16(1-\delta)\delta Q_{2max}^2}{8(1-\delta)}$, which is a quadratic convex function of Q_{2max}. Therefore, the solution has to be at one of the boundaries. In addition, both of the boundaries are continuous. First boundary is the one with part (i), which is dominated. We analyze the second boundary in the next part.

For the fifth part of the proposition 13, $\Pi_S = (1-4Q_{2max})c_1 + 2\delta Q_{2max}^2$, which is a quadratic convex function of Q_{2max}. Hence, the solution has to be on the boundary. The boundary with part (ii) of the proposition 13 is continuous. Since there is an interior solution in that part no solution can come from this boundary. The boundary with the fourth part of proposition 13 is also continuous, however never produces a solution.

The sixth and seventh parts of the proposition 13 are functions of c_1 so we can consider only the solutions at the Q_{2max} boundaries, however both are dominated at those regions.

Therefore the solution in Proposition 14 follows.

Corollary 2 Following Propositions 10 and 14 we have four cutoff points for δ. Below we present the proof separately for each region. For every region $\Pi_S^{SQC2} = \frac{2-\delta^2}{4(4-3\delta)}$.

- When $\delta < \frac{1}{2}$, $\Pi_S^{SQC} = \frac{1}{4(2-\delta)}$ and $\Pi_S^{SQC2} - \Pi_S^{SQC} = \frac{(1-\delta)^2\delta}{4(2-\delta)(4-3\delta)}$, which is clearly positive.
- When $1/2 \leq \delta < \frac{199-3\sqrt{137}}{218}$, $\Pi_S^{SQC2} - \Pi_S^{SQC} = \frac{\delta(82-\delta(85-27\delta))-22}{12(7-5\delta)(4-3\delta)}$, which is an increasing function. In addition it is positive at both end points of the feasible region.
- When $\frac{199-3\sqrt{137}}{218} \leq \delta < \bar{\delta}_7$ $\Pi_S^{SQC2} - \Pi_S^{SQC} = \frac{\delta(12-7\delta)}{64(4-3\delta)}$, which is clearly positive.
- When $\bar{\delta}_7 \leq \delta$, $\Pi_S^{SQC2} - \frac{1}{5} = \frac{\delta(12-5\delta)-6}{20(4-3\delta)} > 0$ since $\delta > \frac{1}{5}\left(6-\sqrt{6}\right)$. For every Q_{max} and for every c_1 in the region defined by the case (iii).b.2 of Proposition 9 ie. $K_c < c_1 \leq \frac{3\delta(4-3\delta)}{2(16-15\delta)}$ in order for $\Pi_S^{SQC} - \frac{1}{5} > 0$ we need $\delta > \frac{1}{210}\left(265-\sqrt{9745}\right)$ and $Q_{max} > 1 - \frac{3c_1}{\delta} + \frac{3\sqrt{\frac{10c_1^2+\delta-5c_1\delta}{\delta^2}}}{\sqrt{10}} > \frac{9}{20}$. However $K_2 < \frac{9}{20}$ in that region. Therefore $\Pi_S^{SQC} < \frac{1}{5}$ and the result follows.

Lemma 12 This proof is very similar to the proof of Lemma 1. The constraint given by the Set (38) binds thus $p_2 = 1 - Q_1 - Q_2$. With this p_2, the following becomes a concave optimization problem of Q_2.

$$(p_2^*, Q_2^*) = \arg \max_{(p_2,Q_2)\in\{(p_2,Q_2):Q_1+Q_2\geq 1-p_2\}} [p_2(\bar{\theta}-p_2) - c_2Q_2]$$

First order condition yields the optimal values in part (i) of the Lemma 12.

Similarly with above optimal values the following becomes a concave optimization problem in c_2.

$$c_2^* = \arg\max_{c_2} c_2 Q_2^*,$$

where the first order condition gives the optimal c_2 in part (ii) of the Lemma 12.

Proposition 16 This proof is very similar to the proof of Proposition 1. Following Lemma 12 retailer's profit function can be written in terms of Q_1 and $\bar{\theta}$, which has an Hessian equal to $\begin{pmatrix} -\frac{3}{8}(5-4d) & -\frac{3}{4} \\ -\frac{3}{4} & -\frac{3}{2} \end{pmatrix}$. Therefore, the objective function is jointly concave in $(Q_1, \bar{\theta})$ and KKT conditions fully characterize the equilibrium presented in the Proposition.

Proposition 17 Each part of the supplier's optimization problem corresponding to the cases presented in Proposition 16 are concave with respect to c_1. Since each of these subproblems has one variable and one parameter, δ, defining the optimal solution is straighforward. However, the overall objective function is not quasiconcave.Therefore one needs to compare the solutions from each case, where the result follows from comparing the cases (i) and (ii).

Lemma 13 See proof of Lemma 5.

Proposition 18 This proof is very similar to the proof of Proposition 3. Retailer's profit function can be written as the following

$$(Q_{max} - 1 + \bar{\theta})(Q_{max} + (1-\delta)(1-\bar{\theta})) - Q_1(\bar{\theta} + c_1 + 2(Q_{max} - 1)),$$

which is a quadratic convex function of Q_{max}. Hence the optimal Q_{max} must be at one of the two end points of the objective function. Following the proof of Proposition 3 we separate the problem into two case where

Case A refers to the case when Q_{max} is at lowerbound i.e. $Q_{max} = Q_1$ and *Case B* refers to the case when Q_{max} is at uppebound i.e. $Q_{max} = \frac{Q_1+1}{2} - \frac{\bar{\theta}}{4}$. In both cases the resulting functions are jointly concave in $(Q_1, \bar{\theta})$ so KKT conditions characterize the local solutions. To find the global solution we compare the local solutions, where the threshold K_{c_2} is the positive root of the difference function of the local interior solutions of the Cases A and B.

Proposition 19 This proof is very similar to the proof of Proposition 4. We follow the same approach. Since overall supplier profit function coressponding to the strategies presented in Propostion 18 is not quasi-concave we need to analyze each case separately. To find the global solution we compare the solutions from each case. Note that in each case supplier profit function is concave in c_1, which makes the subproblems easy to characterize. We skip the details and present only the result. Solutions from cases (*ii*) and (*iii*) of Propostion 18 dominate the rest. Specifically when delta is smaller than the first root of the following equation, which characterizes $\bar{\delta}_8$ specifically,

$$261 - 2\delta(307 - 2\delta(98 - \delta(15 - 2\delta))) = 0, \tag{48}$$

optimal solution comes from the second case of Proposition 18.

Lemma 14 The proof of this result is similar to the proof of Lemma 1 and it is omitted.

Proposition 20 This proof is very similar to the Proof of Proposition 6. There are two strategies for the retailer depending on the value of Q_2^*, which is a function of Q_1 and $\bar{\theta}$.

When $Q_2^* > 0$, the retailer profit in (25) and Lemma 14 lead to the following,

$$\Pi_R = -\frac{1}{4}(3 - 2\delta)\bar{\theta}^2 + \left(\frac{1}{2}(2 + c_2 - (1 + c_2)\delta)\right)\bar{\theta} + \frac{1}{4}(c_2(c_2 + 2\delta - 4(1 - Q_1)) - 4c_1Q_1). \tag{49}$$

When $Q_2^* = 0$, the retailer profit in (25) becomes $\Pi_R = (1 - \bar{\theta})\bar{\theta} - c_1 Q_1$.

In both of the regions Π_R is jointly concave in $(\bar{\theta}, Q_1)$ and Π_R is quasiconcave across these two regions. However, no solution comes from the second part and KKT conditons on the first part gives the regions presented in the Proposition 20.

Proposition 21 To find the supplier's optimal policy we need to analyze the supplier function in each case of the Proposition 20. Supplier function from the first case is a function of c_2 since $\bar{\theta}^* = 1$ in that case. Supplier function from the second case is jointly concave in c_1 and c_2 therefore overall optimization problem is quasiconcave. KKT conditions give the result in Proposition 21.

Lemma 15 This proof is the same as the proof of Lemma 1 given $Q_1 = 1 - \bar{\theta}$.

Proposition 22 Following Lemma 15 and the fact that $Q_1 = 1 - \bar{\theta}$, retailer's profit funtion is concave in $\bar{\theta}$ and the KKT conditions yield the result.

Proposition 23 Overall supplier's profit function is quasiconcave for both cases of Propostion 22. Hence KKT conditions yield the result.

Lemma 9 Similar to proof of lemma 1 both of the profit functions are concave and first order conditions yield the results.

Proposition 11 Retailer's objective function is jointly concave in Q_1, p_1 and KKT conditions yield the optimal values and the regions.

Proposition 12 Supplier's objective function is concave in each part and quasiconcave overall, hence the first order condition yield the result.

Corollary 1 Following propositions 2 and 12 there is one cutoff point for δ hence we have two cases to consider. Below we present the proof for the result for supplier's proft. Proof for the supply chain and the retailer profits are similar. When $\delta \leq \frac{16}{21}$ the difference $\Pi_S^{NC^M} - \Pi_S^{NC} = \frac{(8+\delta)^2}{64(8-\delta)} - \frac{8+3\delta}{64} = \frac{d^2}{16(8-d)}$, which is positive. When $\delta > \frac{16}{21}$ $\frac{d\left(\Pi_S^{NC^M} - \Pi_S^{NC}\right)}{d\delta} = \frac{2028}{25(32-25d)^2} + \frac{4}{(8-d)^2} - \frac{3}{7(8-7d)^2} - \frac{4531}{11200} > 0$, so the difference function is increasing and $\left(\Pi_S^{NC^M} - \Pi_S^{NC}\right)\Big|_{\delta=\frac{16}{21}} = \frac{2}{399} > 0$, hence $\Pi_S^{NC^M} > \Pi_S^{NC}$.

Lemma 16 Since all the market is served $Q_2 = 1 - Q_1$. Then, $\Pi_{R,2} = p_2(1 - \delta) - c_2(1 - Q_1)$, which is linear with repsect to p_2. Therefore, $p_2^* = \delta\theta$, which is the maximum it can get. Since Q_2 is independent from c_2 for any $Q_2 > 0$ supplier has no incentive to set c_2 less than optimal p_2. Then $\Pi_{R,2} = (Q_1 - \delta)\delta\theta$ and $\Pi_{S,2} = (1 - Q1)\delta\theta$.

Proposition 24 Following the marginal customer equation Π_{1R} can be written as $(\delta\theta - c_1)Q_1 + (1 - \delta)\delta\theta$, first order condition gives $Q_1 = \delta = 1$. Substituting for supplier's objective yields the optimal values given in Proposition 24.

Theorem 12 When retailer commits to both price and quantity retailer sets p_2 before supplier sets second period wholesale price. Therefore supplier has no incentive to set c_2 smaller than p_2. Then $Q_2^* = Q_{max} - Q_1$, $p_2^* = \delta(1-Q_{max})$ and $c_2^* = p_2^*$, which yields $\Pi_{R,2} = \delta(1-Q_{max})(Q_1+\theta-1)$ and $\Pi_{S,2} = \delta(1-Q_{max})(Q_{max}-Q_1)$.

Given above second period optimal values and following the marginal customer equation retailer's first period objective is $\Pi_{R,1} = Q_1(\delta(1-Q_{max}) - c_1) + (1-\delta)\theta - (1-\delta)\theta^2$, which is a linear decreasing function with respect to Q_{max}. Hence, Q_{max} should be at the lower bound. Substituting Q_{max} with Q_1; $\Pi_{1,R}$ becomes jointly concave with respect to Q_1 and θ, and first order condition yields $Q_{max}^* = Q_1^* = \frac{1-c_1}{2}$ and $p_1^* = \frac{1+c_1}{2}$ in period 1.

For supplier's problem; following retailer's best response $\Pi_{S,1} = \frac{1}{2}(1-c_1)c_1$, where first order condition yields $c_1^* = \frac{1}{2}$. Note that equilibrium is the same as retailer's price commitment, therefore Theorem 3 follows.

Theorem 13 The following characterizes the optimal values when supplier commits to both a maximum total quantity and the wholesale prices: $Q_{max}^* = \frac{5-4\delta}{16-13\delta}$, $c_1^* = \frac{16-\delta(14-\delta)}{2(16-13\delta)}$, $c_2^* = \frac{2\delta(5-4\delta)}{16-13\delta}$, and $\Pi_S^{SPQC} = \frac{4-\delta(2+\delta)}{2(16-13\delta)}$.

First let's show that this profit is not better than what supplier gets from a quantity commitment only. We do this by showing $\Pi_S^{SPQC} < \Pi_S^{NC}$, since by Theorem 4 $\Pi_S^{NC} \leq \Pi_S^{SQC}$. Following Proposition 2 when $\delta \leq \frac{16}{21}$; $\Pi_S^{NC} - \Pi_S^{SPQC} = \frac{\delta(8-7\delta)}{64(16-13\delta)}$, which is clearly positive. When $\delta > \frac{16}{21}$; $\Pi_S^{NC} = \frac{d(4-3\delta)(14-15\delta)}{(16-15\delta)^2}$ where second derivative is $\frac{48(32-75\delta)}{(16-15\delta)^4}$, which is negative since $\delta > \frac{32}{75}$. Hence, Π_S^{NC} is concave in this region. In addition it is equal to $\frac{9}{56}$ and $\frac{4}{25}$ at the boundaries respectively. Therefore $\Pi_S^{NC} \geq \frac{4}{25}$ but $\Pi_S^{SPQC} < \frac{4}{25}$ since $\delta < \frac{1}{25}\left(27-\sqrt{29}\right)$.

Second let's show how we determined above optimal values. Without loss of generality we can consider that the supplier makes an overall quantity commitment on top of it's price commitment. Instead of characterizing the full equilibrium we determine when quantity commitment binds. Following Lemma 7 in Section 7.2 this outcome happens when $\frac{\delta(2-2Q_1-\bar{\theta})-c_2}{2\delta} = Q_{max} - Q_1$. Furthermore, combining Equation (25) and Lemma 7 leads to the following retailer objective function.

$$\Pi_R = -(1-\frac{3\delta}{4})\bar{\theta}^2 - (c_1-c_2)Q_1 - \frac{1}{4}(2c_2 - 2\bar{\theta}(2-\delta) - \frac{c_2^2}{\delta}).$$

We find the corresponding retailer optimal prices and the regions: $\bar{\theta} = 2(1-Q_{max}) - \frac{c_2}{\delta}$, $Q_1 = 1-\bar{\theta}$ when $c_2 \leq \delta$, $\frac{1}{2}(1-\frac{c_2}{\delta}) \leq Q_{max} \leq 1-\frac{c_2}{\delta}$ and $c_1 \leq 3 - \frac{5\delta}{2} + \left(\frac{5}{2}-\frac{2}{\delta}\right)c_2 - (4-3\delta)Q_{max}$.

Substituting these expressions into the supplier objective and maximizing in the given region yields the values presented above.

Appendix L: Equilibrium results for $\delta > 4/5$

In our main model, we assume $\delta \leq 4/5$. This assumption preserves concavity of profit functions and helps us avoid considering many boundary cases. This assumption essentially implies that the product's value decreases at least by 20% in the second period, which is a reasonable assumption for many product categories.

We can still characterize the equilibrium for $\delta > 4/5$, but this analysis requires considering many additional equilibrium regions (including the unrealistic regions that no sales are made in period 1). Having these additional regions in the main body of our paper would needlessly complicate our exposition without contributing to our insights. Our goal in this paper is to show that decentralization can make commitments significantly less valuable than one would expect based on the insights of a centralized model. We believe restricting our analysis to $\delta \leq 4/5$ case enables us to keep our focus on our key message without having to deal with many additional (and unrealistic) cases which may take the focus away from this key message.

In this appendix, we illustrate how these additional regions emerge by stating the equilibrium results for no-commitment and the retailer's quantity commitment scenarios. We also consider the retailer's price commitment scenario (equilibrium results do not change at all in that case). We show how our results on the value of commitments extend to $\delta > 4/5$ case for these examples. The proofs of these results are quite similar to the proofs of our results in the main body of the paper, therefore, we do not replicate those proofs here.

Let us begin with no-commitment scenario. The retailer's and the supplier's period 2 choices do not change and they are stated in Lemma 1. Proposition 1 gets extended as follows.

PROPOSITION 25. *The SPNE order quantity Q_1^* and retail price p_1^* and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.*

(i) When $c_1 < \min(\frac{3\delta(4-3\delta)}{2(16-15\delta)}, 6(1-\delta))$, the retailer sells the product in both periods, and it carries inventory between periods, where $Q_1^* = \frac{11}{14} - \frac{c_1(112-91\delta)+6\delta}{21\delta(8-7\delta)}$, $p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-7\delta)}$, and $\bar{\theta}^* = \frac{3(4-3\delta)+2c_1}{3(8-7\delta)}$.

(ii) When $\frac{3\delta(4-3\delta)}{2(16-15\delta)} \le c_1 < \frac{8-3\delta}{8}$*, the retailer sells the product in both periods and it does not carry inventory between periods, where* $Q_1^* = \frac{8(1-c_1)-3\delta}{16-9\delta}$, $p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}$*, and* $\bar{\theta}^* = \frac{8(1+c_1)-6\delta}{16-9\delta}$.

(iii) When $c_1 \ge \max(\frac{8-3\delta}{8}, \frac{3\delta}{4})$*, the retailer sells the product only in period 2, and it does not carry inventory between periods, where* $Q_1^* = 0$*, and* $\bar{\theta}^* = 1$.

(iv) When $6(1-\delta) \le c_1 < 3\delta/4$*, the retailer sells the product only in period 2, but it does carry inventory between periods, where* $Q_1^* = \frac{1}{2} - \frac{2c_1}{3\delta}$, $p_1 = 1 - (\frac{\delta}{2} - \frac{c_1}{3})$*, and* $\bar{\theta}^* = 1$.

There is now an additional region (region iv) in which retailer buys inventory in period 1 but it does not sell any units in period 1, it sells only in period 2. Please also note that the threshold of regions (i) and (iii) now have min and max expressions which get simplified to the corresponding expressions in Proposition 1 when $\delta \le 4/5$.

Proposition 2 that describes the supplier's equilibrium prices also gets additional regions.

PROPOSITION 26. *The SPNE wholesale price* c_1 *in period 1 is as follows.*
(i) When $\delta \le \frac{16}{21}$, $c_1^* = \frac{16-3\delta}{32}$.
(ii) When $\frac{16}{21} < \delta \le \frac{4}{5}$, $c_1^* = \frac{3\delta(4-3\delta)}{2(16-15\delta)}$.
(iii) When $\frac{4}{5} < \delta \le \frac{4(79-\sqrt{61})}{309}$, $c_1^* = \frac{9\delta(12-11\delta)}{4(32-35\delta)}$.
(iv) When $\delta > \frac{4(79-\sqrt{61})}{309}$, $c_1^* = \frac{9\delta}{16}$.

Specifically, regions (iii) and (iv) are added to Proposition 2. These regions map to regions (i) and (iv) of Proposition 25 respectively.

Now let us consider retailer's quantity commitment. The retailer's and the supplier's period 2 choices do not change and they are stated in Lemma 5. Proposition 3 describes the retailer's period 1 decisions. When $\delta > 4/5$, we need to add two additional regions to describe the retailer's choices. Here is the revised statement of this proposition.

PROPOSITION 27. *Suppose the retailer can make a quantity commitment. Its SPNE order quantity* Q_1^**, quantity commitment* Q_{max}^**, retail price* p_1^* *and the corresponding marginal customer* $\bar{\theta}^*$ *in period 1 are as follows.*

(i) The retailer sells the product only in period 1 and the quantity commitment binds strictly when $c_1 < \frac{\sqrt{16-5\delta}-1}{5}$ & $\delta \le \frac{95-\sqrt{385}}{9}$*;* $c_1 < \frac{\delta(2-2\delta+\sqrt{2(18-45\delta+37\delta^2-10\delta^3)})}{16-25\delta+10\delta^2}$ & $\frac{95-\sqrt{385}}{9} < \delta \le \frac{85-\sqrt{313}}{72}$ *and* $c_1 < \frac{\sqrt{3\delta(1-\delta)}}{\sqrt{4-3\delta}}$ & $\frac{85-\sqrt{313}}{72} < \delta \le 1$ *Furthermore,* $Q_1^* = \frac{1-c_1}{2}$, $Q_{max}^* = \frac{1-c_1}{2}$, $p_1^* = \frac{1+c_1}{2}$ *and* $\bar{\theta}^* = \frac{1+c_1}{2}$.

(ii) The retailer sells the product in both periods, it does not carry inventory between periods and the quantity commitment binds weakly when $\max(\frac{\sqrt{16-5\delta}-1}{5}, \frac{2\delta}{16-15\delta}) \le c_1 < \frac{8-3\delta}{8}$*. Furthermore,* $Q_1^* = \frac{8(1-c_1)-3\delta}{16-5\delta}$, $Q_{max}^* = \frac{2(10-6c_1)-7\delta}{2(16-5\delta)}$, $p_1^* = \frac{(4(1+c_1)-\delta)(4-\delta)}{2(16-5\delta)}$*, and* $\bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta}$.

(iii) The retailer sells the product only in period 2, it does not carry inventory between periods and the quantity commitment binds weakly when $c_1 \ge \max(\frac{8-3\delta}{8}, \frac{3\delta}{4})$*. Furthermore,* $Q_1^* = 0$, $Q_{max}^* = \frac{1}{4}$*, and* $\bar{\theta}^* = 1$.

(iv) The retailer buys and sells the product in both periods, it carries inventory between periods and the quantity commitment binds weakly when $\frac{\delta(2-2\delta+\sqrt{2(18-45\delta+37\delta^2-10\delta^3)})}{16-25\delta+10\delta^2} \le c_1 < \min(\frac{2\delta}{16-15\delta}, 6(1-\delta))$*. Furthermore,* $Q_1^* = \frac{1}{60}(36 - \frac{40c_1}{\delta} - \frac{6+5c_1}{6-5\delta})$, $Q_{max}^* = \frac{\delta(8-7\delta)-c_1(4-3\delta)}{2\delta(6-5\delta)}$, $p_1^* = \frac{6+5c_1-6\delta-4c_1\delta+\delta^2}{12-10\delta}$*, and* $\bar{\theta}^* = \frac{6-4\delta+c_1}{12-10\delta}$.

(v) The retailer sells the product only in period 2, it carries inventory between periods and the quantity commitment binds weakly when $\max(\frac{\sqrt{3\delta(1-\delta)}}{\sqrt{4-3\delta}}, 6(1-\delta)) \le c_1 < \frac{3\delta}{4}$.

Note that in addition to having two new regions (regions iv and v), the thresholds for c_1 now have min and max expressions. These thresholds reduce to their corresponding expressions in Proposition 3 when $\delta \le 4/5$.

The following proposition characterizes the supplier's equilibrium prices, which has two more regions than Proposition 4.

PROPOSITION 28. *Suppose the retailer can make a quantity commitment. The supplier's SPNE wholesale price* c_1 *in period 1 is as follows.*
(i) when $\delta \le \bar{\delta}_1$, $c_1^* = \frac{1}{2}$,
(ii) when $\bar{\delta}_1 < \delta \le 8/9$, $c_1^* = \frac{\sqrt{16-5\delta}-1}{5}$,
(iii) when $8/9 < \delta \le 148/161$, $c_1 = \frac{2\delta(50-86\delta+37\delta^2)}{128-204\delta+81\delta^2}$*, and*
(iv) when $148/161 < \delta \le 1$, $c_1 = \frac{9\delta}{16}$.

The new two regions, (iii) and (iv), map to regions (iv) and (v) of Proposition 27.

We can now study the value of retailer's quantity commitment for the retailer, the supplier and the supply chain. The following theorem extends Theorem 2 and states what happens when we relax our $\delta \leq 4/5$ assumption.

THEOREM 14. *There exists* $\bar{\delta}_2 \approx 0.799$, $\bar{\delta}_9 \approx 0.838$, *and* $\bar{\delta}_{10} \approx 0.878$. *such that*
(i) $\Pi_S^{NC} < \Pi_S^{RQC}$ *if and only if* $\bar{\delta}_{10} < \delta < 8/9$,
(ii) $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{RQC} + \Pi_R^{RQC}$,
(iii) $\Pi_R^{NC} < \Pi_R^{RQC}$ *if and only if* $\bar{\delta}_2 < \delta < \bar{\delta}_9$,

This theorem shows that the retailer's quantity commitment can continue to hurt itself, the supplier and the entire supply chain even when we allow $\delta > 4/5$.

Finally, we find that allowing $\delta > 4/5$ does not lead to any changes in our equilibrium results for the retailer's price commitment. Proposition 5 stays as is. Furthermore, when we compare equilibrium payoffs of the retailer, the supplier and the supply chain profits in no-commitment and retailer's price commitment scenarios we find that Theorem 3 continues to hold as is even when we allow $\delta > 4/5$.