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Competitive Customization - Online Appendix

A. Addendum to Lemma 1:

There are two cases depending on whether one of the firms dominates the market. The case where no firm dominates the market is described in Lemma 1. The following table states the equilibrium prices and profits when one of the firms dominates the market. We describe the equilibrium only when Firm 1 dominates the market for (T,T),(CU,CU) and (CM,CM) as the results are symmetric when Firm 2 dominates.

Strat.	Condition	Equilibrium prices and profits
(T,T)	$m_1 - m_2 \geq 3r$	$\Pi_1 = (m_1 - m_2 - r)$, $p_1 = m_1 - m_2 - r + c_1$, $\Pi_2 = 0,\ p_2 = c_2$
(CU,T)	$m_1 - m_2 \geq (3 - 2k_1)r$	$\Pi_1 = (m_1 - m_2 - (1 - k_1)r)$, $p_1 = m_1 - m_2 - (1 - k_1)r + c_1$, $\Pi_2 = 0,\ p_2 = c_2$
	$m_1 - m_2 \leq -(3 - k_1)r$	$\Pi_1 = 0,\ p_1 = c_1$, $\Pi_2 = (m_2 - m_1 - r)$, $p_2 = m_2 - m_1 - r + c_2$
(CU,CU)	$m_1 - m_2 \geq (3 - 2k_1 - k_2)r$	$\Pi_1 = (m_1 - m_2 - (1 - k_1)r)$, $p_1 = m_1 - m_2 - (1 - k_1)r + c_1$, $\Pi_2 = 0,\ p_2 = c_2$
(CM,T)	$m_1 - m_2 \geq (1 - k_1)r$, $m_2 \geq r$	$\Pi_1 = (m_1 - m_2 + k_1 r/2)$, $p_1(\theta) = w_1 - m_2 + r - (2 - k_1)\theta r$, $\Pi_2 = 0,\ p_2 = c_2$
	$m_1 - m_2 \geq (1 - k_1)r$, $m_2 < r$	$\Pi_1 = (m_1 - m_2^2/(2r) - (1 - k_1)r/2)$, $p_1(\theta) = w_1 - (1 - k_1)\theta r - [m_2 - (1 - \theta)r]^+$, $\Pi_2 = 0,\ p_2 = c_2$
	$m_1 - m_2 \leq -(3 - k_1)r$	$\Pi_1 = 0,\ p_1 = c_1$, $\Pi_2 = (m_2 - m_1 - r)$, $p_2 = m_2 - m_1 - r + c_2$
(CM,CM)	$m_1 - m_2 \geq (1 - k_1)r$, $m_2 \geq (1 - k_2)r$	$\Pi_1 = (m_1 - m_2 + (k_1 - k_2)r/2)$, $p_1(\theta) = w_1 - m_2 + (1 - k_2)r - (2 - k_1 - k_2)\theta r$, $\Pi_2 = 0,\ p_2 = c_2$
	$m_1 - m_2 \geq (1 - k_1)r$, $m_2 < (1 - k_2)r$	$\Pi_1 = (m_1 - m_2^2/(2(1 - k_2)r) - (1 - k_1)r/2)$, $p_1(\theta) = w_1 - (1 - k_1)\theta r - [m_2 - (1 - \theta)(1 - k_2)r]^+$, $\Pi_2 = 0,\ p_2 = c_2$

B. Proofs

Proof of Lemma 1. We prove the pricing equilibrium for each game.

(CU, CU) game: In the following, we describe the best response prices, and the equilibrium prices are then given by intersecting the best response prices. When Firm i sets price p_i, the proportion of customers who derive positive utility from Firm i's product is given by

$$\phi_i(p_i) = \frac{w_i - p_i}{(1-k_i)r}, \tag{A-1}$$

and the firms compete when

$$\phi_1 + \phi_2 > 1. \tag{A-2}$$

Assuming (A-2) holds (we will state when it holds at the end), the marginal customer θ^m who is indifferent between the two firms, if in the interior, is given by $w_1 - p_1 - \theta^m(1-k_1)r = w_2 - p_2 - (1-\theta^m)(1-k_2)r$. Hence,

$$\theta^m(p_1, p_2) = \min(1, \max(0, \frac{p_2 - p_1 + w_1 - w_2 + (1-k_2)r}{(2-k_1-k_2)r})).$$

Firm 2's profit is equal to

$$\Pi_2 = (p_2 - c_2)(1 - \theta^m(p_1, p_2)). \tag{A-3}$$

The FOC leads to the best response price for Firm 2,

$$p_2^{BR}(p_1) = \begin{cases} c_2 & \theta^m = 1 \\ c_2 + (p_1 + m_2 - w_1 + (1-k_1)r)/2 & 0 < \theta^m < 1 \\ c_2 + p_1 + m_2 - w_1 - (1-k_2)r & \theta^m = 0 \end{cases}.$$

The best response price for Firm 1 is given by a symmetric expression. The equilibrium prices are given by the intersection of best response prices. The condition (A-2) holds at the equilibrium prices when $m_1 + m_2 > 3r$, which is satisfied under our parametric assumption on r. The marginal customer in equilibrium is

$$\theta^m(m_1, m_2) = \min(1, \max(0, \frac{m_1 - m_2 + (3 - 2k_2 - k_1)r}{3(2-k_1-k_2)r})).$$

Thus, Firm i dominates the market, i.e., $\theta^m = 1$, when $m_i - m_j \geq (3 - 3k_i - k_j)r$. On the other hand, both firms earn positive market shares when $-(3-k_i-2k_j)r < m_i - m_j < (3-2k_i-k_j)r$.

(CU, T) game: The equilibrium prices are given by plugging $k_2 = 0$ in the (CU, CU) game.

(T, T) game: The equilibrium prices are given by plugging $k_1 = k_2 = 0$ in the (CU, CU) game.

$\underline{(CM, T) \text{ game:}}$ We first describe the *CM*-firm's best response price menu. This helps us characterize the *T*-firm's equilibrium price, which is plugged into the *CM*-firm's best response price menu to characterize its equilibrium prices.

As the *CM*-firm sets a price menu, it is optimal to set the maximum price that leaves each customer indifferent to buying from the competitor subject to two conditions: The firm does not set the price below its unit cost, and it cannot charge beyond the net reservation price (reservation value minus customer sacrifice). So, the customizing firm's best response price menu is given by

$$p_1^{BR}(p_2, \theta) = \max(c_1, \min(w_1, p_2 + (1-\theta)r) - (1-k_1)\theta r). \tag{A-4}$$

In particular, the *CM*-firm lowers its price as low as its unit cost to gain every possible customer. The proportion of customers who derive positive utility from the *CM*-firm's offers is given by $\phi_1(c_1)$ defined in (A-1), and the firms compete when

$$\phi_1(c_1) + \phi_2(p_2) > 1. \tag{A-5}$$

Assuming (A-5) holds, the marginal customer, if in the interior, is given by $w_1 - c_1 - (1-k_1)\theta^m r = w_2 - p_2 - (1-\theta^m)r$, since the *CM*-firm's price for the marginal customer is always equal to its unit cost, i.e., $p_1(\theta^m) = c_1$. So, the marginal is $\theta^m(p_1) = \frac{p_2+m_1-w_2}{(2-k_1)r}$. Firm 2's profit is as in (A-3) and the FOC leads to

$$p_2^* = \begin{cases} c_2 & \theta^m = 1 \\ c_2 + (m_2 - m_1 + (1-k_2)r)/2 & 0 < \theta^m < 1 \\ c_2 + m_2 - w_1 - r & \theta^m = 0 \end{cases}. \tag{A-6}$$

Plugging (A-6) into (A-4) gives the customizing firm's equilibrium price menu. Thus, the *CM*-firm sets price $p_1^{BR}(p_2^*, 0)$ when base product prices are set.[16] It is straightforward to check that the condition (A-5) holds at the equilibrium prices when $(1-k_1)m_1 + m_2 > (3-k_1)r$, which is satisfied by our parametric assumption on r. The marginal customer in equilibrium is given by

$$\theta^m(m_1, m_2) = \min(1, \max(0, \frac{m_1 - m_2 + (3-k_1)r}{2(2-k_1)r})).$$

Hence, Firm 1 dominates, i.e., $\theta^m = 1$, when $m_1 - m_2 > (1-k_1)r$, similarly Firm 2 dominates, i.e., $\theta^m = 0$, when $m_2 - m_1 > (3-k_1)r$, and both firms earn positive market shares when $-(3-k_1)r < m_1 - m_2 < (1-k_1)r$.

[16] Any price higher than $p_1^{BR}(p_2^*, 0)$ would also work.

$\underline{(CM, CM)\text{ game:}}$ Each firm sets the maximum price that leaves its customers indifferent to buying from the competitor subject to customers' reservation values and the firm's unit cost. In particular, Firm 1's best response price menu is given by

$$p_1^{BR}(p_2, \theta) = \max(c_1, \min(w_1, p_2(\theta) + (1-\theta)(1-k_2)r) - (1-k_1)\theta r),$$

and Firm 2's best response menu is given by a symmetric expression. Then the firms compete when $\phi_1(c_1) + \phi_2(c_2) > 1$, or in other words when $m_1/(1-k_1) + m_2/(1-k_2) > r$, which is satisfied by our parametric assumption on r. Each firm's price is equal to its unit cost for the marginal customer. Thus, $w_1 - c_1 - (1-k_1)\theta^m r = w_2 - c_2 - (1-(1-k_2)\theta^m)r$ and the marginal customer is given by

$$\theta^m(m_1, m_2) = \frac{m_1 - m_2 + (1-k_2)r}{(2-k_1-k_2)r}.$$

The rest follows similar to the other cases. ■

Proof of Proposition 2. Let $\gamma = m_i - m_j$. By inspection, $d\Delta_i^v/d\gamma \geq 0$ for $v : CM, CU$. Therefore, both Δ_i^{CM} and Δ_i^{CU} are nondecreasing in $m_i - m_j$, and $f(\cdot)$ and $g(\cdot)$ follow by the solution of $\Delta_i = 0$. ■

Proof of Proposition 3. Part (i) follows immediately. Part (ii) follows from the fact that $d\delta_i^v/d\gamma \geq 0$ for $v : CM, CU$ and $h(\cdot)$ solves $\delta_i^{CU} = 0$. ■

Proof of Proposition 4.

(i) Following Propositions 2 and 3, only Firm 1 can adopt MC. If $\hat{\Delta}_1^{CU} \geq 0$, Firm 1 adopts MC as it is profitable, leading to (CU, T), otherwise the firm does not adopt leading to (T, T).

(ii) The following Lemma is useful for the proof.

Lemma A-1 *$\Delta_i^{CM} < \delta_j^{CM}$ if and only if $m_i - m_j < \bar{m}$, where*

$$\bar{m} = \frac{3(1-k)(7+k-2\sqrt{8-k^3})r}{17-3k+4k^2}.$$

Furthermore, $0 \leq \bar{m} \leq (1-k)r$.

Proof. By inspection, Δ_i^{CM} increases and δ_j^{CM} decreases in $m_i - m_j$, thus $\Delta_i^{CM} - \delta_j^{CM}$ increases in $m_i - m_j$, and $\bar{m}$ is given by the solution of $\Delta_i^{CM} = \delta_j^{CM}$. ■

First, we show that Firm 2 never unilaterally adopts MC in the equilibrium. It follows from the Lemma that

$$\hat{\delta}_1^{CM}(k_2^{T,CM}) \geq \delta_1^{CM}(k_2^{T,CM}, k_2^{T,CM}) - S(k_2^{T,CM}) \geq \Delta_2^{CM}(k_2^{T,CM}) - S(k_2^{T,CM}) = \hat{\Delta}_2^{CM}.$$

In other words, when Firm 2 adopts MC, Firm 1 earns at least as much as Firm 2 by adopting MC at the same customization level.

We now go over the first column of the table. When $\hat{\Delta}_1^{CM} < 0$, Firm 1 does not adopt MC leading to (T, T) equilibrium. Notice that -$\hat{\delta}_i^{CM}(k_i)$ is Firm i's best payoff from breaking the (CM, CM) equilibrium with $(k_i, k_j) \in K^{CM,CM}$. When $\hat{\delta}_{min}^{CM} = \sup_{k \in K^{CM,CM}} \min(\hat{\delta}_1^{CM}(k_2), \hat{\delta}_2^{CM}(k_1)) \geq 0$, both firms adopt MC leading to (CM, CM), in this case the firms are trapped in a prisoner's dilemma. Notice that each firm's profit is supermodular and continuous in $(k_1, -k_2)$, and $[0, 1]$ is compact in (CM, CM), hence, the set of equilibrium (pure strategy) customization levels $K^{CM,CM}$ is nonempty (Cachon & Netessine 2004).

We now go over the second column. When $\hat{\Delta}_1^{CM} \geq 0$, it is optimal for Firm 1 to unilaterally adopt MC. Notice that $\hat{\delta}_2^{CM}(k_1^{CM,T})$ is Firm 2's best payoff from breaking the (CM, T) equilibrium. Thus, if $\hat{\delta}_2^{CM}(k_1^{CM,T}) < 0$, Firm 2 does not follow Firm 1 in adopting MC, leading to the (CM, T) equilibrium. On the other hand, if $\hat{\delta}_2^{CM}(k_1^{CM,T}) \geq 0$, Firm 2 earns a positive profit by following: (CM, T) can not be an equilibrium, we look for the (CM, CM) equilibrium. However, if $\hat{\delta}_{min}^{CM} < 0$, at least one firm will deviate from (CM, CM), and there is no pure strategy equilibrium, otherwise (CM, CM) is the equilibrium.

Finally, we show that $\hat{\delta}_{min}^{CM} \leq \hat{\delta}_2^{CM}(k_1^{CM,T})$, so the condition in the second row of the table is well-defined. Notice that $d\Pi_1^{CM,T}/dk_1 \leq d\Pi_1^{CM,CM}/dk_1$ for all parameters, therefore $k_1^{CM,T} \leq \inf\{k_1 : k \in K^{CM,CM}\}$. So, $\sup_{k \in K^{CM,CM}} \hat{\delta}_2^{CM}(k_1) \leq \hat{\delta}_2^{CM}(k_1^{CM,T})$ following the fact that $\Pi_2^{CM,CM}$ decreases in k_1. Hence, the result follows.

■

Proof of Corollary 5. For uniform prices, the result immediately follows from Propositions 2 and 3, as MC never has positive returns. For menu prices, it is straightforward to check that when $m_1 = m_2$, $\delta_i^{CM}(k, k) \geq \Delta_j^{CM}(k)$ for any k such that $\Delta_j^{CM}(k) \geq 0$. In other words, whenever it is attractive for a firm to unilaterally adopt MC, it is also attractive for its competitor to follow and adopt MC. Hence, the result follows. ■

Proof of Proposition 6. Let $m = m_1 - m_2$. We will first show that $k_1^{CU,T} \leq k_1^{CM,T}$ if and only if $k_1^{CU,T} \leq L(m)$ where $L(m) = 3/2 - m/(2r)$. We will then show that $k_1^{CU,T}$ is non-decreasing in m, thus $k_1^{CU,T} - L(m)$ increases in m, which leads to the result.

Now, we show that $k_1^{CU,T} \leq k_1^{CM,T}$ if and only if $k_1^{CU,T} \leq L(m)$. Suppose $k_1^{CU,T} \leq L(m)$. If $k_1^{CU,T} = 0$, then the result is immediate. In the following, we show that if $k_1^{CU,T} > 0$ then $\min(k_1^{CU,T}, k_1^{CM,T}) \geq 1-m$. Notice that $d\Pi_1^{CU,T}/dk_1 \leq d\Pi_1^{CM,T}/dk_1$ when $1-m \leq k_1 \leq L(m)$. Thus, $k_1^{CU,T} > k_1^{CM,T}$ would lead to a contradiction, as the CM-firm would then benefit increasing $k_1^{CM,T}$ up to $k_1^{CU,T}$. To complete the argument, we need to show that $k_1^{CU,T} > 0$ implies $\min(k_1^{CU,T}, k_1^{CM,T}) \geq 1 - m$. This is immediate when $m \geq 1$, so suppose $m < 1$. Notice that $d\Pi_1^{CU,T}/dk_1 \geq 0$ if and only if $k_1 \geq 1 - m$. Thus, $k_1^{CU,T} > 0$ implies $k_1^{CU,T} > k_1^* \geq 1 - m$ where k_1^* is given by $\Pi_1^{CU,T}(k_1^*) = \Pi_1^{CU,T}(0)$.[17] Observe that $\Pi_1^{CM,T}(k_1^*) - \Pi_1^{CM,T}(0) \geq \Pi_1^{CU,T}(k_1^*) - \Pi_1^{CU,T}(0) = 0$ and $d\Pi_1^{CM,T}/dk_1 \geq d\Pi_1^{CU,T}/dk_1$ for $1 - m \leq k_1 \leq L(m)$. Thus, $\Pi_1^{CM,T}(k_1) - \Pi_1^{CM,T}(0) \geq \Pi_1^{CU,T}(k_1) - \Pi_1^{CU,T}(0)$ for all $k_1^* \leq k_1 \leq L(m)$. Therefore, $k_1^{CU,T} > k_1^*$ requires $k_1^{CM,T} > k_1^* \geq 1 - m$.

Similarly, $k_1^{CU,T} \geq L(m)$ implies $k_1^{CM,T} \geq k_1^{CU,T}$, as $d\Pi_1^{CU,T}/dk_1 > d\Pi_1^{CM,T}/dk_1$ when $k_1 > L(m)$.

Finally, we show that $k_1^{CU,T}$ is non-decreasing in m: This follows from the observation that $d^2\Pi_1/dk_1 dm \geq 0$. ■

Proof of Proposition 7. Following the proof of Proposition 4 and Topkis (1979), $(\bar{k}_1^{CM,CM}, \underline{k}_2^{CM,CM}) \in K^{CM,CM}$.

Note that $k_1^{CM,CM} = \text{argmax}_k(\Pi_1^{CM,CM}(k, k_2^{CM,CM}) - S(k))$. Imposing the FOC leads to

$$\frac{(m_1 - m_2 + (1 - k_2^{CM,CM})r)^2}{2(2 - k_1^{CM,CM} - k_2^{CM,CM})^2 r} - S'(k_1^{CM,CM}).$$

Comparing the FOCs for both firm gives

$$\frac{S'(k_1^{CM,CM})}{S'(k_2^{CM,CM})} = \frac{(m_1 - m_2 + (1 - k_2^{CM,CM})r)}{(m_2 - m_1 + (1 - k_1^{CM,CM})r)}.$$

When $m_1 - m_2 \geq r/2$ the right hand side is always greater than 1, and the result follows by the convexity of $S(\cdot)$. ■

[17] To be concrete, $k_1^* = 11/4 + 5m/(6r) - m^2/(36r^2)$.

Proof of Proposition 8. The result immediately follows from the following equations.

$$\Gamma^{CU,T}(m_1-m_2) = \begin{cases} 0 & m_1-m_2 < -(3-k_1)r \\ \frac{m_1-m_2-(1-k_1)r}{2(2-k_1)} & -(3-k_1)r < m_1-m_2 \le (3-2k_1)r \\ 1 & m_1-m_2 > (3-2k_1)r \end{cases}$$

$$\Gamma^{CM,T}(m_1-m_2) = \begin{cases} 0 & m_1-m_2 < -(3-k_1)r \\ \frac{m_1-m_2-(1-k_1)r}{2(2-k_1)} & -(3-k_1)r < m_1-m_2 \le (1-k_1)r \\ 1/2 & m_1-m_2 > (1-k_1)r \end{cases}$$

$$\Gamma^{CM,CM}(m_1-m_2) = \begin{cases} 0 & m_1-m_2 < -(1-k_2)r \\ \frac{m_1-m_2-(1-k_1)r}{2(2-k_1-k_2)} & -(1-k_2)r < m_1-m_2 \le (1-k_1)r \\ 1/2 & m_1-m_2 > (1-k_1)r \end{cases}.$$

■

Proof of Proposition 9.

(i) Uniform prices: A type-θ customer buys the product when $w-p-|1/2-\theta|(1-k)r \ge 0$. Thus, the proportion of customer's who buy the product is equal to $\theta^m(p) = \min(1, 2\frac{w-p}{1-k})$, where the firm's profit is $\Pi(p) = (p-c)\theta^m(p)$. The optimal price follows from the first order condition (FOC).

Menu prices: It is optimal to set the maximum price for each product type that leaves customers indifferent to not buying as long as this price is above the unit cost. Thus, the optimal price menu is $p(\theta) = \max(c, w-|1/2-\theta|(1-k)r|)$.

(ii) The result follows by definition.

■

Proof of Lemma 10. We will derive the result for (CM, CM) pricing game. The proof follows similarly for (CM, T) and (T, CM) games (Notice that $k_1+k_2 \le 1$ is always true for these games).

Consider (CM, CM) pricing game. ***(i)*** We will show that a pure strategy equilibrium exists in regions *(a)* $k_1+k_2 \le 1$ and *(b)* $2k_1+k_2 > 2, k_1+2k_2 > 2$ by construction. ***(ii)*** We will also show that a pure strategy equilibrium does not exists outside these regions.

We first state a lemma which will be helpful for characterizing the equilibrium price menus.

Lemma A-2 • *If $k_i > 0$ then Firm i never sells a customer a product customized for another type, that is, if for price menu $p_i(\cdot)$ a customer buys a product customized for another type, then there exist $\tilde{p}_i(\cdot)$ that dominates $p_i(\cdot)$ yielding higher profits.*

- *No customer buys a products customized for another type from Firm i if and only if Firm i's price menu $p_i(\cdot)$ is continuous with its slope's absolute value smaller than or equal to k_i over its customer segment.*

Proof.

- Let Firm i be at $\zeta_i = 0$. The following observation is useful for the proof: If a type-θ customer chooses to buy the product customized for type-θ' then all customer types between θ and θ' also choose the product customized for type-θ'.

  Following the above observation, suppose that for price menu $p_i(\cdot)$ there exist an interval of customers $[\theta', \theta' + \epsilon]$ who buy the product customized for type-θ' at price $p_i(\theta')$. We show that $p_i(\cdot)$ is dominated by $\tilde{p}_i(\cdot)$, where

$$\tilde{p}_i(\theta) = \begin{cases} p_i(\theta), & \theta \le \theta' \\ p_i(\theta') + k_i r(\theta - \theta'), & \theta' < \theta \le \theta' + \epsilon \\ p_i(\theta), & \theta > \theta' + \epsilon \end{cases},$$

  so $p_i(\cdot)$ cannot be an equilibrium strategy. Under $\tilde{p}_i(\cdot)$ customers in $[\theta', \theta' + \epsilon]$ buy their designated customized product and pay higher prices than $p_i(\cdot)$. Notice that buying their designated customizations under $\tilde{p}_i$ and buying the product customized for θ' under p_i yields the same utility for these customers. Furthermore, $\tilde{p}_i(\cdot)$ does not introduce any new attractive alternatives for customers in $[0, \theta']$ or $[\theta' + \epsilon, 1]$, hence their choices do not change. Hence $\tilde{p}_i$ does not alter their utilities.

  Similarly, one can show that if price menu $p_i(\cdot)$ induces customers in $[\theta', \theta' + \epsilon]$ to buy the product customized for type-$(\theta' + \epsilon)$, then it is dominated by another price menu which induces these customers to buy their designated customizations.

- Notice that the slope of disutility of miscustomization in (C-1) is rk_i. When the slope of price menu is larger (or when there is a discontinuity), some customers would not buy their designated customizations, and vice versa.

■

Lemma A-2 shows that without loss of generality we can restrict ourselves to the class of price menus under which customers choose their designated customizations. This class is characterized by price menus whose slopes are smaller than or equal to rk_i over the firm's customers. In other words, when the slope of price menu is smaller than rk_i, incentive compatibility is satisfied for customers and each customer buys the product specifically customized for her type.

(i) Now, we show that a pure strategy equilibrium exists in regions *(a)* $k_1 + k_2 \leq 1$ and *(b)* $2k_2 + k_1 > 2, 2k_1 + k_2 > 2$ by construction.

(i.a) Consider region $k_1 + k_2 \leq 1$. Following Lemma A-2, each firm's price menu is characterized by its price for the marginal customer. Firm 1's price menu is given by,

$$p_1(\theta) = \begin{cases} p_1(\theta^m) + k_1 r(\theta^m - \theta), & \theta \leq \theta^m + (p_1(\theta^m) - c_1)/(rk_1) \\ c_1, & \theta > \theta^m + (p_1(\theta^m) - c_1)/(rk_1) \end{cases}.$$

Firm 2's price menu is also characterized by its price for the marginal customer. Thus, deriving the equilibrium amounts to finding the marginal customer θ^m and the prices charged to marginal customer $p_1(\theta^m), p_2(\theta^m)$.

For Firm 1, deviating to gain (decreasing prices) or lose (increasing prices) market share via

$$p_1^d(\theta) = \begin{cases} p_1(\theta^m) + k_1 r(\theta^m - \theta) + \phi, & \theta \leq \theta^m + (p_1(\theta^m) + \phi - c_1)/(rk_1) \\ c_1, & \theta > \theta^m + (p_1(\theta^m) + \phi - c_1)/(rk_1) \end{cases},$$

should not be attractive. For $\phi < 0$, the loss due to price decrease $\theta^m \phi$ should not be smaller than the gain due to market share expansion $p_1(\theta^m)\frac{\phi}{2(1-k_1-k_2)}$. Similarly, for $\phi > 0$, the gain due to price increase $\theta^m \phi$ should not be larger than the loss due to market share loss $p_1(\theta^m)\frac{\phi}{2(1-k_1-k_2)}$. Thus,

$$p_1(\theta^m) - c_1 = 2(1 - k_1 - k_2)\theta^m. \tag{A-7}$$

Applying the same arguments to Firm 2 leads to

$$p_2(\theta^m) - c_2 = 2(1 - k_1 - k_2)(1 - \theta^m). \tag{A-8}$$

Finally, by the definition of θ^m,

$$p_1(\theta^m) + (1 - k_1)r\theta^m = p_2(\theta^m) + (1 - k_2)r(1 - \theta^m). \tag{A-9}$$

Solving (A-7-A-9) simultaneously gives $p_1(\theta^m), p_2(\theta^m)$ and θ^m, which are then used to find the equilibrium payoffs.

(i.b) Consider region $2k_2 + k_1 > 2, 2k_1 + k_2 > 2$. When $2k_i + k_j > 2$, incentive compatibility for Firm i's price menu is not binding. That is, the price menu formed by the highest prices that leave customers indifferent to the competitor's best alternative (the product customized to their type and priced at cost) is incentive compatible. This is because of Lemma A-2 and the fact that the slope of this price menu is smaller than $r((1 - k_i) + (1 -$

$k_j)$), which in turn is smaller than rk_i (Note that $r((1-k_i)+(1-k_j))$ is equal to the rate of change in the gap between customer's sacrifice from the two firms' products as customer types varies). Therefore, in this region, the equilibrium price menus are same as in Lemma 1 (perfect information case) and yield the same payoffs. Clearly, the firms do not have an attractive deviation regardless of customers' reactions as a firm cannot do any better than its payoff with perfect information and the ability to set differential prices.

(ii) Now, we show that there is no pure strategy equilibrium in the remaining regions, mainly in *(a)* $2k_2+k_1>2, 2k_1+k_2\leq 2$, *(b)* $2k_2+k_1\leq 2, 2k_1+k_2>2$ and *(c)* $2k_2+k_1\leq 2, 2k_1+k_2\leq 2, k_1+k_2>1$. We prove the result for one region, that is for region *(a)*. The proofs for the other regions follow similarly. The outline of the proof is as follows.

1. There cannot be a pure strategy equilibrium in which a firm's product customized for the marginal customer is priced above its unit cost. Therefore, an equilibrium price menu should set the price of the product customized for the marginal customer at unit cost.

2. For each firm, among the class of price menus that offers the product customized for the marginal customer at unit cost, we identify the one that dominates all others.

3. Finally, we show that this price menu is dominated by another price menu that does not offer the product customized for the marginal customer at unit cost. Thus, there cannot be an equilibrium.

We now prove 1: Let θ^m show the marginal customer who is indifferent between the two firms. Suppose that $p_1(\theta^m)>c_1$. We will seek contradiction. Let $\epsilon=(p_1(\theta^m)-c_1)/(rk_1)$. We claim that

$$p_2(\theta)<p_2(\theta^m)+r[(1-2k_1)+(1-k_2)](\theta-\theta^m) \text{ for } \theta\in[\theta^m,\theta^m+\epsilon], \tag{A-10}$$

otherwise Firm 1 has an attractive deviation:

$$\tilde{p_1}(\theta)=\begin{cases} p_1(\theta)-k_1r\phi, & \theta\leq\theta^m \\ p_1(\theta^m)-k_1r(\theta-\theta^m+\phi), & \theta^m<\theta\leq\theta^m+\epsilon-\phi \\ c_1, & \theta>\theta^m+\epsilon-\phi \end{cases},$$

where ϕ is arbitrarily small. Observe that $\tilde{p_1}$ does not affect the choices of Firm 1's customers in $[0,\theta^m]$, and it attracts new customers in $[\theta^m,\theta^m+\epsilon]$ increasing the firm's profit. On the other hand, in the following, we show that when (A-10) holds, Firm 2 has also an attractive deviation and hence we reach contradiction. Following Lemma A-2, $p_1(\theta) \geq p_1(\theta^m) - k_1 r(\theta - \theta^m)$ for $\theta \in [\theta^m, \theta^m + \epsilon]$, thus p_2 can get arbitrarily close to the right hand side in (A-10) to increase Firm 2's profit leading to contradiction. Notice that following Lemma A-2 this does not affect incentives of Firm 2's customers as $k_2 > (1-2k_1) + (1-k_2)$. The last inequality requires $k_1 + k_2 > 1$.

We now prove 2: Following Lemma A-2,

$$p_1^*(\theta) = \begin{cases} c_1 + k_1 r(\theta^m - \theta), & \theta \leq \theta^m \\ c_1, & \theta > \theta^m \end{cases}$$

dominates all price menus for given θ^m with $p_1(\theta^m) = c_1$. Notice that customers in $[0, \theta^m]$ prefer buying from Firm 1 at prices $p_1^*(\cdot)$ to their best alternative from Firm 2, its product customized for their type and priced at cost. This is because $k_1 < (1-k_1) + (1-k_2)$ in this region, where the left hand side shows the rate of increase in $p_1^*(\cdot)$ and the right hand side shows the rate of increase in the gap between the customer's sacrifice from the two firms' products as customer type varies. We define p_2^* similarly.

We now prove 3: Thus if a pure strategy equilibrium exists, Firm 1 and 2 should follow p_1^* and p_2^*. But then p_1^* is dominated by

$$\bar{p_1}(\theta) = \begin{cases} c_1 + k_1 r(\theta^m - \theta + \phi), & \theta \leq \theta^m + \phi \\ c_1, & \theta > \theta^m + \phi \end{cases},$$

when ϕ is sufficiently small. Notice that $\bar{p_1}$ leads to the loss of customer segment $[\theta^m - \phi k_1/((2-k_1-k_2)), \theta^m]$ compared to p_1^* resulting in a profit loss in the order of ϕ^2, but it increases the profit extracted from existing customers by $\phi k_1 r(\theta^m - \phi k_1/((2-k_1-k_2)))$. Thus, the gain is larger than the loss when ϕ is sufficiently small. ■

Proof of Proposition 11. It is straightforward to show that price menus of Lemma 1 (stated in the addendum to Lemma 1) are incentive compatible i.e., no customer wants to deviate to buy a product customized for another type and these are indeed the optimal price menus for the firms. Thus, the payoffs in the pricing subgame are same as in Lemma 1. The remaining steps closely follows the proof of Proposition 4, hence it is omitted. ■