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RESEARCH NOTE

Segmentation Opportunities for a Social Planner: Impact of Limited Resources

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ABSTRACT

In this research note, we investigate segmentation opportunities for social planners such as government agencies, nonprofits, and public organizations. These opportunities arise when the potential products are vertically (quality) differentiated and the consumers are heterogeneous in their preferences toward quality. In these cases, whether to offer quality differentiated products and what quality level to choose are important decisions for a social planner. In this research note, we identify the conditions where it is socially optimal to offer either one homogenous or two quality differentiated products. We find that the resource limitations may result in a single product offering and that the quality of the product depends on the maximum surplus per unit resource consumed by the products. We also compare our findings to a profit-maximizing firm. We find that the resource limitations may cause a profit-maximizing firm to provide a better service to some consumers than the social planner. Contrary to common wisdom, we also show that the capacity limitations may force the social planner to act like a profit-maximizing firm in terms of its pricing and product mix choice.

Subject Areas: Limited Capacity, Marketing Manufacturing Interface, Product Line Design, Public Sector, and Segmentation.

INTRODUCTION

Quality-based segmentation is a common practice in private sectors such as the automobile, consumer electronics, travel, and clothing industries. Firms offer multiple products with different quality levels within the same product line. Different consumers with different willingness to pay for quality would buy these differentiated products at different prices. In the end, consumers self-select based on their preferences and private business owners enjoy improved profits through this second degree price discrimination among the consumers.

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In this article, we investigate the vertically (quality) differentiated product line design and segmentation problem for organizations with different goals from those of the private sector. These organizations (government agencies, nonprofits, and public organizations) are commonly referred to as “social planners” and they aim to increase the overall welfare of the communities they serve. Satisfaction of the consumers’ needs is the ultimate objective for these establishments.

For example, at the University of California at Berkeley, there are many dorm room options for the students to choose from, with prices ranging from \$11,600 (for standard triple rooms) to \$17,600 (for suite-type single rooms) per academic year (UC-Berkeley, 2010). The solution to the student housing problem is similar around the globe. At the University of Winnipeg, a public university in Manitoba, Canada, there are seven different types of dorm rooms with different features (UWinnipeg, 2010). As a high-end dorm room, furnished one-bedrooms (\$2,536 per semester) have both a single bed and a queen size futon bed, and private bathrooms with bathtubs and private kitchens. At the lower end, type A dorm rooms (\$1,740 per semester) have no living room (just the bedroom) and occupants use shared baths and kitchens. Pryor (2006) reports that the universities, in particular those located in urban areas or expensive suburbs, are severely limited by space to expand housing options for the students. Hence, public or private, many universities are constrained by space for student housing, while they offer a range of dorm room types with different sizes and amenities at various prices.

Another example is the different seat types offered by the airlines. In the air, there are many state-owned and operated airlines. Moreover, these airlines are shown to do at least as well as their private counterparts in terms of customer satisfaction (Backx, Carney, & Gedajlovic, 2002; Lopez-Bonilla, & Lopez-Bonilla, 2008). According to SkyTrax, an airline quality consulting company, of the World’s Best 34 airlines (5-star and 4-star combined), seven are state-owned: Air New Zealand, Emirates, Etihad Airways, Finnair, Garuda Indonesia, Thai Airways International, and Turkish Airlines (Skytrax, 2010). All of these airlines offer different cabin and seating options to their passengers (ranging from spacious bedroom suites to undersized economy class seats) and they are able to do this within the very limited and fixed space of an aircraft (SeatGuru, 2010).

Quality differentiation is also observed within government services. For example, the United States Postal Service (USPS) offers various delivery speed options (with various pricing) for mailing letters (Express Mail, Priority Mail, First-Class Mail, etc.) (USPS, 2010). In addition, the sender can choose from a variety of “Extra Options” like registered mail, certified mail, and return receipt, for an additional fee. Another example is the premium processing option offered by the U.S. Citizenship and Immigration Services (USCIS) for the Immigrant Petition for Alien Worker (USCIS, 2009). An applicant can go through a regular procedure for a nominal fee or choose to pay more and get an expedited service. We also note that the regular procedure is a more efficient operation due to economies of scale compared to the premium processing where an employee is dedicated to a single file.

The health care industry provides us with another example. Today, many hospitals including the University of Chicago Medical Center (UCMC, 2010), offer their patients the option of staying in a private room. In a private room a patient has more space and privacy compared to a semiprivate room (Chaudhury, Mahmood, & Valente 2003). However, it is also shown that it costs a lot more to build hospitals with private rooms (Davis-Langdon-Adamson, 2003), routine maintenance such as cleaning also takes more time-per-patient, and more walking is required for the nurses to attend to the private rooms (Chaudhury et al., 2003).

Even within private hospital rooms there may be upgrade options for patients who are willing to pay for them. Mayo Clinic, the first and largest integrated, not-for-profit group practice in the world, offers its patients “the option of private, world-class accommodations” within Saint Mary’s Hospital (Mayo Clinic, 2010). “Patients who choose this service receive the same high-quality care unique to all Mayo Clinic patients, while enjoying special amenities and services,” such as fine linens, personalized meal consultation and preparation, and concierge services. The additional costs associated with these accommodations are not covered by insurance.

In all of these examples, social planners ensure that the society as a whole benefits from a differentiated menu of offerings. Because the consumers are heterogeneous in terms of their income levels, the high-end consumers are happier with premium products and they are willing to pay the premium pricing associated with the product. This, in return, generates financial backing for the products targeted at the low-end consumers. Hence, the segmentation and price discrimination strategies may in fact improve social welfare.

In this research note, we are interested in understanding under what conditions the vertical (quality) differentiation is a socially optimal strategy and how a social planner’s policies would differ from a profit maximizing firm. Understanding these conditions and their implications would help social planners to make better decisions and serve the public more effectively, especially when there are resource limitations.

In our stylized analytical model, we consider a social planner who is in charge of a producer. The producer may be an actual production facility as well as a service provider depending on the context. The planner takes the consumers’ surplus into account while it makes sure that the producer does not operate on a loss. The planner has the option to provide different qualities of products that may vary in their unit production/operating costs and resource consumptions. The consumer base is heterogeneous in their willingness to pay for quality. The producer has limited capacity that needs to be allocated among these products.

There is a rich variety of literature in marketing and economics on the design of vertically differentiated product lines (e.g., Mussa & Rosen, 1978; Itoh, 1983; Moorthy, 1984; Gabszewicz, Shaked, Sutton, & Thisse, 1986; Desai, 2001; Fountain, 2001; Kim & Chhajed, 2002; Johnson & Myatt, 2003; Lutz & Pezzino, 2010). This literature has studied product choice, pricing, market segmentation, social welfare, and cannibalization among other things. However, it has ignored the effects of capacity limitations on the firm’s product line choice. Indeed, a firm (especially a social planner) does not have unlimited resources for offering its products and often faces capacity constraints in the form of time, labor, equipment, space, and inventory. With this note, we aim to contribute to the above literature by incorporating such a capacity constraint. We also contribute to the emerging literature in the marketing-operations interface that looks at the effects of operational elements on a firm's product line. Heese and Swaminathan (2006), Desai, Kekre, Radhakrishnan, and Srinivasan (2001), and Kim and Chhajed (2000) study the effects of component commonality on a profit maximizing firm's product line. Netessine and Taylor (2007) characterize the effect of production technology (production to order vs. production to stock). Dobson and Yano (2002) and Chayet, Kouvelis, and Yu (2009) consider a shared resource used for offering a product line. However, in Dobson and Yano (2002), products have independent demands, there is no cannibalization, and the firm is profit maximizing. In Chayet et al. (2009), high- and low-quality products have equal resource consumptions (equal production time in their context) in a profit maximizing firm; while the social welfare optimization and possible difference in resource consumption of product types are some of the key elements of our model.

Our main model considers two product types, high- and low-quality products with fixed quality levels. We later extend our model to allow for the social planner to make quality decisions. The low- and high-quality products may differ in their unit production/operating costs and resource consumptions. The social planner decides how to allocate the producer's limited capacity to these products and their prices. Consumer preference is private information, but the planner knows its distribution. Each consumer self-selects from the menu of offerings and the no purchase option, and chooses the one that maximizes his/her utility. We solve for the resulting socially efficient equilibrium, study its characteristics, and compare it to a profit maximizing firm's assignment.

Overall, we find that capacity plays a critical role in the social planner's decisions. Limitations in capacity can lead to expanding the menu of offerings with an additional product type, and can also lead to serving fewer product types. Among other results, our key findings are as follows.

When capacity is limited for the social planner, we find that it would be better for society to have a single homogenous product (i.e., either the low- or the high-quality product). The optimal product type depends on the maximum surplus as well as the resource consumptions of each product type. This is in contrast to the existing literature (Mussa & Rosen, 1978; Moorthy, 1984; Desai, 2001), which argues that the social planner should serve both products, offering a differentiated product line, when there are increasing costs to quality (i.e., when the unit cost to quality ratio of high-quality product is greater than that of the low-quality product). When there are decreasing costs to quality, the existing literature (disregarding the capacity constraint) shows that the firm should always focus on the high-quality product (Bhargava & Choudhary 2001; Johnson & Myatt 2003). We show that society might be better off with an exact opposite policy focusing on the low-quality product when the producer's limited capacity is taken into consideration.

Limited capacity may induce a social planner to degrade the quality level for high-end consumers compared to a profit maximizing firm's assignment. This result is in line with the empirical observations that state that the quality has improved in many countries after privatization of once-government operated industries (Poole, 1988). However, the existing literature fails to explain this phenomenon and claims that lower-end consumers get a better-quality product while the higher-end consumers may get the same level of quality from a social planner compared to a profit maximizing firm's assignment (Mussa & Rosen, 1978; Moorthy, 1984; Desai, 2001; Johnson & Myatt, 2006). We also show that when the capacity is limited, it is always optimal for the social planner to act like a profit maximizing firm which is also in contrast to the existing literature.

The rest of this article is organized as follows. Our basic model is presented in the following section. Next, we solve the social planner's problem, discuss our findings and compare it to the profit maximizing firm's solution. Building on these findings, we then study what happens when the quality of products are part of the decision making process. We provide our conclusions in the final section.

MODEL

We consider a social planner managing a producer with limited capacity. The social planner has the option of serving vertically differentiated products in a heterogeneous society. There are two product types, high- and low-quality products. Our main model assumes fixed quality levels $q_h > q_l$. We later relax this assumption and allow for the social planner to make quality decisions as well. Each unit of product i costs c_i and it requires s_i units of the capacity for production. We assume $c_h \geq c_l$ and $s_h \geq s_l$ to avoid trivial cases.

The planner decides how to allocate the producer's limited capacity K to the product types. To serve x_i units of product i, the producer needs to allocate $s_i x_i$ units of its capacity. We assume that the parameters are such that the trivial case ($x_l = 0$ and $x_h = 0$) is not optimal. The planner sets prices p_i, and this in turn determines the demand of each product $D_i(p_l, p_h)$. Clearly, the producer sells the minimum of allocated capacity x_i and the demand $D_i(p_l, p_h)$ for product i. Without loss of generality, we can restrict the analysis to $x_i = D_i(p_l, p_h)$. If $D_i(p_l, p_h) > x_i$, the social planner can increase price p_i and achieve a higher producer's profit, similarly if $D_i(p_l, p_h) < x_i$, the planner can decrease x_i without affecting the surplus. Following $x_i = D_i(p_l, p_h)$, producer's earnings can be simplified to

$$\Pi^P = (p_h(x_h, x_l) - c_h)x_h + (p_l(x_h, x_l) - c_l)x_l. \tag{1}$$

We adopt the classical vertical differentiation demand model (Tirole, 1988). The consumers vary in their willingness to pay for quality. Specifically, the consumer types θ are uniformly distributed in the unit interval [0, 1] with unit total mass. Because the market size is normalized to 1 with this assumption, all other parameters should be interpreted carefully. For example, the capacity parameter (K) should be interpreted as the capacity-to-market size ratio. Hence, high levels of K mean that the capacity is enough to cover most of the demand, whereas low levels of K mean that the resources are limited compared to the demand. Similarly, the production quantity variables ($x_i = D_i(p_l, p_h)$) should also be interpreted as the proportion of the market purchasing the product.

When type θ consumer buys product i at price p_i, his/her utility is equal to $U(q_i, p_i, \theta) = \theta q_i - p_i$. If the consumer does not buy a product, his/her utility is zero. Thus, each consumer has three options, buying the high-quality product, buying the low-quality product and not buying a product, and he/she chooses the one that maximizes his/her utility. Hence, the consumers' surplus is given as follows:

$$CS = \int_{1-x_h-x_l}^{1-x_h} (\theta q_l - p_l)\, d\theta \;+\; \int_{1-x_h}^{1} (\theta q_h - p_h)\, d\theta. \tag{2}$$

SOCIAL PLANNER'S SOLUTION

In this section, we solve for the social planner's problem and compare it to the existing literature. The social planner maximizes the total surplus (sum of consumers' surplus (2) and producer's earnings (1)), which is given by,

$$\Pi^S = \int_{1-x_h-x_l}^{1-x_h} (\theta q_l - c_l)\, d\theta \;+\; \int_{1-x_h}^{1} (\theta q_h - c_h)\, d\theta. \tag{3}$$

Thus, the social planner solves

$$\max_{x_h, x_l \geq 0} \Pi^S \text{ subject to } s_h x_h + s_l x_l \leq K. \tag{4}$$

The solution to this problem is fully characterized in Appendix A. In the following passages, we characterize the socially optimal product line and contrast it to the existing literature. Propositions 1 and 2 describe the optimal policy when increasing cost to quality (i.e., $c_l/q_l < c_h/q_h$) and decreasing cost to quality (i.e., $c_l/q_l \geq c_h/q_h$) respectively. We define the threshold capacities $\bar{K}_1^S - \bar{K}_5^S$ given in Appendix A (A1)–(A5) to facilitate the presentation of the propositions. All proofs appear in Appendix C.

***Proposition 1*:** Suppose $c_l/q_l < c_h/q_h$. The socially optimal product line is as follows:

(i) If $q_l - c_l \geq q_h - c_h$, then the social planner should supply only the low-quality product for all capacity levels.

(ii) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then

- if $K \leq \bar{K}_1^S$, the social planner should supply only the low-quality product.
- if $K > \bar{K}_1^S$, the social planner should follow a segmentation strategy and supply both products.

(iii) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, then

- if $K \leq \bar{K}_2^S$, the social planner should supply only the high-quality product.
- if $K > \bar{K}_2^S$, the social planner should follow a segmentation strategy and supply both products.

Proposition 1a describes the trivial case where it is always better to provide only the low-quality product. Note that the willingness to pay of the highest valuation consumer is normalized ($\theta = 1$). Thus, in this case, the maximum surplus for the low-quality product is $q_l - c_l$ and it is larger than that of the high-quality product ($q_h - c_h$).

Proposition 1b and 1c describe what happens when the maximum surplus for the high-quality product $(q_h - c_h)$ is larger. In this case, the socially optimal product line depends on the producer's capacity. The proposition shows that when the producer has a large capacity compared to the demand (which is normalized to 1), the planner prefers providing a differentiated product line offering both products, which is in line with the existing literature (Mussa & Rosen, 1978; Moorthy, 1984; Desai, 2001). Serving a differentiated product line helps the planner segment the market, which in turn enables attracting a larger demand with a smaller sacrifice in the surplus of the high-quality product. However, when the producer is already capacity constrained, a larger demand has little value and the society may not benefit from offering both product types. Indeed, in contrast to existing literature, we show that when the producer has a small capacity, society is better off with one homogeneous product. The right focus depends on the *capacity adjusted surplus* $(\frac{q_i - c_i}{s_i})$, that is, the maximum surplus per unit resource consumed. In Proposition 1b, the lower-quality product has a greater capacity adjusted surplus and the planner provides only the low-quality product when its capacity is smaller than $\bar{K}_1^S$. On the other hand, in Proposition 1c, the high-quality product has a greater capacity adjusted surplus and the planner provides only the high-quality product when its capacity is smaller than $\bar{K}_2^S$.

This may correspond to the case of public transportation in many countries. Public bus or metro systems provide a homogenous offering to all consumers. In most cases, it is customary for the passengers to stand up inside the vehicle during travel. In this context, a higher-quality product might be providing a private compartment for every passenger. However, a private compartment occupies a lot of space, while the additional comfort/happiness/surplus it provides to the passengers is not that much. Hence, it is optimal for the social planners to provide a homogeneous, low-quality service in the context of public transportation.

***Proposition 2*:** Suppose $c_h/q_h \leq c_l/q_l$. The socially optimal product line is as follows:

(i) If $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then
- if $K \leq \bar{K}_1^S$, the social planner should supply only the low-quality product.
- if $\bar{K}_1^S < K \leq \bar{K}_2^S$, the social planner should follow a segmentation strategy and supply both products.
- if $K > \bar{K}_2^S$, the social planner should supply only the high-quality product.

(ii) If $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, then the social planner should supply only the high-quality product for all capacity levels.

Proposition 2 characterizes the socially optimal policy when there are decreasing costs to quality, i.e., $c_h/q_h \leq c_l/q_l$. For this case, the existing literature (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003) has shown that a profit maximizing firm should offer only the high-quality product when there are no capacity limitations. We extend this result for the social planner's context when the producer has a large capacity or when the high-quality product has a larger capacity adjusted surplus. In that case, a social planner with no capacity limitations only supplies the better-quality product.

However, when the low-quality product has a larger capacity adjusted surplus (i.e., maximum surplus per unit capacity ($\frac{q_i - c_i}{s_i}$)), it is more beneficial to society than the high-quality product and the optimal product line choice critically depends on the producer's capacity. Specifically, the society is better off with a homogeneous low-quality product when the producer's capacity is small (i.e., $K \leq \bar{K}_1^S$). In this case, a social planner that disregards the capacity constraint would be offering the high-quality (i.e., diametrically opposite) product. In addition, for intermediate capacity levels (i.e., $\bar{K}_1^S < K \leq \bar{K}_2^S$), the social planner supplies both products. It is interesting that producer's limited capacity induces the social planner to expand its product line, offering more product types in this case. The capacity constraint gives incentive to the social planner to allocate more resources to the product that has a greater capacity adjusted surplus. This leads to adding the low-quality product to the menu of offerings.

State owned airlines are good examples of this case. Even within the same airlines' aircrafts, the cabin and seat allocations might differ due to capacity differences. For example, Thai Airways offers 279 economy class seats (seat pitch (p) is 32" and seat width (w) is 17") and 30 business class seats (p60", w21.5") in their Boeing 777-200. However, they offer 325 economy class seats (p32", w17"), 40 business class seats (p60", w20"), and 10 first class seats (p78", w21.8") in their Boeing 747-400. The difference in these two designs is the particular aircraft's capacity. The airline offers a first class option only when it has enough capacity.

Our results show that scarcity of resources forces a social planner to drop the product with the smaller capacity adjusted surplus from its product line and dedicate the entire capacity to the more beneficial product. This is formally stated in the following corollary.

***Corollary 1*:** Suppose $\frac{q_i - c_i}{s_i} < \frac{q_j - c_j}{s_j}$. There exist $\hat{K} > 0$ such that x_i is nondecreasing in K with $x_i = 0$ for $K < \hat{K}$ in the socially optimal solution.

COMPARISON TO A PROFIT MAXIMIZING FIRM

In this section, we solve the profit maximizing firm's problem and discuss our findings. The firm chooses the production quantities x_i and prices p_i to maximize its profit $\Pi^M = \Pi^P$ given in (1) subject to the capacity constraint. Specifically the profit maximizing firm solves,

$$\max_{x_l, x_h \geq 0} \Pi^M \text{ subject to } s_h x_h + s_l x_l \leq K. \tag{5}$$

The solution to this problem is fully characterized in Appendix B. Here, we discuss our findings and contrast them to the existing literature. We define additional capacity thresholds $\bar{K}_1^M - \bar{K}_5^M$ given in Appendix B (B2)–(B6) that will be helpful in stating our results.

In the standard vertical differentiation literature, it is well known that consumers get a lower-quality product or nothing at all from a profit maximizing firm compared to a social planner except for the high-end consumer who gets the same assignment in both cases (e.g., Mussa & Rosen, 1978; Moorthy, 1984; Desai, 2001; Johnson & Myatt, 2006). In contrast, we show that a profit maximizing firm may offer a higher-quality product than that of a social planner to some consumer segments due to the capacity constraint. Specifically, when the maximum surplus of the high-quality product is larger, but its capacity adjusted surplus is smaller, the high-end consumers get a better-quality product from the profit maximizing firm than the social planner's assignment for a range of capacity levels. This is formally stated in the following proposition.

Proposition 3**:** When $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the profit maximizing firm supplies the high-quality product whereas the social planner does not supply it for $\bar{K}_1^M < K \leq \bar{K}_1^S$.

Under the condition in Proposition 3, the profit maximizing firm and the social planner prefer offering only the low-quality product for small capacities (i.e., $K \leq \bar{K}_1^M$). Similarly, they both supply both product types for large capacity levels (i.e., $K > \bar{K}_1^S$). However, when $\bar{K}_1^M < K \leq \bar{K}_1^S$, the social planner does not provide the high-quality product in order to increase its market coverage whereas the profit maximizing firm driven by the higher profit margins offers the high-quality product. In this case, while the higher-end consumer segment is getting better quality under the private ownership, the lower-end consumer segment is getting nothing. The better quality at the high end comes at the expense of the lower-end consumer segment.

The conventional wisdom in the literature states that the social planner serves a larger portion of the market compared to a profit maximizing firm (e.g., Mussa & Rosen, 1978). This is indeed the case when one ignores the capacity constraint. However, we show that depending on the capacity level, a profit maximizing firm may serve a greater portion of the market than a social planner. Specifically, this happens when there are increasing costs to quality and the high-quality product has a greater capacity adjusted surplus.

Proposition 4**:** When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_l/q_l \leq c_h/q_h$, the profit maximizing firm covers a greater portion of the market than the social planner for $\bar{K}_2^M < K < \min\{\bar{K}_2^S, \bar{K}_3^M\}$.

Under the condition in Proposition 4, both the profit maximizing firm and the social planner offer only the high-quality product for small capacity levels (i.e., $K \leq \bar{K}_2^M$) and similarly they both offer both product types for large capacity levels $K > \bar{K}_2^S$ (the profit maximizing firm does not use all of its capacity for $K > \bar{K}_3^M$). However, when $\bar{K}_2^M < K < \min\{\bar{K}_2^S, \bar{K}_3^M\}$, only the profit maximizing firm serves the low-quality product. By offering the low-quality product, the profit maximizing firm serves the high-quality product to a smaller market segment and this in turn keeps its price higher. Whereas the social planner is not concerned about prices and sells only the high-quality product. This results in a larger market coverage under the private ownership because both the profit maximizing firm and the social planner utilize all the capacity and the high-quality product consumes a greater amount of resources per unit.

***Proposition 5*:** When the capacity is small, all consumers get their socially efficient assignment from the profit maximizing firm.

It is well-established in the literature that the profit maximizing firm degrades the quality level offered to low valuation consumers (e.g., Mussa & Rosen, 1978; Moorthy, 1984). However, we show that when the capacity is limited compared to the demand, the optimal policy for both the profit maximizing firm and the social planner is to dedicate all capacity to the most beneficial product type (in terms of maximum surplus per unit resource consumed, $\frac{q_i - c_i}{s_i}$). Because all capacity is dedicated to the same homogeneous product, the segment of consumers who get the high-quality product, the low-quality product, and nothing are the same for both the profit maximizing firm and the social planner.

ENDOGENOUS QUALITY LEVELS

In this section, through numerical examples, we make observations about what happens when quality levels are part of the decision making process. It would be interesting to see whether the introduction of quality level as a decision variable would eliminate the results discussed so far or not. By the following counterexamples, we prove our point that the capacity limitation makes a difference on the optimal product line strategies of all types of organizations even when the quality is endogenous to the problem.

The social planner has two decisions: whether to offer one or two products in a quality differentiated product line and allocation of the producer's capacity to the products. In addition, the planner determines the quality levels $q_i \in [0, \overline{q}]$, where $\overline{q}$ is the maximum technologically feasible quality level. Functions $c(\cdot)$ and $s(\cdot)$ show how unit production/operating costs and unit resource consumptions depend on the quality decision. Specifically, each product with quality q_i costs $c(q_i)$ and requires $s(q_i)$ amount of the resource. The social planner solves

$$\begin{aligned} \max_{x_l, x_h \geq 0,\ q_l, q_h \in [0, \overline{q}]} \quad & \int_{1-x_h-x_l}^{1-x_h} (\theta q_l - c(q_l))\, d\theta \ + \int_{1-x_h}^{1} (\theta q_h - c(q_h))\, d\theta \\ \text{subject to} \quad & s(q_h)x_h + s(q_l)x_l \leq K. \end{aligned} \tag{6}$$

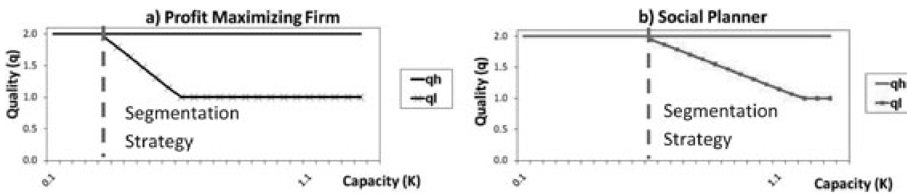
On the other hand, to maximize its profit, the private firm solves

$$\begin{aligned} \max_{x_l, x_h \geq 0,\ q_l, q_h \in [0, \overline{q}]} \quad & (p_h(x_h, x_l) - c(q_h))x_h + (p_l(x_h, x_l) - c(q_l))x_l \\ \text{subject to} \quad & s(q_h)x_h + s(q_l)x_l \leq K. \end{aligned} \tag{7}$$

In our examples, we use polynomial unit costs $c(q) = \alpha q^{\beta}$ as commonly assumed in the literature (e.g., Moorthy, 1984; Desai, 2001). Notice that a strictly convex unit cost function (i.e., $\beta > 1$), results in increasing costs to quality ($c_l/q_l < c_h/q_h$). Similarly, a strictly concave unit cost (i.e., $\beta < 1$), results in decreasing costs to quality.

Here, we discuss how our results in earlier sections may carry over when the quality levels become endogenous decisions. Remember that the capacity

Figure 1: Optimal quality levels: profit maximizing firm versus social planner ($c(q) = 0.1q^2$, $s(q) = q^{1/2}$, and $\overline{q} = 2$).



parameter, K, should be interpreted as the capacity-to-market-size ratio because the market size is normalized to 1 in these analyses.

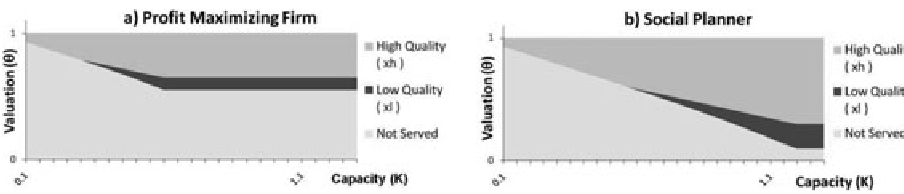
Figures 1a and 1b show the optimal quality levels chosen by a profit maximizing firm and a social planner as a function of their capacities when there are increasing costs to quality. Recall that the existing literature (e.g., Mussa & Rosen, 1978; Moorthy, 1984; Johnson & Myatt, 2003), while disregarding the limited capacity, has shown that the decision maker should offer a quality differentiated product line in this case. Indeed, Figure 1 is consistent with the existing literature when the producer's capacity is large compared to the demand (i.e., when $K > 0.6$ for the social planner and $K > 0.3$ for the profit maximizing firm).

However, when the producer's capacity is small, the decision maker prefers following a homogenous product strategy as in Proposition 1. The organization is better off dedicating all of its capacity to the high-quality product, this enables it to provide the high-quality product to more consumers. The low-quality product should be introduced to differentiate only if the producer's capacity is high.

A similar scenario has been observed during the social housing policy implementation in Turkey. During the 1980s, the Turkish Government stepped up to solve the housing shortage in the country and established Emlak Konut A.S. (Emlak Konut, 2010), which is governed under the Housing Development Administration of Turkey (TOKI) (TOKI, 2010). Emlak Konut A.S. initially built and sold affordable housing targeted at medium and high income residents. In 2003, after construction of more than 30,000 such homes, TOKI undertook a second wave of social housing projects called The Emergency Action Plan for Housing and Urban Development. During this wave, TOKI has built and sold almost 180,000 low-end, smaller homes targeted at low-income residents; about 190,000 standard size homes targeted at medium income residents; and almost 70,000 high-end homes targeted at high-income residents. The 1980s were a low capacity phase and TOKI has sold high-end homes. When its responsibilities and capabilities increased in 2003, TOKI expanded its product line and started supplying a variety of social homes including low-end homes targeted at low-income residents.

The space travel industry provides another example. Space programs are developed and carried out by government agencies such as the U.S. National Aeronautics and Space Administration and the Russian Federal Space Agency. Initially, space travel was open only to astronauts that have completed a rigorous training. During the 1980s, "payload specialists" who were selected and trained by

Figure 2: Market coverage: profit maximizing firm versus social planner ($c(q) = 0.1q^2$, $s(q) = q^{1/2}$ and $\overline{q} = 2$).



commercial or research organizations were also allowed on some flights (NASA, 2010). In 1984, Charles David Walker became the first nongovernment astronaut to fly, with his flight commercially sponsored by his employer, McDonnell Douglas. In 2001, the first space tourist, Dennis Tito, visited the International Space Station for seven days (Karash, 2001). Today, travel to space is an ultra-luxurious product with a price tag of US \$40 million (Olsen, 2007). As the capacity of the shuttles improves, space agencies may begin offering affordable, lower-quality space flights as well. However, until that day, space travel is a high-end product provided solely by government agencies to high-end consumers.

Figure 3a and 3b plot the optimal quality levels for a profit maximizing firm and for a social planner when there are decreasing costs to quality. In this case, the existing literature suggests that the optimal product line would be to supply only the high-quality product to avoid cannibalization (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003). Figure 3a coincides with the existing literature when the firm's capacity is large (when $K > 2$). However, the firm prefers the differentiation strategy, supplying a high- and a low-quality product when the capacity is scarce. Producing the low-quality product enables the firm to serve a larger market in this case, as the low-quality product consumes less capacity. The same argument is also true for the social planner. Notice that this observation is consistent with Proposition 2.

The decision regarding private rooms within hospitals is a good example of this case. While there are some hospitals that provide private rooms to all patients (Hoholik, 2008), there are also urban location hospitals that continue to offer semiprivate rooms as a norm (UCMC, 2010). Our analysis provides insights as to how the social planner's optimal decision might differ in different settings. Although patient preferences may be similar in all of these settings, the space limitations of the hospitals and expansion opportunities are certainly different for different locations. If the capacity (compared to the demand) is small, hospitals may not have the option of offering all-private rooms to their patients. If the capacity is really low, then offering all semiprivate rooms may be the best alternative for the welfare of the whole society in terms of access to medical attention.

Now, we compare the social planner's decisions with those of the profit maximizing firm. Figures 2 and 4 show the optimal market coverage chosen by a profit maximizing firm (Figures 2a and 4a) and by a social planner (Figures 2b and 4b) as a function of their capacities. The existing literature shows that compared to

Figure 3: Optimal quality levels: profit maximizing firm versus social planner ($c(q) = 0.01q^{1/2}$, $s(q) = q^2$, and $\overline{q} = 2$).

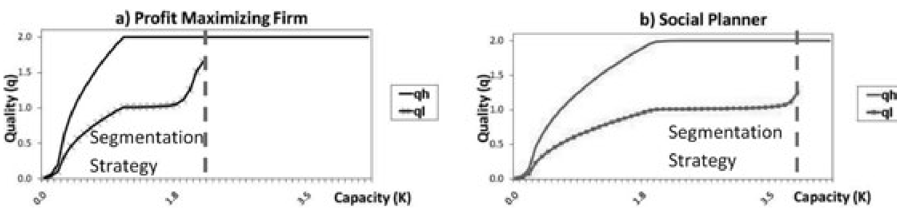
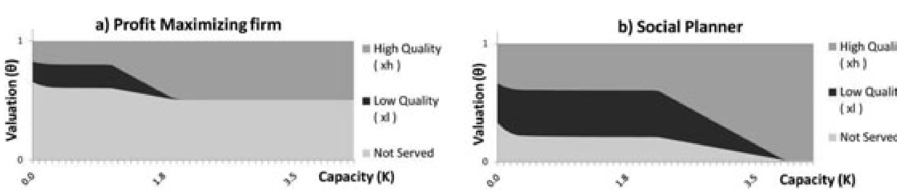


Figure 4: Market coverage: profit maximizing firm versus social planner ($c(q) = 0.01q^{1/2}$, $s(q) = q^2$, and $\overline{q} = 2$).



a social planner, a profit maximizing firm degrades the quality of products offered except to the high-end consumers. However, Figures 3 and 4 show an example where a profit maximizing firm offers a higher-quality product than a social planner due to the capacity limitation as in Proposition 3. For example, when capacity $K = 1$, the consumers that are at the high end ($\theta \geq 0.80$, Figure 4a and 4b) get a better-quality product from a profit maximizing firm ($q_h = 2.0$, Figure 3a) than from a social planner ($q_h = 1.423$, Figure 3b). This happens because for a given capacity, the social planner can serve more consumers at the low end by degrading the quality level at the high end.

The existing literature also shows that the profit maximizing firm chooses to serve a smaller market than that of a social planner. Figures 2a and 2b, similar to Proposition 4, show that this result does not necessarily hold when the capacity constraint is taken into account. In this example, when the capacity is small, the profit maximizing firm serves a larger market compared to a social planner. For example, in Figure 2 at $K = 0.50$, both organizations utilize all their capacity and the profit maximizing firm serves the consumers with valuation $\theta \geq 0.635$ whereas the social planner serves only the consumers with valuation $\theta \geq 0.646$. In this case, the social planner offers only high-quality product type while the profit maximizing firm offers both a high and a low-quality product type. This helps the profit maximizing firm to limit the amount of high-quality product offered in the market and keep its prices high. On the other hand, the social planner prefers offering only the high-quality product type to a larger consumer segment and consequently generating greater surplus for the consumers. Although this leads to smaller market coverage under the social planner, the total surplus of the consumers is greater than the private ownership outcome.

In addition, Figures 1 and 2 show that a social planner's decisions may be exactly the same as a profit maximizing firm's decisions due to limited capacity. For example, when capacity $K = 0.2$, both the social planner and the profit maximizing firm offer only a single product type at quality $q_h = 2$, they both utilize their whole capacity and serve the same consumer segment (i.e., $\theta \geq 0.86$). Notice that these observations are consistent with Proposition 5, although the quality levels are chosen endogenously.

Finally, Figures 1 and 3 also show that the quality of the products supplied depend on the capacity level in a nontrivial way. The quality of the lower-quality product can be decreasing in capacity as in Figure 1 or it can be increasing in capacity as in Figure 3.

CONCLUSIONS AND MANAGERIAL INSIGHTS

In this article, we studied the segmentation problem for a social planner and compared the results with those of a profit maximizing firm. Contrary to basic intuition, our findings imply that a government office or a nonprofit organization that aims to maximize social welfare might consider following a segmentation strategy.

Public organizations should take the relationships between quality levels, unit operating costs, and unit resource consumptions of the products into account before making decisions about their product offerings. In particular, organizations should pay special attention to the surplus per unit resource consumed for each product and the available resources. When resources are limited for the organization, they should not pursue a segmentation strategy and should instead dedicate all available resources to one homogenous product with the highest surplus per unit resource consumed, which can be either the high- or the low-quality product. However, if the organization has abundant resources compared to the demand in their area, and the cost to quality ratio is increasing, they may consider supplying both the high- and the low-quality products. On the other hand, when the cost to quality ratio is decreasing, for medium levels of capacity (compared to the demand), society as a whole would be better off if the public organization provides differentiated options. Ignoring the capacity limitations and resource consumptions of the products may lead to fundamentally wrong strategies.

Another finding is that public organizations may or may not provide a better-quality service than a profit maximizing firm depending on the available capacity. When resources are abundant, a social planner offers services at least as good as those offered by a profit maximizing firm. However, when resources are limited, a profit maximizing firm might offer a better service for the high-end consumers. Our analysis shows that this improvement comes at the expense of the low-end consumers who may not get any service from a profit maximizing firm in such a case.

This article has its limitations. Following the literature, we assumed that consumer preferences are distributed uniformly. This makes the analysis tractable, enabling us to keep our focus on the effects of capacity limitation on the socially efficient product line choice and to derive our insights. While we verified that these insights can carry over to nonuniform distributions through some numerical examples (which are not reported in this article), it would be worthwhile for future work to further study what happens under nonuniform distributions in general. Furthermore, we consider a single provider setting. It would be fruitful to extend our work to a competitive context and study how capacity constraints affect the product line choice in the presence of competing providers. [Received: December 2009. Accepted: September 2010.]

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APPENDIX A: SOCIAL PLANNER'S SOLUTION

The social planner's problem in (4) leads to the following after solving for the integral in the objective function (3):

$$\max \Pi^S = x_h \left(q_h \left(1 - \frac{x_h}{2} - c_h\right)\right) + x_l \left(q_l \left(1 - x_h - \frac{x_l}{2}\right) - c_l\right)$$

subject to $s_h x_h + s_l x_l \leq K \quad x_h \geq 0 \quad x_l \geq 0$

$$Hessian(\Pi^S) = \begin{bmatrix} -q_h & -q_l \\ -q_l & -q_l \end{bmatrix}.$$

Given that $q_h > q_l$, the $Hessian(\Pi^S)$ is negative definite. Because the objective function of this problem is jointly concave on a convex set defined by linear constraints, the optimal solution can be obtained by solving the first order conditions together with the feasibility conditions (Bazaraa, Sherali, & Shetty, 2006). First order conditions are as follows for this problem: $\{-c_h + q_h - q_h x_h - q_l x_l - s_h\lambda + \mu_h = 0\}$, $\{-c_l - q_l(-1 + x_h + x_l) - s_l\lambda + \mu_l = 0\}$, $\{(K - s_h x_h - s_l x_l)\lambda = 0\}$, $\{x_l \mu_l = 0\}$, $\{x_h \mu_h = 0\}$ where the feasibility conditions are given as follows: $x_h \geq 0, x_l \geq 0, \lambda \geq 0, \mu_h \geq 0, \mu_l \geq 0, K \geq x_h s_h + x_l s_l$.

We define the following threshold capacities to facilitate the presentation of the solution:

$$\bar{K}_1^S = \frac{-(s_l(c_h s_l - q_h s_l + (-c_l + q_l)s_h))}{q_l(s_l - s_h)}. \tag{A1}$$

$$\bar{K}_2^S = \frac{s_h(-c_h s_l + q_h s_l + (c_l - q_l)s_h)}{q_h s_l - q_l s_h}. \tag{A2}$$

$$\bar{K}_3^S = \frac{q_l(q_l - q_h)s_h + c_h q_l(-s_l + s_h) + c_l(q_h s_l - q_l s_h)}{q_l(q_l - q_h)}, \tag{A3}$$

$$\bar{K}_4^S = \frac{(q_h - c_h)s_h}{q_h}, \tag{A4}$$

$$\bar{K}_5^S = \frac{(q_l - c_l)s_l}{q_l}. \tag{A5}$$

Then, the solution for the social planner's problem is characterized as follows:

(i) For the parameters $q_l - c_l \geq q_h - c_h$, the solution is as follows: For $K < \bar{K}_5^S$, $x_h = 0$ and $x_l = \frac{K}{s_l}$; for $K \geq \bar{K}_5^S$, $x_h = 0$ and $x_l = \frac{q_l - c_l}{q_l}$.

(ii) For the parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$:

(a) If $(c_h/c_l > q_h/q_l)$, the solution is as follows: For $K < \bar{K}_1^S$, $x_h = 0$ and $x_l = \frac{K}{s_l}$; for $\bar{K}_1^S \leq K < \bar{K}_3^S$, $x_h = \frac{K q_l(-s_l+s_h)+s_l(-c_h s_l+q_h s_l+c_l s_h-q_l s_h)}{q_h s_l^2+q_l s_h(-2s_l+s_h)}$ and $x_l = \frac{K(q_h s_l-q_l s_h)+s_h(c_h s_l-q_h s_l-c_l s_h+q_l s_h)}{q_h s_l^2+q_l s_h(-2s_l+s_h)}$; for $K \geq \bar{K}_3^S$, $x_h = \frac{(q_h-c_h)-(q_l-c_l)}{q_h-q_l}$ and $x_l = \frac{q_l c_h - q_h c_l}{q_l(q_h-q_l)}$.

(b) If $(c_h/c_l \leq q_h/q_l)$, the solution is as follows: For $K < \bar{K}_1^S$, $x_h = 0$ and $x_l^{SP} = \frac{K}{s_l}$; for $\bar{K}_1^S \leq K < \bar{K}_2^S$, $x_h = \frac{K q_l(-s_l+s_h)+s_l(-c_h s_l+q_h s_l+c_l s_h-q_l s_h)}{q_h s_l^2+q_l s_h(-2s_l+s_h)}$ and $x_l = \frac{K(q_h s_l-q_l s_h)+s_h(c_h s_l-q_h s_l-c_l s_h+q_l s_h)}{q_h s_l^2+q_l s_h(-2s_l+s_h)}$; for $\bar{K}_2^S \leq K < \bar{K}_4^S$, $x_h = \frac{K}{s_h}$ and $x_l = 0$; for $K \geq \bar{K}_4^S$, $x_h = \frac{q_h - c_h}{q_h}$ and $x_l = 0$.

(iii) For the parameters $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$:

(a) If $(c_h/c_l > q_h/q_l)$, the solution is as follows: For $K < \bar{K}_2^S$, $x_h = \frac{K}{s_h}$ and $x_l = 0$; for $\bar{K}_2^S \leq K < \bar{K}_3^S$, $x_h = \frac{K q_l(-s_l+s_h)+s_l(-c_h s_l+q_h s_l+c_l s_h-q_l s_h)}{q_h s_l^2+q_l s_h(-2s_l+s_h)}$ and $x_l = \frac{K(q_h s_l-q_l s_h)+s_h(c_h s_l-q_h s_l-c_l s_h+q_l s_h)}{q_h s_l^2+q_l s_h(-2s_l+s_h)}$; for $K \geq \bar{K}_3^S$, $x_h = \frac{(q_h-c_h)-(q_l-c_l)}{q_h-q_l}$ and $x_l = \frac{q_l c_h - q_h c_l}{q_l(q_h-q_l)}$.

(b) If $(c_h/c_l \leq q_h/q_l)$, the solution is as follows: For $K < \bar{K}_4^S$, $x_h = \frac{K}{s_h}$ and $x_l = 0$; for $K \geq \bar{K}_4^S$, $x_h = \frac{q_h - c_h}{q_h}$ and $x_l = 0$.

APPENDIX B: PROFIT MAXIMIZING FIRM'S SOLUTION

The demand model yields $0 \leq \theta_l \leq \theta_h \leq 1$ such that consumers in $[0, \theta_l)$ do not buy a product, consumers in $[\theta_l, \theta_h)$ buy the low-quality product and consumers in $[\theta_h, 1]$ buy the high-quality product. So, the demand for the high- and the low-quality products are $D_h = 1 - \theta_h$ and $D_l = \theta_h - \theta_l$. It is straightforward to show that the marginal consumer θ_h who is indifferent between buying the high- and the low-quality products is given by $\theta_h = (p_h - p_l)/(q_h - q_l)$ and similarly, the marginal consumer θ_l who is indifferent between buying the low-quality product and not buying a product at all is given by $\theta_l = p_l/q_l$. Thus, we can express the demands for the two product types as follows:

$$D_h(p_l, p_h) = 1 - \frac{p_h - p_l}{q_h - q_l} \qquad D_l(p_l, p_h) = \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}. \tag{B1}$$

Following the fact that $x_i = D_i(p_h, p_l)$, the price-quantity equations (B1) for the firm can be solved for prices as follows: $p_h = q_h(1 - x_h) - q_l x_l$; $p_l = q_l(1 - x_l - x_h)$. Then, the formulation takes the following form:

$$\max \Pi^M = x_h(q_h(1 - x_h) - q_l x_l - c_h) + x_l(q_l(1 - x_l - x_h) - c_l)$$
$$\text{subject to } x_h s_h + x_l s_l \leq K;\ x_h \geq 0;\ x_l \geq 0$$

$$\text{Note that } Hessian\ (\Pi^M) = \begin{bmatrix} -2q_h & -2q_l \\ -2q_l & -2q_l \end{bmatrix}.$$

Given that $q_h > q_l$, the $Hessian(\Pi^M)$ is negative definite. Because the objective function of this problem is jointly concave on a convex set defined by linear constraints, the optimal solution can be obtained by solving the first order conditions together with the feasibility conditions (Bazaraa et al., 2006). First order conditions are as follows for this problem: $\{-c_h + q_h - 2q_h x_h - 2q_l x_l - s_h\lambda + \mu_h = 0\}$, $\{-c_l + q_l - 2q_l x_h - 2q_l x_l - s_l\lambda + \mu_l = 0\}$, $\{(K - s_h x_h - s_l x_l)\lambda = 0\}$, $\{x_l\mu_l = 0\}$, $\{x_h\mu_h = 0\}$ where the feasibility conditions are as follows: $x_h \geq 0, x_l \geq 0, \lambda \geq 0, \mu_h \geq 0, \mu_l \geq 0, K \geq x_h s_h + x_l s_l$. The following threshold capacities are defined to facilitate the presentation of the solution:

$$\bar{K}_1^M = \frac{s_l(s_h(q_l - c_l) - s_l(q_h - c_h))}{2q_l(s_h - s_l)}, \tag{B2}$$

$$\bar{K}_2^M = \frac{s_h(s_l(q_h - c_h) - s_h(q_l - c_l))}{2q_h s_l - 2q_l s_h}, \tag{B3}$$

$$\bar{K}_3^M = \frac{q_l(q_l - q_h)s_h + c_h q_l(-s_l + s_h) + c_l(q_h s_l - q_l s_h)}{2q_l(q_l - q_h)}, \tag{B4}$$

$$\bar{K}_4^M = \frac{(q_h - c_h)s_h}{2q_h}, \tag{B5}$$

$$\bar{K}_5^M = \frac{(q_l - c_l)s_l}{2q_l}. \tag{B6}$$

When $(c_h/c_l > q_h/q_l)$, for a profit maximizing firm the optimal product line configuration is as follows:

(i) For parameters $q_l - c_l \geq q_h - c_h$, the solution is characterized as follows: For $K < \bar{K}_5^M$, $x_h = 0$ and $x_l = \frac{K}{s_l}$. For $K \geq \bar{K}_5^M$, $x_h = 0$ and $x_l = \frac{q_l - c_l}{2q_l}$. Hence, the result follows.

(ii) For parameters $q_l - c_l < q_h - c_h$,

(a) For $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the solution is characterized as follows: For $K < \bar{K}_1^M$, $x_h = 0$ and $x_l = \frac{K}{s_l}$. For $\bar{K}_1^M \leq K < \bar{K}_3^M$, $x_h = \frac{2Kq_l(-s_l+s_h)+s_l(-c_hs_l+q_hs_l+c_ls_h-q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$ and $x_l = \frac{2K(q_hs_l-q_ls_h)+s_h(c_hs_l-q_hs_l-c_ls_h+q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$. For $K \geq \bar{K}_3^M$, $x_h = \frac{(q_h-c_h)-(q_l-c_l)}{2(q_h-q_l)}$ and $x_l = \frac{q_lc_h - q_hc_l}{2q_l(q_h-q_l)}$. Hence, the result follows.

(b) For $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, the solution is characterized as follows: For $K < \bar{K}_2^M$, $x_h = \frac{K}{s_h}$ and $x_l = 0$. For $\bar{K}_2^M \leq K < \bar{K}_3^M$, $x_h = \frac{2Kq_l(-s_l+s_h)+s_l(-c_hs_l+q_hs_l+c_ls_h-q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$ and $x_l = \frac{2K(q_hs_l-q_ls_h)+s_h(c_hs_l-q_hs_l-c_ls_h+q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$. For $K \geq \bar{K}_3^M$, $x_h = \frac{(q_h-c_h)-(q_l-c_l)}{2(q_h-q_l)}$ and $x_l = \frac{q_lc_h - q_hc_l}{2q_l(q_h-q_l)}$. Hence, the result follows.

When $(c_h/c_l \leq q_h/q_l)$, for a profit maximizing firm the optimal product line configuration is as follows:

(i) For parameters $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the solution is characterized as follows: For $K < \bar{K}_1^M$, $x_h = 0$ and $x_l = \frac{K}{s_l}$. For $\bar{K}_1^M \leq K < \bar{K}_2^M$, $x_h = \frac{2Kq_l(-s_l+s_h)+s_l(-c_hs_l+q_hs_l+c_ls_h-q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$ and $x_l = \frac{2K(q_hs_l-q_ls_h)+s_h(c_hs_l-q_hs_l-c_ls_h+q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$. For $\bar{K}_2^M \leq K < \bar{K}_4^M$, $x_h = \frac{K}{s_h}$ and $x_l = 0$. For $K \geq \bar{K}_4^M$, $x_h = \frac{q_h - c_h}{2q_h}$ and $x_l = 0$. Hence, the result follows.

(ii) For parameters $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, the solution is characterized as follows: For $K < \bar{K}_4^M$, $x_h = \frac{K}{s_h}$ and $x_l = 0$. For $K \geq \bar{K}_4^M$, $x_h = \frac{q_h - c_h}{2q_h}$ and $x_l = 0$. Hence, the result follows.

APPENDIX C: PROOFS

In this Appendix, we provide the necessary proofs for the propositions and the corollary mentioned in this article.

***Proof of Proposition 1*:** Proof directly follows from the social planner's solution provided in Appendix A.

***Proof of Proposition 2*:** Proof directly follows from the social planner's solution provided in Appendix A.

***Proof of Corollary 1*:** Proof directly follows from the proofs of Propositions 1 and 2. It is also straightforward to show that the derivative with respect to K is nonnegative in each case.

***Proof of Proposition 3*:** Following Propositions 1 and 2 and Appendix B, when $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, both the profit maximizing firm and the social planner offer only the low-quality product below a threshold capacity, but they offer both product types (high and low quality) above that threshold. The threshold for the profit maximizing firm and the social planner are $\bar{K}_1^M$ and $\bar{K}_1^S$ respectively where $\bar{K}_1^S = 2\bar{K}_1^M$. Thus, for all $\bar{K}_1^M < K < \bar{K}_1^S$, the profit maximizing firm serves the high-quality product while the social planner does not serve it, hence the result follows. □

***Proof of Proposition 4*:** Following Propositions 1 and 2 and Appendix B, when $c_l/q_l > c_h/q_h$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the strategy for both the profit maximizing firm and the social planner is to offer only the high-quality product below a threshold capacity, but to offer both product types above that threshold. The threshold for the profit maximizing firm is $\bar{K}_2^M$ and it is $\bar{K}_2^S$ for the social planner where $\bar{K}_2^S = 2\bar{K}_2^M$. For $\bar{K}_2^M < K < \min\{\bar{K}_2^S, \bar{K}_3^M\}$, the optimal strategy for the profit maximizing firm is to offer both high- and low-quality product types ($x_h^M > 0$ and $x_l^M > 0$), while the social planner serves only the high-quality product ($x_h^{SP} > 0$ and $x_l^{SP} = 0$). Notice that both the profit maximizing firm and the social planner use their whole capacity in this case. Thus, $s_h x_h^M + s_l x_l^M = K = s_h x_h^{SP} \Rightarrow x_h^M + \frac{x_l^M}{s_h/s_l} = x_h^{SP} \Rightarrow x_h^M + x_l^M > x_h^{SP}$. □

***Proof of Proposition 5*:** The proof follows from Propositions 1 and 2 and Appendix B:

(i) When $q_l - c_l \geq q_h - c_h$, both the profit maximizing firm and social planner assignments are the same ($x_h = 0$ and $x_l = K/s_l$) for all $K < \bar{K}_5^M$.

(ii) When $q_l - c_l < q_h - c_h$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, both the profit maximizing firm and social planner assignments are the same ($x_h = 0$ and $x_l = K/s_l$) for all $K < \bar{K}_1^M$.

(iii) When $\frac{q_h - c_h}{s_h} = \frac{q_l - c_l}{s_l}$ and $c_h/c_l > q_h/q_l$, both the profit maximizing firm and social planner assignments are the same

$$\left(x_h^M = \frac{2Kq_l(-s_l + s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{2(q_h s_l^2 + q_l s_h(-2s_l + s_h))} \right.$$
$$= \frac{Kq_l(-s_l + s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)} = x_h^{SP}$$

and

$$x_l^M = \frac{2K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{2(q_h s_l^2 + q_l s_h(-2s_l + s_h))}$$
$$\left. = \frac{K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)} = x_l^{SP} \right)$$

for all $K < \bar{K}_3^M$.

(iv) When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_h/c_l > q_h/q_l$, both the profit maximizing firm and social planner assignments are the same ($x_h = K/s_h$ and $x_l = 0$) for all $K < \bar{K}_2^M$.

(v) When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_h/c_l < q_h/q_l$, both the profit maximizing firm and social planner assignments are the same ($x_h = K/s_h$ and $x_l = 0$) for all $K < \bar{K}_4^M$. □

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