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Online Supplement to Product-Line Competition: Customization vs. Proliferation

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Appendix D: Customizing Firm's Capacity Choice

Here, we supplement our discussion in Section 5 to show how the results in Section 4 apply to endogenous customization capacity in a specific setting. Specifically, we consider the results of Propositions 4, 5, 6 and 8 (Proposition 7 already describes what happens as customization capacity varies). We use numerical examples with a linear capacity cost function $C(\mu) = a\mu$ and the parameters in Table 3 of Section 5 for w, S, k, v, h, l and r as our base case.

Proposition 4 extends to endogenous capacity trivially. Part (i) follows from the fact that a monopoly that sells only standard products always allocates a greater market share to standard products than a duopoly.[1] Part (ii) follows because a dual monopoly that can offer both standard and custom products always serves only either standard or custom products under linear capacity costs (this is shown in Appendix F)[2] and when $c_c - c_t$ is large, it is more likely to serve only standard products. It follows that a dual monopoly will allocate a lager market share to standard products and offer more standard product variants compared to duopoly. Similarly, when $c_c - c_t$ is small, a dual monopoly is more likely to serve only custom products, so a duopoly results in more standard product variants in this case.

Proposition 5 characterizes the effect of r on firm profits. Table 4 shows the results of a numerical study in which the customizing firm is allowed to choose its capacity μ at cost $C(\mu)$. The table reports firm profits at various levels of r, c_c and c_t, which confirms the finding of Proposition 5. When the unit cost differential is sufficiently favorable for the traditional firm ($c_c = 3.5, c_t = 2$), a

[1] Recall that the optimal market share per standard variant is the same in all cases (duopoly, monopoly) and for all capacity levels of the customizing firm. This follows from Propositions 1 and 3 and it is discussed following Proposition 4.

[2] This is not necessarily true for strictly convex costs. For a strictly convex cost function, a dual monopoly can serve both product types and in Appendix F we also show that our monopoly result (Proposition 2) continues to hold for such a cost function.

Table 4 **The effect of intensity of preferences r and unit cost differential $c_c - c_t$ on firm profits ($w = 8, S = 3, h = 0.15, l = 6, k = 0.75, \lambda = 10, v = 20, C(\mu) = 0.25\mu$).**

c_c, c_t	r	$\Pi_c - C$	Π_t	μ	n	p_c	p_t	λ_c	$\mathbb{E}[W_c]$
$c_c = 0.5$, $c_t = 2$	60	35.75	0.151	34.09	4.2	6.25	3.85	7.70	0.76
	80	39.48	0.154	34.07	5.1	6.74	4.08	7.69	0.76
	100	42.63	0.155	34.05	6.0	7.16	4.28	7.67	0.76
$c_c = 3.5$, $c_t = 2$	60	9.12	0.650	28.24	7.3	6.20	3.95	5.99	0.90
	80	12.64	0.540	29.26	8.4	6.71	4.15	6.22	0.87
	100	15.65	0.481	29.86	9.3	7.14	4.34	6.36	0.85

Table 5 **The effect of holding cost h and unit cost differential $c_c - c_t$ on firm profits ($w = 8, S = 3, r = 80, l = 6, k = 0.75, \lambda = 10, v = 20, C(\mu) = 0.25\mu$).**

c_c, c_t	h	$\Pi_c - C$	Π_t	μ	n	p_c	p_t	λ_c	$\mathbb{E}[W_c]$
$c_c = 0.5$, $c_t = 2$	0.10	33.83	0.150	34.10	5.9	5.99	3.61	7.71	0.76
	0.15	39.48	0.154	34.07	5.1	6.74	3.90	7.69	0.76
	0.20	44.10	0.156	34.04	4.6	7.36	4.15	7.67	0.76
$c_c = 3.5$, $c_t = 2$	0.10	7.35	0.739	27.49	10.7	5.94	3.72	5.84	0.92
	0.15	12.64	0.540	29.26	8.4	6.71	3.98	6.22	0.87
	0.20	17.06	0.459	30.09	7.1	7.33	4.21	6.41	0.84

larger r makes the traditional firm worse off. However, when the unit cost differential is sufficiently unfavorable for the traditional firm ($c_c = 0.5, c_t = 2$), it benefits from a larger r. In this case, although a larger r makes standard products less attractive, the traditional firm benefits from a milder price competition due to a larger gap between standard and custom products. Notice that endogenous capacity strengthens this result: Here, a larger r results in a smaller capacity choice by the customizing firm, further reducing the intensity of price competition.

Proposition 6 describes the effect market size. In Section 5, we discuss in detail how its insights can extend to endogenous capacity and Table 3 shows the results of our corresponding numerical study.

Proposition 8 shows how changes in the holding cost h and order lead time l affect firm profits. Tables 5 and 6 report our corresponding numerical examples under endogenous customization capacity. The tables show that the insights of Proposition 8 continue to hold when the customizing firm can choose its customization capacity. In particular, although a larger holding cost h and longer lead time l would normally make the traditional firm less profitable, these can increase the traditional firm's profit in a duopoly competition against a customizing firm when the traditional firm has a sufficiently large unit cost disadvantage ($c_c = 0.5, c_t = 2$). In this case, larger h and l help the traditional firm, as they result in a milder price competition. Note that endogenous customization capacity strengthens this result, as the customizing firm chooses a smaller capacity for larger values of h and l, further reducing the intensity of competition.

Table 6 **The effect of order lead time l and unit cost differential $c_c - c_t$ on firm profits ($w = 8, S = 3, r = 80, h = 0.15, k = 0.75, \lambda = 10, v = 20, C(\mu) = 0.25\mu$).**

c_c, c_t	l	$\Pi_c - C$	Π_t	μ	n	p_c	p_t	λ_c	$\mathbb{E}[W_c]$
$c_c = 0.5$, $c_t = 2$	4	34.26	0.150	34.10	5.8	6.05	3.94	7.71	0.76
	6	39.48	0.154	34.07	5.1	6.74	3.90	7.69	0.76
	8	44.27	0.156	34.04	4.6	7.38	3.86	7.67	0.76
$c_c = 3.5$, $c_t = 2$	4	7.74	0.716	27.68	10.4	6.00	4.05	5.87	0.92
	6	12.64	0.540	29.26	8.4	6.71	3.98	6.22	0.87
	8	17.23	0.457	30.11	7.1	7.36	3.93	6.42	0.84

Appendix E: Other Extensions

Uniform vs. Menu prices: Our base model has restricted the firms to uniform prices. Here, we describe what happens when the firms are allowed to set different prices for each product configuration, i.e., the traditional firm sets a vector of prices $(p_1, p_2, ..., p_n)$ and the customizing firm sets a price menu $p(\theta)$ for $\theta \in \Theta$. The analysis of this alternative model requires assuming that the customizing firm sets its price menu after observing the traditional firm's prices, otherwise a pure strategy equilibrium with *undominated* strategies (firms not setting their prices below their costs) need not exist.[3] The following Proposition characterizes the equilibrium.

PROPOSITION 11. *When traditional Firm t competes with customizing Firm c under menu prices, it offers $n = (1 - \lambda_c/\lambda)/\gamma$ product variants positioned symmetrically at $\zeta_i = (2i-1)/(2n)$ for $i = 1, 2, ..., n$ and it sets a uniform price*

$$p_t = c_c + \frac{v\mu}{(\mu - \lambda_c)^2} - \frac{\gamma r}{2} - \frac{lv[(1+k^2)(1-\Phi(k)) - k\phi(k)]}{2Q^*(\gamma\lambda)}$$

for all variants, where $Q^*(\cdot)$ is given by (9) and λ_c is given by the solution of

$$\frac{v\mu(\mu + \lambda_c - 2\lambda)}{(\mu - \lambda_c)^3} = c_t - c_c + \frac{3\gamma r}{2},$$

and the customizing firm's price menu is given by

$$p_c(\theta) = c_c + \frac{v\lambda_c}{(\mu - \lambda_c)^2} - \frac{\gamma r}{2} + \max(\min_{i \le n} |\theta - \zeta_i|, \frac{\gamma}{2})r.$$

In equilibrium, customers in $[\zeta_i - \gamma/2, \zeta_i + \gamma/2]$ buy standard product i for $i : 1, 2, ..., n$ and the remaining customers buy the product customized for their own types. The customizing firm sets the maximum prices that leave its customers indifferent to buying their best alternative from the

[3] The order of events is same as in Thisse and Vives (1988), where the firm that sets a continuous set of prices follows the firm that sets a discrete set of prices. Thisse and Vives (1988) also point the same problem with simultaneous pricing for this case (p. 128).

Table 7 Comparison of comparative statics and the firms' policies in monopoly and duopoly - Menu prices.

Traditional Channel

	Single Monopoly	Dual Monopoly	Duopoly
n	Largest	Middle	Smallest
$d\Pi_t/d\lambda$	$+$	$+$	$+$
$d\Pi_t/dr$	$-$	$-$	$-$
$d\Pi_t/d\mu$	N/A	$-$	$-$
$d\Pi_t/dh$	$-$	$-$	$-$
$d\Pi_t/dl$	$-$	$-$	$-$

Customizing Channel

	Single Monopoly	Dual Monopoly	Duopoly
λ_c	Largest	Smallest	Middle
$d\Pi_c/d\lambda$	$+$ or 0	$-$	$+$ or $-$
$d\Pi_c/dr$	0	$+$	$+$
$d\Pi_c/d\mu$	$+$	$+$	$+$ or $-$
$d\Pi_c/dh$	N/A	$+$	$+$
$d\Pi_c/dl$	N/A	$+$	$+$

traditional firm whereas the traditional firm sets a uniform price. Notice that symmetric positions of standard products is the *unique* equilibrium in menu price model unlike uniform price model (this assumes the traditional firm breaks ties among its optimal policies to the disadvantage of its opponent). Basically, symmetric locations minimizes the aggregate premium earned by the customizing competitor for any demand rate λ_c. However, in the uniform price model, the customizing competitor extracts the same profit from each customer, therefore its total profit for any demand rate λ_c is the same regardless.

Table 7 summarizes our results for the price menu model.[4] We find that many of our findings carry over to the price menu model. In particular, the customizing firm's profit does not monotonically increase in its customization rate μ and the market size λ. Furthermore, the customizing firm's unit cost disadvantage plays a critical role in determining the effect of these parameters.

Inventory Approximation: We used the standard approximation (6) and (7) (Hadley and Whitin (1963)) to express the expected cost and backorders associated with a (Q,R) policy. The exact expression

$$\hat{B}(Q_i) = \frac{\sigma_i^2}{2Q_i} \begin{aligned}[t] &([(1+k^2)(1-\Phi(k)) - k\phi(k)] - \\ &[(1+(k+Q_i/\sigma_i)^2)(1-\Phi(k+Q_i/\sigma_i)) - (k+Q_i/\sigma_i)\phi(k+Q_i/\sigma_i)]) \end{aligned} \tag{22}$$

is not analytically tractable, and the Hadley-Whitin approximation drops the last term in square brackets. The difference between (6) and (22) is not significant when Q_i/d_i (the ratio of order quantity to lead time demand) is sufficiently large (Zipkin 2000, Axsäter 1995, Hadley and Whitin 1963), which holds for example when the fixed order cost S is large. Zipkin (1986) has shown that (22) is convex in Q_i and the optimal policy can be computed numerically. Thus, we can numerically compute the equilibrium for (22) and the corresponding expression of the expected cost

$$\hat{C}(\lambda_i, Q_i) = S\lambda_i/Q_i + h(Q_i/2 + \sigma_i k + \hat{B}(Q_i)).$$

Figure 3 compares the equilibrium demand allocations to our main model at various setup costs.

[4] The proofs are available from the authors upon request.

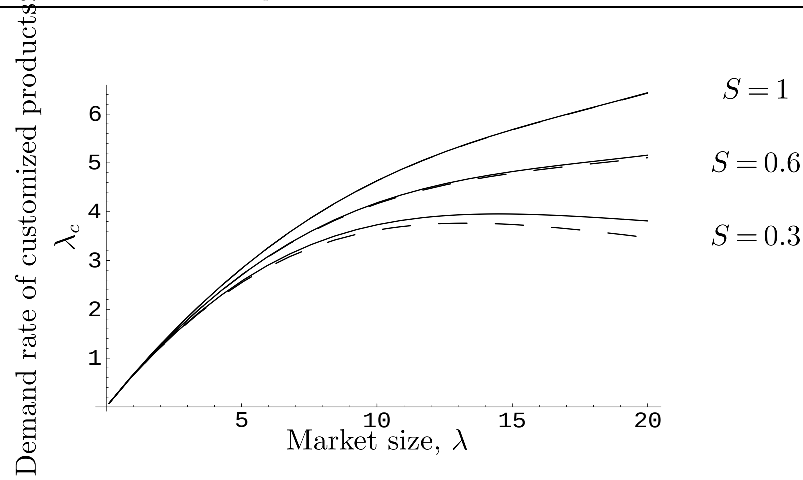

Figure 3 **Comparison of demand rates, straight lines are obtained by using (6), and dashed lines by (22) where** $c_c - c_t = 1.5, h = 0.5, k = 0.5, l = 1, v = 4, r = 20, \mu = 20$**.**

In general, the demand rate of customized products is slightly higher under (6), especially when the market size is large. This is because the approximation tends to overestimate the traditional firm's holding cost. The Figure shows that the difference between the equilibrium demand rates is not significant when the setup cost is sufficiently high, as expected. Furthermore, numerical studies show that our structural insights carry over even for small setup costs.

Appendix F: Dual Monopoly with Endogenous Capacity

We first show that with a linear capacity cost function i.e., $C(\mu) = a\mu$, a dual monopoly always chooses one of the extremes offering only either custom or standard products. Then for a convex cost example $C(\mu) = a\mu^{3/2}$, we show that the findings of Proposition 2 continue to hold.

Following (34), when it sells both standard and custom products (i.e., $0 < \lambda_c < \lambda$), a dual monopoly's profit for a given capacity μ is as follows (excluding capacity investment cost)

$$\Pi(\mu) = \lambda_c\left(\frac{3\gamma r}{2} + c_t - c_c - \frac{v}{\mu - \lambda_c}\right) + \lambda\left(w - \frac{3\gamma r}{2} - c_t\right). \tag{23}$$

Let us define

$$\alpha = \frac{3\gamma r}{2} + c_t - c_c. \tag{24}$$

Plugging (13) into (23) and some basic algebra yield

$$\begin{aligned}
\Pi(\mu) &= \mu\Big(1 - \sqrt{\frac{v}{\mu\alpha}}\Big)\alpha\Big(1 - \sqrt{\frac{v}{\mu\alpha}} + \lambda\Big(w - \frac{3\gamma r}{2} - c_t\Big)\\
&= \mu(\sqrt{\alpha} - \sqrt{v/\mu})^2 + \lambda\Big(w - \frac{3\gamma r}{2} - c_t\Big)\\
&= \mu\alpha - 2\sqrt{\alpha v \mu} + v + \lambda\Big(w - \frac{3\gamma r}{2} - c_t\Big).
\end{aligned} \tag{25}$$

Thus, for $C(\mu)=a\mu$, the firm's total profit $\Pi(\mu)-C(\mu)$ is convex in μ and hence the optimal μ is always at the extreme, either $\mu=0$ or μ is such that $\lambda_c=\lambda$.

Now, for $C(\mu)=a\mu^{3/2}$, we will show that the findings of Proposition 2 carry over. Note that

$$\frac{d}{d\mu}[\Pi(\mu)-C(\mu)]=\alpha-\sqrt{\frac{\alpha v}{\mu}}-\frac{3}{2}a\sqrt{\mu}. \tag{26}$$

It is straightforward to show that (26) is quasiconcave in μ. Thus, if the optimal solution is in the interior (i.e., serving both standard and custom products), it is given by the larger root of (26) which is given by

$$\mu=\frac{2(\alpha^2-3a\sqrt{\alpha v}+\sqrt{\alpha^4-6a\alpha^2\sqrt{\alpha v}})}{9a^2} \tag{27}$$

Furthermore, if a is not very large or very small, the optimal solution is indeed in the interior. For example, this is ensured by $a=0.5$ for $c_t=1, c_c=1$ and the parameters used in Table 3: $h=0.15, r=80, l=6, v=10, k=0.75, S=3$. Thus, we assume $a\in[\underline{a},\bar{a}]$ so that it is optimal to serve both product types.

We show that $d\lambda_c/d\alpha>0$ and the results (i) and (ii) in Proposition 2 immediately follow by the fact that α decreases in λ and increases in h, l and r. Following (13),

$$\lambda_c=\mu(1-\sqrt{\frac{v}{\alpha\mu}}). \tag{28}$$

Thus,

$$\frac{d\lambda_c}{d\mu}=(1-\frac{1}{2}\sqrt{\frac{v}{\alpha\mu}})\frac{d\mu}{d\alpha}+\frac{\sqrt{\mu v}}{2\alpha\sqrt{\alpha}}.$$

The sum of terms in the first parenthesis is positive following (28) and the last term is also positive, therefore $\frac{d\mu}{d\alpha}>0$ would yield $\frac{d\lambda_c}{d\alpha}>0$ proving the result. Finally, (27) can be expressed as follows,

$$\mu=\frac{2\alpha^2(1-3a\sqrt{v/\alpha^3}+\sqrt{1-6a\sqrt{v/\alpha^3}})}{9a^2},$$

which shows that indeed $\frac{d\mu}{d\alpha}>0$.

Appendix G: Proofs

Proof of Lemma 1 The optimal order size is given by minimizing the total cost of holding and ordering costs in (7). $\square$

Proof of Proposition 1 The firm aims to maximize the total profit

$$\Pi=(p_c-c_c)\lambda_c+(p_t-c_t)\sum_i^n\lambda_i-\sum_i^n C(\lambda_i), \tag{29}$$

where $\lambda_c + \sum_i^n \lambda_i = \lambda$. We first characterize the optimal prices and then show that it is optimal for standard products to have equal market shares. Finally, we derive the number of standard products and the optimal market share allocated to customized products which completes the proof.

Let us define

$$\Phi^*(k) = [(1+k^2)(1-\Phi(k)) - k\phi(k)] \tag{30}$$

We describe the firm's policy in terms its choice of market share allocations λ_c and λ_i for $i: 1, 2, ..., n$ and then find the optimal allocations. Clearly, it is optimal for the firm to charge the maximum price that leaves the marginal customers indifferent to their outside option (not buying in this case). Thus, it follows from (2) and (5) the unit price for customized products is $p_c = w - v/(\mu - \lambda_c)$. When standard product i has demand allocation λ_i, following (3) and (4), its distance with its marginal customers is $\lambda_i/(2\lambda)$. So, it follows from (1), (8) and (9) that the unit price for standard products is

$$p_t = w - \max_{i \in \{1,2,...,n\}} [r\lambda_i/(2\lambda) + vl\Phi^*(k)/(2\sqrt{\lambda_i(2S/h + \Phi^*(k))l})]. \tag{31}$$

Now, we show that it is optimal to allocate equal market share to each standard product. Note that the inventory holding and fixed order cost for product i given in (7) calculated at the optimal order quantity (9) is equal to

$$C(\lambda_i) = \sqrt{\lambda_i h(2S + hl\Phi^*(k))} + kh\sqrt{\lambda_i l}. \tag{32}$$

It is straightforward to show that when the total demand of standard products $\sum_i^n \lambda_i$ is kept constant (i.e., $\lambda - \lambda_c$ kept constant) , allocating $\lambda_i = (\lambda - \lambda_c)/n$ to each standard product maximizes $(p_t - c_t)\sum_i^n \lambda_i - \sum_i^n C(\lambda_i)$, where p_t is as in (31). Note that when $\sum_i^n \lambda_i$ is kept constant, its allocation among standard products has no effect on profit of the customization channel $p_c(\lambda - \lambda_c)$.

Thus, the unit price of standard products in (31) can be expressed as

$$p_t = w - \frac{(1-\lambda_c/\lambda)r}{2n} - vl\Phi^*(k)/\left(2\sqrt{(\frac{\lambda - \lambda_c}{n})(2S/h + \Phi^*(k)l)}\right),$$

and the firm's total profit given in (29) turns into

$$\begin{aligned}\Pi(\lambda_c, n) = \lambda_c(w - \frac{v}{\mu - \lambda_c} - c_c) \;+\; (\lambda - \lambda_c)(w - \frac{(1-\lambda_c/\lambda)r}{2n} - \frac{vl\Phi^*(k)}{2\sqrt{\frac{(\lambda-\lambda_c}{n})(2S/h + \Phi^*(k)l)}} - c_t) \\ -\; \sqrt{n}(\sqrt{(\lambda - \lambda_c)h(2S + hl\Phi^*(k))} + kh\sqrt{(\lambda - \lambda_c)l}).\end{aligned} \tag{33}$$

This is concave in n, and it is maximized at $n = (1 - \lambda/\lambda_c)/\gamma$. Recall that we ignore integrality of n for tractability. Plugging this back into (33) gives

$$\Pi(\lambda_c, n^*(\lambda_c)) \;=\; \lambda_c(w - \frac{v}{\mu - \lambda_c} - c_c) + (\lambda - \lambda_c)(w - \frac{\gamma r}{2} - \frac{vl\Phi^*(k)}{2\sqrt{\gamma\lambda(2S/h + \Phi^*(k)l)}} - c_t - \sqrt{\frac{h(2S + hl\Phi^*(k))}{\gamma\lambda}} + kh\sqrt{\frac{l}{\gamma\lambda}})$$

$$= \lambda_c(w - \frac{v}{\mu - \lambda_c} - c_c) + (\lambda - \lambda_c)(w - \frac{3\gamma r}{2} - c_t), \tag{34}$$

where the second equality follows by simple algebra. (34) is concave in λ_c and it is maximized by (13). Note that $\frac{3\gamma r}{2} - \frac{v\mu}{(\mu-(1-\gamma)\lambda)^2} < c_c - c_t < \frac{3\gamma r}{2} - \frac{v}{\mu}$, which is stated in Section 3, indeed guarantees that $0 < \lambda_c < \lambda$ and $n \geq 1$. It is straightforward to verify to that Π in (34) is positive by plugging (13) back into (34) and making use of $\frac{3\gamma r}{2} - \frac{v\mu}{(\mu-(1-\gamma)\lambda)^2} < c_c - c_t < \frac{3\gamma r}{2} - \frac{v}{\mu}$ and $w > \frac{3\gamma r}{2} + c_t$. $\square$

Proof of Proposition 2 The results follow from (13) and (10). $\square$

Proof of Proposition 3 It is straightforward to show that it is optimal for the traditional firm to allocate equal market shares to each standard product as in the Proof of Proposition 1. Note that choosing symmetric product locations guarantees this allocation.

We start with characterizing the firms' market shares given their price and variety choices. When customized products have demand rate λ_c, following (4) and the fact that standard products have equal market shares, the size of each set Θ_i is equal to $(\lambda - \lambda_c)/(2n)$. Thus, following (3), the marginal customer who is indifferent between buying a standard and a customized product is at $(\lambda - \lambda_c)/(2n)$ distance from its nearest standard product. Thus, the indifference condition in (2) leads to

$$p_t + \frac{(1 - \lambda_c/\lambda)r}{2n} + \frac{vl\Phi^*(k)}{2Q^*} = p_c + \frac{v}{\mu - \lambda_c}, \tag{35}$$

where left and right hand sides are the full cost (price + misfit disutility + delay disutility) of buying a standard and a customized product respectively. Note that Q^* and $\Phi^*(k)$ are defined in (9) and (30).

Now, we derive the traditional firm's best response. Let $\lambda_t = \lambda - \lambda_c$. The traditional firm's profit is

$$\Pi_t(p_t, n) = \lambda_t(p_t - c_t) - \sqrt{n\lambda_t}(\sqrt{h(2S + hl\Phi^*(k))} + kh\sqrt{l}), \tag{36}$$

where the fulfillment costs in the above follows from (32). The firm's objective is to maximize (36) for given p_c. We express the firm's profit in terms of its choice of target market share λ_t and the number of product n, which makes it easier to derive its optimal policy. We define $\Pi_t(\lambda_t, n) = \Pi_t(p_t(\lambda_t, n), n)$, where $p_t(\lambda_t, n)$ is given by (35). Thus,

$$\begin{aligned}\Pi_t(\lambda_t, n) &= \lambda_t(p_c + \frac{v}{\mu - \lambda + \lambda_t} - \frac{r\lambda_t}{2n\lambda} - \frac{vl\Phi^*(k)}{2Q^*} - c_t) - \sqrt{n\lambda_t h}(\sqrt{2S + hl\Phi^*(k)} + k\sqrt{hl}) \\ &= \lambda_t(p_c + \frac{v}{\mu - \lambda + \lambda_t} - \frac{r\lambda_t}{2n\lambda} - c_t) - \sqrt{n\lambda_t h}(\frac{2S + \Phi^*(k)l(v/2 + h)}{\sqrt{2S + \Phi^*(k)lh}} + k\sqrt{lh}),\end{aligned} \tag{37}$$

where the first equality is given by replacing p_t using (35) and the second equality follows from plugging in (9) for Q^*.

We show that (37) is quasi-concave in n. Note that

$$\frac{d\Pi_t}{dn} = \frac{\sqrt{\lambda_t}}{2\sqrt{n}}\left[\frac{r\sqrt{\lambda_t^3}}{\sqrt{n^3}\lambda} - \sqrt{h}(\frac{2S+\Phi^*(k)l(v/2+h)}{\sqrt{2S+\Phi^*(k)lh}} + k\sqrt{lh})\right],$$

and the expression in square brackets decreases in n. Hence, (37) is indeed quasi-concave in n, and it is maximized by

$$n = \lambda_t/(\gamma\lambda). \tag{38}$$

Recall that we ignore integrality of n for tractability. Plugging this back into (37) and some straightforward algebra leads to

$$\Pi_t(\lambda_t, n^*(\lambda_t)) = \lambda_t(p_c + \frac{v}{\mu-\lambda+\lambda_t} - c_t - \frac{3\gamma r}{2}). \tag{39}$$

(39) is concave in λ_t and the First Order condition (FOC) and replacing $\lambda_c = \lambda - \lambda_t$ lead to

$$p_c - c_t + \frac{v}{\mu-\lambda_c} - \frac{3\gamma r}{2} - \frac{v(\lambda-\lambda_c)}{(\mu-\lambda_c)^2} = 0. \tag{40}$$

Thus, the traditional firm's best response (p_t, n) to p_c is given by the solution of (35), (38) and (40).

Now, we describe the customizing firm's best response.

$$\Pi_c(p_c) = \lambda_c(p_c - c_c), \tag{41}$$

where λ_c is given by (35). The following is useful for deriving the firm's optimal policy

$$\frac{d\lambda_c}{dp_c} = -1/(\frac{v}{(\mu-\lambda_c)^2} + \frac{r}{2n\lambda}), \tag{42}$$

and it is straightforward to show $d^2\lambda_c/dp_c^2 < 0$. Note that $dQ^*/dp_c = 0$ as the traditional firm cannot react. Π_c is concave in p_c as $d^2\Pi_c/dp_c^2 = 2d\lambda_c/dp_c + (p_c - c_c)d^2\lambda_c/dp_c^2 < 0$. Hence, the FOC gives the firm's best response.

$$\frac{d\Pi_c}{dp_c} = \lambda_c + (p_c - c_c)\frac{d\lambda_c}{dp_c}.$$

Setting this equal to zero and plugging in (42) leads to

$$\frac{v\lambda_c}{(\mu-\lambda_c)^2} + \frac{r\lambda_c}{2n\lambda} - (p_c - c_c) = 0. \tag{43}$$

Thus, the customizing firm's best response p_c to (p_s, n) is given by the solution of (35) and (43).

The equilibrium is given by intersecting the best responses, that is, by solving (35), (38), (40) and (43) together. Specifically, the equilibrium demand rate of customized products λ_c is given by the solution of (38), (40) and (43) leading to (15), where an explicit solution for λ_c is provided in (B1). The number of standard product is given by (38) and $\lambda_t = \lambda - \lambda_c$ leading to (16). The customizing firm's unit price p_c is given by (43) and finally, the traditional firm's unit price p_t is given by (35) leading to (14). Note that the condition on $c_c - c_t$ (listed in Table 1) indeed guarantees that $0 < \lambda_c < \lambda$ and $n \geq 1$.

Finally, the expressions for equilibrium profits in (17) are obtained by plugging in (40) into (39) for the traditional firm and plugging in (38) and (43) into (41) for the customizing firm. Clearly, (17) and the fact that $0 < \lambda_c < \lambda$ show that $\Pi_c > 0$ and $\Pi_t > 0$ in the equilibrium. $\square$

Proof of Proposition 4 Part (i) immediately follows by comparing $n^{duo} = (1 - \lambda_c^{duo}/\lambda)/\gamma$ given by Proposition 3 and $n^{s-mon} = 1/\gamma$ given by Proposition 10 in Appendix A.

Now we prove part (ii). Note that $\lambda_c^{d-mon} < \lambda_c^{duo}$ if and only if $n^{d-mon} > n^{duo}$. So, it suffices to show the result only for λ_c. We can rearrange (13) as

$$\frac{v\mu}{(\mu - \lambda_c)^2} = \frac{3\gamma r}{2} - (c_c - c_t). \tag{44}$$

Observe that the left hand-side increases in λ_c both in (15) for duopoly and in (44) for dual monopoly. Therefore,

$$\lambda_c^{d-mon} < \lambda_c^{duo} \text{ if and only if } [\frac{v(\mu + \lambda_c^{duo} - \lambda)}{(\mu - \lambda_c^{duo})^2} + \frac{\lambda_c^{duo}\gamma r}{2(\lambda - \lambda_c^{duo})}] - [\frac{v\mu}{(\mu - \lambda_c^{duo})^2}] < 0, \tag{45}$$

which is same as

$$\frac{\gamma r}{2} < \frac{v(\lambda - \lambda_c^{duo})^2}{\lambda_c^{duo}(\mu - \lambda_c^{duo})^2}. \tag{46}$$

The right side in (46) decreases in λ_c^{duo}, and λ_c^{duo} decreases in $c_c - c_t$. Note that $\lambda_c^{duo} = (1-\gamma)\lambda$ for $c_c - c_t = \underline{c}$ and $\lambda_c^{duo} = 0$ for $c_c - c_t = \overline{c}$. For $\frac{(1-\gamma)r}{\gamma} > \frac{3v\lambda}{(\mu-(1-\gamma)\lambda)^2}$, it is straightforward to verify that (46) holds when $c_c - c_t = \overline{c}$, and it does not hold when $c_c - c_t = \underline{c}$: Thus, there exists $c^* \in (\underline{c}, \overline{c})$ such that $\lambda_c^{d-mon} < \lambda_c^{duo}$ when $c_c - c_t > c^*$, proving (ii.a). For $\frac{(1-\gamma)r}{\gamma} \leq \frac{3v\lambda}{(\mu-(1-\gamma)\lambda)^2}$, (46) holds both when $c_c - c_t = \overline{c}$, and when $c_c - c_t = \underline{c}$, thus (ii.b) follows.

Note that (44) is valid only when

$$\frac{3\gamma r}{2} - \frac{v\mu}{(\mu - (1-\gamma)\lambda)^2} \leq c_c - c_t \leq \frac{3\gamma r}{2} - \frac{v}{\mu}, \tag{47}$$

that is, when custom products have a positive (weakly) market share[5] and there is at least one standard product variant offered in dual monopoly. So, to complete our proof, we will show that $\frac{3\gamma r}{2} - \frac{v\mu}{(\mu-(1-\gamma)\lambda)^2} < c^* < \frac{3\gamma r}{2} - \frac{v}{\mu}$ for part (ii.a) and we will verify that (ii.b) continues to hold even

[5] That is, the customized products would have a strictly positive market share for any $\epsilon > 0$ decrease in their price.

when (47) does not hold. For part (ii.a), suppose $\overline{c} > c^* > \frac{3\gamma r}{2} - \frac{v}{\mu}$ and we seek contradiction. Note that (A): $\lambda_c^{d-mon} > \lambda_c^{duo}$ for $c_c - c_t < c^*$ and (B): when $c_c - c_t = \frac{3\gamma r}{2} - \frac{v}{\mu}$, $\lambda_c^{d-mon} = 0$ and $\lambda_c^{duo} > 0$. Clearly (A) and (B) contradict with each other. Now, suppose that $\frac{3\gamma r}{2} - \frac{v\mu}{(\mu-(1-\gamma)\lambda)^2} > c^* > \underline{c}$ and we seek contradiction. Note that (C): $\lambda_c^{d-mon} < \lambda_c^{duo}$ for $c_c - c_t > c^*$ and (D): when $c_c - c_t = \frac{3\gamma r}{2} - \frac{v\mu}{(\mu-(1-\gamma)\lambda)^2}$, $\lambda_c^{d-mon} = (1-\gamma)\lambda$ and $\lambda_c^{duo} < (1-\gamma)\lambda$. Clearly (C) and (D) contradict with each other.

With regard to (ii.b), it is straightforward to verify that when $\frac{(1-\gamma)r}{\gamma} \leq \frac{3v\lambda}{(\mu-(1-\gamma)\lambda)^2}$ -which is the condition in part (ii.b)-, $\underline{c} > \frac{3\gamma r}{2} - \frac{v\mu}{(\mu-(1-\gamma)\lambda)^2}$. So, we only need to show the result for $\overline{c} > c_c - c_t > \frac{3\gamma r}{2} - \frac{v}{\mu}$. But this immediately follows, as $\lambda_c^{d-mon} = 0 < \lambda_c^{duo}$ in this case. □

Proof of Proposition 5 (i) Π_t in (17) decreases in λ_c and it depends on r only through λ_c. But $d\lambda_c/dr = \gamma(1 - \frac{\lambda_c}{3(\lambda-\lambda_c)})/(\frac{2v(3\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} + \frac{\lambda\gamma r}{(\lambda-\lambda_c)^2})$ following (15), where the denominator is always positive, and the numerator is positive if and only if $\lambda_c < 3\lambda/4$. Hence, the result follows by the fact that λ_c decreases in $c_c - c_t$ and $\lambda_c = 0$ when $c_c - c_t = \overline{c}$ and $\lambda_c = (1-\gamma)\lambda$ when $c_c - c_t = \underline{c}$. Here, we make use of the assumption $\gamma \leq 1/4$.

(ii) It follows from (17) that

$$\begin{aligned}\frac{d\Pi_c}{dr} &= [\frac{2v\mu\lambda_c}{(\mu-\lambda_c)^3} + \frac{(2\lambda-\lambda_c)\lambda_c\gamma r}{2(\lambda-\lambda_c)^2}]\frac{d\lambda_c}{dr} + \frac{\lambda_c^2\gamma}{6(\lambda-\lambda_c)} \\ &= \left(\gamma\lambda_c(\frac{v((\lambda-\lambda_c)(5\mu+\lambda-\lambda_c)+\lambda(\mu-\lambda))}{3(\mu-\lambda_c)^3(\lambda-\lambda_c)} + \frac{(3\lambda-2\lambda_c)\gamma r}{3(\lambda-\lambda_c)^3})\right) / \left(\frac{2v(3\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} + \frac{\lambda\gamma r}{(\lambda-\lambda_c)^2}\right) > 0\end{aligned}$$

The second equality is given by plugging in $d\lambda_c/dr$ from part (i) and rearranging terms.

□

Proof of Proposition 6 (i) Following (17), $\frac{d\Pi_t}{d\lambda} = \frac{2v(\lambda-\lambda_c)}{(\mu-\lambda_c)^2}[1 - \frac{\mu-\lambda}{\mu-\lambda_c}\frac{d\lambda_c}{d\lambda}]$. To complete the proof we show that $\frac{d\lambda_c}{d\lambda} < 1$. Notice that following (15), $d\lambda_c/d\lambda = (\frac{2v}{(\mu-\lambda_c)^2} + (\frac{(4\lambda-\lambda_c)\lambda_c}{3\lambda(\lambda-\lambda_c)^2} - \frac{1}{\lambda})\gamma r)/(\frac{2v(3\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} + \frac{\lambda\gamma r}{(\lambda-\lambda_c)^2})$, where the first and second terms in the denominator are greater than the corresponding terms in the numerator. Thus, $\frac{d\lambda_c}{d\lambda} < 1$.

(ii) It follows from (17) that

$$\begin{aligned}\frac{d\Pi_c}{d\lambda} &= [\frac{2v\mu\lambda_c}{(\mu-\lambda_c)^3} + \frac{(2\lambda-\lambda_c)\lambda_c\gamma r}{2(\lambda-\lambda_c)^2}]\frac{d\lambda_c}{dr} - \frac{\lambda_c^2(4\lambda-\lambda_c)\gamma r}{6\lambda(\lambda-\lambda_c)^2} && (48)\\ &= [-\lambda_c(\frac{v(\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} - \frac{\gamma r}{2(\lambda-\lambda_c)}) + \lambda_c(\frac{v(3\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} + \frac{\lambda\gamma r}{2(\lambda-\lambda_c)^2})]\frac{d\lambda_c}{d\lambda} - \frac{\lambda_c^2(4\lambda-\lambda_c)\gamma r}{6\lambda(\lambda-\lambda_c)^2} \\ &= -\lambda_c(\frac{v(\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} - \frac{\gamma r}{2(\lambda-\lambda_c)})\frac{d\lambda_c}{d\lambda} + \lambda_c(\frac{v}{(\mu-\lambda_c)^2} - \frac{\gamma r}{2\lambda}) \\ &= \lambda_c(\frac{\lambda_c\gamma r}{2\lambda(\lambda-\lambda_c)} + \frac{2v(\lambda-\lambda_c)}{(\mu-\lambda_c)^3})\frac{d\lambda_c}{d\lambda} + \lambda_c(\frac{v}{(\mu-\lambda_c)^2} - \frac{\gamma r}{2\lambda})(1 - \frac{d\lambda_c}{d\lambda}). && (49)\end{aligned}$$

The second and fourth equalities follow simply by rearranging the terms, and the third equality follows by invoking $d\lambda_c/d\lambda$ derived in part (i).

We now prove part (ii.a). First, we show that when $\frac{2v\lambda}{\mu^2} \geq \gamma r$, $d\lambda_c/d\lambda \geq 0$. This follows from the facts that the numerator of $d\lambda_c/d\lambda$ increases in λ_c, its denominator is always positive and $d\lambda_c/d\lambda \geq 0$ at $\lambda_c = 0$. Now, consider (49): All terms are positive and the result follows, since $0 \leq d\lambda_c/d\lambda < 1$ and $\frac{2v\lambda}{\mu^2} \geq \gamma r$.

We now prove part (ii.b). Note that $d\Pi_c/d\lambda_c < 0$ when $d\lambda_c/d\lambda \leq 0$ following (48). Thus, showing that $d\lambda_c/d\lambda \leq 0$ for $c_c - c_t > c^*$ where $c^* \in (\underline{c}, \overline{c})$ would suffice. Recall that the denominator of $d\lambda_c/d\lambda \leq 0$ is always positive and its numerator increases in λ_c and hence decreases in $c_c - c_t$. It is easy to verify that its numerator is negative when $c_c - c_t = \overline{c}$ (i.e. when $\lambda_c = 0$) and positive when $c_c - c_t = \underline{c}$ (i.e. when $\lambda_c = (1-\gamma)\lambda$). Here, we make use of the assumption $\gamma \leq 1/4$.

□

Proof of Proposition 7 (i) Following (17)

$$\frac{d\Pi_t}{d\mu} = -\frac{2v(\lambda-\lambda_c)(\mu-\lambda)}{(\mu-\lambda_c)^3}\frac{d\lambda_c}{d\mu} - \frac{2v(\lambda-\lambda_c)^2}{(\mu-\lambda_c)^3} \tag{50}$$

The result is immediate when $d\lambda_c/d\mu \geq 0$. Suppose not. $d\lambda_c/d\mu = \frac{v(\mu-2\lambda+3\lambda_c)}{(\mu-\lambda_c)^3} / (\frac{2v(\mu-\lambda)}{(\mu-\lambda_c)^3} + \frac{v(\mu+\lambda_c)}{(\mu-\lambda_c)^3} + \frac{\lambda\gamma r}{2(\lambda-\lambda_c)^2})$ following (15). Clearly, $d\lambda_c/d\mu > \frac{v(\mu-2\lambda+3\lambda_c)}{(\mu-\lambda_c)^3} / \frac{2v(\mu-\lambda)}{(\mu-\lambda_c)^3}$, when $d\lambda_c/d\mu < 0$. Plugging this into (50) leads to $d\Pi_t/d\mu < -\frac{v(\lambda-\lambda_c)^2(\mu+\lambda_c)}{(\mu-\lambda_c)^3} < 0$.

(ii) It follows from (17) that

$$\frac{d\Pi_c}{d\mu} = [\frac{2v\mu\lambda_c}{(\mu-\lambda_c)^3} + \frac{(2\lambda-\lambda_c)\lambda_c\gamma r}{2(\lambda-\lambda_c)^2}]\frac{d\lambda_c}{d\mu} - \frac{2v\lambda_c^2}{(\mu-\lambda_c)^3}. \tag{51}$$

Clearly, $\frac{d\Pi_c}{d\mu} < 0$ when $\frac{d\lambda_c}{d\mu} \leq 0$. Notice that the denominator in $d\lambda_c/d\mu$ is always positive and $d\lambda_c/d\mu < 0$ if and only if

$$\lambda_c < (2\lambda - \mu)/3. \tag{52}$$

The left hand side in (52) decreases in $c_c - c_t$, it is straightforward to verify that (52) holds when $c_c - c_t = \overline{c}$ and it does not hold when $c_c - c_t = \underline{c}$. Hence, the result follows.

Now, we show part (ii.a). Suppose $\mu \geq 2\lambda$. Note that $d\lambda_c/d\mu \geq 0$ in this case. Furthermore, it is straightforward to show that $d\lambda_c/d\mu < 1$. We can express (51) as

$$\begin{aligned}
\frac{d\Pi_c}{d\mu} &= [-\lambda_c(\frac{v(\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} - \frac{\gamma r}{2(\lambda-\lambda_c)}) + \lambda_c(\frac{v(3\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} + \frac{\lambda\gamma r}{2(\lambda-\lambda_c)^2})]\frac{d\lambda_c}{d\mu} - \frac{2v\lambda_c^2}{(\mu-\lambda_c)^3} \\
&= -\frac{v\lambda_c(\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3}\frac{d\lambda_c}{d\mu} + \frac{\lambda_c\gamma r}{2(\lambda-\lambda_c)}\frac{d\lambda_c}{d\mu} + \frac{v\lambda_c(\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3} \\
&\geq \frac{v\lambda_c(\mu-2\lambda+\lambda_c)}{(\mu-\lambda_c)^3}(1-\frac{d\lambda_c}{d\mu}) > 0
\end{aligned}$$

The second equality follows by invoking $d\lambda_c/d\mu$ derived in part (i). The third inequality is due to dropping the second term and the last inequality follows because $d\lambda_c/d\mu < 1$.

□

Proof of Proposition 8 The proof is similar to the proof of Proposition 5 and it is omitted. □

Proof of Propositions 9 For any target demand rate $\lambda_t = \lambda - \lambda_c$, given the optimal unit price $p_t(\lambda_t)$ and the optimal number of standard products $n(\lambda_t)$, the traditional firm's unit profit is $(w - \frac{3\gamma r}{2} - c_t)$ for dual monopoly following (34) and $(p_c + \frac{v}{\mu-\lambda+\lambda_t} - \frac{3\gamma r}{2} - c_t)$ for duopoly following (39). Thus, the optimal value of k minimizes γ in both cases (a similar argument applies to simple monopoly). Notice that minimizing γ is same as minimizing $\frac{2S+l(v/2+h)\Phi^*(k)}{\sqrt{2S+lh\Phi^*(k)}} + k\sqrt{lh}$ and we show that an optimal value k^* exists in the following.[6] The remaining steps in the proofs of Propositions 1 and 3 follows same as before plugging in k^*.

Now, we show that an optimal k^* indeed exists. Let

$$\Phi^*(k) = (1+k^2)(1-\Phi(k)) - k\phi(k)$$

and

$$\Gamma(k) = \frac{2S + l(v/2+h)\Phi^*(k)}{\sqrt{2S + lh\Phi^*(k)}}.$$

Note that we are looking for $\arg\min$ of $\Gamma(k) + k\sqrt{lh}$, which is a continuous function of k. We will show that we can restrict k to $[\bar{K}, \underline{K}]$ for some $\bar{K}$, $\underline{K}$ without loss of generality, hence the result follows by the fact that continuous functions achieve their minima and maxima over compact intervals. It is straightforward to show that $\lim_{k\to\infty} \Phi^*(k) = 0$, therefore $\lim_{k\to\infty} \Gamma(k) = \sqrt{2S}$. This implies that for an ϵ, $\sqrt{2S} > \epsilon > 0$, there exists $\bar{k}$ such that $|\Gamma(k) - \sqrt{2S}| < \epsilon$ for all $k > \bar{k}$. But then $\Gamma(\bar{k}) + \bar{k}\sqrt{lh} < \Gamma(k) + k\sqrt{lh}$ for all $k > \bar{k} + 2\epsilon/\sqrt{lh}$. Therefore, restricting our analysis to $k \le \bar{K}$ where $\bar{K} = \bar{k} + 2\epsilon/\sqrt{lh}$ is without loss of generality. Now, we will show the other direction. It is straightforward to show that $\lim_{k\to-\infty} \sqrt{\Phi^*(k)}/k = -1$ and $\lim_{k\to-\infty} 1/\Phi^*(k) = 0$, therefore $\lim_{k\to-\infty} \Gamma(k)/k = -l(v/2+h)/\sqrt{lh}$. This implies that for an ϵ, $v/(2\sqrt{lh}) > \epsilon > 0$, there exists $\underline{K} < 0$ such that $-l(v/2+h)k/\sqrt{lh} + \epsilon k < \Gamma(k) < -l(v/2+h)k/\sqrt{lh} - \epsilon k$ for all $k < \underline{K}$. But then $\Gamma(\underline{K}) + \underline{K}\sqrt{lh} < \Gamma(k) + k\sqrt{lh}$ for all $k < \underline{K}$. Therefore, restricting our analysis to $k \ge \underline{K}$ is without loss of generality. □

Proof of Proposition 10 The monopoly case where the firm sells either only standard or only custom products is trivial and the proof is omitted. □

Proof of Proposition 11 We first characterize the customizing firm's optimal price menu (Lemma EC-1). Then we characterize the traditional firm's optimal product positions and prices (Lemmas EC-2-EC-4). Finally, we characterize the equilibrium.

[6] When customers do not observe k, then the traditional firm minimizes $\sqrt{h(2S + hl\Phi^*(k))} + k\sqrt{hl}$, and in this case the optimal solution does not exist, as this expression always increases in k.

Note that the full cost of buying a standard product (price+misfit+delay) for a type-θ customer is

$$f(\theta) = min_{i:1,2,\ldots n}(p_i + r|\theta - \zeta_i| + v\mathbb{E}[W_t(\lambda_i)]). \tag{53}$$

This also shows the customer's willingness to pay for a perfectly customized product. The customizing firm subtracts the expected delay disutility due to customization from (53) to set its price menu. For demand rate λ_c, the customizing firm targets customer types

$$\Theta_c(\bar{f}) = \{\theta : f(\theta) \geq \bar{f}\}, \tag{54}$$

such that

$$P(\Theta_c(\bar{f})) = \lambda_c/\lambda. \tag{55}$$

Notice that $\bar{f}$ is a function of λ_c, p and ζ. Lemma EC-1 follows from the above discussion.

Lemma EC-1. *For any effective demand rate λ_c, the customizing firm's optimal pricing policy is given by*

$$p_c(\theta) = \begin{cases} f(\theta) - v\mathbb{E}[W_c(\lambda_c)], & \theta \in \Theta_c(\bar{f}) \\ \bar{f} - v\mathbb{E}[W_c(\lambda_c)], & \theta \notin \Theta_c(\bar{f}) \end{cases}. \tag{56}$$

Given the above prices, it is straightforward to verify that when a type-θ customer buys a customized product, she chooses to buy the product customized for her type. Lemma EC-1 enables us to characterize the customizing firm's optimal price menu by its choice of target demand rate and the traditional firm's product positions and prices.

We call customer types with $f(\theta) = \bar{f}$ *marginal customer types*, as customers with $f(\theta) \geq \bar{f}$ (that is, $\theta \in \Theta_c(\bar{f})$) buy customized products and customers with $f(\theta) < \bar{f}$ (that is $\theta \notin \Theta_c(\bar{f})$) buy standard products; $\bar{f}$ is the full cost (price+misfit+delay) to the marginal customer. We next show that it is optimal for the traditional firm to set equal prices and place its product variants symmetrically, that is, at equal distances from each other. We derive this result in three steps (Lemmas EC-2-EC-4). First, there should be at least one marginal customer between any two product variants of the traditional firm.

Lemma EC-2. *$max_{\theta \in [\zeta_i, \zeta_{i+1}]} f(\theta) \geq \bar{f}$ for all $i : 0, 1, \ldots, n$ where $\zeta_0 = 0$ and $\zeta_{n+1} = 1$.*

Proof: If $max_{\theta \in [\zeta_i, \zeta_{i+1}]} f(\theta) < \bar{f}$, for $i : 1, 2, \ldots, n-1$, the traditional firm can increase p_i and p_{i+1} by $r\delta$, decrease ζ_i by δ, and increase ζ_{i+1} by δ such that $max_{\theta \in [\zeta_i, \zeta_{i+1}]} f(\theta) = \bar{f}$. This increases the profit of the firm because the prices p_i and p_{i+1} increase and yet each product variant retains the same market share as $f(\theta)$ stays the same for $\theta \leq \zeta_i - \delta$ and $\theta \geq \zeta_{i+1} + \delta$. Similarly, there should be also a marginal customer in $[0, \zeta_1]$ and $[\zeta_n, 1]$. $\square$

Following Lemma EC-2, the full cost of marginal customer, $\bar{f}$, and a product's market share determine its price. Second, all product variants should have equal market shares.

LEMMA EC-3. *Any optimal policy of the traditional firm sets equal prices and has equal market shares for its product variants.*

Proof: We consider a policy (p, ζ) with $p_i \neq p_j$ for some product variants i and j (with positive market shares), and we show that there exist a policy (p^*, ζ^*) with $p_1^* = p_2^* = ... = p_{n^*}^*$ and $n^* \leq n$ that retains the same market share but yields higher profits to the traditional firm.

Suppose that product variant i has a market share κ_i under policy (p, ζ), it follows from Lemma EC-2 that

$$p_i = \bar{f} - r\kappa_i/2 - v\mathbb{E}[W_t(\kappa_i\lambda)], \tag{57}$$

so, if $p_i \neq p_j$ then $\kappa_i \neq \kappa_j$. The profit of the traditional firm is

$$\Pi_t = \sum_i^n [\lambda\kappa_i(\bar{f} - r\kappa_i/2 - v\mathbb{E}[W_t(\kappa_i\lambda)]) - \sqrt{\lambda\kappa_i h(2S + hl\Phi^*(k))} + kh\sqrt{\lambda\kappa_i l}]. \tag{58}$$

In (58), $\kappa_i\lambda(\bar{f} - r\kappa_i/2 - v\mathbb{E}[W_t(\kappa_i\lambda)])$ is the revenue: In particular, $\lambda\kappa_i(r\kappa_i/2)$ is the loss due to customer misfit costs, $\lambda\kappa_i v\mathbb{E}[W_t(\kappa_i\lambda)]$ is the loss due to backorders and $\sqrt{\lambda\kappa_i h(2S + hl\Phi^*(k))} + kh\sqrt{\lambda\kappa_i l}$ is the fulfillment cost of product variant i. Note that the optimal target demand rate for the customizing firm λ_c stays constant as long as the full cost of the marginal customer, $\bar{f}$, does not change. Thus, the optimal λ_c stays constant for any other policy (p, ζ) that satisfies (57) and

$$\sum_i^n \kappa_i = 1 - \lambda_c/\lambda. \tag{59}$$

Therefore, any optimal policy of the traditional firm should maximize (58) subject to (59) over $\kappa \geq 0$ for a given number of standard product variants n. It follows from the Karush-Kuhn-Tucker conditions that any such κ^* should satisfy $\kappa_i^* = \kappa_j^*$ whenever $\kappa_i^*, \kappa_j^* > 0$ (where some κ_k^* may be equal to zero, so $n^* \leq n$). Thus, there exists a policy (p^*, ζ^*) that yields market share κ_i^* for each product variant i, and increases the profit of the traditional firm while keeping the market shares of both firms constant. Therefore, any optimal policy of the traditional firm should have equal market shares across product variants, and this together with Lemma EC-2 necessitate equal prices. $\square$

Intuitively, equal market shares spreads out the fulfillment and misfit costs evenly to each product variant and minimizes the total cost. Finally, we show that it is optimal to place the product variants symmetrically, that is, at equal distances from each other.

LEMMA EC-4. *It is optimal for the traditional firm to place its product variants symmetrically, i.e.,* $\zeta_i^* = (2i-1)/(2n)$ *and set equal prices, i.e.,* $p_i^* = p_t$ *for* $i = 1, 2, ..., n$*. This is the unique optimal policy, if the traditional firm breaks the ties among its optimal policies to the disadvantage of the customizing firm.*

Intuitively, when the traditional firm positions its products symmetrically, it minimizes the aggregate premium earned by the customizing firm.

Proof: Following Lemmas EC-2 and EC-3, let the traditional firm sell n product variants each at price p_t and consider its optimal product positions ζ. For a given λ_c and ζ, we define the full cost of the marginal customer $\bar{f}(\lambda_c)$ by (54) and (55) with $\bar{f}(0) = \max_{\theta\in[0,1]} f(\theta)$. The profit of the customizing firm for effective demand rate λ_c is

$$\Pi_c = \int_0^{\lambda_c} \bar{f}(\lambda_x)\, d\lambda_x - v\mathbb{E}[W_c(\lambda_c)]\lambda_c. \tag{60}$$

Since $\bar{f}(\cdot)$ is decreasing in λ_c, the First Order Condition (FOC) is sufficient and the optimal demand rate solves $\bar{f}(\lambda_c) = v\mathbb{E}[W_c(\lambda_c)] + \lambda_c v \frac{d}{d\lambda_c}(\mathbb{E}[W_c(\lambda_c)])$. We show that ζ^*, that is, the symmetric choice of product variants, minimizes $\bar{f}(\lambda_c)$ pointwise at each λ_c and therefore, it minimizes the optimal target demand rate of the customizing firm as well as its profit, thereby maximizing the market share of the traditional firm. We prove this result using the following observation: Any ζ that minimizes $P(\Theta_c(\bar{f}))$ at each $\bar{f}$, also minimizes $\bar{f}(\lambda_c)$ at each λ_c. We now show that ζ^* minimizes $P(\Theta_c(\bar{f}))$ at each $\bar{f}$. Define $z_0 = \zeta_1, z_i = \zeta_{i+1} - \zeta_i$ for $i = 1, 2, ..., n-1$, and $z_n = 1 - \zeta_n$ (Clearly, $\sum_i^n z_i = 1$). It follows from (53-55) that

$$P(\Theta_c(\bar{f})) = [z_0 - (\frac{\bar{f} - p_t - v\mathbb{E}[W_t]}{r})]^+ + \sum_{i=1}^{n-1}[z_i - 2(\frac{\bar{f} - p_t - v\mathbb{E}[W_t]}{r})]^+ + [z_n - (\frac{\bar{f} - p_t - v\mathbb{E}[W_t]}{r})]^+, \tag{61}$$

which is minimized by $z_0 = 1/(2n), z_i = 1/n$ for $i = 1, 2, ..., n-1$ and $z_n = 1/(2n)$, that is by ζ^* for all $\bar{f}$. Therefore, ζ^* is optimal for the traditional firm. But, this need not be the unique solution. Any ζ with $z_0, z_n \geq (\frac{\bar{f}-p_t-v\mathbb{E}[W_t]}{r})$ and $z_i \geq 2(\frac{\bar{f}-p_t-v\mathbb{E}[W_t]}{r})$ for $i = 1, 2, ..., n-1$ would also minimize (61). However, among the optimal solutions, ζ^* minimizes the profit of the customizing competitor as we show that $\bar{f}(\lambda_c, \zeta^*) < \bar{f}(\lambda_c, \zeta)$ for any $\zeta \neq \zeta^*$ when λ_c is sufficiently small. This follows from the fact that $P(\Theta_c(\bar{f}, \zeta^*)) < P(\Theta_c(\bar{f}, \zeta))$ when $P(\Theta_c(\bar{f}, \zeta^*)) > 0$ and $\bar{f}$ is sufficiently large.

Lemma EC-4 allows us to restrict the analysis of the traditional firm to symmetric strategies without loss of generality. We can thus characterize its strategy by the number of product variants n and its price p_s rather than by an array of prices and product types.

The profit of the traditional firm is given by

$$\Pi_t = (p_t - c_t)(\lambda - \lambda_c) - \sqrt{n(\lambda-\lambda_c)}\left(\sqrt{h(2S + hl\Phi^*(k))} + kh\sqrt{l}\right) \tag{62}$$

When the demand intensity of the customizing firm is λ_c, the marginal customers of the standard product i (positioned at $\zeta_i = (2i-1)/2n$) who are indifferent to buying the customized products are given by

$$\theta^m = (2i-1)/2n \pm (1-\lambda_c/\lambda)/2n.$$

Then the profit of the customizing firm is

$$\begin{aligned}\Pi_c &= \lambda_c(p_t + \mathbb{E}[W_t] - c_c - \mathbb{E}[W_c]) + 2n\lambda\int_{\frac{1}{2n}(1-\lambda_c/\lambda)}^{\frac{1}{2n}} r\theta\, d\theta \\ &= \lambda_c\left(p_t + \frac{vl\Phi^*(k)}{2Q^*} - \frac{v}{\mu-\lambda_c} - c_c\right) + \frac{\lambda r}{4n}[1-(1-\lambda_c/\lambda)^2].\end{aligned} \tag{63}$$

The FOC for λ_c is given by setting the following equal to zero,

$$\frac{d\Pi_c}{d\lambda_c} = p_t + \frac{vl\Phi^*(k)}{2Q^*} - c_c + r(1-\lambda_c/\lambda)/(2n) - \frac{v\mu}{(\mu-\lambda_c)^2}. \tag{64}$$

Note that $dQ^*/dp_c = 0$ meaning that $dQ^*/d\lambda_c = 0$, as the traditional firm cannot react. Π_c is concave in λ_c and the FOC is sufficient.

Now, we derive the traditional firm's best response. We express the firm's profit in terms of its choice of target market share $\lambda_t = \lambda - \lambda_c$ and the number of product n, which makes it easier to derive its optimal policy. We define $\Pi_t(\lambda_t, n) = \Pi_t(p_t(\lambda_t, n), n)$, where $p_t(\lambda_t, n)$ is given by setting (64) equal to zero. Thus,

$$\begin{aligned}\Pi_t(\lambda_t, n) &= \lambda_t\left(c_c + \frac{v}{(\mu-\lambda+\lambda_t)^2} - \frac{r\lambda_t}{2n\lambda} - \frac{vl\Phi^*(k)}{2Q^*} - c_t\right) - \sqrt{n\lambda_t h}\left(\sqrt{2S + hl\Phi^*(k)} + k\sqrt{hl}\right) \\ &= \lambda_t\left(c_c + \frac{v}{\mu-\lambda+\lambda_t} - \frac{r\lambda_t}{2n\lambda} - c_t\right) - \sqrt{n\lambda_t h}\left(\frac{2S + \Phi^*(k)l(v/2+h)}{\sqrt{2S+\Phi^*(k)lh}} + k\sqrt{lh}\right),\end{aligned} \tag{65}$$

where the first equality is given by replacing p_t using (64) and the second equality follows from plugging in Q^* given by equation (9) of the paper.

We show that (65) is quasi-concave in n. Note that

$$\frac{d\Pi_t}{dn} = \frac{\sqrt{\lambda_t}}{2\sqrt{n}}\left[\frac{r\sqrt{\lambda_t^3}}{\sqrt{n^3}\lambda} - \sqrt{h}\left(\frac{2S+\Phi^*(k)l(v/2+h)}{\sqrt{2S+\Phi^*(k)lh}} + k\sqrt{lh}\right)\right],$$

and the expression in square brackets decreases in n. Hence, (65) is indeed quasi-concave in n, and it is maximized by

$$n = \lambda_t/(\gamma\lambda). \tag{66}$$

Plugging this back into (65) and some straightforward algebra leads to

$$\Pi_t(\lambda_t, n^*(\lambda_t)) = \lambda_t(c_c + \frac{v}{(\mu-\lambda+\lambda_t)^2} - c_t - \frac{3\gamma r}{2}). \tag{67}$$

Notice that

$$\frac{d\Pi_t}{d\lambda_t} = \frac{1}{(\mu-\lambda+\lambda_t)^2}[v + (c_c - c_t - 3\gamma r/2)(\mu-\lambda+\lambda_t)^2 - \frac{2v\lambda_t}{\mu-\lambda+\lambda_t}],$$

and the terms in square brackets decreases in λ_t as $(c_c - c_t - 3\gamma r/2) < 0$ (see Table 1 in the paper) and $\frac{2v\lambda_t}{\mu-\lambda+\lambda_t}$ increases in λ_t. So (67) is quasi-concave in λ_t and FOC and replacing $\lambda_c = \lambda - \lambda_t$ lead to

$$c_c - c_t + \frac{v}{(\mu-\lambda_c)^2} - \frac{3\gamma r}{2} - \frac{2v(\lambda-\lambda_c)}{(\mu-\lambda_c)^3} = 0. \tag{68}$$

Thus, we completely characterized the equilibrium, where λ_c is given by (68), n is given by (66), p_t is given by setting (64) to zero, and finally $p_c(\cdot)$ is given by (56). $\square$