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The Value of Product Variety When Selling to Strategic Consumers

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We consider a firm that sells two vertically (quality) differentiated products to strategically forward-looking consumers over two periods, setting the prices dynamically in each period. The consumers are heterogeneous in their evaluations of quality, and strategic in that they decide not only whether and which product variant to buy, but also when to buy, choosing the option that maximizes their utility. We derive the equilibrium of the pricing-purchasing game between the firm and the consumers. We find that the loss due to strategic customer behavior can be less with two product variants compared to the single-product benchmark, which indicates that product variety can serve as a lever when dealing with strategic customers. This benefit exists when the additional product has an inferior cost-to-quality ratio. Because of this benefit, a firm may find it attractive to sell a product variant that would be unprofitable otherwise. However, product variety can also hurt profitability due to strategic customer behavior: A product variant that would be profitable absent strategic customers can in fact be unprofitable. This can happen when customer impatience and firm costs are moderate.

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1. Introduction

Firms want consumers to buy their products immediately and choose the most expensive product they can afford. Consumers, on the other hand, often hold off for price markdowns, willing to substitute less expensive alternatives for better value. In particular, firms face two related problems due to customer choice. First, they have trained customers to wait for markdowns (O'Donnell 2006); thus, full price sales today are cannibalized by marked-down sales tomorrow. In fact, a larger portion of sales are made at discount prices each year causing even more pressure for markdowns the following year (Rozhon 2004). A recent study finds that a naive pricing policy that ignores such strategic customer behavior can result in 30% less profit in the video game industry (Nair 2007). Second, firms seek to benefit from segmentation and thus include lower-priced variants in their product lines. However, this may cannibalize demand for higher-end variants: A customer who might otherwise buy the more expensive higher-end variant may instead switch to buying the lower-priced variant. The resulting loss can be severe, and in some cases firms simply decided to stop selling lower-end variants (*The Economic Times* 2001a, b). These two problems have been studied extensively but independently in the literature. Here we consider them concurrently, marrying the two streams of literature.

For example, books are sold in several print formats such as hardcover, paperback, and large prints. Most books originally published in hardcover later become available in both hardcover and paperback formats. Some publishers also release their books in hardcover and paperback simultaneously. For instance, Tyrus Books, a subsidiary of F + W Media, and Tor Books, a subsidiary of Macmillan Publishers, release many of their books in hardcover and paperback at the same time. Similarly, Apple offers its iPhone in multiple versions with varying levels of flash drive capacity. In these examples, Offering the lower-end alternatives may cannibalize the higher-end product, but it helps segment the market.

Traditionally, when the segmentation benefit outweighs cannibalization loss for a product variant, firms included it in their assortments. How does forward-looking strategic customer behavior affect the added-value of a product variant for the assortment? On one hand, having another product variant may allow taking advantage of segmentation. On the other hand, the opportunity of having another product option can make waiting for markdowns even more attractive, which would encourage the strategic behavior. Overall, the added value of a product for the assortment depends on how it affects the loss due to strategic customer behavior, its cannibalization of other products, and its segmentation benefit. Accordingly, we study the interplay between intertemporal and variety substitution and aim to characterize the value of product variety in the presence of strategic customers.

In this paper, we study dynamic pricing of multiple product variants sold to forward-looking strategic customers over multiple periods. We consider the most parsimonious model that is rich enough to capture the interactions between product variety and strategic customer behavior. Specifically, we consider a two-period model with two product variants. Our model is closest to Besanko and Winston (1990). They consider only a single product, whereas we extend their model to include multiple product variants. The two products are vertically differentiated, that is, the firm offers a high- and low-quality product. The firm and the consumers are involved in a pricing-purchasing game. While the firm sets its product prices dynamically, which also determines its effective product portfolio in each period, the customers decide which product to buy (if any) and time their purchases strategically to maximize their utility.

Customers vary in their valuations of quality and, all else being equal, are willing to pay more to purchase a product in the first period rather than waiting to buy it in the second period. In particular, a product's value decreases over time. This may be because of declining popularity or seasonality. For example, the value of the new generation iPhone is higher when it is first introduced compared to when the next generation is about to be released. We solve for the resulting subgame perfect Nash equilibrium in which the firm sets prices optimally in each period given the remaining consumers in the market, and consumers make their purchase decisions optimally given rational expectations of future prices. In addition to our base model, we study the single-product model benchmark wherein the firm can offer only one product variant, as well as the benchmark with myopic consumers who consider only the options available in that period in their purchasing decisions. These benchmarks help us tease out the effects of product variety and strategic customer behavior.

Let us now summarize our key findings:

- Our results demonstrate that, when carefully chosen, product variety can serve as a lever to deal with strategic customer behavior. Specifically, firm loss due to strategic customers can be lower with more product variants (Proposition 2). This happens because offering an additional product may enable the firm to attract a larger consumer segment early on, thereby decreasing the size of the consumer segment that waits for a markdown. Because of this benefit the firm finds it attractive to include a product variant in its product line that would otherwise be unprofitable when strategic customer behavior is disregarded (Corollary 1). Although product variety may introduce intratemporal cannibalization (i.e., customers downgrading to the lower-quality product), it can reduce the loss due to intertemporal cannibalization (i.e., customers strategically waiting to buy in period 2 rather than buying in period 1).
- However, we also demonstrate that the firm's profit with two product variants can be lower than the single-product benchmark with only the high-end product (Proposition 3). This occurs because the firm may find it attractive to serve a larger segment in period 2 due to the additional lower-end product. It necessitates leaving a larger surplus to the consumers in period 2, which in turn increases the payoff from strategic waiting for a markdown. Thus, the firm needs to incur a larger loss to convince customers to buy in period 1. This can happen when it becomes optimal for the firm to sell only the lower-end product in period 2, which requires the customers' impatience and the firm's unit cost to be moderate. In this case, even though the additional product variant would be attractive if the customers were myopic, its introduction hurts profitability because of forward-looking strategic customer behavior.
- The existing work has indicated that ignoring forward-looking strategic customer behavior can lead to significantly suboptimal inventory and pricing decisions. We take these findings one step further by showing that disregard for strategic customer behavior can lead to critically incorrect decisions about product assortment. Our numerical studies indicate that the additional loss due to suboptimal product selection can be quite significant; it could be up to 2/3 of the loss due to suboptimal pricing. Furthermore, our results illustrate that the error in product portfolio can flow in either direction. A product variant that is unprofitable (profitable) to include in the product line when strategic customer behavior is ignored can be profitable (unprofitable) when strategic consumers are accounted for (Proposition 3 and Corollary 1). We explicitly characterize when these situations happen and identify their key drivers. Thus, our results underscore the significant impact of considering strategic customer behavior on product assortment.

Through numerical studies, we illustrate that our results can continue to hold when customers differ in their impatience levels. Another numerical study demonstrates that the firm prefers a lesser quality differentiation in its product line when temporal differentiation is large, and it is faced with strategic customers compared to myopic customers. In contrast, when temporal separation is small, the firm prefers a greater quality differentiation when faced with strategic customers compared to myopic customers. Finally, we show that the firm always benefits from its ability to offer an additional product variant when it can make price or quantity commitments.

2. Literature Review

A number of researchers have studied dynamic pricing of a product in the presence of forward-looking strategic customers. Besanko and Winston (1990) show that disregarding strategic customer behavior can result in substantial losses in profitability. Su (2007) considers a mix of strategic and myopic consumers with heterogeneous evaluations and shows that the optimal pricing policy can involve a markup or markdown depending on the proportion of each customer segment in the population.

Aviv and Pazgal (2008) study optimal timing and the extent of discounts assuming consumers with declining evaluations. Elmaghraby et al. (2008) study optimal markdown mechanisms when customers have multiunit demands. Su (2010) studies the firm's pricing problem when customers can strategically stockpile for future consumption. Lai et al. (2010) study the effectiveness of price matching policies to discourage intertemporal cannibalization.

Su and Zhang (2008) illustrate that commonly used supply chain contracts can serve as commitment devices and decentralization can improve supply chain performance because of its impact on strategic customer behavior. Cachon and Swinney (2009) find that the value of a quick response (midseason replenishment opportunity) is much higher in the presence of strategic consumers. Liu and van Ryzin (2008) study quantity decisions instead of price decisions in a capacity rationing model with strategic customers. They note that when customers are risk neutral and prices are set endogenously, as in our model, capacity rationing is not optimal. Indeed, it is optimal for the firm to fulfill all of the demand at the equilibrium prices in our model.

Yin et al. (2009) study the efficacy of inventory display formats (display one versus display all) in inducing strategic customers to buy at full price rather than waiting for sales. Swinney (2011) considers a consumer population with uncertain, heterogeneous values for a product. Each consumer decides whether to buy the product before or after the uncertainty is resolved. Jerath et al. (2009) consider competing single-product firms and show that when facing strategic customers, selling through an opaque intermediary that hides the exact product specifications (e.g., departure times for airlines) can enhance profitability.

All of the above studies are confined to single-product firms. A key difference in our paper is that we study a firm that offers more than one product variant. Thus, we contribute to the above stream of papers by studying the impact of product variety when dealing with forward-looking strategic customers, which is the key theme of our paper.

We refer the reader to Shen and Su (2007) for a survey of papers studying the impact of strategic customer behavior in various other contexts. There is also a rich literature on dynamic pricing beyond the strategic customer context; Talluri and van Ryzin (2004) provide an in-depth review.

Our work is also related to the literature on product variety management. We refer the reader to Ho and Tang (1998) and Kök et al. (2008) for a comprehensive review of this literature. Our work differs from the papers in this stream of literature in that we consider dynamic pricing of products in a multiperiod model; thus, customers have the option of not only variety substitution but also intertemporal substitution, giving opportunity to strategic behavior. Dong et al. (2009) study dynamic pricing and inventory control of substitute products; however, they do not consider forward-looking strategic customer behavior, which is the key driver of the pricing policy in our model.

Our model considers forward-looking strategic consumers. In the context of textbooks, Chevalier and Goolsbee (2009) empirically demonstrate students' willingness to pay indeed depends on their conjectured future resale value of a textbook.

We consider a firm that offers vertically (quality) differentiated products. Netessine and Taylor (2007) study the interplay between a firm's production technology and its product line that is composed of vertically differentiated products. Desai (2001) studies the design of vertically differentiated product lines in a competitive context. Unlike our paper these studies do not consider strategic customers.

The papers that consider vertically differentiated products in the durable goods literature are also relevant for our research. Moorthy and Png (1992) consider a firm selling a high- and low-quality product in two periods. They examine whether the firm should introduce the two products simultaneously or sequentially finding that sequential introduction can be attractive in some cases because of its impact on cannibalization. Moorthy and Png (1992) do not address our key research question, the value of product variety; the ability to offer an additional product never harms profitability in their model. Using a demand model similar to Moorthy and Png (1992), Hahn (2006) shows that a firm selling a durable good can benefit from introducing a damaged good (Deneckere and McAfee 1996), that is, a stripped-down lower-quality variant. Similarly, Takeyama (2002) shows that selling a lower-quality variant can help mitigate a time inconsistency problem for durable goods. Unlike our model, the low-quality product can be upgraded to a higher quality in Takeyama (2002). Furthermore, both Takeyama (2002) and Hahn (2006) assume that the lower-quality variant and the original (higher-quality) product have the same unit cost, whereas the cost differential between the product variants is a key element of our model: A lower-quality product is never sold in our model when its unit cost is the same as that of a higher-quality product.

3. Model

In the following, we first introduce the supply side of our model, discuss how customers make their buying decisions resulting in strategic customer behavior, and describe the structure of the equilibrium. We consider a two-period model with two product variants. Products are vertically differentiated, that is, the firm offers a high- and low-quality product, which will be referred to hereafter as products H and L. The quality levels are exogenous to our model. The quality of product H is normalized to 1 and the quality of product L is equal to $\beta < 1$. At the beginning of each period t, the firm sets prices p_{it} for that period where i: H, L is the index for products, and it procures the products at unit cost c for product H and γc for product L, where $\gamma < 1$. The seller's objective is to maximize its total profit over the two periods. Note that prices effectively determine the firm's product line in each period because a sufficiently high price for a product variant implies it is not sold.

All consumers are present at time zero and leave the market when they make a purchase. Each consumer purchases at most one unit of a product. The consumers consider various alternatives including the option of not making a purchase. They decide whether to buy a product, which product to buy, and when to buy it, choosing the option that maximizes their utility. Thus, in period 1 each customer decides whether to buy a product in that period or wait for period 2. The customer valuations decrease over time. Each customer is willing to pay more to buy a product in period 1 rather than in period 2. The assumption that valuations decline over time seems to be common in the sales of fashion, technology, and seasonal products (Aviv and Pazgal 2008, Cachon and Swinney 2009, Lai et al. 2010, Desiraju and Shugan 1999). Specifically, the value of a product decreases by $1-\delta$ in period 2 where $\delta \in [0, 1]$. Thus, $1-\delta$ indicates perishability of the product value for consumers and is a measure of customer impatience. Hence, its complement δ shows the degree of customer patience.

Customers differ in their valuation of quality. Specifically, when type θ customer buys a product i at price p_{ti} in period t, her utility is equal to

$$U_t(\theta, q_i, p_{it}) = \delta^{t-1}\theta q_i - p_{it}, \quad t\colon 1, 2,\ i\colon L, H.$$

Thus, the value of product quality q_i for customer type θ is θq_i and $\delta\theta q_i$ in periods 1 and 2, respectively. Not buying a product at all yields zero utility. We require $\gamma < \beta + (1-\delta)(1-\beta)/(2-\delta)$ to avoid trivial cases, otherwise the relative cost of product L is too high, the firm never sells product L, and the model reduces to a single-product model. We also assume that unit cost c is sufficiently small to avoid trivial cases in which no product is sold in a period: Assuming $c < \beta(2-\delta)\delta/(\gamma(4-3\delta))$ for $\beta > \gamma$ and $c < (2\beta-\delta)\delta/(4\beta-2\gamma\delta-\delta)$ for $\beta \le \gamma$ is necessary and sufficient to ensure that at least one product is sold in each period in equilibrium. These conditions are obtained by solving for the equilibrium. Nevertheless, we relax these assumptions and describe the equilibrium for the trivial cases as well in Appendix A for the sake of completeness. Furthermore, we assume $\delta \le \beta$, meaning customers prefer buying product L in period 1 to waiting for product H in period 2 when these two options are priced equally. This implies for $\theta' > \theta$, if type θ customer purchases a product in period 1, then type θ' customer will also purchase a product in the same period. Note that we also study what happens when $\delta > \beta$ in Appendix B.

Customer types θ are uniformly distributed on the unit interval $[0, 1]$ and have a total mass of 1. The distribution of customer types is common knowledge. However, the firm does not know the type of any particular customer, hence perfect price discrimination is not feasible. Note that as is common in models of strategic customer behavior (e.g., Besanko and Winston 1990, Su 2007, Liu and van Ryzin 2008, Elmaghraby et al. 2008), the total demand is deterministic in our model, however its allocation to product choices, including the no-purchase option, over the two periods depends on the firm's pricing policy.

We analyze the above game between the firm and consumers seeking a subgame perfect Nash equilibrium (SPNE) (Selten 1975) in which the firm sets prices optimally in each period, provided the remaining consumers in the market make their purchase decisions optimally given their rational expectation of future prices. We need to introduce additional notation to explicitly define the equilibrium. Let $p_t = (p_{tL}, p_{tH})$ show the price vector in period t, Π_t show the firm's profit in period t, D_{ti} show the demand for product i in period t, and $\hat{U}_t(\theta, p_t)$ denote type-θ consumer's payoff from her best option (including no purchase) in period t given prices p_t, namely,

$$\hat{U}_t(\theta, p_t) = \max(U_t(\theta, q_L, p_{tL}), U_t(\theta, q_H, p_{tH}), 0). \quad (1)$$

Furthermore, let $\bar{\theta}$ show the type of the marginal consumer who is indifferent between buying in periods 1 and 2 such that consumers $\theta \ge \bar{\theta}$ buy a product in period 1 and consumers $\theta < \bar{\theta}$ wait for period 2. This uses the fact that $\hat{U}_1 - \hat{U}_2$ is monotonically increasing in θ.

An SPNE in our model is defined by the solutions of the following three equations:

$$p_2^*(\bar{\theta}) = \arg\max_{p_2} \Pi_2(p_2, \bar{\theta}), \quad (2)$$

$$\bar{\theta}(p_1) = \inf\{\theta\colon \hat{U}_1(\theta, p_1) \ge \hat{U}_2(\theta, p_2^*(\bar{\theta}))\}, \quad (3)$$

$$p_1^* = \arg\max_{p_1} \Pi(p_1), \quad (4)$$

where the firm's profit in periods 1 and 2 and their total are given by

$$\Pi_1(p_1) = D_{1L}(p_1)[p_{1L} - \gamma c] + D_{1H}(p_1)[p_{1H} - c], \quad (5)$$

$$\Pi_2(p_2, \bar{\theta}) = D_{2L}(p_2, \bar{\theta})[p_{2L} - \gamma c] + D_{2H}(p_2, \bar{\theta})[p_{2H} - c], \quad (6)$$

$$\Pi(p_1) = \Pi_1(p_1) + \Pi_2(p_2^*(\bar{\theta}(p_1)), \bar{\theta}(p_1)), \quad (7)$$

and the demand for each product i in period t, D_{ti}, can be characterized as follows:

$$D_{1i}(p_1) = \int_{\{\theta:\, U_1(\theta, q_i, p_{1i}) \geq \max(U_1(\theta, q_k, p_{1k}),\, \hat{U}_2(\theta, p_2^*(\bar{\theta}(p_1)))),\, k \neq i\}} d\theta, \quad (8)$$

$$D_{2i}(p_2, \bar{\theta}) = \int_{\{\theta:\, \theta < \bar{\theta} \text{ and } U_2(\theta, q_i, p_{2i}) \geq \max(U_2(\theta, q_k, p_{2k}), 0),\, k \neq i\}} d\theta. \quad (9)$$

Basically, (2) states that the firm sets product prices p_2 in period 2 to maximize its profit in that period given the remaining consumer segment $[0, \bar{\theta})$. Each consumer chooses her best option in period 1 in (3) by comparing period 1 prices p_1 with her conjecture of period 2 prices $p_2^*(\bar{\theta})$. The marginal consumer $\bar{\theta}$ is indifferent between buying in period 1 and waiting for period 2. Finally, (4) states that the firm chooses period 1 prices p_1 to maximize its total profit over the two periods. Notice that period 1 prices affect the consumers' conjecture about period 2 prices through $\bar{\theta}$ as shown in (2) and (3). In defining the equilibrium, we require firm and consumer behavior to be optimal not only when they follow the equilibrium path, but also when they are off the equilibrium path.

The demand for each product in (8) and (9) results from our discussion of customer choice. In period 2, products L and H compete among themselves for the remaining consumers. For product H to win a consumer, its utility should be higher than that of product L and vice versa. On the other hand, in period 1, products also compete with the consumers' option to wait for period 2. In this case, for product H to win the type θ customer in period 1, it needs to yield a higher utility than product L in period 1 and also the consumer's conjectured utility from her best option in period 2.

4. The Equilibrium

Equilibrium is derived by applying backward induction. Specifically, we follow the order in Equations (2)–(4), first, determining the firm's optimal prices in period 2, then deriving the consumers' optimal choices. Finally, we solve for the firm's optimal prices in period 1 and characterize the equilibrium. We also discuss the equilibrium of single-product and myopic consumers benchmarks at the end.

In period 2, the firm sets prices to extract the maximum profit from the remaining consumers $[0, \bar{\theta})$. The consumers do not have the strategic option of delaying their purchases further because it is the last period. Thus, the solution of the second period is the same as a single-period static model (see Moorthy 1984). The following lemma describes the firm's optimal policy in period 2.

Lemma 1. *Suppose consumers $[0, \bar{\theta})$ remain in the market in period 2.*

(i) *For $\beta \leq \gamma$ and $\bar{\theta} > c/\delta$, the firm sells only product H in period 2 at price $p_{2H}(\bar{\theta}) = \frac{1}{2}(\delta\bar{\theta} + c)$ yielding profit $\Pi_2(\bar{\theta}) = (\delta\bar{\theta} - c)^2/(4\delta)$ in equilibrium.*

(ii) *For $\beta > \gamma$, and $(1-\gamma)c/((1-\beta)\delta) \geq \bar{\theta} > \gamma c/(\beta\delta)$, the firm sells only product L in period 2 at price $p_{2L}(\bar{\theta}) = \frac{1}{2}(\delta\beta\bar{\theta} + \gamma c)$ yielding profit $\Pi_2(\bar{\theta}) = (\delta\beta\bar{\theta} - \gamma c)^2/(4\delta\beta)$ in equilibrium.*

(iii) *For $\beta > \gamma$, and $\bar{\theta} > (1-\gamma)c/((1-\beta)\delta)$, the firm sells both products H and L in period 2 at prices $p_{2L}(\bar{\theta}) = \frac{1}{2}(\delta\beta\bar{\theta} + \gamma c)$ and $p_{2H}(\bar{\theta}) = \frac{1}{2}(\delta\bar{\theta} + c)$ yielding profit $\Pi_2(\bar{\theta}) = (\delta\bar{\theta} - c)^2/(4\delta) + (\beta - \gamma)^2 c^2/(4(1-\beta)\beta\delta)$ in equilibrium.*

(iv) *For $\min(c/\delta, \gamma c/(\beta\delta)) \geq \bar{\theta}$, the firm does not sell any products in period 2.*

Proofs of all results are provided in Online Appendix F (available in the online supplement provided at http://msom.journal.informs.org/). Note that increasing β relative to γ favors product L and as expected, the firm's product portfolio moves from selling only product H in part (i) to both products in part (ii) and to only product L in part (iii). In addition, as time adjusted unit cost c/δ increases, the firm chooses to sell fewer or no products.

Now, consider the consumers' choices in period 1, given prices p_1. There exist $1 \geq \theta_1 \geq \bar{\theta}$ such that consumers in $[\theta_1, 1]$ buy product H, consumers in $[\bar{\theta}, \theta_1)$ buy product L and the remaining consumers wait for period 2. This is illustrated in Figure 1. The consumer θ_1 who is indifferent between buying products L and H in period 1 satisfies

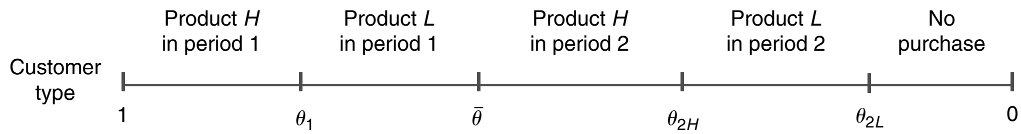
$$\beta\theta_1 - p_{1L} = \theta_1 - p_{1H}. \quad (10)$$

Similarly, the marginal consumer $\bar{\theta}$, who is indifferent between buying in period 1 and waiting for period 2, is given by the solution of

$$\max(\bar{\theta} - p_{1H}, \beta\bar{\theta} - p_{1L}) = \hat{U}_2(\bar{\theta}, p_2^*(\bar{\theta})). \quad (11)$$

Essentially, (11) simply states the marginal customer obtains the same utility from her best options in periods 1 and 2 where she conjectures period 2 prices p_2^* as given in Lemma 1.

Figure 1 Consumer Segments When Both Products Are Sold in Both Periods



To determine the firm's optimal prices in period 1, we can equivalently solve for the firm's choice of consumers segments in period 1 characterized by θ_1 and $\bar{\theta}$. To achieve this segmentation, the firm needs to set the following prices based on (10) and (11):

$$\begin{aligned} p_{1L} &= \beta\bar{\theta} - \hat{U}_2(\bar{\theta}, p_2^*(\bar{\theta})),\\ p_{1H} &= \theta_1 - \beta(\theta_1 - \bar{\theta}) - \hat{U}_2(\bar{\theta}, p_2^*(\bar{\theta})). \end{aligned} \tag{12}$$

Note product H could command a price of θ_1 in period 1 when it is sold to segment $[\theta_1, 1]$ if it were competing against *only* the no purchase option, i.e., consumers without the option of buying product L or waiting for period 2. Similarly, product L could command a price of $\beta\bar{\theta}$ in isolation. The firm suffers margin losses from these prices due to inter- and intratemporal cannibalization. The term $\hat{U}_2(\bar{\theta}, p_2^*(\bar{\theta}))$ in both p_{1L} and p_{1H} in (12) shows the *margin loss due to intertemporal cannibalization*, i.e., customers strategically waiting for period 2 instead of buying in period 1 (intertemporal substitution). In other words, if the customers were myopic, not considering future options in their decision making, the firm would not incur this loss when targeting the same segments. Let $\psi(\bar{\theta}) = \hat{U}_2(\bar{\theta}, p_2^*(\bar{\theta}))$ denote the margin loss due to intertemporal cannibalization, which is equal to the surplus of the marginal customer. It follows from Lemma 1 and the definition of $\hat{U}_t$ in (1) that

$$\psi(\bar{\theta}) = \begin{cases} \dfrac{\delta\bar{\theta} - c}{2} & \text{for } \bar{\theta} > \max\left(\dfrac{(1-\gamma)c}{(1-\beta)\delta}, \dfrac{c}{\delta}\right),\\[2ex] \dfrac{\delta\beta\bar{\theta} - \gamma c}{2} & \text{for } \dfrac{(1-\gamma)c}{(1-\beta)\delta} \geq \bar{\theta} > \dfrac{\gamma c}{\beta\delta} \text{ and } \beta > \gamma,\\[2ex] 0 & \text{for } \min\left(\dfrac{\gamma c}{\beta\delta}, \dfrac{c}{\delta}\right) \geq \bar{\theta}. \end{cases} \tag{13}$$

Let us define the total loss due to intertemporal cannibalization as

$$\Psi(\bar{\theta}) = [1 - \bar{\theta}]\psi(\bar{\theta}). \tag{14}$$

Notice the loss from intertemporal cannibalization in period 1 depends on the size of the consumer segment who waits for period 2 as well as the optimal product line in period 2 (whether products H and L are sold).

In addition to the intertemporal cannibalization, product H also suffers from intratemporal cannibalization (consumers downgrading to product L), and $\beta(\theta_1 - \bar{\theta})$ in (12) shows the *margin loss due to intratemporal cannibalization* in period 1. This loss is because of the consumers' option of switching to product L (variety substitution) and does not exist when product L is not sold in period 1, i.e., when $\theta_1 = \bar{\theta}$.

We can now reformulate the firm's optimal pricing problem in (4) as its optimal choice of consumer segments in period 1:

$$\max_{0 \leq \bar{\theta} \leq \theta_1 \leq 1} \big[(1-\theta_1)(\theta_1 - c) + (\theta_1 - \bar{\theta})(\beta\bar{\theta} - \gamma c) - \beta(1-\theta_1)(\theta_1 - \bar{\theta}) - \Psi(\bar{\theta}) + \Pi_2(\bar{\theta})\big]. \tag{15}$$

There is a nice interpretation of (15): The first two terms show the profit the firm could generate in period 1 absent customer choice, the third and fourth terms show the loss due to intra- and intertemporal cannibalization in period 1 respectively, and the last term is the profit in period 2. The firm chooses θ_1 and $\bar{\theta}$ to maximize its total profit over the two periods. The solution of (15) combined with (12) give the firm's optimal prices in period 1. Proposition 1 reports the resulting equilibrium. Recall that Lemma 1 states the firm's optimal prices p_2 in period 2. Hence, the equilibrium of the game between the firm and consumers is fully characterized by Proposition 1 together with Lemma 1.

Proposition 1 shows that the firm chooses not to sell products H and L in certain periods in equilibrium depending on cost (c and γ), quality (β), and customer patience (δ) parameters. We say the firm sells *weakly* only product L in period 2 when even a tiny increase in period 1 prices, any $\epsilon > 0$, will result in selling both product types in period 2.

Proposition 1. *The SPNE is characterized as follows. Corresponding period 1 prices p_1 and the marginal customer $\bar{\theta}$ are given in Appendix A.*

(i) *Suppose $\beta > \gamma$.*

(H, HL): *For $0 \leq c < (1-\beta)\delta/((1-\gamma)(4-3\delta))$, the firm sells only product H in period 1 and both products H and L in period 2.*

(HL, HL): *For $(1-\beta)\delta/((1-\gamma)(4-3\delta)) \leq c < (1-\beta)(2\beta-\delta)\delta/((1-\gamma)(2\beta(2-\delta)-\delta))$, the firm sells both products H and L in period 1 and both products H and L in period 2.*

(HL, L_w): *For* $(1-\beta)(2\beta-\delta)\delta/((1-\gamma)(2\beta(2-\delta)-\delta)) \leq c < (1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta))$, *the firm sells both products H and L in period* 1 *and weakly only product L in period* 2.

(HL, L): *For* $(1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta)) \leq c < (1-\beta)/(1-\gamma)$, *the firm sells both products H and L in period* 1 *and only product L in period* 2.

(L, L): *For* $(1-\beta)/(1-\gamma) \leq c$, *the firm sells only product L in both periods* 1 *and* 2.

(ii) *Suppose* $\beta \leq \gamma$.

(H, H): *For* $0 \leq c < (1-\beta)\delta/((1-\gamma)(4-3\delta))$, *the firm sells only product H in both periods.*

(HL, H): *For* $(1-\beta)\delta/((1-\gamma)(4-3\delta)) \leq c$, *the firm sells both products H and L in period* 1 *and only product H in period* 2.

Note that the proposition describes the equilibria when at least one product type is sold in each period. The equilibria for the remaining trivial cases are stated in Appendix A. When $\beta > \gamma$, cost-to-quality ratio favors product L, and product H is not sold in period 2 unless product L is also sold. This is because period 2 is the last period. There is no opportunity for forward-looking strategic behavior; thus cost-to-quality ratio becomes the only factor determining the firm's portfolio. As γ increases, the firm's portfolio shifts from selling only product L to both products L and H in period 2. Note that all cost thresholds in the proposition increase in γ. In contrast, when $\beta \leq \gamma$, cost-to-quality ratio favors product H and the firm never sells product L in period 2. At one extreme, when $\beta = 1$, that is when product L is as valuable as product H, product H is never sold. At the other extreme, when $\gamma = 1$, that is when product L is as costly as product H, product L is never sold.

The products retain more of their value in period 2 as δ increases. Consequently, this expands offered product variety in period 2 while reducing it in period 1. Specifically, the equilibrium moves from the (HL, L) region toward the (H, HL) region as δ increases. Note that all cost thresholds in the proposition weakly increase in δ. At the extreme, when customers become perfectly patient ($\delta = 1$), there are no sales in period 1, i.e., $\bar{\theta} = 1$. At the other extreme, when $\delta = 0$, products do not retain any value and there are no sales in period 2. In these two instances, the model effectively reduces to a single-period model. Finally, how does unit cost c affect product offerings? The proposition shows increasing c results in dropping the product with the cost-to-quality disadvantage (e.g., product L when $\beta > \gamma$) from product offerings or adding the alternative product to the product line.

In addition to our main model, we also consider single-product and myopic consumers benchmarks. The equilibria of these two benchmarks are stated in Appendix C. The equilibrium of the single-product benchmark where the firm can offer only product H can be obtained by simply setting $\beta = \gamma = 1$ in our two-product model (i.e., by eliminating the quality and cost differentiation). We define the margin loss ψ and total loss Ψ due to intertemporal cannibalization for this benchmark similar to our main model, as explicitly stated in (C1) and (C2) in Appendix C.

The myopic consumers benchmark assumes consumers only consider the options available in the current period in their purchasing decisions. The equilibrium of this benchmark is given by setting $\Psi = 0$ in (15), by eliminating the loss due to intertemporal cannibalization in the firm's problem. These benchmarks help us tease out the effects of product variety and strategic consumer behavior, which are discussed in the following section.

5. Product Variety and Forward-Looking Strategic Behavior

Absent product variety, when the firm sells only product H, it suffers from only intertemporal cannibalization. Product variety, serving product L in addition to product H, also introduces intratemporal cannibalization. The key question becomes how does product variety affect intertemporal cannibalization? This is addressed by the following proposition. Notice that we use superscript "1" for the single-product benchmark and no superscripts for our main model.

Proposition 2. *Consider the margin loss* ψ *and the total loss* Ψ *due to intertemporal cannibalization in equilibrium in our main model and the single-product benchmark.* $\Psi > \Psi^1$ *and* $\psi > \psi^1$ *if and only if the firm sells strictly only product L in period* 2 *in equilibrium in our main model.*[1]

What are the drivers of the above result? On one hand, serving product L in addition to product H enables the firm to attract a larger consumer segment in period 1, thereby decreasing the size of the consumer segment the firm serves in period 2. This allows the firm to leave less surplus to consumers in period 2, making the waiting option less attractive, which in turn decreases the loss due to intertemporal cannibalization. On the other hand, if only product L is sold in period 2, for example as in the (HL, L) region, the firm serves a larger segment in period 2 compared to the single-product benchmark which sells only product H. Doing so necessitates leaving

[1] Recall that we say the firm sells weakly only product L in period 2 if any $\epsilon > 0$ increase in period 1 prices will result in selling both product types in period 2. Similarly, we say that the firm sells strictly only product L in period 2 if the firm will continue to sell only product L in period 2 even when period 1 prices increase by ϵ for some $\epsilon > 0$.

a larger surplus to the marginal customer, which increases the attractiveness of the waiting option. Thus, the firm incurs a larger loss to convince customers to buy in period 1. Following Proposition 1, this happens when cost-to-quality ratio favors product L, i.e., $\beta > \gamma$, and customer patience δ is sufficiently low (note thresholds in Proposition 1 increase in δ), that is, customer evaluations decrease significantly over time. Otherwise, when customer patience δ is very high, or when cost-to-quality ratio favors product H, the firm never sells *only* product L in period 2 and product variety decreases the loss due to intertemporal cannibalization.

Proposition 2 shows that although product variety introduces intratemporal cannibalization, it can reduce the loss due to intertemporal cannibalization, and the proposition explicitly characterizes when this benefit exists. In fact, we will show that because of this benefit, the firm may prefer selling a product variant that it would not sell otherwise. It is well known that when $\beta \le \gamma$, the cost-to-quality ratio favors product H, and it is unprofitable to sell product L in classical models of vertical product differentiation (e.g., Tirole 1988). Indeed, the firm sells only product H in the myopic consumers benchmark as shown in Appendix C.

In contrast, the firm finds it profitable to sell product L to strategic consumers, and having product L in the product line yields a strictly higher profit than selling only product H in this case, as shown in the following corollary.

Corollary 1. *Suppose $\beta \le \gamma$. The firm sells product L in addition to product H in the main model and $\Pi > \Pi^1$ in equilibrium when $(1-\beta)\delta/((1-\gamma)(4-3\delta)) < c$.*

The corollary gives an example in which intertemporal substitution changes the added value of a product from negative to positive. It is straightforward to show if the firm could commit to future prices or quantities, it would not sell product L in this case (see Appendix D). Intuitively, when the firm already has another lever to control strategic customers (i.e., commitments), it does not need to sell product L just to deal with forward-looking strategic behavior. Note that when $c < (1-\beta)\delta/((1-\gamma)(4-3\delta))$, the profit margin is too high and the firm is better off offering only product H in both periods. Overall, Proposition 2 combined with Corollary 1 show firms can take advantage of product variety to reduce their losses due to strategic customers.

Intuitively, one would expect a firm to always earn a higher profit when it can offer more product variants. However, Proposition 2 demonstrates product variety can intensify the loss due to intertemporal cannibalization in some cases. Can this loss dominate the segmentation benefit of product variety? Can the firm suffer because of its ability to offer more product variants? These questions are addressed in Proposition 3.

Proposition 3. (i) *Suppose $\beta \le \gamma$, then $\Pi \ge \Pi^1$.*

(ii) *Suppose $\frac{1}{2}(5-\sqrt{25-16\beta}) \ge \gamma$, then there exist $\bar{\delta}(\beta,\gamma)$, $\underline{c}(\delta,\beta,\gamma)$, and $\bar{c}(\delta,\beta,\gamma)$ such that $0 < \bar{\delta}(\beta,\gamma) < 1$ and $\Pi < \Pi^1$ if and only if $\delta < \bar{\delta}(\beta,\gamma)$ and $\underline{c}(\delta,\beta,\gamma) < c < \bar{c}(\delta,\beta,\gamma)$. Furthermore, $\underline{c}(\delta,\beta,\gamma) > (1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta))$.*

Part (i) of the proposition shows that adding a product with an inferior cost-to-quality ratio (i.e., $\beta \le \gamma$) never hurts as far as strategic customer behavior is concerned. This result is consistent with Proposition 2 which indicated that the loss due to strategically forward-looking customers is always smaller with two products when $\beta \le \gamma$. In this case, the firm can always match the outcome of a model without the additional product: Because it is never optimal to sell the additional product in period 2, the firm can eliminate the demand for this product through its pricing. However, the same is not true for an additional product with better cost-to-quality ratio, which is discussed in part (ii) of the proposition.

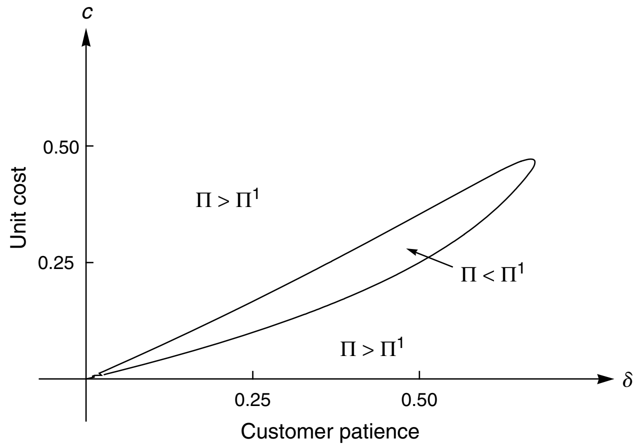
In part (ii), note that $\frac{1}{2}(5-\sqrt{25-16\beta})$ is very close to β: It is straightforward to show $\beta = \frac{1}{2}(5-\sqrt{25-16\beta})$ at the two ends $\beta = 0$ and $\beta = 1$ and $0 \le \beta - \frac{1}{2}(5-\sqrt{25-16\beta}) \le 1/16$ for $\beta \in [0,1]$. Thus, the condition $\frac{1}{2}(5-\sqrt{25-16\beta}) \ge \gamma$ is slightly stronger than the condition $\beta > \gamma$ and this is for analytical tractability. Our numerical studies show that the profit of the single-product benchmark can be higher even when $\frac{1}{2}(5-\sqrt{25-16\beta}) < \gamma \le \beta$, for example, $\Pi < \Pi^1$ when $\delta = 0.4$, $\beta = 0.5$, $\gamma = 0.45$, and $c = 0.25$.

The proof of part (ii) is quite involved, but, essentially we show that $\Pi^1 - \Pi$ is quasiconcave in c in equilibrium, and its peak is positive if and only if $\delta < \bar{\delta}$. We explicitly characterize $\bar{\delta}$ in the proof of this result. However, it cannot be expressed in a compact form: it is given by the unique solution of (EC-2) in Online Appendix F.

Part (ii) of the proposition demonstrates that product variety does not necessarily increase the firm's profit. To the contrary, it can harm profitability. This provides an example where intertemporal substitution changes the added value of a product from positive to negative. Figure 2 shows the region in which a single product generates a higher profit than two products when $\beta = 0.75$ and $\gamma = 0.6$. Note that this is not due to the classical (intratemporal) cannibalization (cf. Desai 2001), but to the intertemporal cannibalization arising from forward-looking strategic customers.

Basically, the added value of a product depends on its effect on the loss due to intertemporal substitution and its segmentation benefit. Proposition 2 shows that the addition of product L increases the loss due to

Figure 2 **Comparison Between the Profits of Single Product and Two Products** ($\beta = 0.75$ **and** $\gamma = 0.6$)



intertemporal substitution when the firm sells only product L in period 2 in equilibrium for example as in (HL, L) and (L, L) regions. This happens when $\beta > \gamma$ and $(1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta)) < c < \beta(2-\delta)\delta/(\gamma(4-3\delta))$, which requires customer patience δ and unit cost c not to be too high or too low. If c is too high or δ is too low, the firm does not sell any products in period 2. Similarly, if c is too low or δ is too high, the firm sells both products in period 2. Recall we assume $c < \beta(2-\delta)\delta/(\gamma(4-3\delta))$ for $\beta > \gamma$ throughout the paper to avoid outcomes in which no product is sold in a period.

Because of the segmentation benefit, the added value of a product can be positive even when it increases the loss due to intertemporal substitution. Thus, $\underline{c} > (1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta))$, and the loss due to intertemporal substitution dominates only for $\underline{c} < c < \bar{c}$. Indeed, Figure 2 is consistent with our discussion: The two-product model yields a lower profit than the single-product benchmark when the customer patience δ and unit cost c are moderate. Observe that customer patience δ is critical for this result. When customers are perfectly patient, i.e., $\delta = 1$, offering two products is always better than a single product since $\bar{\delta} < 1$ in the proposition.

Note for the cases in which having two products yields a lower profit, the firm cannot imitate the single-product scenario. Essentially, the firm cannot replicate the outcome of second period because it becomes optimal for the firm to sell product L in that period, and this is foreseen by the consumers.

Table 1 summarizes our comparison of optimal product portfolios for myopic and strategic customers. The table illustrates that ignoring intertemporal cannibalization in product assortment can lead to critically incorrect decisions. Specifically, an unprofitable (profitable) product variant included in the product line when intertemporal substitution is ignored can in fact be profitable (unprofitable).

Table 1 **Optimal Product Portfolio Among Products *H* and *L***

	Myopic customers	Strategic customers
$\beta \le \gamma$	Only *L*	*H* and *L* *or* only *L*
$\beta > \gamma$	*H* and *L*	*H* and *L* *or* only *H*

6. Loss Due to Forward-Looking Strategic Behavior

We have shown that ignoring strategic customer behavior results in both suboptimal pricing and suboptimal product portfolio choices. In this section, through numerical studies, we quantify the magnitude of the loss from such suboptimal decisions. To this end, we consider alternative models in which the firm does not take into account forward-looking strategic behavior in its decision making and assumes consumers behave myopically, when they continue to behave strategically. Details of the analysis are provided in Online Appendix H. We compare these alternative models with our main model to quantify the loss from ignoring strategic behavior.

For brevity, our main model has focused on $\beta \ge \delta$ scenario. To carry out our numerical study, we need to specify the equilibrium for $\beta < \delta$ as well, which is stated in Appendix B. In our numerical studies, we consider all combinations of the following parameters, $c \in \{0.2, 0.3, 0.4\}$, $\delta \in \{0.4, 0.5, 0.6, 0.7\}$, $\gamma \in \{0.2, 0.4, 0.6, 0.8\}$, and $\beta \in \{0.3, 0.45, 0.6, 0.75, 0.9\}$. Here, we discuss our findings using a representative subset.

Table 2 compares the loss from ignoring intertemporal substitution in pricing for single-product and two-product models. It presents the percentage loss due to suboptimal pricing relative to attainable profit with optimal pricing. The key observation is that product variety makes suboptimal pricing more costly. Specifically, the table shows that suboptimal pricing that ignores intertemporal substitution results in larger losses with two products than with a single product. Intuitively, ignoring intertemporal substitution results in a smaller than expected buying

Table 2 **Loss Due to Suboptimal Pricing that Ignores Forward-Looking Strategic Behavior**

$\beta = 0.75$		$c = 0.3$		$c = 0.4$	
δ	γ	Single product (%)	Two products (%)	Single product (%)	Two products (%)
0.5	0.2	3.0	6.4	0.0	6.1
	0.4		5.1		4.5
	0.6		3.5		1.7
	0.8		3.1		0.0
0.6	0.2	6.7	9.7	4.0	9.4
	0.4		8.5		8.1
	0.6		5.7		5.7
	0.8		6.7		4.0

Table 3 Total Loss From Ignoring Forward-Looking Strategic Behavior Both in Pricing and Product Portfolio Choice

$\beta = 0.75$		$c = 0.3$			$c = 0.4$		
δ	γ	Relative impact of pricing (%)	Relative impact of product portfolio (%)	Total loss (%)	Relative impact of pricing (%)	Relative impact of product portfolio (%)	Total loss (%)
0.5	0.2	100	0	6.4	100	0	6.1
	0.4	100	0	5.1	100	0	4.5
	0.6	60	**40**	5.7	100	0	1.7
	0.8	97	**3**	3.1	100	0	0.0
0.6	0.2	100	0	9.7	100	0	9.4
	0.4	100	0	8.5	100	0	8.1
	0.6	100	0	5.7	74	**26**	7.6
	0.8	100	0	6.7	100	0	4.0

customer segment in the first period. This creates suboptimal relative pricing of product variants, which leads to suboptimal management of intratemporal substitution, causing additional losses.

Table 2 shows the impact of suboptimal pricing, but does not address the effect of suboptimal product portfolio. To separate the effects of pricing and product portfolio, we consider three models: (i) the firm takes into account strategic behavior both in its pricing and product selection (our main model); (ii) the firm ignores strategic behavior in its product selection, but takes into account strategic behavior in its pricing; (iii) the firm ignores strategic behavior both in its pricing and product selection. Comparing models (i), (ii), and (iii) allows us to quantify the total loss from ignoring forward-looking strategic behavior and the relative impact of suboptimal pricing and product selection. Specifically, the total loss shows the gap in models (i) and (iii), and the relative impact of suboptimal product portfolio is given by the ratio of profit gaps in models (i) and (ii), and models (i) and (iii). Table 3 shows the results of these comparisons. When product portfolios in models (i) and (ii) do not differ, ignoring intertemporal substitution does not lead to any error in product portfolio and relative impact of product portfolio is obviously zero.

However, in a few instances, models (i) and (ii) result in different portfolios and ignoring intertemporal substitution produces a suboptimal product portfolio that has more or fewer product variants than what is optimal. These appear in bold in Table 3. Specifically, when $c = 0.3$, $\beta = 0.75$, $\delta = 0.5$, $\gamma = 0.6$, or $c = 0.4$, $\beta = 0.75$, $\delta = 0.6$, $\gamma = 0.6$, the suboptimal portfolio has more product variants. In contrast, when $c = 0.3$, $\beta = 0.75$, $\delta = 0.5$, $\gamma = 0.8$, the suboptimal portfolio has fewer product variants. Table 3 shows that when ignoring strategic behavior results in an incorrect product portfolio, the relative impact of suboptimal product portfolio can be anywhere from 3% to 40%. The existing literature has focused mainly on the loss due to suboptimal pricing without considering product selection. Our study demonstrates that whereas suboptimal pricing has a bigger impact on the loss, additional loss due to suboptimal product selection can be quite significant—up to 2/3 of the loss due to suboptimal pricing.

Furthermore, Table 3 leads to another important observation. It shows that offering suboptimally more product variants results in a bigger relative loss compared to offering suboptimally fewer product variants (26%–40% versus 3% in this example). This finding holds in general for the entire set in our numerical study.

7. Alternative Models

Three different modeling extensions are considered in this section.

7.1. Customer Impatience

Customers are uniform in their degree of impatience, $1 - \delta$, in our main model. Here, we consider alternative models to discuss what happens when we relax this assumption. First, we suppose each customer's impatience level is drawn from a distribution independent of her type θ, that is, her sensitivity to quality. We present some numerical examples and describe our findings. Then, we discuss scenarios in which customers' impatience levels are correlated to their sensitivity to quality and provide a structural result.

Suppose each customer's patience level, δ, is determined by one of $\delta_1, \delta_2, \ldots, \delta_n$ according to a probability distribution, which is independent of customer type. In this case, each patience segment δ_i will have a different partitioning of customer types θ into products L and H and periods 1 and 2, which could make it more difficult to exploit segmentation efficiently. The firm determines its pricing based on the expected demand over all patience segments, otherwise, the equilibrium is derived similarly to our main model. The analysis of such a model analytically becomes intractable because of the numerous equilibrium regions it generates as a result of different patience segments. However, we solve numerical examples, which allows us to illustrate that our key finding can continue to hold with varying levels of patience.

Proposition 3 shows that a two-product firm can have a smaller profit than a single-product firm in our main model with a uniform patience level. Having (HL, L) equilibrium outcome is critical for this result as explained in §5. Thus, we expect this result to continue to hold with a distribution of patience levels when a sufficient mass of customers fall into patience segments with (HL, L) equilibrium outcome. Indeed, the following numerical example confirms this conjecture. When there are two segments $\delta_h = 0.8$ and $\delta_l = 0.6$ with equal probabilities, and $\beta = 0.75$, $\gamma = 0.3$, $c = 0.1$, single-product benchmark yields a higher profit than the two-product model. In this case, $\delta_h = 0.8$ segment has (H, HL) while $\delta_l = 0.6$ segment has (HL, L) equilibrium outcomes. Furthermore, in the single-product benchmark, $\delta_h = 0.8$ segment has (H, H) while $\delta_l = 0.6$ segment has $(H, 0)$ equilibrium outcomes.

Corollary 1 demonstrates that the firm may find it optimal to sell product L to forward-looking strategic consumers even when it is unprofitable to sell it to myopic consumers. This result also can be replicated with a distribution of patience levels. When δ is $\delta_h = 0.5$ with probability 0.2 and $\delta_l = 0.4$ otherwise, and $\beta = 0.75$, $\gamma = 0.8$, $c = 0.2$, product L is sold to $\delta_l = 0.4$ segment. Specifically, $\delta_h = 0.5$ segment has (H, H) while $\delta_l = 0.4$ segment has (HL, H) equilibrium outcomes. Note that it is never profitable to sell product L to myopic consumers in this case as $\beta < \gamma$.

Now suppose that each customer's patience level δ is a function of her type θ. We consider two alternative models: $\delta(\theta) = 1 - \theta/2$, in which high-type customers are less patient, and $\delta(\theta) = \theta/2$, in which high-type customers are more patient. The next lemma indicates that both of these examples would result in similar temporal partitioning of customers leading to the same set of equilibrium regions as in our main model (e.g., (H, HL), (HL, HL), (H, HL), etc.).

Lemma 2. *There exist $\bar{\theta}$, $\bar{\bar{\theta}}$ such that $0 \geq \bar{\bar{\theta}} \geq \bar{\theta} \geq 1$ and customers in $[0, \bar{\bar{\theta}})$ do not buy a product at all, customers in $[\bar{\bar{\theta}}, \bar{\theta})$ and $[\bar{\theta}, 1]$ buy a product in periods 2 and 1, respectively in equilibrium when the following two conditions hold*: (i) *the utility of a product increases in customer type θ*; (ii) *the gap between period 1 and 2 utilities of a product increases in customer type θ*.

It is straightforward to show that examples $\delta(\theta) = 1 - \theta/2$ and $\delta(\theta) = \theta/2$ satisfy both conditions (i) and (ii), as does our main model. Although the lemma shows that the same equilibrium structure (as in our main model) can emerge when there is correlation between sensitivity to quality and time, we should note that explicit characterization of the equilibrium is not easily tractable for these examples.

7.2. Endogenous Quality Differentiation

In §5, we characterize when the firm benefits from adding a lower-quality variant to its product line, where the quality of this product is exogenously given. In this section, through a numerical study, we examine what happens when the firm can determine the quality of this additional product endogenously. Details of this study are presented in Appendix E. As expected, the firm never chooses a quality level that would make the additional product detrimental in this case. Table 4 reports the results showing the optimal quality level β for product L both in myopic and strategic customers scenarios as unit c and customer patience δ vary. Quality of product H is normalized to 1. Thus, by choosing the quality of product L, the firm in turn chooses the level of quality differentiation in its product line.

Table 4 Optimal Quality Level β^*

	$c = 0.3$		$c = 0.4$	
δ	Myopic	Strategic	Myopic	Strategic
0.3	0.53	0.76	0.59	0.66
0.4	0.54	0.89	0.55	0.77
0.5	0.56	0.47	0.56	0.87
0.6	0.59	0.35	0.57	0.60
0.7	0.62	0.41	0.59	0.56
0.8	0.65	0.57	0.62	0.55

Table 4 demonstrates that strategic customers lead to a lower-quality level β when customers are patient (high δ), whereas strategic customers results in a higher-quality level β when customers are relatively impatient (medium to low δ). *Our numerical study shows that when the temporal differentiation is large (i.e., small δ), the firm prefers a smaller quality differentiation (i.e., large β) with strategic customers compared to myopic customers.* In this case, products have little value in period 2, hence intertemporal cannibalization is less of a concern. *In contrast, when temporal separation is small (i.e., large δ), the firm chooses a larger quality differentiation (i.e., small β) with strategic customers compared to myopic customers.* We considered various other parameters and cost functions and found this insight to be robust.

7.3. Price and Quantity Commitments

In this section, we discuss what happens when the firm can make price or quantity commitments. All of our assertions here adhere to the analytical results in Appendix D. Proposition 3 has shown that an additional product variant can hurt the firm's profitability in our base model. In contrast, when the firm can make price or quantity commitments, we find it always benefits from offering an additional product variant. Specifically, for the same parameters that make it unprofitable to include product L in the assortment in Proposition 3, the firm finds it attractive to sell product L when it can limit its quantity or price markdown through commitments.

Interestingly, committing to price or quantity of only product L (without any commitments for product H) is sufficient to avoid the peril of product variety due to strategic customers. In this case, the firm's profit will always be higher with two products. This stems from the fact that the two-product model results in a lower profit than the single-product benchmark only when the firm sells only product L in period 2. On the other hand, making commitments only for product H does not suffice to prevent product L from hurting profitability. The firm's profit can still be lower than the single-product benchmark.

8. Concluding Remarks

Firms are increasingly concerned about customer tendencies to hold off their purchases strategically for price markdowns. The impact of this behavior has attracted growing attention in academic circles. We contribute by considering this problem in the presence of product variety. We consider a firm facing forward-looking strategic customers who are heterogeneous in their evaluations of product quality. The firm sells two vertically differentiated products: a higher- and lower-quality product. Thus, the customers decide what to buy in addition to whether and when to buy a product. The introduction of product variety in this context makes the firm vulnerable to intratemporal cannibalization (variety substitution) in addition to intertemporal cannibalization (intertemporal substitution). We solve for the firm's optimal dynamic pricing policy by characterizing the equilibrium of the pricing-purchasing game between the firm and the strategic customers.

Our results show that ignoring strategic customer behavior results not only in suboptimal pricing but also in critically incorrect suboptimal product portfolio choices. Including a product variant in the assortment can yield a negative value without strategic customers while having a positive value with strategic customers and vice versa. While the existing literature has focused mainly on the loss due to suboptimal pricing ignoring the impact of product selection, we find the additional loss due to suboptimal product selection can be quite significant—up to 2/3 of the loss due to suboptimal pricing.

We also show that product variety can decrease the loss due to strategic customers by explicitly characterizing when this benefit exists and identifying its drivers. We illustrate that this benefit encourages a firm to sell product variants that it would not sell otherwise, and demonstrate how carefully selected product assortment, in addition to pricing and stocking, can serve as another lever for dealing with strategic customers.

We contrast our main model with the single-product benchmark in which the firm can sell only the higher-quality product to allow characterization of the impact of product variety. Alternatively, one can also consider a single-product benchmark in which the firm can sell only the lower-quality product as shown in Appendix C. We can show that this alternative benchmark always results in a smaller profit compared to our main model. This indicates that the impact on the forward-looking strategic customer behavior, while requiring careful consideration when adding a lower-quality variant, is not an issue when adding a higher-quality variant to the product line.

We consider deterministic demand like many others studying strategic customer behavior (e.g., Besanko and Winston 1990, Su 2007, Liu and van Ryzin 2008, Elmaghraby et al. 2008) and ignore any capacity limitation. However, the fear of product unavailability due to demand-capacity imbalance would discourage strategic waiting for a markdown. An additional product variant, on the other hand, can weaken this dynamic as customers can rely on product substitution when their first choice is not available. Thus, as far as forward-looking strategic behavior is concerned, we expect capacity scarcity due to stochastic demand to make product variety less attractive.

Our model considers two products with exogenous quality levels and compares profits when the firm can sell both products versus only one product. This quantifies the value of adding a given product variant to the assortment in the face of strategic customers. In this sense, our work addresses the impact of strategic customer behavior on product assortment extending previous studies about its impact on product pricing and stocking. Future research can extend our work further and address the impact of strategic customer behavior on product design by studying how it affects the quality levels that would be chosen when quality is endogenous. Our numerical study in §7.2 suggests some interesting questions in that direction. Finally, our work shows that the interplay between product variety and strategic customer behavior leads to interesting dynamics. This shows that future research on the optimal level of product variety taking into account its effect on forward-looking strategic behavior promises to be fruitful.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://msom.journal.informs.org/.

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Appendix A. Addendum to Proposition 1

In the main body of the paper, we have assumed that $c < \beta(2-\delta)\delta/(\gamma(4-3\delta))$ when $\beta > \gamma$ and $c < (2\beta-\delta)\delta/(4\beta - 2\gamma\delta - \delta)$ when $\beta \leq \gamma$ as stated in §3. This is necessary and sufficient to ensure that there is at least one product type sold in each period. Proposition 1 describes the equilibrium with this assumption. Here, we relax this assumption and state the equilibrium for the remaining cases.

Proposition 1(i) describes the equilibrium for $\beta > \gamma$ when $c < \beta(2-\delta)\delta/(\gamma(4-3\delta))$. The following describes the equilibrium for $\beta > \gamma$ when $c \geq \beta(2-\delta)\delta/(\gamma(4-3\delta))$.

$(HL, 0_w)$: For $\beta(2-\delta)\delta/(\gamma(4-3\delta)) \leq c < \min(\beta\delta/(\gamma(2-\delta)), (1-\beta)/(1-\gamma))$, the firm sells both products H and L in period 1 and weakly nothing in period 2, where $p_{1L} = \gamma c/\delta$, $p_{1H} = (1-\beta+(1-\gamma)c)/2 + \gamma c/\delta$, and $\bar{\theta} = \gamma c/(\beta\delta)$.

$(HL, 0)$: For $\beta\delta/(\gamma(2-\delta)) \leq c < (1-\beta)/(1-\gamma)$, the firm sells both products H and L in period 1 and nothing in period 2, where $p_{1L} = \gamma c/\delta$, $p_{1H} = (1-\beta+(1-\gamma)c)/2 + \gamma c/\delta$, and $\bar{\theta} = \gamma c/(\beta\delta)$.

$(L, 0_w)$: For $\max((1-\beta)/(1-\gamma), \beta(2-\delta)\delta/(\gamma(4-3\delta))) \leq c < \beta\delta/(\gamma(2-\delta))$, the firm sells product L in period 1 and weakly nothing in period 2, where $p_{1L} = \gamma c/\delta$ and $\bar{\theta} = \gamma c/(\beta\delta)$.

$(L, 0)$: For $\max((1-\beta)/(1-\gamma), \beta\delta/(\gamma(2-\delta))) \leq c < \beta/\gamma$, the firm sells product L in period 1 and nothing in period 2, where $p_{1L} = (\beta+\gamma c)/2$ and $\bar{\theta} = \frac{1}{2} + \gamma c/(2\beta)$.

$(0, 0)$: For $c \geq \beta/\gamma$, the market is infeasible, the firm does not sell any product.

Recall we say that the firm sells *weakly* only product L in period 2 when even a tiny increase in period 1 prices, any $\epsilon > 0$, will result in selling both product types in period 2. Similarly, in the following, we say the firm sells *weakly* nothing in period 2 when any $\epsilon > 0$ increase in period 1 prices will result in selling a product in period 2.

Proposition 1(ii) describes the equilibrium for $\beta \leq \gamma$ when $c < (2\beta-\delta)\delta/(4\beta-2\gamma\delta-\delta)$. The following describes the equilibrium for $\beta \leq \gamma$ when $c \geq (2\beta-\delta)\delta/(4\beta-2\gamma\delta-\delta)$.

$(HL, 0_w)$: For $(2\beta-\delta)\delta/(4\beta-2\gamma\delta-\delta) \leq c < (1-\beta)\delta/(2(1-\beta)-\delta(1-\gamma))$, the firm sells both products H and L in period 1 and weakly nothing in period 2, where $p_{1L} = \gamma c/\delta$, $p_{1H} = (1-\beta+(1-\gamma)c)/2 + \gamma c/\delta$, and $\bar{\theta} = \gamma c/(\beta\delta)$.

$(H, 0_w)$: For $(1-\beta)\delta/(2(1-\beta)-\delta(1-\gamma)) \leq c < \delta/(2-\delta)$, the firm sells product H in period 1 and weakly nothing in period 2, where $p_{1H} = c/\delta$ and $\bar{\theta} = c/\delta$.

$(H, 0)$: For $\delta/(2-\delta) \leq c < 1$, the firm sells product H in period 1 and nothing in period 2, where $p_{1H} = (1+c)/2$ and $\bar{\theta} = (1+c)/2$.

$(0, 0)$: For $c \geq 1$, the market is infeasible, the firm does not sell any product.

For Proposition 1, the SPNE period 1 prices p_1 and the marginal customer who is indifferent between buying in period 1 and 2, $\bar{\theta}$, are as follows.

(H, HL): $p_{1H} = (1+c)/2 - \delta(1-\delta)/(2(4-3\delta))$, and $\bar{\theta} = (2-\delta)/(4-3\delta)$.

(H, H): $p_{1H} = (1+c)/2 - \delta(1-\delta)/(2(4-3\delta))$, and $\bar{\theta} = (2-\delta)/(4-3\delta)$.

(HL, H): $p_{1L} = (\beta+\gamma c)/2 - (\beta-\delta+(1-\gamma)c)\delta/(2(4\beta-3\delta))$, $p_{1H} = (1+c)/2 - (\beta-\delta+(1-\gamma)c)\delta/(2(4\beta-3\delta))$, and $\bar{\theta} = \frac{1}{2} - (4(1-\gamma)c-\delta)/(2(4\beta-3\delta))$.

(HL, HL): $p_{1L} = (\beta+\gamma c)/2 - (\beta-\delta+(1-\gamma)c)\delta/(2(4\beta-3\delta))$, $p_{1H} = (1+c)/2 - (\beta-\delta+(1-\gamma)c)\delta/(2(4\beta-3\delta))$, and $\bar{\theta} = \frac{1}{2} - (4(1-\gamma)c-\delta)/(2(4\beta-3\delta))$.

(HL, L_w): $p_{1L} = (\beta(1-\gamma+1-\delta) - (\beta-\delta)\gamma)c/(2(1-\beta)\delta)$, $p_{1H} = (1-\beta+c)/2 + (1-\gamma)(2-\delta)\beta c/(2(1-\beta)\delta)$, and $\bar{\theta} = (1-\gamma)c/((1-\beta)\delta)$.

(HL, L): $p_{1L} = (\beta+\gamma c)/2 - \beta(1-\delta)\delta/(2(4-3\delta))$, $p_{1H} = (1+c)/2 - \beta(1-\delta)\delta/(2(4-3\delta))$, and $\bar{\theta} = (2-\delta)/(4-3\delta)$.

(L, L): $p_{1L} = (\beta+\gamma c)/2 - \beta(1-\delta)\delta/(2(4-3\delta))$ and $\bar{\theta} = (2-\delta)/(4-3\delta)$.

$(HL, 0_w)$: $p_{1L} = \gamma c/\delta$, $p_{1H} = (1-\beta+(1-\gamma)c)/2 + \gamma c/\delta$, and $\bar{\theta} = \gamma c/(\beta\delta)$.

$(HL, 0)$: $p_{1L} = (\beta+\gamma c)/2$, $p_{1H} = (1+c)/2$, and $\bar{\theta} = \frac{1}{2} + \gamma c/(2\beta)$.

Appendix B. $\beta < \delta$ Equilibrium

Our main model focuses on $\beta \geq \delta$ scenario, and §4 describes the equilibrium with this assumption. Here, we specify the equilibrium for $\beta < \delta$ as well. Note that $\beta < \delta$ implies customers prefer buying product H in period 2 rather than buying product L in period 1 when these two options are priced equally. The equilibrium for this case is stated in the next proposition.

Proposition 4. *Suppose $\beta < \delta$. The SPNE period 1 prices p_1 and the marginal customer $\bar{\theta}$ are as follows (Period 2 prices are given in Lemma 1).*

(H, HL): For $\beta > \gamma$ and $0 \leq c < (1-\beta)\delta[\beta(2-\delta) - (1-\delta)\cdot\delta\sqrt{(1-\beta)/(4-3\delta)}]/((1-\gamma)(\beta(2-\delta)^2 + (1-\delta)\delta))$, the firm sells only H in period 1 and both products H and L in period 2, where $p_{1H} = (1+c)/2 - \delta(1-\delta)/(2(4-3\delta))$, and $\bar{\theta} = (2-\delta)/(4-3\delta)$ in equilibrium.

(HL, L_w): For $\beta > \gamma$ and $(1-\beta)\delta[\beta(2-\delta) - (1-\delta)\delta\sqrt{(1-\beta)/(4-3\delta)}]/((1-\gamma)(\beta(2-\delta)^2 + (1-\delta)\delta)) \leq c < (1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta))$, the firm sells both products H and L in period 1 and weakly only product L in period 2, where $p_{1L} = (\beta(1-\gamma+1-\delta) - (\beta-\delta)\gamma)c/(2(1-\beta)\delta)$, $p_{1H} = (1-\beta+c)/2 + (1-\gamma)(2-\delta)\beta c/(2(1-\beta)\delta)$, and $\bar{\theta} = (1-\gamma)c/((1-\beta)\delta)$ in equilibrium.

For $\beta > \gamma$ and $c \geq (1-\beta)(2-\delta)\delta/((1-\gamma)(4-3\delta))$ the equilibrium is same as in Proposition 1. Finally, for $\beta \leq \gamma$, product L is never sold and the equilibrium is same as in the single-product benchmark given in Proposition 5 in Appendix C.

Note that although $\beta < \delta$ requires considering different regions, the result that single-product benchmark can yield a higher profit than the two-product model continues to hold. For example, when $\beta = 0.6$, $\gamma = 0.5$, and $c = 0.5$, single-product benchmark yields a higher profit than the two-product model for $0.65 < \delta < 0.73$.

Appendix C. Benchmarks

C.1. Single-Product Benchmark

The equilibrium for the single-product benchmark where the firm can offer only product L or H is given by the following proposition. Note that we present the equilibrium for selling only product L, however, plugging $\gamma = 1$ and $\beta = 1$ immediately gives the equilibrium for selling only product H.

Proposition 5. *The SPNE prices p and the marginal customer who is indifferent between buying in period 1 and 2, $\bar{\theta}$, are given by the following.*

$(H,H)^1$: *For $c < \delta(2-\delta)\beta/((4-3\delta)\gamma)$, the firm sells the product in both periods where $p_1 = (\beta+c\gamma)/2 - (1-\delta)\delta\beta/(2(4-3\delta))$, $p_2 = (\delta\beta+c\gamma)/2 - (1-\delta)\delta\beta/(4-3\delta)$, and $\bar{\theta} = (2-\delta)/(4-3\delta)$ in equilibrium.*

$(H,0_w)^1$: *For $\delta(2-\delta)\beta/((4-3\delta)\gamma) \le c < \delta\beta/((2-\delta)\gamma)$, the firm sells the product in period 1 and weakly nothing in period 2 where $p_1 = c\gamma/\delta$, and $\bar{\theta} = c\gamma/(\delta\beta)$ in equilibrium.*

$(H,0)^1$: *For $\delta\beta/((2-\delta)\gamma) \le c < \beta/\gamma$, the firm sells the product in period 1 and nothing in period 2 where $p_1 = (\beta+c\gamma)/2$, and $\bar{\theta} = (\beta+c\gamma)/(2\beta)$ in equilibrium.*

$(0,0)^1$: *For $c > \beta/\gamma$, the firm does not sell the product in both periods.*

Note that we say that the firm sells weakly nothing in period 2 when any $\epsilon > 0$ increase in period 1 prices will result in selling the product in period 2.

We can characterize the margin loss due to intertemporal cannibalization for this benchmark similar to (13), and it is given by,

$$\psi^1(\bar{\theta}) = \begin{cases} \dfrac{\delta\beta\bar{\theta} - c\gamma}{2} & \text{for } \bar{\theta} > \dfrac{c\gamma}{\delta\beta}, \\ 0 & \text{for } \dfrac{c\gamma}{\delta\beta} \ge \bar{\theta}. \end{cases} \tag{C1}$$

Similarly, the total loss due to intertemporal cannibalization is

$$\Psi^1(\bar{\theta}) = [1-\bar{\theta}]\psi^1(\bar{\theta}). \tag{C2}$$

C.2. Myopic Consumers Benchmark

The following proposition describes the equilibrium when consumers are myopic in that they do not consider future options in their decision making. They consider only the options available in the current period. Notice we use superscript "m" to denote the myopic consumers benchmark.

Proposition 6. *The equilibrium for the myopic consumers benchmark in which the consumers do not consider their future options in their purchasing decisions is as follows:*

(i) *For $\beta \le \gamma$:*

$(H,H)^m$: *For $c < \delta/(2-\delta)$, the firm sells product H in both periods, where $p_{1H} = (2+c)/(4-\delta)$, and $\bar{\theta} = (2+c)/(4-\delta)$ in equilibrium.*

$(H,0)^m$: *For $1 > c \ge \delta/(2-\delta)$, the firm sells only product H in period 1 and nothing in period 2, where $p_{1H} = (1+c)/2$, and $\bar{\theta} = (1+c)/2$ in equilibrium.*

(ii) *For $\beta > \gamma$:*

$(H,HL)^m$: *For $0 \le c < f_1^m(\delta,\beta,\gamma)$, the firm sells only product H in period 1 and both products H and L in period 2, where $p_{1H} = (2+c)/(4-\delta)$, and $\bar{\theta} = (2+c)/(4-\delta)$ in equilibrium.*

$(HL,HL)^m$: *For $f_1^m(\delta,\beta,\gamma) \le c < f_2^m(\delta,\beta,\gamma)$, the firm sells both products H and L in period 1 and both products H and L in period 2, where $p_{1L} = (2\beta - (1-2\gamma)c)\beta/(4\beta-\delta)$, $p_{1H} = (2+c)/4 + (2\beta-(1-2\gamma)c)\delta/(4(4\beta-\delta))$, and $\bar{\theta} = (2\beta - (1-2\gamma)c)/(4\beta-\delta)$ in equilibrium.*

$(HL,L)^m$: *For $f_2^m(\delta,\beta,\gamma) \le c < \min(f_3^m(\delta,\beta,\gamma), (1-\beta)/(1-\gamma))$, the firm sells both products H and L in period 1 and only product L in period 2, where $p_{1L} = (2\beta+\gamma c)/(4-\delta)$, $p_{1H} = (1+c)/2 + (\beta\delta - (2-\delta)\gamma c)/(2(4-\delta))$, and $\bar{\theta} = (2\beta+\gamma c)/((4-\delta)\beta)$ in equilibrium.*

$(HL,0)^m$: *For $f_3^m(\delta,\beta,\gamma) \le c < (1-\beta)/(1-\gamma)$, the firm sells both products H and L in period 1 and nothing in period 2, where $p_{1L} = (\beta+\gamma c)/2$, $p_{1H} = (1+c)/2$, and $\bar{\theta} = (\beta+\gamma c)/(2\beta)$ in equilibrium.*

$(L,L)^m$: *For $(1-\beta)/(1-\gamma) \le c < f_3^m(\delta,\beta,\gamma)$, the firm sell product L in both periods 1 and 2, where $p_{1L} = (2\beta+\gamma c)/(4-\delta)$ and $\bar{\theta} = (2\beta+\gamma c)/((4-\delta)\beta)$.*

$(L,0)^m$: *For $\max((1-\beta)/(1-\gamma), f_3^m(\delta,\beta,\gamma)) \le c < \beta/\gamma$, the firm sell product L in period 1 and nothing in period 2, where $p_{1L} = (\beta+\gamma c)/2$ and $\bar{\theta} = \frac{1}{2} + \gamma c/(2\beta)$.*

$(0,0)^m$: *For $c \ge \beta/\gamma$, the market is infeasible, the firm does not sell any product.*

The thresholds are given by

$$f_1^m(\delta,\beta,\gamma) = \frac{(1-\beta)\delta}{(2-\delta)(1-\gamma)+2(\beta-\gamma)},$$

$$f_2^m(\delta,\beta,\gamma) = \frac{2(1-\beta)\beta\delta}{(1-\gamma)\beta(4-\delta)-(1-\beta)\gamma\delta},$$

$$f_3^m(\delta,\beta,\gamma) = \frac{\beta\delta}{(2-\delta)\gamma}.$$

The firm's optimal period 2 prices are the same as in Lemma 1. This is because customers do not have the option of strategically delaying their purchases in period 2 because it is the last period. Therefore, the optimal prices in period 2 are the same for myopic and strategic customer models.

Appendix D. Commitments

Here, we summarize our results concerning what happens when the firm can make price or quantity commitments. Proofs of these results and more detailed discussion of commitments can be found in Online Appendix G.

The price and quantity commitments result in the same equilibrium outcome. This is because both of these commitments effectively reduce the model to a single period. The following proposition describes the resulting equilibrium.

Proposition 7. *Suppose the firm can commit to future prices or it can make quantity commitments. In equilibrium, the firm does not sell any products in period 2. Furthermore,*

(i) *for $\beta > \gamma$ and $c < (1-\beta)/(1-\gamma)$, the firm sells both products H and L in period 1;*

(ii) *for $\beta \le \gamma$ and $c < 1$, the firm sells only product H in period 1;*

(iii) *for $c \ge \min((1-\beta)/(1-\gamma), 1)$, the firm never sells product H.*

The proposition shows that the firm limits its sales to only period 1 to eliminate its loss due to intertemporal cannibalization.

The following corollary shows how commitments affect the value of product variety. Let superscripts "$pc,1$" and "$qc,1$" denote price and quantity commitments for the single-product benchmark.

Corollary 2. $\Pi^{qc} \ge \Pi^{qc,1}$ *and* $\Pi^{pc} \ge \Pi^{pc,1}$ *in equilibrium.*

The inequalities in the corollary are strict for $\beta > \gamma$. The corollary shows that when the firm can make price or quantity commitments, it always benefits from offering an additional product variant.

If the firm can commit to price or quantity of only product L (without any commitments for product H), its profit will always be higher with two products. On the other hand, if the firm can make commitments only for product H, this may not suffice to prevent product L from hurting profitability. This is formally stated in the next proposition. Let superscript "pch" ("qch") denote the scenario in which the firm can commit to price (quantity) of only product H.

Proposition 8. *If* $\Pi^1 > \Pi$ *then* $\Pi^1 > \Pi^{pch}$ *and* $\Pi^1 > \Pi^{qch}$.

The proposition demonstrates that whenever two product variants yield a lower profit than the single-product benchmark (as in Proposition 3), the ability to make commitments for only product H does not suffice, and the firm's profit still remains lower than the single-product benchmark. So, although the ability to make commitments for the lower-quality product ensures that product variety always results in a higher profit, commitments only for the higher-quality product may still result in a lower profit than the single-product benchmark.

Appendix E. Endogenous Quality Differentiation Numerical Study

In our numerical study, the firm first chooses the quality level β and then makes its pricing decisions. We assume $\gamma = \beta^3$, which corresponds to a convex relation between cost and quality. Furthermore, we require the firm to choose a quality level above a minimum quality, i.e., $\beta \geq \beta_{\min}$. This might be the result of market expectations. Furthermore, this prevents the firm from choosing a quality level near zero rather than not selling an additional product. We choose $\beta_{\min} = 0.1$ for our study.

Table 4 shows the optimal quality level β for product L both in myopic and strategic customers scenarios as unit c and customer patience δ vary. Note that when β is endogenously chosen, the firm avoids quality levels that will make offering two products worse than a single product as in Proposition 3. Indeed, in the examples considered in Table 4, offering two products always yields a higher profit than a single product.

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