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The Value of Commitments When Selling to Strategic Consumers: A Supply Chain Perspective

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We consider a decentralized supply chain consisting of a retailer and a supplier that serves forward-looking consumers in two periods. In each period, the supplier and the retailer dynamically set the wholesale and retail price to maximize their own profits.The consumers are heterogeneous in their evaluations of the product and are strategic in deciding whether and when to buy the product, choosing the option that maximizes their utility, including waiting for a price mark-down. We derive the equilibrium and study the value of price and quantity commitments from both the retailer's and the supplier's perspective. We find that, while a centralized system always benefits from making price and quantity commitments, this is not true for a firm in a decentralized supply chain due to how the other firm responds to these commitments. We show that the retailer suffers from making a price or quantity commitment and that, similarly, the supplier does not benefit from making a price commitment. In these cases, commitments can harm not only the firm itself but also profitability of the other firm in the supply chain, thereby disadvantaging the entire supply chain. This happens because such commitments aggravate double-marginalization inefficiency in the supply chain. Furthermore, we show eliminating this inefficiency through a coordinating contract (e.g., a two part tariff or quantity discount) makes commitments beneficial.

Key words: Strategic consumer behavior, dynamic pricing, decentralization, coordination

1. Introduction

Firms are dealing with increasingly sophisticated and better informed consumers who are becoming more prudent in their purchasing decisions. In one example of consumer savvy, many consumers strategically postpone their purchases, anticipating price mark-downs. Indeed recent reports show that a larger portion of sales are made at discount prices in each succeeding year (Rozhon 2004, O'Donnell 2006).

Existing work has shown that making commitments can be effective in dealing with strategic customer behavior (cf. Tirole 1988, Bulow 1982, Butz 1990). These studies show that committing to future prices (e.g., no mark-downs) or availability (e.g., no more than a limited quantity) can deter customers from waiting for price mark-downs. However, in spite of their assumed benefits, such commitments are not too prevalent in practice.

In this paper, we study the value of commitments to all parties in a decentralized supply chain. We find that supply chain relations constitute one reason why firms may shy away from making these commitments. It is well known that supply chains suffer from a lack of coordination (Cachon 2003). Wholesale price contracts, commonly used on account of their simplicity, are especially vulnerable to double-marginalization inefficiency (Arrow 1985, Lariviere and Porteus 2001, Cachon 2003, Iyer and Villas-Boas 2003). We show that commitments can further exacerbate the inefficiency of wholesale price contracts and hurt firm profitability, illustrating how commitments can be undesirable.

Our work demonstrates that the inefficiencies that result from a lack of coordination and of strategic customer behavior are related. When double marginalization inefficiency is not addressed, commitments, which aim to address strategic customer behavior, generally backfire and intensify double marginalization inefficiency. However, when double marginalization inefficiency is eliminated through a more sophisticated contract (e.g., a two-part tariff), commitments can reduce the loss due to strategic customer behavior and improve supply chain profits.

We explicitly model the interaction between the retailer and its supplier in addition to the relation between consumers and the retailer. Our model is closest to that of Besanko and Winston (1990). They consider a single seller; we extend their model to a supply chain. Specifically, we consider a supply chain that consists of a supplier and a retailer selling a product to consumers over two periods. In each period, first, the supplier sets its wholesale price; the retailer then decides its procurement quantity and sets its retail price; finally, the consumers make their purchasing decisions. While the supplier and the retailer set wholesale and retail prices dynamically to maximize their individual profits given the remaining consumers in the market, each consumer decides whether and when to buy the product to maximize her own utility based on her expectation of future prices. Customers are heterogeneous in their valuations of the product and they are willing to pay more to purchase a product in the first period, rather than wait to buy it in the second period. In particular, the product's value decreases over time. This may result, for example, from declining popularity (common for technology and fashion products) or seasonality (common for apparel). In addition to our base (no-commitment) model, we study alternative models in which the retailer or the supplier can make price or quantity commitments. We also consider the vertically integrated centralized system as a benchmark. Comparing these models allows us to tease out the value of commitments to both the supplier and the retailer; we also characterize the impact of these commitments on the other supply chain partner, as well as on the whole supply chain.

As expected, the centralized system benefits from its ability to make price and quantity commitments. Using these commitments, the centralized system limits the sale of its product to a single period, which in turn eliminates the opportunity for strategic customer behavior. In contrast, we show that, in a decentralized supply chain, the impact of commitments is more intricate because of how they affect the interactions between the retailer and the supplier. In particular, ignoring the relations within a supply chain can lead to overestimating the benefits of commitments.

We find that the retailer can become worse off from its ability to make price and quantity commitments. This happens because of how these commitments affect the supplier's pricing. For example, had the supplier's wholesale price stayed the same, the retailer would have benefited from making a quantity commitment at the expense of the supplier by limiting sales to only a single period, as in the centralized system. However, the supplier responds to this threat by increasing its period 1 wholesale price to force the retailer to sell the product in both periods, which in turn harms the retailer's profitability. Similarly, when the retailer can make a price commitment, the supplier sets a higher period 1 wholesale price, hurting the retailer. Interestingly, the retailer's commitments hurt not only itself but also the supplier, thus leaving the whole supply chain worse off.

In our model the power lies with the supplier, as it moves first and sets a take-it-or- leave-it wholesale price. Therefore, one would expect the supplier to benefit from its commitments. Indeed, the supplier benefits from its ability to make a quantity commitment. Intuitively, the supplier can always nullify its commitment by choosing an arbitrarily large commitment quantity, ensuring that it never becomes worse off. In contrast, however, we find that the supplier, too, can become worse off by making a price commitment because of how this commitment affects the retailer's ordering policy. When the supplier does not commit to a period 2 wholesale price, the retailer has an incentive to buy more in period 1, as it results in a lower period 2 wholesale price. However, price commitment eliminates this incentive. We find that the supplier's price commitment hurts the retailer and the whole supply chain as well.

In these examples, commitments hurt profitability as they amplify existing double marginalization inefficiency. However, we show that eliminating this inefficiency through a two-part tariff or a quantity discount contract can make the commitments beneficial.

2. Literature Review

A growing body of research dating back to Coase (1972) examines the effects of the forward-looking behavior of consumers. Coase (1972) points out that a durable goods monopolist loses monopoly power and ends up pricing at marginal cost when faced with strategic customers. Coase (1972) notes that the monopolist can avoid this problem if it can make some contractual agreements that include price and quantity commitments. Bulow (1982) notes that the ability to make binding quantity commitments alleviates the loss to the monopolist caused by strategic customer behavior.

Likewise, Stokey (1979) finds that a firm benefits from its ability to make quantity commitments. In this case, the firm gives up the opportunity to exercise inter-temporal price discrimination and limits the sale of its product to a single period to eliminate the opportunity for strategic customers to await for mark-downs.

Butz (1990) shows that making price commitments in the form of best-price provisions increases a firm's profitability. With a best-price provision, the firm promises to refund the price difference between the original price and the sale price, if ever the firm decreases the price of its product. Similarly, Besanko and Winston (1990) and Aviv and Pazgal (2008) show that a firm facing strategic consumers can benefit from a commitment to a price path, as opposed to setting prices dynamically. Liu and Zhang (2013) find that committing to a price path has even greater benefits in a competitive environment.

In contrast to the findings of the above papers, we show that firms may not necessarily benefit from their ability to make quantity and price commitments; we find that, on the contrary, such commitments can hurt firms' profitability. The key difference between these papers and ours is that, while these papers consider only the interactions between consumers and a single seller, ours, in addition, considers the vertical relationship between supplier and retailer.

We should note that Aviv and Pazgal (2008), Cachon and Swinney (2009), Dasu and Tong (2010), Aflaki et al. (2016) and Papanastasiou and Savva (2017) also observed that commitments can hurt a firm's profitability. However, the drivers of their results are entirely different. Inability to respond to demand variability (or supply-demand imbalance) is the key driver in Aviv and Pazgal (2008), Cachon and Swinney (2009), Dasu and Tong (2010) and Papanastasiou and Savva (2017). Customers incur a costly effort such as monitoring prices to engage in strategic behavior in Aflaki et al. (2016) and price commitment can lower the cost of this effort. Therefore, it may not be desirable. These papers do not consider the relation between retailer and supplier. In contrast, we consider deterministic demand and the vertical relation between retailer and supplier drives our result.

Strategic consumer behavior has attracted a lot of interest in operations management literature. The various mechanisms studied in the context of strategic customers include capacity rationing (Liu and van Ryzin 2008, Zhang and Cooper 2008, Levin et al. 2010); quick response (Cachon and Swinney 2009, Swinney 2011); inventory display format (Yin et al. 2009); resale (Su 2010); product variety (Parlaktürk 2012, Bernstein and de Albéniz 2017); availability guarantees (Su and Zhang 2009); advance demand information (Li and Zhang 2013); reservations (Alexandrov and Lariviere

2012, Çil and Lariviere 2013); and price and quantity commitments (Elmaghraby et al. 2008, Aviv and Pazgal 2008, Lai et al. 2009).

Caldentey et al. (2016) study intertemporal pricing under minimax regret criteria. Özer and Zheng (2016) consider the impact of behavioral motives on this problem. When consumers incur search cost to purchase the product, Cachon and Feldman (2015) find that the seller prefers committing to offer the discount price with a higher likelihood than demand-contingent pricing. Aflaki et al. (2016) look at whether consumers collectively benefit from forward-looking strategic behavior. While the above papers study the optimal policy of a single seller facing strategic consumers, our work differs in that we consider a decentralized supply chain consisting of a supplier and a retailer, and we explicitly model the interactions between them. This approach allows us to address our key research question –the value of commitments to both the retailer and the supplier.

Su and Zhang (2008), Arya and Mittendorf (2006) and Desai et al. (2004) find that a decentralized supply chain can do better than a centralized supply chain due to strategic consumers. Su and Zhang (2008) show that in a decentralized supply chain some contracts can mimic the price and quantity commitments of a centralized supply chain. Our work complements this result in that we find actual price and quantity commitments in a decentralized supply chain may not be beneficial with a wholesale price contract. Our model differs from Su and Zhang (2008) in important ways. In Su and Zhang (2008), wholesale price and marked-down retail price are exogenously set. Supply-demand mismatch due to demand uncertainty –which, in turn, creates availability risk at marked-down price– generates the strategic customer behavior. When demand uncertainty is removed, the strategic behavior vanishes because the retailer never has available units at marked-down price. In contrast, demand is deterministic, and endogenously chosen wholesale and retail prices are critical in our model, because how commitments affect these prices drive our finding. These modeling differences can lead to opposite results: While we show that price and quantity commitments may harm profitability, such commitments are always beneficial in Su and Zhang (2008). Furthermore, customers have the same valuation of the product in Su and Zhang (2008); therefore, they all either buy at full price or wait for mark-down. In contrast, heterogeneity in customers' valuations in our model may cause them to choose different actions (buy now or wait).

Both Arya and Mittendorf (2006) and Desai et al. (2004) consider durable goods with secondhand markets. They show that decentralization through the addition of a retailer in a distribution channel can increase a manufacturer's profit if the product is sufficiently durable, i.e., lasting more than two periods (Arya and Mittendorf 2006) or if the manufacturer can commit to future wholesale prices (Desai et al. 2004). In other words, they show that a decentralized system can outperform a centralized system. These papers, however, do not address our key research question about the value of commitments in a decentralized supply chain.

The supply side of our model is similar to Anand et al. (2008) in that both papers wholesale prices are set dynamically which provides the retailer the opportunity to manipulate them by holding strategic inventory. Arya and Mittendorf (2013), Arya et al. (2015) and Roy et al. (2018) also study strategic inventory in various other contexts. However, there is a key difference in our demand model. These papers do not consider forward-looking customers, therefore, the two period demands are independent of each other in their models. In contrast, forward looking customers decide when to buy in our model and the two period demands are interlinked. This difference has an important effect on the use of strategic inventory. While the supplier sets a sufficiently high wholesale price in period 1 and deters the retailer from carrying strategic inventory in our model, this strategy is not attractive in these papers. When customers are not forward looking, unserved demand in period 1 is not carried over to period 2. It would be more costly to prevent strategic inventory by increasing the wholesale price in these models. Arya and Mittendorf (2013) shows that manufacturer-to-consumer rebates can also reduce (but not eliminate) the retailers strategic inventories.

Anand et al. (2008) shows that supplier's price commitment can hurt both the supplier itself and the retailer. Roy et al. (2018) illustrates that the impact on the retailer critically depends on whether the supplier can observe the retailer's inventory. We also show that the supplier's price commitment can hurt both parties and we find that the impact on the supplier critically depends on the retailer's capability to carry strategic inventory. However, the retailer continues to suffer even when holding inventory is not possible. In addition, we show that supplier's quantity commitment and retailer's price and quantity commitments can also hurt their profitability as well. Furthermore, we illustrate that these results are primarily driven by strategic customer behavior: they continue to hold even when retailer cannot carry inventory.

3. Model

In the following, we first introduce the supply side of our model; we then discuss how customers make their buying decisions resulting in strategic customer behavior. We finally describe the structure of the equilibrium. We consider a supply chain consisting of a supplier and a retailer selling a product over two periods. The supplier and the retailer set their prices dynamically to maximize their individual profits over two periods. Specifically, before the start of each period i, the supplier sets its wholesale price c_i, and the retailer decides its procurement quantity Q_i, which is delivered at the beginning of the period. The retailer then sets the retail price p_i, and consumers decide whether to buy the product in that period. The sequence of events is summarized in Figure 1. We assume that the supplier has unlimited capacity and its production costs are normalized to zero. Furthermore, the retailer does not incur any cost for holding inventory, however the salvage value of any unsold unit at the end of period 2 is zero.

All consumers are present at time zero and they leave the market when they make a purchase. Each consumer buys at most one unit of inventory. Dynamic retail prices provide an opportunity for strategic consumers. They decide whether to buy the product and when to buy it, choosing the option that maximizes their utility. Thus, in period 1 each consumer decides whether to buy a product in that period or wait for period 2. The consumers do not observe the retailer's procurement quantities Q_i, thus they rely on their beliefs $\hat{Q}_i$ when making their decisions. In most settings, consumers indeed do not observe a firm's order quantities. Note, however, that all of our key insights carry over when consumers can observe order quantities.

The consumer evaluations decrease over time, that is, everything else being equal, each customer prefers buying the product sooner rather than later. Specifically, the value of the product decreases by $1-\delta$ in period 2. Thus $1-\delta$ indicates perishability of the product value for consumers and is a measure of customer impatience. Hence, its complement δ shows the degree of customer patience. The assumption that valuations decline over time seems to be common in the sales of fashion, technology, and seasonal products (Aviv and Pazgal 2008, Cachon and Swinney 2009, Lai et al. 2009, Desiraju and Shugan 1999). Throughout the paper, we assume that $0 < \delta \leq 4/5$, that is, the value of a product decreases by at least 20% in period 2. This approach preserves the concavity of profit functions and helps us focus on more interesting cases. Our analysis carries over to $4/5 < \delta \leq 1$ as well; however, this extension requires considering many additional equilibrium regions that would needlessly complicate our exposition without contributing to our insights.[1]

The customers differ in their willingness to pay. Specifically, when type θ customer buys the product at price p_i in period i, her utility is equal to

$$U_i(\theta, p_i) = \delta^{i-1}\theta - p_i, \ \ i : 1, 2. \tag{1}$$

Thus, the value of the product for type θ customer is θ and $\delta\theta$ in periods 1 and 2 respectively. Not buying a product yields zero utility. An alternative discounting assumption is discussed in Section 9, where the discount factor δ applies to both the product value and the payment.

[1] We state the equilibrium results for no-commitment and retailer's commitments when $\delta > .8$ in the online supplement to illustrate how equilibrium regions look in that case.

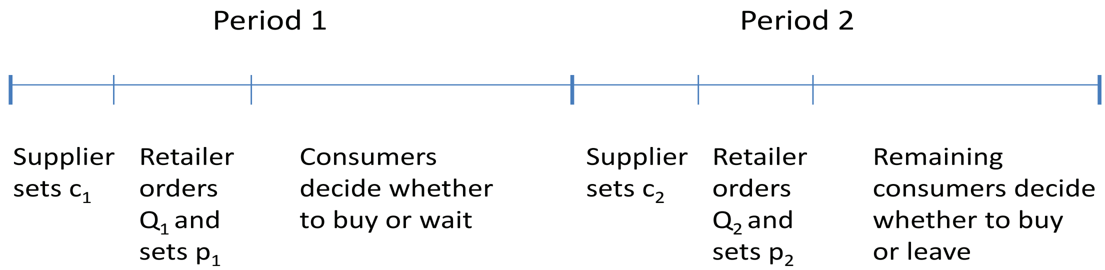

Figure 1 Sequence of events.

Customer types θ are uniformly distributed on the unit interval $[0,1]$ and have a total mass of 1. The distribution of customer types is common knowledge. However, the retailer does not know the type of any particular customer, hence perfect price discrimination is not feasible. Note that as is common in models of strategic customer behavior (e.g. Besanko and Winston 1990, Su 2007, Liu and van Ryzin 2008, Elmaghraby et al. 2008, Liu and Zhang 2013), the total market size is deterministic in our model, however its allocation to no purchase option and demand in each period depends on the retailer's pricing policy.

We analyze the above game between the supplier, the retailer, and consumers by looking for a subgame perfect Nash equilibrium (SPNE) (Selten 1975).

We restrict our analysis to only pure strategies. To explicitly define the equilibrium, we introduce additional notation. Let $\bar{\theta}$ show the type of marginal consumer who is indifferent between buying in periods 1 and 2. We make use of the fact that consumers follow a threshold type policy: if type-$\bar{\theta}$ consumer finds it attractive to buy in period 1 then all consumers with higher valuations also find it attractive to buy in period 1.

Even though the demand is deterministic in our model, the retailer may choose to carry inventory because of its impact on the supplier's period 2 wholesale price c_2. This is called strategic inventory in Anand et al. (2008). An alternative model in which the retailer cannot carry over inventory is considered in Section 9.4. This extension disentangles the effect of strategic inventory from our results.

An SPNE in our model is defined by the solutions of the following equations. Note that * denotes SPNE strategies.

$$(p_2^*, Q_2^*) = \arg \max_{(p_2,Q_2)\in S_2} [p_2(\bar{\theta} - p_2/\delta) - c_2 Q_2], \tag{2}$$

$$c_2^* = \arg\max_{c_2} c_2 Q_2^*, \tag{3}$$

$$\bar{\theta}^* = \inf\{\theta : \theta - p_1 \geq \delta\theta - p_2^*(\hat{Q}_1)\}, \tag{4}$$

$$(p_1^*, Q_1^*) = \arg \max_{(p_1,Q_1)\in S_1} [p_1(1-\bar{\theta}^*) - c_1 Q_1 + p_2^*(\bar{\theta}^* - p_2^*/\delta) - c_2^* Q_2^*], \tag{5}$$

$$\hat{Q}_1 = Q_1^*, \tag{6}$$
$$c_1^* = \arg\max_{c_1}[c_1 Q_1^* + c_2^* Q_2^*]. \tag{7}$$

where the retailer's feasible strategy sets S_i in each period are given by

$$S_1 = \{(p_1, Q_1) : Q_1 \geq 1 - \bar{\theta}^*(p_1) \geq 0\}, \tag{8}$$
$$S_2 = \{(p_2, Q_2) : Q_1 + Q_2 \geq 1 - p_2/\delta\}. \tag{9}$$

Basically, (2) states that the retailer chooses its order quantity Q_2 and price p_2 in period 2 to maximize its profit in that period, given the remaining consumer segment $[0, \bar{\theta})$, the wholesale price c_2, and its carry-over inventory. The constraint $(p_2, Q_2) \in S_2$ ensures that the retailer does not sell more than it has on-hand. Because the demand is deterministic, the retailer always sells all of its remaining inventory in period 2 in equilibrium. The supplier chooses its wholesale price c_2 to maximize its profit in period 2 in (3). The optimal c_2^* takes into account its impact on the retailer's optimal order quantity Q_2^*. Each consumer chooses her best option in period 1, given her conjecture of period 2 price, $p_2^*(\hat{Q}_1)$, which depends on her belief $\hat{Q}_1$ about how many units the retailer procured in period 1. Because the difference between period 1 and 2 utilities is monotone in θ, a threshold-type purchasing policy emerges in equilibrium. In particular, (4) states that it is optimal to wait for period 2 for all consumers with lower valuations than the marginal customer $\bar{\theta}$. As a result of monotonicity, (4) also implies that it is optimal to buy in period 1 for all consumers with higher valuations than the marginal customer $\bar{\theta}$. Therefore, the marginal customer $\bar{\theta}$ shows indifference between buying in period 1 and waiting for period 2.

Furthermore, (5) shows that the retailer chooses its period 1 order quantity Q_1 and price p_1 to maximize its total profit over two periods given the wholesale price c_1 in period 1. The condition $(p_1, Q_1) \in S_1$ states that the retailer's sales quantity in period 1 cannot exceed Q_1 units, its on-hand inventory.

The equilibrium requires the consumers to share the same belief on retailer period 1 order quantity and this belief to be consistent with the actual outcome as stated in (6). Note that consumers' beliefs on period 2 order quantity is not relevant since period 2 is the last period. Finally, (7) states that the supplier chooses its period 1 wholesale price, c_1, to maximize its total profit, taking into account its impact on the retailer's period 1 and 2 actions.

4. Equilibrium

We solve for the equilibrium using backward induction. Essentially, we follow the order in equations (2-7). First, we find the retailer's optimal price in period 2, then we derive the supplier's optimal wholesale price in that period. Next, we characterize the consumers' optimal choice in period 1 (buy in period 1 vs. wait for period 2). Then, we solve for the retailer's optimal price in period 1. Finally, we determine the supplier's optimal wholesale price in period 1, which fully characterizes the equilibrium. Note that all proofs are provided in our online supplement.

In period 2, the supplier sets its wholesale price c_2 to extract maximum profit from the retailer while the retailer sets its price p_2 to extract maximum profit from the remaining consumers $[0,\bar{\theta})$. Because period 2 is the last period, the consumers do not have the strategic option of delaying their purchases further. They decide whether or not to buy the product. Specifically, there exist $\theta_2 \le \bar{\theta}$, such that consumers in $[\theta_2,\bar{\theta})$ buy the product and the remaining consumers do not buy it. The following Lemma describes the retailer's and supplier's optimal policy in period 2.

LEMMA 1. *Suppose the retailer orders Q_1 units in period 1, and consumer segment $[0,\bar{\theta})$ remains in the market in period 2.*

(i) For any given wholesale price c_2, the retailer orders $Q_2^* = \max(\frac{\delta(2-2Q_1-\bar{\theta})-c_2}{2\delta},0)$ units and sets $p_2^* = \delta(1-Q_1-Q_2^*)$ in period 2.

(ii) The supplier sets $c_2^* = \frac{\delta(2-2Q_1-\bar{\theta})}{2}$ in period 2.

This yields profits $\Pi_{R,2} = \frac{\delta(2-2Q_1-\bar{\theta})^2}{16} + \delta(1-Q_1)(Q_1-1+\bar{\theta})$ and $\Pi_{S,2} = \frac{\delta(2-2Q_1-\bar{\theta})^2}{8}$ for the retailer and supplier respectively.

Next, we consider the consumers' choices in period 1. The consumers conjecture period 2 price, given their beliefs about the retailer's inventory level $\hat{Q}_1$. Specifically, following Lemma 1, they expect $p_2^*(\hat{Q}_1) = \frac{\delta(2-2\hat{Q}_1+\bar{\theta})}{4}$. The marginal consumer $\bar{\theta}$, who is indifferent about whether to buy in period 1 or wait for period 2, is given by the solution of $\bar{\theta} - p_1 = \delta\bar{\theta} - p_2^*(\hat{Q}_1)$, where the left and right hand sides correspond to the utility of buying in periods 1 and 2, respectively. This leads to

$$\bar{\theta} = \frac{4p_1 - 2\delta(1-\hat{Q}_1)}{4-3\delta}. \tag{10}$$

Thus, consumers in $[\bar{\theta},1]$ buy the product in period 1, and the remaining consumers wait for period 2.

Now, consider the retailer's period 1 problem. To determine the retailer's optimal price, we can equivalently solve for the optimal consumer segment that it should entice in period 1. Following (10), in order to induce consumers in $[\bar{\theta},1]$ to buy the product in period 1, the retailer needs to set the following price:

$$p_1(\bar{\theta},\hat{Q}_1) = \bar{\theta} - \frac{\delta(2(\hat{Q}_1-(1-\bar{\theta}))+\bar{\theta})}{4}. \tag{11}$$

Setting price p_1 equal to $\bar{\theta}$ would be sufficient to attract consumer segment $[\bar{\theta}, 1]$ if the consumers were myopic, i.e., if they were not considering a future option in their decision making. The terms in parentheses are non-negative in (11) since $\hat{Q}_1 - (1-\bar{\theta})$ corresponds to the consumers' belief about the retailer's inventory level at the end of period 1. Therefore, the retailer suffers a margin loss from price $\bar{\theta}$ that results from strategic customer behavior. Observe that this loss increases in $\hat{Q}_1$, that is, when customers believe that the retailer has a higher inventory level, the retailer needs to set a lower price to convince the customer to buy in period 1 rather than to wait for sales in period 2. Because the actual order quantity Q_1 is unobservable, it does not have a direct impact on this loss. By definition, the retailer can conjecture the equilibrium choices. Therefore, it predicts $\hat{Q}_1$, which is equal to the equilibrium order quantity in period 1.

We can now reformulate the retailer's total profit in terms of its order quantity Q_1 and target consumer segment $[\bar{\theta}, 1]$ in period 1:

$$\Pi_R(Q_1, \bar{\theta}, \hat{Q}_1) = [1-\bar{\theta}]p_1(\bar{\theta}, \hat{Q}_1) - Q_1 c_1 + \Pi_{R,2}(Q_1, \bar{\theta}). \tag{12}$$

The retailer's optimal policy in equilibrium then satisfies the following:

$$(Q_1^*, \bar{\theta}^*) = \arg \max_{(Q_1, \bar{\theta}) \in S_1'} \Pi_R(Q_1, \bar{\theta}, \hat{Q}_1) \tag{13}$$

$$\hat{Q}_1 = Q_1^*, \tag{14}$$

$$S_1' = \{(Q_1, \bar{\theta}) : Q_1 \geq 1 - \bar{\theta} \geq 0\}. \tag{15}$$

Here, the restriction to set S_1' ensures that the retailer does not sell a negative quantity, or more than its on-hand inventory in period 1. The solution of (13-14) gives the retailer's optimal order quantity and price in period 1, which are stated in the following Proposition.

PROPOSITION 1. *The SPNE order quantity Q_1^* and retail price p_1^* and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.*

(i) When $c_1 < \frac{3\delta(4-3\delta)}{2(16-15\delta)}$, the retailer sells the product in both periods, and it carries inventory between periods, where $Q_1^* = \frac{11}{14} - \frac{c_1(112-91\delta)+6\delta}{21\delta(8-7\delta)}$, $p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1 - 9\delta(1-\delta)}{6(8-7\delta)}$, and $\bar{\theta}^* = \frac{3(4-3\delta)+2c_1}{3(8-7\delta)}$.

(ii) When $\frac{3\delta(4-3\delta)}{2(16-15\delta)} \leq c_1 < \frac{8-3\delta}{8}$, the retailer sells the product in both periods and it does not carry inventory between periods, where $Q_1^* = \frac{8(1-c_1)-3\delta}{16-9\delta}$, $p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}$, and $\bar{\theta}^* = \frac{8(1+c_1)-6\delta}{16-9\delta}$.

(iii) When $c_1 \geq \frac{8-3\delta}{8}$, the retailer sells the product only in period 2, and it does not carry inventory between periods, where $Q_1^* = 0$, and $\bar{\theta}^* = 1$.

The regions characterized in parts (i)-(iii) are depicted in Figure 2. Part (i) corresponds to the interior of S_1' since $Q_1^* > 1 - \bar{\theta}^* > 0$. In contrast, in parts (ii) and (iii), $Q_1^* = 1 - \bar{\theta}^*$, so the upper bound in (15) binds. Furthermore, in part (iii), $\bar{\theta}^* = 1$, thus, the lower bound in (15) also binds.

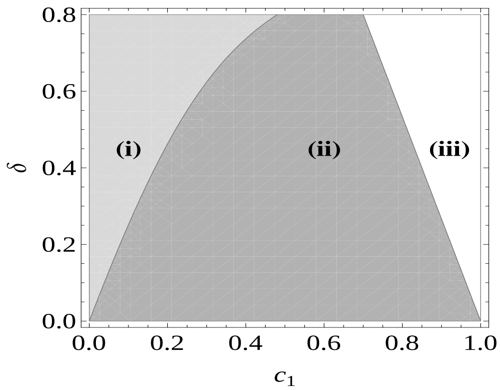

Figure 2 Equilibrium Regions in Proposition 1.

Finally, let us consider the supplier's pricing problem in period 1. The supplier chooses c_1 to maximize its total profit:

$$\Pi_S = c_1 Q_1^* + \Pi_{S,2}(c_1, Q_1^*, \bar{\theta}^*), \tag{16}$$

where Q_1^* and $\bar{\theta}^*$ are given by Proposition 1 and $\Pi_{S,2}$ follows from Lemma 1. The following Proposition describes the supplier's optimal policy.

Proposition 2. *The SPNE wholesale price c_1 in period 1 is as follows.*

(i) When $\delta \leq \frac{16}{21}$, $c_1^* = \frac{16-3\delta}{32}$.

(ii) When $\frac{16}{21} < \delta$, $c_1^* = \frac{3\delta(4-3\delta)}{2(16-15\delta)}$.

Propositions 1 and 2 state the firms' optimal policy in period 1. Recall that Lemma 1 describes their optimal policy in period 2 and (10) identifies the marginal customer. Therefore, the equilibrium of the game between the supplier, retailer and consumers is fully characterized by Propositions 1 and 2, Lemma 1 and (10). Because our analysis follows backward induction, period 2 equilibrium decisions are stated in terms of period 1 outcomes in all scenarios. It is straightforward to confirm that $p_1^* \geq p_2^*$ and $c_1^* \geq c_2^*$ in all scenarios, reflecting the fact the value of product decreases in period 2.

Propositions 1 and 2 show that the supplier's pricing in period 1 makes carrying inventory into the next period unattractive for the retailer. In other words, the supplier sets its period 1 wholesale price c_1 sufficiently high so that the retailer can not use inventories to get a better period 2 wholesale price c_2. In part (ii) of Proposition 2, the solution is at the boundary; any lower wholesale price c_1 will result in the retailer carrying inventory into period 2.

It is well known that strategic customer behavior leads to profit loss (cf. Besanko and Winston 1990, Cachon and Swinney 2009, Su and Zhang 2008). Indeed, it is straightforward to show that strategic customer behavior hurts the profits of both the supplier and retailer in our setup; that is, their profits would be higher if the customers were myopic, i.e., not considering their future pay-offs. In the following, we study the value of price and quantity commitments both from the supplier's and the retailer's perspectives in a supply-chain setting. We also consider a centralized system as a benchmark.

5. Benchmark: Centralized System

Similarly to our main model, the centralized system sets the retail price p_i in each period $i:1,2$ and consumers decide whether to buy the product in that period. The resulting equilibrium is given by the following Lemma.

LEMMA 2. *Consider a centralized supply chain. The SPNE is as follows. The centralized firm sells the product in both periods, setting* $p_1^* = \frac{(2-\delta)^2}{2(4-3\delta)}$ *in period 1 and* $p_2^* = \frac{(2-\delta)\delta}{2(4-3\delta)}$ *in period 2, and the marginal customer is given by* $\bar{\theta}^* = \frac{2-\delta}{4-3\delta}$.

Now, suppose that the firm can make price or quantity commitments. When the firm credibly commits to a future price p_2, which can be different than p_1, it does not change its committed price in period 2 even when this deviation would increase its profit. Similarly, when the firm commits to sell a limited quantity, it does not sell more than the committed quantity even when doing so would increase its profit.

LEMMA 3. *Suppose a centralized firm can make price or quantity commitments. In equilibrium, the firm sells its product only in period 1 at price* $p_1^* = \frac{1}{2}$ *in either case.*

When the firm can make a commitment, it limits its sale to a single period to eliminate the strategic behavior. Let Π_C^j $j:NC,QC,PC$ show the centralized system's profit when it cannot make any commitments (NC) and when it can make quantity (QC) and price (PC) commitments. The following Theorem compares the centralized firm's profit in these scenarios.

THEOREM 1. $\Pi_C^{NC} < \Pi_C^{QC} = \Pi_C^{PC}$.

The Theorem shows that both price and quantity commitments increase the centralized system's profit. In the rest of the paper, we study the impact of such commitments for a decentralized system.

6. Retailer's Commitments

First, we consider the retailer's commitments in a decentralized supply chain. We consider quantity commitment in Section 6.1 and price commitment in Section 6.2. Following from Section 5, we know that a centralized system always benefits from the ability to make quantity or price commitments.

What is the impact of such commitments for the supplier, the retailer itself, and the whole supply chain?

6.1 Retailer's Quantity Commitment

Suppose that the retailer can credibly convince customers that it will not sell more units than its committed quantity. The order of events is similar to our no-commitment model.

When determining its period 1 procurement quantity Q_1, the retailer commits not to sell more than a total of Q_{max} units in two periods. Therefore, the retailer's period 2 procurement quantity Q_2 needs to satisfy $Q_2 \leq Q_{max} - Q_1$. Otherwise, the order of events is the same as in our no-commitment model shown in Figure 1. We exploit the following Lemma for deriving the equilibrium.

Lemma 4. *For any SPNE in which retailer's quantity commitment does not bind, i.e., $Q_1^* + Q_2^* < Q_{max}^*$, there is an equivalent SPNE with the same outcome (wholesale and retail prices, quantities and supplier and retailer profits) in which the retailer's quantity commitment binds, i.e., $Q_1^* + Q_2^* = Q_{max}^*$.*

Therefore, we can assume that the retailer's quantity commitment binds in equilibrium without loss of generality. This is because the retailer can always match the profit of a non-binding commitment with a binding commitment by simply setting $Q'_{max} = Q_1^* + Q_2^*$.

Let us define the critical period 2 wholesale price $\bar{c}_2$ below which the retailer finds it attractive to procure all of its committed quantity. Thus, $\bar{c}_2$ solves $Q_2^*(c_2) = Q_{max} - Q_1$. In order for the retailer's quantity commitment to bind, the following condition needs to be satisfied

$$\left.\frac{d\Pi_{R,2}}{dQ_2}\right|_{Q_2 = Q_{max} - Q_1} \geq 0, \text{ and } \left.\frac{d\Pi_{S,2}}{dc_2}\right|_{c_2 = \bar{c}_2} \leq 0. \tag{17}$$

Basically, (17) ensures that it is optimal for the retailer to procure all of its committed quantity given the supplier's period 2 wholesale price, and it is not optimal for the supplier to increase this price. Note that decreasing its wholesale price does not benefit the supplier in this case either, as the retailer cannot procure more than its committed quantity. The solution of (17) leads to the following Lemma, which characterizes the firms' equilibrium strategies in period 2.

Lemma 5. *When the retailer's quantity commitment binds, $Q_2^* = Q_{max} - Q_1$, $p_2^* = \delta(1 - Q_{max})$ and $c_2^* = \delta(2(1 - Q_{max}) - \bar{\theta})$ in a SPNE in period 2. This yields $\Pi_{R,2} = p_2^*(Q_{max} - (1 - \bar{\theta})) - c_2^* Q_2^*$ and $\Pi_{S,2} = c_2^* Q_2^*$. Furthermore, the retailer quantity commitment binds if and only if $Q_{max} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4}$.*

The Lemma shows that for the supplier to set a low enough wholesale price to persuade the retailer to procure all of its remaining committed quantities in period 2, the retailer's period 1 price, quantity and commitment choices should satisfy $Q_{max} \le \frac{Q_1+1}{2} - \frac{\bar{\theta}}{4}$. Otherwise, given the supplier's optimal wholesale price, the retailer's quantity commitment does not bind.

When the retailer's commitment binds, it essentially dictates period 2 retail price $p_2^* = \delta(1 - Q_{max})$; therefore, customers do not need to form beliefs about the retailer's order quantities to conjecture its p_2. The marginal customer $\bar{\theta}$, who is indifferent between buying in period 1 and waiting for period 2 then solves $\bar{\theta} - p_1 = \delta\bar{\theta} - p_2^*$ leading to

$$p_1(\bar{\theta}, Q_{max}) = \bar{\theta} - \delta(Q_{max} - (1-\bar{\theta})). \tag{18}$$

The retailer's margin loss resulting from strategic customer behavior, which is given by $\delta(Q_{max} - (1-\bar{\theta}))$, increases in the number of units the firm will sell in the second period, that is, $Q_{max} - (1-\bar{\theta})$. Following Lemmas 4 and 5 and (18), we can now state the retailer's period 1 problem in terms of its order quantity Q_1, commitment Q_{max} and target consumer segment $[\bar{\theta}, 1]$ in period 1:

$$\max_{Q_1, Q_{max}, \bar{\theta}} \; [1-\bar{\theta}] \; p_1(\bar{\theta}, Q_{max}) - Q_1 c_1 + \Pi_{R,2}(Q_1, Q_{max}, \bar{\theta}) \tag{19}$$

$$\text{s.t.} \quad Q_{max} \le \frac{Q_1+1}{2} - \frac{\bar{\theta}}{4}, \tag{20}$$

$$Q_1 \le Q_{max}, \tag{21}$$

$$(Q_1, \bar{\theta}) \in S_1^{'}. \tag{22}$$

Lemmas 4 and 5 leads to condition (20) which ensures that the quantity commitment binds in period 2. Condition (21) ensures that the quantity commitment is honored in period 1. Note that $S_1^{'}$ referred (22) in is defined in (15). The solution of the retailer's problem in (19-22) leads to the following Proposition.

Proposition 3. *Suppose the retailer can make a quantity commitment. Its SPNE order quantity Q_1^*, quantity commitment Q_{max}^*, retail price p_1^* and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.*

(i) When $c_1 < \frac{\sqrt{16-5\delta}-1}{5}$, the retailer sells the product only in period 1 and the quantity commitment binds strictly, where $Q_1^* = \frac{1-c_1}{2}$, $Q_{max}^* = \frac{1-c_1}{2}$, $p_1^* = \frac{1+c_1}{2}$ and $\bar{\theta}^* = \frac{1+c_1}{2}$.

(ii) When $\frac{\sqrt{16-5\delta}-1}{5} \le c_1 < \frac{8-3\delta}{8}$, the retailer sells the product in both periods, it does not carry inventory between periods and the quantity commitment binds weakly, where $Q_1^* = \frac{8(1-c_1)-3\delta}{16-5\delta}$, $Q_{max}^* = \frac{2(10-6c_1)-7\delta}{2(16-5\delta)}$, $p_1^* = \frac{(4(1+c_1)-\delta)(4-\delta)}{2(16-5\delta)}$, and $\bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta}$.

(iii) When $c_1 \ge \frac{8-3\delta}{8}$, the retailer sells the product only in period 2, it does not carry inventory between periods and the quantity commitment binds weakly, where $Q_1^* = 0$, $Q_{max}^* = \frac{1}{4}$, and $\bar{\theta}^* = 1$.

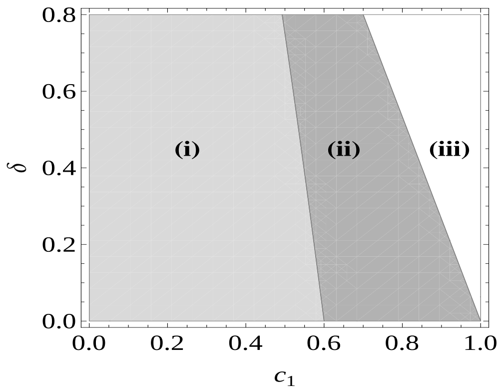

Figure 3 **Equilibrium Regions in Proposition 3.**

We say that the quantity commitment binds strictly (weakly) when the first inequality in (17) holds strictly (weakly). In other words, when the commitment binds strictly, ordering more units than its committed quantity would increase the retailer's profit. In contrast, when the commitment binds weakly, the retailer does not want to exceed its committed quantity. Figure 3 shows the regions characterized by parts (i)-(iii). When period 1 wholesale price c_1 is small enough as in part (i), the quantity commitment enables the retailer to limit sales of the product to only period 1 similar to the centralized benchmark. However, when c_1 is sufficiently high, the retailer sells the product in period 2 as well and the quantity commitment does not bind strictly.

Finally, the supplier chooses c_1 to maximize its total profit: $\Pi_S = c_1 Q_1^* + \Pi_{S,2}(c_1, Q_1^*, Q_{max}^*, \bar{\theta}^*)$, where Q_1^*, Q_{max}^* and $\bar{\theta}^*$ are given by Proposition 3 and $\Pi_{S,2}$ follows from Lemma 5. The following Proposition describes the supplier's optimal policy and completes the characterization of the equilibrium.

Proposition 4. *Suppose the retailer can make a quantity commitment. The supplier's SPNE wholesale price c_1 in period 1 is as follows. There exists $\bar{\delta}_1 \approx 0.249$ such that*

(i) when $\delta \le \bar{\delta}_1$, $c_1^* = \frac{1}{2}$, and

(ii) when $\delta > \bar{\delta}_1$, $c_1^* = \frac{\sqrt{16-5\delta}-1}{5}$.

Note that $\bar{\delta}_1$ is explicitly characterized in the proof of the proposition. Parts (i) and (ii) of Proposition 4 map to parts (i) and (ii) of Proposition 3 respectively. Specifically, when δ is small, the retailer's quantity commitment binds strictly and the product is sold only in period 1, while when δ is big, the commitment binds weakly and the product is sold in both periods.

These findings prepare us to address our key research question: How does the commitment affect the retailer, the supplier and the whole supply chain? Note that superscripts *NC* and *RQC* in the following theorem stand for no commitment and retailer's quantity commitment respectively.

Theorem 2. *There exists* $\bar{\delta}_2 \approx 0.799$ *such that*
(i) $\Pi_S^{NC} > \Pi_S^{RQC}$,
(ii) $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{RQC} + \Pi_R^{RQC}$,
(iii) $\Pi_R^{NC} > \Pi_R^{RQC}$ *when* $\delta < \bar{\delta}_2$.

Here, $\bar{\delta}_2$ is explicitly characterized in the proof. The Theorem shows that, unlike the centralized benchmark, the retailer can become worse off when it makes a quantity commitment. Furthermore, this hurts the supplier as well as the whole supply chain. Similar to the centralized system benchmark, the retailer would have benefited from its quantity commitment if the supplier's wholesale prices were given exogenously, that is, if the supplier's wholesale price stayed the same after the retailer's commitment.[2] However, the retailer would benefit at the expense of the supplier in this case as it will decrease its order quantity.

Comparisons of Propositions 2 and 4 show that the supplier increases its period 1 wholesale price when the retailer can make a quantity commitment, which exacerbates double marginalization hurting both the retailer and supplier. In this case, if the supplier sets a relatively low price, the retailer would restrict its sales to only period 1 using the quantity commitment and not buy any units from the supplier in period 2. Because of this threat, the supplier sets a high price in period 1 to force the retailer to sell the product in both periods. As a result, both the supplier and the retailer lose compared to no-commitment scenario. A high wholesale price in period 1 means that the retailer can serve only the higher end of the market making the remaining customer segment in period 2 attractive, which in turn, prevents the retailer from limiting its sale to only period 1.

Note, when $\delta > \bar{\delta}_2$, the retailer can benefit from its quantity commitment. Everything else being equal, a high δ encourages customers to wait for period 2. Therefore, commitment can become more attractive in that case as it discourages customers from waiting for a price mark-down.

6.2 Retailer's Price Commitment

Suppose that the retailer can credibly commit to a future retail price. This means that when setting the retail price p_1 in period 1, the retailer also commits to period 2 retail price p_2. Otherwise, the order of events is the same as in our no-commitment model shown in Figure 1. We derive the equilibrium using backward induction. The following Lemma characterizes the equilibrium in period 2.

[2] It is straightforward to show that the retailer can always replicate the no-commitment outcome in this case, thus it can never get worse off.

Lemma 6. *When the retailer commits to a future retail price, it does not procure any units in period 2 in equilibrium, that is,* $Q_2^* = 0$.

Because the retailer already commits to period 2 price p_2 before the supplier sets its period 2 wholesale price c_2, the supplier has no incentive to set a wholesale price c_2 smaller than p_2. Because the retailer's sales quantity in period 2 and its order quantity Q_2 do not depend on c_2 as long as $c_2 \leq p_2$. Anticipating this response, the retailer procures all of its needed units in period 1.

Following Lemma 6, similar to the centralized system benchmark, the retailer limits its sales to only period 1 by committing that period 2 retail price will not be attractive. The next proposition characterizes the resulting equilibrium.

Proposition 5. *Suppose the retailer can commit to a future retail price. The SPNE is as follows.*

(i) Given the wholesale price c_1, *the retailer orders* $Q_1^* = \frac{1-c_1}{2}$, *sets* $p_1^* = \frac{1+c_1}{2}$ *in period 1 and it does not sell the product in period 2 by committing to a sufficiently high* p_2 *in period 1.*

(ii) The supplier chooses $c_1^* = \frac{1}{2}$.

It is straightforward to show that the retailer would benefit from limiting its sales to only period 1, which eliminates the strategic customer behavior, if the supplier's wholesale prices stay the same as in the no-commitment scenario. How do the changes in supplier's wholesale prices impact the value of commitment? This is addressed by the next Theorem. Note that superscripts *NC* and *RPC* stand for no commitment and retailer's price commitment respectively.

Theorem 3. *(i)* $\Pi_S^{NC} > \Pi_S^{RPC}$.

(ii) $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{RPC} + \Pi_R^{RPC}$.

(iii) $\Pi_R^{NC} > \Pi_R^{RPC}$.

Similar to quantity commitment, committing to a future retail price hurts the retailer's profitability due to how it affects supplier's pricing. This also makes the supplier as well as the whole supply chain worse off. The retailer's price commitment amplifies double marginalization inefficiency as a result of two factors. First, absent any commitments the retailer procures units in both periods where period 2 wholesale price is always cheaper than that of period 1. In contrast, with price commitment, the retailer procures units only in period 1 at a single wholesale price. When the supplier sells the product at two different prices, double marginalization is alleviated. This benefit vanishes with retailer's price commitment. Second, a comparison of Propositions 2 and 5 shows that the supplier sets a higher wholesale price c_1 when the retailer can commit to prices. The supplier sets a higher c_1 anticipating that the retailer is going to limit its sales to only period 1.

Overall, Theorems 2 and 3 show that retailer's price and quantity commitments can hurt profitability of both itself and its supplier. The vertical relation between the retailer and the supplier, which leads to double-marginalization, is the key driver of this result. In particular, endogenous wholesale price is critical. Commitments would be beneficial if the wholesale price was exogenous (so the supplier does not react to retailer's commitments) or if the model did not consider the supplier at all as in the centralized benchmark (see Theorem 1).

It is worthwhile to contrast retailer's price and quantity commitments. The retailer's price commitment directly determine the sales quantity in each period. In contrast,when the retailer commits to a maximum quantity, the supplier can manipulate the total sales quantity and its allocation to two periods as well as the future retail price through its choice of wholesale prices. In that sense, price commitment is a stronger commitment for the retailer. Similarly, supplier's price commitment determines all of its future actions whereas when it commits to a maximum quantity, the retailer can still manipulate the supplier's future wholesale price through its order quantity in period 1.

7. Supplier's Commitments

We now consider the supplier's commitments. In Section 6, we have seen that the retailer's commitments always hurts the supplier. The next question becomes whether the supplier can benefit from its own commitments in a decentralized supply chain. The supplier moves first and sets a take-it-or-leave-it wholesale price; in other words the power lies with the supplier in our model. One would expect the supplier to benefit, therefore, from its commitments. Indeed, in Section 7.1, we show that the supplier always benefits from a quantity commitment. In contrast, however, in Section 7.2, we show that the supplier too can become worse off from making price commitments because of how this commitment affects the interactions between the supplier and the retailer.

7.1 Supplier's Quantity Commitment

Suppose that the supplier can commit to not selling more than a committed quantity. Specifically, before setting its wholesale price in period 1, the supplier announces that it will not sell more than Q_{max} units in two periods. The order of remaining events is the same as our no-commitment model shown in Figure 1. The equilibrium and its analysis for this scenario are provided in Appendix A. Here, we state how the commitment affects the supplier's and the retailer's profitability. Let superscripts *NC* and *SQC* denote no commitment and supplier's quantity commitment respectively.

THEOREM 4. *(i)* $\Pi_S^{NC} \leq \Pi_S^{SQC}$.
(ii) $\Pi_S^{NC} + \Pi_R^{NC} \leq \Pi_S^{SQC} + \Pi_R^{SQC}$.
(iii) $\Pi_R^{NC} \geq \Pi_R^{SQC}$ *when* $\delta \leq \bar{\delta}_3$.

Note $\bar{\delta}_3 \approx 0.674$ and it is explicitly characterized in the proof of this theorem. Because the supplier moves first, it can always match the outcome of a no-commitment scenario by committing to a non-binding arbitrarily large quantity. Thus, the supplier always benefits from its ability to make a quantity commitment. Furthermore, when the quantity commitment binds strictly, the supplier's profit increases strictly. In this case, the product is sold only in period 1. However, the supplier's commitment can hurt the retailer. This happens because the supplier commits to a smaller quantity than that of the centralized benchmark in order to keep a high wholesale price.

Because quantity commitment discourages customers from waiting for a price mark-down, this benefit would be more valuable when they are more inclined to wait, i.e., when δ is high. Therefore, similar to Theorem 2, supplier's commitment can also help the retailer when δ is sufficiently high. While the supplier's quantity commitment improves the total supply chain profit, it does not eliminate the loss due to forward-looking customers. Similarly, double marginalization inefficiency is not eliminated; the centralized benchmark results in a higher profit.

7.2 Supplier's Price Commitment

In this section, we discuss what happens when the supplier commits to future wholesale prices. In particular, while setting period 1 wholesale price c_1 before the beginning of period 1, the supplier also commits to period 2 wholesale price c_2. Otherwise, the order of events is the same as that in our no-commitment model.

We derive the equilibrium following backward induction. The following Lemma describes the equilibrium in period 2.

LEMMA 7. *Suppose that the supplier commits to wholesale prices c_1 and c_2, the retailer orders Q_1 in period 1 and consumer segment $[0,\bar{\theta})$ remain in the market in period 2. The retailer then orders $Q_2^* = max(\frac{\delta(2-2Q_1-\bar{\theta})-c_2}{2\delta}, 0)$ units and sets $p_2^* = \delta(1-Q_1-Q_2^*)$ in period 2.*

The marginal customer $\bar{\theta}$, who is indifferent about whether to buy in period 1 or to wait for period 2, solves the following equation $\bar{\theta} - p_1 = \delta\bar{\theta} - p_2^*$, where the left and right hand sides correspond to the utility of buying in period 1 and 2 respectively. Following Lemma 7, this leads to

$$\bar{\theta} = \frac{2p_1 - c_2}{2-\delta}. \tag{23}$$

Note that (23) is independent of the retailer's period 1 order quantity Q_1; in other words, in their purchasing decisions in period 1, customers do not need to rely on their beliefs about Q_1. Intuitively, when the supplier already commits to a period 2 wholesale price in period 1, the retailer has no incentive to carry inventory into period 2. Recall that, in our model, the retailer carries inventory only to get a better wholesale price in period 2. Therefore, to conjecture period 2 retail price in period 1, customers do not need to know Q_1. Rearranging (23), we get

$$p_1(\bar{\theta}) = \frac{(2-\delta)\bar{\theta} + c_2}{2}. \tag{24}$$

The retailer then in period 1 solves the following problem to maximize his total profit in two periods

$$\max_{(Q_1,\bar{\theta}) \in S_1^{'}} [(1-\bar{\theta})p_1(\bar{\theta}) - Q_1 c_1 + Q_2^*(p_2^* - c_2)], \tag{25}$$

where Q_2^* and p_2^* are given in Lemma 7 and $S_1^{'}$ is defined in (15), which states that the retailer's sales quantity in period 1 should be non-negative and cannot exceed Q_1. The solution of (25) leads to the following Proposition.

Proposition 6. *Suppose the supplier commits to wholesale prices c_1 and c_2. The retailer's SPNE order quantity Q_1^*, and retail price p_1^* in period 1 are as follows.*

(i) When $c_2 \le c_1 + \delta - 1$, the retailer sells the product only in period 2, where $Q_1^* = 0$ and $p_1^* = 1 + \frac{c_2 - \delta}{2}$.

(ii) When $c_1 + \delta - 1 < c_2 < \frac{\delta(2+2c_1-\delta)}{4-\delta}$, the retailer sells the product in both periods, where $Q_1^* = \frac{2(1-c_1+c_2-\delta)}{4-3\delta}$ and $p_1^* = \frac{4+2c_1(2-\delta)-(4+c_2-\delta)\delta}{2(4-3\delta)}$.

(iii) When $\frac{\delta(2+2c_1-\delta)}{4-\delta} \le c_2 < \frac{\delta(1+c_1)}{2}$, the retailer sells the product only in period 1, where $Q_1^* = 1 - \frac{c_2}{\delta}$ and $p_1^* = \frac{c_2}{\delta}$.

(iv) When $c_2 \ge \frac{\delta(1+c_1)}{2}$, the retailer sells the product only in period 1, where $Q_1^* = \frac{1-c_1}{2}$ and $p_1^* = \frac{1+c_1}{2}$.

The retailer does not carry inventory between periods in all cases, thus the marginal customer is given by $\bar{\theta}^* = 1 - Q_1^*$.

Note that $c_1 - c_2$ difference decreases going from part (i) to (iv), which, as expected, shifts the retailer's sales from the second period to the first period.

Finally, we need to solve the supplier's problem to completely characterize the equilibrium. The supplier maximizes $c_1 Q_1^* + c_2 Q_2^*$ where Q_1^* and Q_2^* are given by Proposition 6 and Lemma 7. The solution of this problem is given by the next Proposition.

Proposition 7. *Suppose the supplier can commit to future wholesale prices. The equilibrium wholesale prices c_1 and c_2 are as follows.*

(i) When $\delta \le 1 - \frac{1}{\sqrt{17}}$, $c_1^* = \frac{16-(14-\delta)\delta}{2(16-13\delta)}$ and $c_2^* = \frac{2\delta(5-4\delta)}{16-13\delta}$.

(ii) When $\delta > 1 - \frac{1}{\sqrt{17}}$, $c_1^* = c_2^* = \frac{\delta(6-5\delta)}{2(4-3\delta)}$.

Both parts (i) and (ii) of Proposition 7 correspond to part (ii) of Proposition 6; that is, in equilibrium the supplier commits to wholesale prices that induces the retailer to buy and sell the product in both periods.

The next theorem addresses our key research question. What is the value of the supplier's commitment for the supplier itself, for the retailer and for the whole supply chain? Note that superscripts *NC* and *SPC* stand for no commitment and supplier's price commitment respectively.

THEOREM 5. *(i)* $\Pi_S^{NC} > \Pi_S^{SPC}$.
(ii) $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{SPC} + \Pi_R^{SPC}$.
(iii) $\Pi_R^{NC} > \Pi_R^{SPC}$.

The Theorem shows that similar to the retailer's commitments, the supplier need not benefit from its ability to make price commitments. Furthermore, the supplier's price commitment not only hurts its own profitability but also that of the retailer, making the whole supply chain worse off. This happens because the supplier's price commitment increases the inefficiency due to decentralization. When the supplier does not make price commitments, its period 2 wholesale price c_2 decreases in the retailer's period 1 procurement quantity Q_1, as seen in Lemma 1.ii. In other words, buying a larger quantity in period 1 allows the retailer to receive a lower wholesale price in period 2. This mimics a quantity discount mechanism and alleviates the double marginalization inefficiency. However, when the supplier already commits to a period 2 wholesale price in period 1, the retailer's incentive to buy more to pay less later vanishes. Knowing this, the supplier sets a higher period 1 wholesale price (as shown in Propositions 2 and 7) which increases the double marginalization inefficiency. Note that the supplier's quantity commitment, on the other hand, maintains the retailer's incentive by preserving the relation between period 1 order quantity and period 2 wholesale price.

The impact of supplier's price commitment on retailer's ordering policy is the key driver of Theorem 5. Retailer's ability to carry strategic inventory along with dynamic wholesale pricing is critical for this result because dynamic wholesale prices encourage the retailer to act strategically by buying a larger quantity in period 1 in order to receive a lower wholesale price in period 2. We show in Section 9.4 that the supplier would benefit from its price commitment if the retailer could not carry over inventory.

Note that the supplier's and retailer's dynamic pricing have different effects. The retailer's dynamic pricing encourages consumers to wait for a mark-down, thereby shifting consumer demand to period 2. In contrast, the supplier's dynamic pricing encourages the retailer to buy more early on, thereby shifting retailer demand to period 1.

8. Commitments under a Coordinating Contract

We have shown that commitments can aggravate existing coordination inefficiency under a wholesale price contract making the supplier and/or the retailer worse off. In contrast, we now consider a coordinating contract that achieves the profit of a centralized system. We show that under a coordinating contract, commitments never hurt the supplier and the retailer and always improve the profitability of a supply chain by eliminating strategic customer behavior. Specifically, we show that a two-part tariff achieves the profit of the centralized benchmark and commitments are always beneficial with a two-part tariff.

When the supplier offers a two-part tariff to the retailer, in each period the supplier charges a lump-sum fee F_i in addition to the wholesale price c_i per unit of product. The following proposition states that a two-part tariff achieves coordination in our setting.[3]

PROPOSITION 8. *With a two part tariff, the supplier matches the profit of a centralized system leaving zero profit to the retailer. The retail prices and sales quantities in each period are same as that of the centralized system. The supplier sets* $c_1^* = 0$, $F_1^* = \frac{(2-\delta)^2}{4(4-3\delta)}$ *and* $c_2^* = 0$ *and* $F_2^* = \frac{\delta(2-2Q_1-\bar{\theta})^2}{4}$ *in periods 1 and 2 respectively.*

The next theorem illustrates that with a two-part tariff, both the supplier's and retailer's commitments are always beneficial.

THEOREM 6. *Suppose the supplier can offer a two-part tariff.*
(i) $\Pi_S^{RQC} > \Pi_S^{NC}$, $\Pi_R^{RQC} = \Pi_R^{NC}$.
(ii) $\Pi_S^{RPC} > \Pi_S^{NC}$, $\Pi_R^{RPC} = \Pi_R^{NC}$.
(iii) $\Pi_S^{SQC} > \Pi_S^{NC}$, $\Pi_R^{SQC} = \Pi_R^{NC}$.
(iv) $\Pi_S^{SPC} > \Pi_S^{NC}$, $\Pi_R^{SPC} = \Pi_R^{NC}$.

In both the supplier's and retailer's price and quantity commitments, the supplier attains the profit of the centralized system leaving no profit to the retailer. Furthermore, the sales are restricted to only period 1 in these examples similar to the commitments of the centralized system.

Note that the supplier can replicate any outcome of a two-part tariff by a quantity discount contract. This can be achieved for example by simply including the lump-sum fee F_i in the wholesale price of the first unit in the quantity discount contract. Therefore, with a quantity discount, the supplier can extract the entire surplus achieving profit of the centralized system and ensure that the commitments are always beneficial as in Theorem 6.

[3] If the retailer incurs inventory holding cost as in Anand et al. (2008), a two-part tariff cannot also achieve coordination in our setting. However, commitments are still always beneficial as long as this holding cost is not excessive.

9. Alternative Models

In this section, we consider alternative commitment models and modeling assumptions and discuss the robustness of our results. The full analyses of these extensions are provided in our online supplement. Here, we will only discuss their key results.

9.1 Period-specific Quantity Commitments

In Sections 6.1 and 7.1, the retailer and the supplier commit to Q_{max}, which limits the aggregate quantity that can be sold over two periods. Alternatively, the retailer and the supplier can commit to Q_{1max} and Q_{2max}, which limit the quantities that can be sold in periods 1 and 2.

It is straightforward to show that aggregate and period-specific quantity commitments by the retailer lead to the same outcome. Simply, any equilibrium outcome with an aggregate quantity commitment Q_{max} can be replicated by period-specific quantity commitments (and vice versa) by setting $Q_{1max} = Q_1$ and $Q_{2max} = Q_{max} - Q_1$. This happens because both Q_1 and Q_{1max} are chosen by the retailer. In contrast, aggregate and period-specific quantity commitments by the supplier can lead to different outcomes. In this case, we find that it is never optimal for the supplier to make a strictly binding commitment in period 1. Essentially, it is more profitable for the supplier to sell in period 1, thus it does not want to limit the sales in that period. On the other hand, the sales quantity in period 2 is critical for consumers' tendency to wait for mark-down, therefore limiting the sales in period 2 can be beneficial. The next theorem states the impact of such a commitment on profits. Let Π_S^{SQC2} show the supplier's profit when it commits to not selling more than Q_{2max} units in period 2.

Theorem 7. *(i)* $\Pi_S^{NC} < \Pi_S^{SQC2}$.
(ii) $\Pi_S^{NC} + \Pi_R^{NC} < \Pi_S^{SQC2} + \Pi_R^{SQC2}$.
(iii) $\Pi_R^{NC} > \Pi_R^{SQC2}$.

Similar to our main model, the supplier's quantity commitment hurts the retailer while benefiting itself (see Theorem 4). The only difference is that the retailer always suffers from the supplier's period-specific quantity commitment whereas the retailer may benefit from the supplier's aggregate quantity commitments when discount factor δ is sufficiently high. This happens because when the supplier commits to the aggregate quantity, the retailer has the flexibility to allocate this quantity over the two periods and it can manipulate period 2 wholesale price through carrying inventory. In contrast the supplier's period-specific quantity commitment takes away the retailer's flexibility and reduces its ability to manipulate wholesale price. In fact, we find that because of this benefit, the supplier achieves a higher profit by committing only to a period 2 quantity instead of making an aggregate quantity commitment. Intuitively, limiting period 2 sales is more aligned with the supplier's objective of pushing sales to period 1, which enables setting a higher wholesale price.

9.2 Concurrent Commitments

In Sections 6 and 7, the supplier and retailer make unilateral price or quantity quantity commitments. We have also considered what happens when both the supplier and the retailer can make independent quantity commitment or when they both can make price commitments. We find that they continue to suffer from commitments showing that the problem is due to lack of coordination and is not resolved by concurrent commitments.

Essentially, we find that when two parties make concurrent commitments to limit their future actions, the more stringent of the two effectively determines the result. Therefore, the outcome is similar to those of the commitments by a single party. Specifically, in the case of concurrent quantity commitments, the supplier's commitment dominates when customers are sufficiently patient, that is when δ is sufficiently high, otherwise, the retailer's commitment dominates. In the case of concurrent price commitments, equilibrium is always determined by the retailer's commitment, that is, it is same as that of the retailer's unilateral price commitment.

9.3 Utility Discounting

In our main model, product value diminishes by $1-\delta$ in period 2, but the payment term in period 2 is not discounted as seen in (1). This approach is appropriate for fashion, technology and seasonal products where the utility discounting is primarily due to the perishable value of the product. In contrast, in this section we study what happens when the product value and the payment are both discounted at the same rate. Specifically, the following replaces the customer utility function in (1):

$$U_i(\theta, p_i) = \delta^{i-1}(\theta - p_i), \ i:1,2. \tag{26}$$

This approach is more appropriate for products that are relatively immune to fashion and technology trends and seasonality. In this case, the loss due to delayed product use is comparable to the benefit of a delayed payment.

The following Theorem shows that the retailer's commitments can continue to hurt the profitability of both the retailer and the supplier.

THEOREM 8. *Suppose customer utility function is as in (26).*
(i) $\Pi_S^{NC} \geq \Pi_S^{RQC}$; $\Pi_S^{NC} + \Pi_R^{NC} \geq \Pi_S^{RQC} + \Pi_R^{RQC}$; $\Pi_R^{NC} \geq \Pi_R^{RQC}$.
(ii) $\Pi_S^{NC} > \Pi_S^{RPC}$; $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{RPC} + \Pi_R^{RPC}$; $\Pi_R^{NC} > \Pi_R^{RPC}$.

Inequalities in part (i) are strict when $\delta < \frac{1}{132}(127 - \sqrt{817}) \approx 0.745$. Similarly, the supplier's price commitments can make the retailer and the supplier worse off.

THEOREM 9. *Suppose customer utility function is as in (26). There exists* $\bar{\delta}_4 \approx 0.659$ *such that* $\Pi_S^{NC} > \Pi_S^{SPC}$ *when* $\delta < \bar{\delta}_4$; $\Pi_R^{NC} > \Pi_R^{SPC}$; $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{SPC} + \Pi_R^{SPC}$.

Note that $\bar{\delta}_4$ is explicitly characterized in the proof of this Theorem.

Finally, it is straightforward to show that the supplier never gets worse off from its quantity commitment as in Theorem 4, since it can always nullify this commitment by choosing an arbitrarily large quantity. However, the retailer may not suffer from the supplier's quantity commitment with the alternative customer utility function in (26). We find that $\Pi_R^{NC} \leq \Pi_R^{SQC}$ when $\delta \in \{0.2, 0.4, 0.6, 0.8\}$. The equilibrium of supplier's quantity commitment involves several regions similar to our main model (see Appendix A), thus instead of characterizing equilibrium for all scenarios, we resort to numerical examples in this extension.

It is worthwhile to highlight the differences between the results of our main model and this extension. While in our main model the supplier's price commitment always hurts its profitability (Theorem 5), the supplier can benefit from its price commitment in this extension when δ is sufficiently high (Theorem 9). Intuitively, when the discount factor applies to both product value and retail price in period 2 as in (26) as opposed to only product value as in (1), consumers are willing to pay higher prices in period 2, which enables the retailer and the supplier to set higher prices in period 2. Thus, the supplier can commit to a higher price in period 2, which allows setting a higher price in period 1. Similarly, the supplier's quantity commitment can be more beneficial to the retailer with the alternative utility function in (26): While the retailer in our main model may suffer from the supplier's quantity commitment, our numerical examples in this extension show that the supplier's quantity commitment does not hurt the retailer. Because the supplier can set a higher wholesale price in period 2, it does not excessively limit its quantity, which in turn helps the retailer.

9.4 No-Inventory Carryover

Our main model assumes the retailer does not incur inventory holding cost. Here, we consider the other extreme assuming that carrying inventory is prohibitively costly or infeasible so the retailer cannot carry over inventory. This extension also disentangles the effects of strategic inventory from our results. The following Theorem shows that the retailer's and the supplier's commitments can continue to hurt either themselves or their supply chain partners.

THEOREM 10. *Suppose the retailer cannot carry over inventory.*

(i) $\Pi_S^{NC} > \Pi_S^{RQC}$; $\Pi_R^{NC} > \Pi_R^{RQC}$.

(ii) $\Pi_S^{NC} > \Pi_S^{RPC}$; $\Pi_R^{NC} > \Pi_R^{RPC}$.

(iii) $\Pi_S^{NC} < \Pi_S^{SQC}$; $\Pi_S^{NC} + \Pi_R^{NC} < \Pi_R^{SQC} + \Pi_R^{RQC}$; $\Pi_R^{NC} > \Pi_R^{SQC}$ *when* $\delta < \bar{\delta}_5$.

(iv) $\Pi_S^{NC} < \Pi_S^{SPC}$; $\Pi_S^{NC} + \Pi_R^{NC} > \Pi_S^{SPC} + \Pi_R^{SPC}$ *when* $\delta < \bar{\delta}_6$; $\Pi_R^{NC} > \Pi_R^{SPC}$.

Note $\bar{\delta}_5 \approx 0.694$ and $\bar{\delta}_6 \approx 0.592$, and they are explicitly characterized in the proof of this theorem.

Different from our main model, the supplier benefits from its price commitment when the retailer cannot carry over inventory. In our main model, the supplier's price commitment hurts its profitability because it eliminates the retailer's incentive to buy more in period 1 to pay less in period 2. However, this incentive is dampened in the no-commitment scenario when the retailer cannot carry over inventory. In other words, the supplier's price commitment does not decrease its profitability mainly because its no-commitment profit is lower when the retailer cannot hold inventory.

10. Conclusions

Previous studies have shown that a firm's ability to make commitments is effective in dealing with strategic customer behavior. However, those studies consider only the interactions between a retailer and consumers. In this paper, we study the value of commitments in a decentralized supply chain by also taking into account how commitments affect the interactions between the retailer and its supplier. We consider a two-period selling season. In our base model, in each period, the supplier sets the wholesale price, the retailer places its order from the supplier and sets its retail price, and then forward-looking strategic customers decide whether to buy the product in that period. We also study the supplier's and retailer's price and quantity commitments and compare them with our base model. Table 1 summarizes the impact of retailer's and supplier's price and quantity commitments on their and total supply chain's profitability.

Table 1 Impact of Commitments on Profitability

		Retailer	Supplier	Supply Chain
Commitment Type	**Retailer's Price**	Negative	Negative	Negative
	Retailer's Quantity	Negative unless customers are sufficiently patient	Negative	Negative
	Supplier's Price	Negative	Negative	Negative
	Supplier's Quantity	Negative unless customers are sufficiently patient	Positive	Positive

While commitments always increase profitability of a centralized system, we find that in a decentralized supply chain, the retailer's and supplier's commitments do not necessarily improve their profitability. Furthermore, they can hurt the other partner in the supply chain, thereby making the whole supply chain worse off. This is because such commitments can exacerbate double-marginalization inefficiency in a supply chain. Without addressing the coordination problem, then, commitments cannot address the strategic customer problem in a decentralized supply chain.

Note that the total market size is deterministic in our model, which enables analytical tractability. Earlier studies (e.g., Aviv and Pazgal 2008, Cachon and Swinney 2009) show that demand uncertainty can make commitments less valuable in a centralized system. This is because making commitments impairs a firm's ability to respond to demand variability. However, in a decentralized supply chain, the effect of demand uncertainty may be more intricate because of the vertical interactions between the retailer and the supplier.

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Appendix A: Analysis of Supplier's Quantity Commitment

Without loss of generality, we can restrict our analysis to $Q_{max} \in [1/4, 1/2]$, as is formally stated in the following Lemma.

LEMMA 8. *Suppose the supplier can commit to sell not more than Q_{max} units in two periods. $Q_{max} \geq \frac{1}{4}$ in all equilibria. Furthermore, for any equilibrium with $Q_{max} > \frac{1}{2}$, there is an equivalent equilibrium with $Q_{max} = \frac{1}{2}$, which yields the same payoffs to all parties (supplier, retailer and customers).*

We follow backward induction to characterize the equilibrium starting with period 2. Following Lemma 5, the quantity commitment binds; that is, the retailer buys all of the supplier's committed quantity in period 2, if and only if

$$Q_{max} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4}. \tag{27}$$

Thus, when (27) holds, period 2 equilibrium is given by Lemma 5. On the other hand, when (27) does not hold, the commitment is ineffective, and period 2 equilibrium is the same as in a no-commitment scenario, which is given by Lemma 1.

Note that the retailer's period 1 choices determine whether the commitment will bind in period 2, that is, whether (27) will hold. When the commitment binds, i.e., (27) holds, the retailer's profit is given by (19). In contrast, when the commitment does not bind, i.e., (27) does not hold, the retailer's profit is given by (12). Solving the retailer's maximum profit for both cases and comparing them, we determine the retailer's optimal policy in period 1. However, overall profit function for the retailer is not quasiconcave. In addition, the profit for the case when commitment binds is not a concave function. This lack of concavity leads to many cases of the following Proposition where we describe the optimal policy in period 1.

PROPOSITION 9. *Suppose the supplier commits to sell not more than $Q_{max} \in [1/4, 1/2]$ units in two periods. The retailer's SPNE policy is characterized as follows.*

(i) The quantity commitment binds strictly, the retailer sells the product in both periods and it carries inventory between periods with $Q_1^* = \frac{1-c_1-3\delta Q_{max}}{2(1-2\delta)}$, $p_1^* = \frac{1+\delta(1-c_1)+\delta(Q_{max}(1+\delta)-3)}{2(1-2\delta)}$ and $\bar{\theta}^* = \frac{1+c_1-\delta(4-3Q_{max})}{2(1-2\delta)}$, when $\delta < 1/2$, $\frac{1-c_1}{2-\delta} < Q_{max} < \frac{5-3c_1-4\delta}{8-7\delta}$.

(ii) The quantity commitment binds strictly, the retailer sells the product only in period 1 with $Q_1^* = Q_{max}$ and $p_1^* = \bar{\theta}^* = 1 - Q_{max}$, when

a) $\delta < 1/2$, $Q_{max} \leq \frac{1-c_1}{2-\delta}$,

b.1) $\delta \geq 1/2$, $c_1 \leq \frac{2(1-\delta)(5\delta-1)}{23-25\delta}$, $Q_{max} < K_1$,

b.2) $\delta \geq 1/2$, $c_1 > \frac{2(1-\delta)(5\delta-1)}{23-25\delta}$, $Q_{max} < \frac{4-3c_1-2\delta}{7-5\delta}$.

(iii) The quantity commitment binds weakly, the retailer sells the product in both periods and it does not carry inventory between periods with $Q_1^* = \frac{1}{3}(4Q_{max} - 1)$*,* $p_1^* = \frac{1}{3}(4-\delta)(1-Q_{max})$*, and* $\bar{\theta}^* = \frac{4}{3}(1-Q_{max})$*, when*

a) $\delta < 1/2$*,* $\frac{5-3c_1-4\delta}{8-7\delta} \le Q_{max} \le \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$*,*

b.1) $\delta \ge 1/2$*,* $c_1 \le K_c$*,* $\frac{4-3c_1-2\delta}{7-5\delta} \le Q_{max} \le \frac{10+3c_1-14\delta}{4(4-5\delta)}$*,*

b.2) $\delta \ge 1/2$*,* $K_c < c_1 \le \frac{3\delta(4-3\delta)}{2(16-15\delta)}$*,* $\frac{4-3c_1-2\delta}{7-5\delta} \le Q_{max} \le K_2$*,*

b.3) $\delta \ge 1/2$*,* $c_1 > \frac{3\delta(4-3\delta)}{2(16-15\delta)}$*,* $\frac{4-3c_1-2\delta}{7-5\delta} \le Q_{max} \le \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$*.*

(iv) The quantity commitment binds weakly, the retailer sells the product in both periods and it carries inventory between periods with $Q_1^* = \frac{4Q_{max}(4-3\delta)-6(1-\delta)}{4(2-\delta)}$*,* $p_1^* = \frac{2+(1-\delta)(4\delta-c_1)-2\delta(4-3\delta)Q_{max}}{2(2-\delta)}$ *and* $\bar{\theta}^* = \frac{2-c_1+2\delta(1-2Q_{max})}{2(2-\delta)}$*, when* $\max(K_1, \frac{10+3c_1-14\delta}{4(4-5\delta)}) < Q_{max} \le K_3$*.*

(v) The quantity commitment does not bind, the retailer sells the product in both periods and it carries inventory between periods with $Q_1^* = \frac{11}{14} - \frac{c_1(112-91\delta)+6\delta}{21\delta(8-7\delta)}$*,* $p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-7\delta)}$*, and* $\bar{\theta}^* = \frac{3(4-3\delta)+2c_1}{3(8-7\delta)}$*, when*

a) $c_1 \le K_c, Q_{max} > K_3$*,*

b) $K_c < c_1 < \frac{3\delta(4-3\delta)}{2(16-15\delta)}$*,* $Q_{max} > K_2$*.*

(vi) The quantity commitment does not bind, the retailer sells the product in both periods and it does not carry inventory between periods with $Q_1^* = \frac{8(1-c_1)-3\delta}{16-9\delta}$*,* $p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}$*, and* $\bar{\theta}^* = \frac{8(1+c_1)-6\delta}{16-9\delta}$*, when*

a) $\delta < 1/2$*,* $c_1 < \frac{8-3\delta}{8}$*,* $Q_{max} > \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$*,*

b) $\delta \ge 1/2$*,* $\frac{3\delta(4-3\delta)}{2(16-15\delta)} \le c_1 < \frac{8-3\delta}{8}$*,* $Q_{max} > \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$*.*

When $c_1 \ge \frac{8-3\delta}{8}$*, the retailer does not procure any units in period 1. The thresholds* K_1, K_2, K_3 *and* K_c *are defined as follows.*

$$K_1 = \frac{4+4c_1-14\delta}{4(2-5\delta)} + \frac{\sqrt{(2-\delta)\left(\frac{3c_1^2}{2(1-\delta)} + \delta - 2c_1\right)}}{2(2-5\delta)} \tag{28}$$

$$K_2 = \frac{20-12c_1-7\delta}{2(16-5\delta)} + \frac{\sqrt{2\delta\left(3c_1\delta(512-\delta(992-(496-13\delta)\delta)) + 9\delta^2(32-\delta(88-\delta(68-13\delta))) - 2c_1^2(3072-\delta(8576-\delta(7980-2479\delta)))\right)}}{2\delta(16-5\delta)(8-7\delta)} \tag{29}$$

$$K_3 = \frac{\delta(6(1-\delta)-c_1)\sqrt{2(2-\delta)(16-13\delta)} + 6\delta(8-7\delta)^2 - c_1(192-\delta(312-126\delta))}{12\delta(6-5\delta)(8-7\delta)} \tag{30}$$

$$K_c = \frac{6\delta\left(1024-2496\delta+2128\delta^2-783\delta^3+130\delta^4+(4-5\delta)(8-7\delta)(8-9\delta)\sqrt{2(2-\delta)(16-13\delta)}\right)}{49152-\delta(161792-\delta(196288-5\delta(20672-3943\delta)))} \tag{31}$$

Next, we study the supplier's optimal wholesale price and commitment quantity in period 1, which completes the analysis of the equilibrium. The next proposition describes the supplier's optimal policy. Let $\Pi_S(Q_{max}, c_1)$ show the supplier's total profit when it commits to Q_{max} and sets the wholesale price c_1 in period 1.

PROPOSITION 10. *The supplier's SPNE wholesale price* c_1^* *and quantity commitment* Q_{max}^* *are as follows.*

(i) When $\delta < 1/2$*,* $Q_{max}^* = \frac{1}{2(2-\delta)}$ *and* $c_1^* = 1/2$*.*

(ii) When $1/2 \le \delta < \frac{199-3\sqrt{137}}{218}$*,* $Q_{max}^* = \frac{2-\delta}{7-5\delta}$ *and* $c_1^* = \frac{2-\delta}{3}$*.*

(iii) When $\frac{199-3\sqrt{137}}{218} \le \delta < \bar{\delta}_7$*,* $Q_{max}^* = \frac{7}{16}$ *and* $c_1^* = \frac{16-3\delta}{32}$*.*

(iv) When $\bar{\delta}_7 \le \delta$*,* $Q_{max}^* = K_2$ *and* $c_1^* = \arg\max_{c_1} \Pi_S(K_2, c_1)$*,*

Note $\bar{\delta}_7 \approx 0.754$ and it is explicitly characterized in the proof of this proposition. Parts (i) and (ii) of Proposition 10 map to part (ii) of Proposition 9; in this case, quantity commitment binds strictly. Similarly, parts (iii) and (iv) of Proposition 10 map to part (iii) of Proposition 9; in this case, quantity commitment binds weakly. Thus, the supplier always finds it attractive to choose a binding (weakly or strictly) quantity commitment. Note that the supplier can always match the outcome of a no-commitment scenario by committing to non-binding arbitrarily large quantity. Thus, the supplier should always benefit from its ability to make a quantity commitment. This is formally stated in Theorem 4.

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