# Online Supplement to The Value of Product Variety When Selling to Strategic Consumers

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In this online Appendix, we provide proofs of the analytical results in Appendix F and we provide detailed analysis of loss due to forward-looking strategic behavior and commitments in Appendices H and G. Proofs are presented in the order their corresponding results appear in the paper.

## Appendix F: Proofs:

To facilitate our discussion, we introduce variables $f_n(\delta,\beta,\gamma)$, $n:1,...,5$ which are explicitly stated in Table 5. These variables correspond to the thresholds in the Propositions. Note that $f_1 \leq f_2 \leq f_3 \leq f_4 \leq f_5$ , each $f_n$, $n:1,...,5$ is increasing in $\delta$, and the inequalities are strict when $1>\delta>0$ and $\beta \neq \gamma$. Furthermore, $\beta=\gamma$ is the critical threshold for *min* and *max* terms in $f_n$.

**Table 5** $f_n(\delta,\beta,\gamma)$ **thresholds.**

| | | |
|---|---|---|
| $f_1$ | $=$ | $\frac{(1-\beta)\delta}{(1-\gamma)(4-3\delta)}$, |
| $f_2$ | $=$ | $\min(\frac{(1-\beta)(2\beta-\delta)\delta}{(1-\gamma)(2\beta(2-\delta)-\delta)}, \frac{(2\beta-\delta)\delta}{4\beta-2\gamma\delta-\delta})$, |
| $f_3$ | $=$ | $\min(\frac{(1-\beta)(2-\delta)\delta}{(1-\gamma)(4-3\delta)}, \frac{(2-\delta)\delta}{4-3\delta})$, |
| $f_4$ | $=$ | $\frac{\beta(2-\delta)\delta}{\gamma(4-3\delta)}$, $\beta>\gamma$,<br>$\frac{(1-\beta)\delta}{2(1-\beta)-\delta(1-\gamma)}$, $\beta\leq\gamma$ |
| $f_5$ | $=$ | $\max(\frac{\beta\delta}{\gamma(2-\delta)}, \frac{\delta}{2-\delta})$. |

*Proof of Lemma 1* Note that $\theta_{2H}$ and $\theta_{2L}$ are given by consumers who are indifferent between product H and L, and product L and not buying a product in period 2. Specifically

$$\theta_{2H}=\frac{p_{2H}-p_{2L}}{(1-\beta)\delta}, \text{ and } \theta_{2L}=\frac{p_{2H}}{\beta\delta} \tag{EC-1}$$

and the firm solves the following problem which leads to the Lemma.

$$\begin{aligned}\max_{p_{2H},p_{2L}} \quad & [(p_{2H}-c)(\bar{\theta}-\theta_{2H})+(p_{2L}-\gamma c)(\theta_{2H}-\theta_{2L})] \\ \text{st.} \quad & \bar{\theta}\geq\theta_{2H}\geq\theta_{2L}\geq 0, \\ & p_{2H},p_{2L}\geq 0.\end{aligned}$$

□

*Proof of Proposition 1* Note that the firm's optimal period 2 prices and the resulting profit is given in Lemma 1. We reduced the firm's problem in period 1 to choosing segments $0 \le \bar{\theta} \le \theta_1 \le 1$ such that it is optimal for consumers in $[\bar{\theta}, \theta_1]$ and $[\theta_1, 1]$ to buy product L and H respectively in period 1. So, the firm chooses $\bar{\theta}$ and $\theta_1$ to maximize its total profit $\Pi = \Pi_1 + \Pi_2$ given in (15) which is a piecewise function with three regions. These regions follow from Lemma 1 and (13), namely $\bar{\theta} \in [0, \frac{\gamma c}{\beta\delta}], (\frac{\gamma c}{\beta\delta}, \frac{(1-\gamma)c}{(1-\beta)\delta}]$, and $(\frac{(1-\gamma)c}{(1-\beta)\delta}, 1]$. We solve for the optimal solution in each of these segments separately and compare against each other which yields the firm's optimal pricing policy leading to the Proposition.

It is straightforward to show that $\Pi$ is jointly strictly concave in $(\bar{\theta}, \theta_1)$ in each of the three regions. The eigenvalues of the Hessian matrix are negative. Note that the firm optimizes $\Pi$ over a convex set of linear constraints shown in (15), namely $g_n \le 0,\ n: 1, 2, 3$ where

$$g_1(\bar{\theta}, \theta_1) = \theta_1 - 1,\ g_2(\bar{\theta}, \theta_1) = \bar{\theta} - \theta_1,\ g_3(\bar{\theta}, \theta_1) = -\bar{\theta}$$

Thus, the Karush Kuhn Tucker (KKT) conditions result in a unique optimal solution for any given parameters $(\delta, \beta, \gamma, c)$ in each region. The KKT solution is given by the following standard equations

$$\begin{aligned} \nabla\Pi(\bar{\theta}, \theta_1) + \sum_{n=1}^{3} \mu_n \nabla g_n(\theta_1, \bar{\theta}) &= 0, \\ g_n(\bar{\theta}, \theta_1) &\le 0, \text{ for all } n: 1, 2, 3, \\ \mu_n g_n(\bar{\theta}, \theta_1) &= 0, \text{ for all } n: 1, 2, 3. \end{aligned}$$

*Proof of Proposition 2* We prove this result in 5 parts.

(i) When $c \le f_1(\delta, \beta, \gamma)$, $\Psi = \Psi^1$ and $\psi = \psi^1$.

(ii) When $f_3(\delta, \beta, \gamma) > c > f_1(\delta, \beta, \gamma)$, $\Psi < \Psi^1$ and $\psi < \psi^1$.

(iii) When $f_4(\delta, \beta, \gamma) > c > f_3(\delta, \beta, \gamma)$, for $\beta > \gamma$, $\Psi > \Psi^1$ and $\psi > \psi^1$.

(iv) When $c \ge f_4(\delta, \beta, \gamma)$, for $\beta > \gamma$, $\Psi = \Psi^1 = 0$ and $\psi = \psi^1 = 0$.

(v) When $c \ge f_3(\delta, \beta, \gamma)$, for $\beta \le \gamma$, $\Psi = \Psi^1 = 0$ and $\psi = \psi^1 = 0$.

Recall that $f_i$ are defined in Table 5, furthermore, part (iii) above corresponds to $(HL, L)$ and $(L, L)$ regions. Parts (iv) and (v) are immediate as no products are sold in period 2 in both models. For $\beta > \gamma$, part (iii) follows because in region $f_4(\delta, \beta, \gamma) > c > \frac{\delta(2-\delta)}{4-3\delta}$, no products are sold in period 2 in a single product model ($\Psi^1 = \psi^1 = 0$) but some products are sold in period 2 in our main model ($\Psi, \psi > 0$), and moreover in region $\frac{\delta(2-\delta)}{4-3\delta} \ge c > f_3(\delta, \beta, \gamma)$, it follows from Propositions 1 (see addendum in the Appendix) and 5 that $\bar{\theta} = \bar{\theta}^1 = \frac{2-\delta}{4-3\delta}$ and $\psi = \frac{1}{2}(\delta\beta\bar{\theta} - \gamma c) > \frac{1}{2}(\delta\bar{\theta} - c) = \psi^1$. In part (ii), when $f_3 > c > f_1$, it is straightforward to show that $\bar{\theta} < \bar{\theta}^1$ and $\psi(\bar{\theta}) < \psi^1(\bar{\theta}^1)$ in equilibrium in this case following Proposition 1 (see addendum in the Appendix) and 5. The total loss $\Psi$ is concave in $\bar{\theta}$ (note that $\Psi^1$ has the same functional form), furthermore $\bar{\theta}$ and $\bar{\theta}^1$ are smaller than $\arg\max_\theta \Psi$ in equilibrium, thus $\bar{\theta} < \bar{\theta}^1$ implies $\Psi(\bar{\theta}) < \Psi^1(\bar{\theta}^1)$. Finally in part (i), only product H is sold in both models and $\bar{\theta} = \bar{\theta}^1$ and $\psi = \psi^1$. $\square$

*Proof of Corollary 1* Note that when $\beta \leq \gamma$, it is never optimal to sell product L in period 2, thus the firm can always imitate the single product outcome by setting a sufficiently high price for Product L in period 1. Therefore, its profit is at least as high as the single-product benchmark. Thus, when its optimal to sell product L, the firm's profit is strictly better than the single-product benchmark, the rest of the result immediately follows from Propositions 1 and 5. $\square$

*Proof of Proposition 3* Recall that $f_i$ are defined in Table 5 at the beginning of this section.

Part (i) is straightforward to prove. Note that when $\beta \leq \gamma$, it is never optimal to sell product L in period 2, thus the firm can always imitate the single product outcome by setting a sufficiently high price for Product L in period 1. Therefore, its profit is at least as high as the single-product benchmark, which proves part (i).

We now move to part (ii). Let us define $\Delta = \Pi^1 - \Pi$, that is, the difference between the profit of the single-product benchmark and the main model. Note that $\Delta$ is continuous in $c$ as both $\Pi^1$ and $\Pi$ are continuous. To complete the proof we will show that $(a)$ $\Delta < 0$ when $c \leq f_3$ and $c \geq f_5$. $(b)$ $\Delta$ is quasiconcave in $c$ for $f_3 < c < f_5$ in equilibrium, and $(c)$ $\max_{c\in(f_3,f_5)} \Delta > 0$ if and only if $\delta < \bar{\delta}$ for some $\bar{\delta} \in (0,1)$, which we will explicitly characterize.

First, we show part $(a)$. Note that $c \geq f_5(\delta,\beta,\gamma)$ corresponds to the $(HL,0)$ region in our main model (see Addendum to Proposition 1 in the Appendix). Furthermore, $f_5(\delta,\beta,\gamma) > \frac{\delta}{2-\delta}$, thus $c \geq f_5(\delta,\beta,\gamma)$ corresponds to the $(H,0)^1$ region in the single-product benchmark (see Proposition 5). It is straightforward to verify that $\Pi > \Pi^1$ in $(HL,0) \cap (H,0)^1$.

For $c \leq f_3(\delta,\beta,\gamma)$, we show that (*1*) a seller with two products (our main model) can imitate the single-product benchmark in period 1 achieving the same profit in that period and (*2*) for a given remaining customer segment in period 2 its profit is always larger than the single product scenario, hence $\Pi > \Pi^1$ in this case. Let us begin with part (*2*): For the same remaining market segment $[0,\bar{\theta})$, $\Pi_2(\bar{\theta}) \geq \Pi_2^1(\bar{\theta})$ as a seller with two products can always replicate the profit of single-product benchmark by setting same price for product H. Now, we show part (*1*). Following Proposition 5 $c \leq f_3(\delta,\beta,\gamma)$ corresponds to $(H,0)^1$ in the single-product benchmark since $f_3(\delta,\beta,\gamma) < \frac{(2-\delta)\delta}{4-3\delta}$ and $\bar{\theta}^1 = \frac{2-\delta}{4-3\delta}$ in this case. A seller with two products that imitates the single-product benchmark in period 1 would achieve the same profit in that period only if its total loss due to inter-temporal cannibalization $\Psi$ is same as the single-product benchmark (see (15)). To complete the proof of part $(a)$, we show that $\Psi(\bar{\theta}^1)=\Psi^1(\bar{\theta}^1)$ in equilibrium. It follows from (13-14) and (C-1-C-2) that $\Psi(\bar{\theta})=\Psi^1(\bar{\theta})$ when $\bar{\theta}\geq\frac{(1-\gamma)c}{(1-\beta)\delta}$. Finally, $\bar{\theta}^1\geq\frac{(1-\gamma)c}{(1-\beta)\delta}$ if and only if $c\leq f_3(\delta,\beta,\gamma)$. Thus, $\Psi(\bar{\theta}^1)=\Psi^1(\bar{\theta}^1)$ for $c\leq f_3(\delta,\beta,\gamma)$ in equilibrium.

Now, we show part $(b)$. We characterize the behavior of $\Delta$ in each region in the following which will be useful for the proof. We do not explicitly state $\Delta$ here as it involves very long expressions in each region. However, following Propositions 1 and 5, it is straightforward to compute $\Delta$ and verify our assertions in this proof. Note that $f_3\leq\frac{(2-\delta)\delta}{4-3\delta}\leq f_4$, $\frac{\delta}{2-\delta}\leq f_5$ where $f_3,f_4$ and $f_5$, are given in Table 5. Furthermore, $f_4>\frac{\delta}{2-\delta}$ if and only if $\frac{\beta}{\gamma}>\frac{4-3\delta}{(2-\delta)^2}$.

(A) $f_3<c<\frac{(2-\delta)\delta}{4-3\delta}$ corresponds to $(HL,L)\cap(H,H)^1$. In this region, $\frac{d\Delta}{dc}<0$ if and only if $\delta>\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. This uses the fact that $\frac{d\Delta}{dc}$ is linear in $c$ without any constants.

(B) $\frac{(2-\delta)\delta}{4-3\delta}\leq c<\min(f_4,\frac{\delta}{2-\delta})$ corresponds to $(HL,L)\cap(H,0_w)^1$. $\Delta$ is concave in $c$ in this region as $\frac{d^2\Delta}{dc^2}<0$. Moreover, $\frac{d\Delta}{dc}<0$ when $\delta>\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. The last point follows because $\frac{d\Delta}{dc}\Big|_{c=\frac{(2-\delta)\delta}{4-3\delta}}$ is decreasing in $\delta$ and it is equal to zero at $\delta=\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$.

(C) $\frac{\delta}{2-\delta}\leq c<f_4$, corresponds to $(HL,L)\cap(H,0)^1$. Note that this region exists if and only if $\frac{\beta}{\gamma}>\frac{4-3\delta}{(2-\delta)^2}$. $\Delta$ monotonically decreases in $c$ in this region. This is because $\frac{d\Delta}{dc}$ is linear in $c$ with a negative coefficient and without any constants.

(D) $f_4\leq c<\frac{\delta}{2-\delta}$, corresponds to $(HL,0_w)\cap(H,0_w)^1$. Note that this region exists if and only if $\frac{\beta}{\gamma}<\frac{4-3\delta}{(2-\delta)^2}$. $\Delta$ is concave in $c$ in this region. This uses the fact that $\frac{d^3\Delta}{d\gamma dc^2}\geq 0$ and when $\gamma=\beta$, $\frac{d^2\Delta}{dc^2}<0$ . Furthermore, $\frac{d\Delta}{dc}<0$ when $\delta>\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. This is because $\frac{d\Delta}{dc}\Big|_{c=f_4}$ decreases in $\delta$ and it is negative when $\delta=\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$.

(E) $\max(f_4,\frac{\delta}{2-\delta})\leq c<f_5$ corresponds to $(HL,0_w)\cap(H,0)^1$. $\Delta$ monotonically decreases in $c$ in this region. This is because $\frac{d^2\Delta}{dc^2}$ is independent of $c$, therefore $\frac{d\Delta}{dc}$ can change its sign only once as $c$ varies, furthermore $\frac{d\Delta}{dc}<0$ at both $c=f_5$ and $c=f_4$.

When $\delta>\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$, $\Delta$ is monotone decreasing in $c$ following (A-E), hence it is quasiconcave and $\Delta<0$ in this case. The last point is due to continuity of $\Delta$ and the fact that $\Delta<0$ at $c=f_3$ following part $(a)$. Later on, we will show that $\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}>\bar{\delta}$.

When $\delta\leq\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$, there are two cases to consider. For $\frac{\beta}{\gamma}<\frac{4-3\delta}{(2-\delta)^2}$,[1] $\Delta$ is increasing in $c\in[f_3,\frac{(2-\delta)\delta}{4-3\delta}]$ corresponding to region $(HL,L)\cap(H,H)^1$; $\Delta$ is concave in $c\in[\frac{(2-\delta)\delta}{4-3\delta},f_4]$ and $c\in[f_4,\frac{\delta}{2-\delta}]$ corresponding to regions $(HL,L)\cap(H,0_w)^1$ and $(HL,0_w)\cap(H,0_w)^1$ respectively, moreover $\frac{d\Delta}{dc}$ is continuous at $c=f_4$; finally, $\Delta$ decreasing in $c$ in $c\in[\frac{\delta}{2-\delta},f_5]$ corresponding to region $(HL,0_w)\cap(H,0)^1$ following (A-E). Thus, $\Delta$ is quasiconcave in $c$ in this case. Similarly, for $\frac{\beta}{\gamma}\geq\frac{4-3\delta}{(2-\delta)^2}$, $\Delta$

[1] Recall that $f_4>\frac{\delta}{2-\delta}$ if and only if $\frac{\beta}{\gamma}>\frac{4-3\delta}{(2-\delta)^2}$.

is increasing in $c \in [f_3, \frac{(2-\delta)\delta}{4-3\delta}]$ corresponding to region $(HL,L)\cap(H,H)^1$; $\Delta$ is concave in $c \in [\frac{(2-\delta)\delta}{4-3\delta}, \frac{\delta}{2-\delta}]$ corresponding to region $(HL,L)\cap(H,0_w)^1$; and, $\Delta$ decreasing in $c$ in $c\in[\frac{\delta}{2-\delta}, f_4]$ and $c\in[f_4,f_5]$ corresponding to regions $(HL,L)\cap(H,0)^1$ and $(HL,0_w)\cap(H,0)^1$ respectively. Thus, $\Delta$ is also quasiconcave in $c$ in this case.

Now, we show part $(c)$. Recall that $\Delta<0$ when $\delta>\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. So, we can restrict our analysis to $\delta \le \frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. We will show that $\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2} > \bar{\delta}$ at the end. Let $c^* = \arg\max_{c\in[f_3,f_5]}\Delta$. We will show that $(I)$ $c^* \in [\frac{(2-\delta)\delta}{4-3\delta}, \min(f_4, \frac{\delta}{2-\delta})]$ corresponding to region $(HL,L)\cap(H,0_w)^1$ when $\gamma \le \frac{1}{2}(5-\sqrt{25-16\beta})$ and $(II)$ $\Delta(c^*)>0$ if and only if $\delta<\bar{\delta}$ for some $0<\bar{\delta}<1$ which we will explicitly characterize. Let us begin with part $(I)$. Note that because of quasiconcavity of $\Delta$ in $c$, $c^*\in[\frac{(2-\delta)\delta}{4-3\delta}, \min(f_4,\frac{\delta}{2-\delta})]$ when the solution of $\frac{d\Delta}{dc}=0$ is in $[\frac{(2-\delta)\delta}{4-3\delta}, \min(f_4,\frac{\delta}{2-\delta})]$. We show that $\frac{d\Delta}{dc}>0$ at the lower end and $\frac{d\Delta}{dc}<0$ at the upper end in $[\frac{(2-\delta)\delta}{4-3\delta}, \min(f_4,\frac{\delta}{2-\delta})]$, hence the result follows. It is straightforward to show that $\frac{d\Delta}{dc}\Big|_{c=\frac{(2-\delta)\delta}{4-3\delta}}>0$ when $\delta\le\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. Similarly, we immediately have $\frac{d\Delta}{dc}\Big|_{c=\frac{\delta}{2-\delta}}<0$. However, showing $\frac{d\Delta}{dc}\Big|_{c=f_4}<0$ is not straightforward. Basically,

$$\frac{d\Delta}{dc}\bigg|_{c=f_4} = G(\delta,\beta,\gamma)H(\delta,\beta,\gamma),$$

where $G(\cdot)$ is always negative and $H(\cdot)$ is quadratic convex in $\delta$ and $H(\cdot)\ge 0$ when

$$X(\gamma) = 16(\beta-\gamma)[(\beta-\gamma)^2-\gamma^2(1-\beta)] - (1-\beta)[\gamma^2+3\gamma-4\beta]^2 \ge 0.$$

This follows from the condition for the solutions of $H(\delta)=0$ to be a real number. So we need to show that $X(\gamma)\ge 0$ to complete the proof of part $(I)$. Note that $\frac{d^3X}{d\gamma^3}\ge 0$, hence $X$ is concave in $\gamma$ up to a point and convex thereafter. Clearly, $X(0)>0$ and $X(\beta)<0$, thus $X(\gamma)=0$ cannot have more than one solution in $\gamma\in[0,\beta]$. Furthermore $X(\frac{1}{2}(5-\sqrt{25-16\beta}))>0$, therefore $X(\gamma)>0$ for $\gamma\le\frac{1}{2}(5-\sqrt{25-16\beta})$.

Here, we show part $(II)$ of part $(b)$. First, we show that $\Delta(c^*)$ is convex in $\delta\in[0,1]$ when $\gamma\le\frac{1}{2}(5-\sqrt{25-16\beta})$. This follows because $\frac{d^3\Delta(c^*)}{d\delta^3}<0$ meaning that $\Delta(c^*)$ is convex in $\delta$ up to some critical $\hat{\delta}$ and concave thereafter where $\hat{\delta}$ solves $\frac{d^2\Delta(c^*)}{d\delta^2}=0$. Furthermore we show that $\hat{\delta}\ge 1$ when $\gamma\le\frac{3-2\beta-\sqrt{9-22\beta+17\beta^2-4\beta^3}}{2-\beta}$. Moreover $\frac{3-2\beta-\sqrt{9-22\beta+17\beta^2-4\beta^3}}{2-\beta}\ge\frac{1}{2}(5-\sqrt{25-16\beta})$ and the result follows. Now, we show the final step of the proof that is $\Delta(c^*)>0$ if and only if $\delta<\bar{\delta}$. Observe that $\Delta(c^*)$ is positive at $\delta=0$ and negative at $\delta=1$. Thus, due to convexity, $\Delta(c^*)=0$ has only one solution $\bar{\delta}$ in $[0,1]$ and $\Delta(c^*)>0$ for $\delta<\bar{\delta}$. Specifically, $\Delta(c^*)=K(\delta,\beta,\gamma)M(\delta,\beta,\gamma)$ where $K(\cdot)$ is always positive and $\bar{\delta}$ solves $M=0$ where

$$M = 2\beta\gamma(\delta(4-3\delta)-\delta^2(3-\gamma)+2\gamma) \;+\; \beta^2(4-\delta(12+\gamma^2)+2\delta^2(4-\gamma+\gamma^2)$$

$$- \delta^3(1-\gamma)^2) - 4\beta^3(1-\gamma)^2 - \gamma^2(4-3\delta). \quad \text{(EC-2)}$$

Finally, we show that $\frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2} > \bar{\delta}$ to complete the proof. This follows as $\Delta(c^*) > 0$ if and only if $\delta < \bar{\delta}$ and $M$ is negative (therefore $\Delta(c^*)$ is negative) at $\delta = \frac{(\beta-\gamma^2)(1-\beta)}{\beta(1-\gamma)^2}$. □

*Proof of Proposition 4* Proof of similar to that of Proposition 1. The only difference is that when $\delta > \beta$, the firm never sells both products in both periods in equilibrium, that is, $(HL, HL)$ is never an equilibrium. We will proof this result at the end. Given this result, for $c \geq f_3$, equilibrium is same as in Proposition 1. For $c < f_3$, we need to compare the profits of $(H, HL)$ and $(HL, L_w)$ strategies. It is straightforward to verify that optimal solution in region $(H, HL)$ is in the interior if and only if $c < f_3$, similarly, optimal solution in region $(HL, L)$ is at the boundary, that is in $(HL, L_w)$ if and only if $c < f_3$. $\Pi_{HL,L_w} - \Pi_{H,HL}$ is concave in $c$, furthermore this difference is positive at $c = f_e$. Therefore, $\Pi_{HL,L_w} > \Pi_{H,HL}$ when $c$ is greater than the smaller root, which is equal to $f_e$. Now, we show that $(HL, HL)$ cannot be an equilibrium when $\delta > \beta$ to complete the proof. We will prove this result by contradiction. Suppose $(HL, HL)$ is the equilibrium region. Because higher valuation customers prefer product H in period 2 to product L in period 1 when $\delta > \beta$, there exists $\theta_1 > \theta_2 > \theta_3 > \theta_4$ such that customers in $[\theta_1, 1]$, $[\theta_2, \theta_1]$, $[\theta_3, \theta_2]$ and $[\theta_4, \theta_3]$ buy product H in period 1, product H in period 2, product L in period 1, and product L in period 2, respectively. $\theta_3$ should be indifferent between buying product L in periods 1 and 2. This implies, $\delta\beta\theta_3 - p_{2L} = \beta\theta_3 - p_{1L}$, thus $\delta\beta\theta_2 - p_{2L} < \beta\theta_2 - p_{1L}$. Similarly, $\theta_2$ should be indifferent between buying product H and L in period 2. So, $\delta\theta_2 - p_{2H} = \delta\beta\theta_2 - p_{2L}$. This would imply $\beta\theta_2 - p_{1L} > \delta\theta_2 - p_{2H}$ which is a contradiction. □

*Proof of Proposition 5* Proof of this proposition is omitted since it is straightforward given the proof of Proposition 1 for the two product scenario. □

*Proof of Proposition 6* The result is obtained by setting $\Psi = 0$ (i.e., loss due to strategic customers) in (15). The proof is similar to the proof of Proposition 1 in essence and it is omitted. □

*Proof of Lemma 2* First, we show that non-purchasing customers have lower types than customer making a purchase in period 2. We seek contradiction. Suppose $\theta'' > \theta'$ and customer $\theta'$ buys a product in period 2 while customer $\theta''$ does not make a purchase. This means utility of customer $\theta'$ from the product is larger than 0. Following condition *(i)* in the Lemma, customer $\theta''$ will get a higher utility from that product, which in turn should be larger than 0, hence buying the product will make customer $\theta''$ better off compared to no-purchase option. Thus, there is a contradiction. Now, we show that customers buying a product in period 1 have higher types than customer buying a product in period 2. Again, we will seek contradiction. Suppose $\theta'' > \theta'$. If customer $\theta''$ buys product L in period 2, and customer $\theta'$ buys product L or H in period 1, the contradiction is obvious following condition *(ii)* in the Lemma. What happens when customer $\theta''$ buys product H in period 2, and customer $\theta'$ buys product L period 1 is not immediately obvious. Let $U_i(\theta, x)$ show the utility of customer $\theta$ from product $x$ in period $i$. Let $\theta^*$ show the customer with the lowest type (marginal customer) who buy product H in period 2, this requires $\theta'' \geq \theta^* \geq \theta'$. By definition $U_2(\theta^*, H) = \max(U_2(\theta^*, L), 0)$. Given that $\theta'$ buys product L in period 1, $U_1(\theta', L) \geq \max(U_2(\theta', L), 0)$ and conditions *(i)* and *(ii)* lead to $U_1(\theta^*, L) > \max(U_2(\theta^*, L), 0)$, but this would imply that $U_2(\theta^*, H) < U_1(\theta^*, L)$, which contradicts the definition of $\theta^*$. □

# Appendix G: Commitments

## G.1. Price Commitment

Suppose that the firm can credibly commit to future prices. Specifically, at the beginning of period 1, the firm commits to period 1 and 2 prices. This implies that the seller will not change its committed prices in period 2 even when the changes would increase its profit. Because now period 2 prices are set upfront, they affect the loss due to inter-temporal cannibalization through their impact on customers' strategic purchasing decisions in period 1 (buy now or wait). In contrast, in our base model (without price commitments) period 2 prices are only concerned with period 2 profit because they are set after customers have already made their strategic choices. In other words, without price commitment period 2 prices maximize only $\Pi_2(\bar{\theta}, p_2)$ as given in Lemma 1, whereas with price commitment period 2 prices maximize $\Pi_2(\bar{\theta}, p_2) - \Psi(\bar{\theta}, p_2)$ as given in the next Lemma (when the firm aims to induce consumers in $[\bar{\theta}, 1]$ to buy in period 1). Let superscript "*pc*" denote price commitment.

LEMMA EC-1. *Suppose the firm can commit to future prices and it wants to induce customers in* $[\bar{\theta}, 1]$ *to buy in period 1 where* $0 \leq \bar{\theta} \leq 1$. *Its optimal period 2 prices are* $p_{2L} = \frac{1}{2}(\delta\beta + \gamma c)$ *and* $p_{2H} = \frac{1}{2}(\delta + c)$ *and the resulting period 2 profit* $\Pi_2^{pc}$ *and the total loss due inter-temporal cannibalization* $\Psi^{pc}$ *are as follows.*

*(i) For* $\beta > \gamma$ *and* $\bar{\theta} > \frac{(1-\gamma)c}{2(1-\beta)\delta} + \frac{1}{2}$, *the firm sells both products H and L in period 2 yielding* $\Pi_2^{pc}(\bar{\theta}) = \frac{(\delta\bar{\theta}-c)^2}{4\delta} + \frac{(\beta-\gamma)^2c^2}{4(1-\beta)\beta\delta} - \frac{\delta(1-\bar{\theta})^2}{4}$ *and* $\Psi^{pc}(\bar{\theta}) = (1-\bar{\theta})(\frac{\delta\bar{\theta}-c}{2} - \frac{\delta(1-\bar{\theta})}{2})$.

*(ii) For* $\beta > \gamma$ *and* $\frac{(1-\gamma)c}{2(1-\beta)\delta} + \frac{1}{2} \geq \bar{\theta} > \frac{\gamma c}{2\beta\delta} + \frac{1}{2}$, *the firm sells only product L in period 2 yielding* $\Pi_2^{pc}(\bar{\theta}) = \frac{(\delta\beta\bar{\theta}-\gamma c)^2}{4\delta\beta} - \frac{\delta\beta(1-\bar{\theta})^2}{4}$ *and* $\Psi^{pc}(\bar{\theta}) = (1-\bar{\theta})(\frac{\delta\beta\bar{\theta}-c}{2} - \frac{\delta\beta(1-\bar{\theta})}{2})$.

*(iii) For* $\beta \leq \gamma$ *and* $\bar{\theta} > \frac{c}{2\delta} + \frac{1}{2}$, *the firm sells only product H in period 2 yielding* $\Pi_2^{pc}(\bar{\theta}) = \frac{(\delta\bar{\theta}-c)^2}{4\delta} - \frac{\delta(1-\bar{\theta})^2}{4}$ *and* $\Psi^{pc}(\bar{\theta}) = (1-\bar{\theta})(\frac{\delta\bar{\theta}-c}{2} - \frac{\delta(1-\bar{\theta})}{2})$.

*(iv) For* $\min(\frac{\gamma c}{2\beta\delta}, \frac{c}{2\delta}) + \frac{1}{2} \geq \bar{\theta}$*, the firm does not sell any products in period 2 yielding* $\Pi_2^{pc} = 0$ *and* $\Psi^{pc} = 0$.

Note that given the optimal prices in the Lemma, there is no demand for product H in part (ii) and no demand for any product in part (iii). It is interesting to compare the firm's period 2 profit and its loss due to strategic customer behavior with our base model in which the firm does not make any commitments (see (13-14) and Lemma 1). This shows that the firm's ability to commit to prices decreases its loss due to inter-temporal cannibalization, and specifically its loss is reduced by $\frac{\delta(1-\bar{\theta})^2}{2}$ or $\frac{\delta\beta(1-\bar{\theta})^2}{2}$ when it sells product H (either by itself or together with product L) or only product L respectively. However, this benefit comes at the expense of the firm's period 2 profit. The firm's price commitments lower its period 2 profit by $\frac{\delta(1-\bar{\theta})^2}{4}$ or $\frac{\delta\beta(1-\bar{\theta})^2}{4}$ when it sells product H or only product L, respectively. In other words, in order to decrease the loss due to strategic customer behavior, the firm sets suboptimal (ex-post) period 2 prices, hence, the need for commitment.

We can now express the firm's optimal pricing problem as its optimal choice of consumer segments in period 1 similar to (15) and solve for the equilibrium where $\Pi_2^{pc}(\bar{\theta})$ and $\Psi^{pc}(\bar{\theta})$ are given in Lemma EC-1. This leads to Proposition 7.

## G.2. Quantity Commitment

Suppose that the firm can credibly convince customers that it will sell no more than $Q_L$ and $Q_H$ units of products L and H in the entire two periods. These commitments dictate period 2 prices and control customers' strategic purchasing decisions (buy now or wait). The following Lemma shows the optimal commitment levels for any desired customer behavior. Let superscript "*qc*" denote quantity commitment.

LEMMA EC-2. *Suppose the firm can make quantity commitments and it wants to induce customers in* $[\bar{\theta}, \theta_1)$ *and* $[\theta_1, 1]$ *to buy products L and H in period 1 respectively where* $0 \leq \bar{\theta} \leq \theta_1 \leq 1$*. The optimal period 2 prices are* $p_{2L} = \delta\beta(1 - Q_L - Q_H)$ *and* $p_{2H} = \delta(1 - \beta Q_L - Q_H) - \delta(1-\beta)(\theta_1 - \bar{\theta})$*. The optimal commitments levels, the resulting period 2 profit* $\Pi^{qc}$ *and the total loss due to inter-temporal cannibalization* $\Psi^{qc}$ *are as follows.*

*(i) For* $\bar{\theta} > \frac{(1-\gamma)c}{2(1-\beta)\delta} + \frac{1}{2}$, $Q_L = \theta_1 - \bar{\theta} + \frac{c(\beta-\gamma)}{2\delta\beta(1-\beta)}$, $Q_H = \frac{1}{2} - (\theta_1 - \bar{\theta}) - \frac{c(1-\gamma)}{2\delta(1-\beta)}$ *and the firm sells both products H and L in period 2 yielding* $\Pi_2^{qc}(\bar{\theta}) = \frac{(\delta\bar{\theta}-c)^2}{4\delta} + \frac{(\beta-\gamma)^2c^2}{4(1-\beta)\beta\delta} - \frac{\delta(1-\bar{\theta})^2}{4}$ *and* $\Psi^{qc}(\bar{\theta}) = (1-\bar{\theta})(\frac{\delta\bar{\theta}-c}{2} - \frac{\delta(1-\bar{\theta})}{2})$.

*(ii) For* $\frac{(1-\gamma)c}{2(1-\beta)\delta} + \frac{1}{2} \geq \bar{\theta} > \frac{\gamma c}{2\beta\delta} + \frac{1}{2}$, $Q_L = \theta_1 - \frac{1}{2} - \frac{c\gamma}{2\delta\beta}$, $Q_H = 1 - \theta_1$ *and the firm sells only product L in period 2 yielding* $\Pi_2^{qc}(\bar{\theta}) = \frac{(\delta\beta\bar{\theta}-\gamma c)^2}{4\delta\beta} - \frac{\delta\beta(1-\bar{\theta})^2}{4}$ *and* $\Psi^{qc}(\bar{\theta}) = (1-\bar{\theta})(\frac{\delta\beta\bar{\theta}-c}{2} - \frac{\delta\beta(1-\bar{\theta})}{2})$.

*(ii) For* $\frac{\gamma c}{2\beta\delta} + \frac{1}{2} \geq \bar{\theta}$, $Q_L = \theta_1 - \bar{\theta}$ *and* $Q_H = 1 - \theta_1$ *and the firm does not sell any products in period 2 yielding* $\Pi_2^{qc} = 0$ *and* $\Psi^{qc} = 0$.

Following Lemmas EC-1 and EC-2, note that quantity and price commitments result in the same period 2 profits and the losses due inter-temporal cannibalization. Thus, the impact of quantity commitment on the loss due to strategic customers and period 2 profit is similar to price commitment: The firm lowers its loss due to strategic customers by committing to a suboptimal profit in period 2. Following Lemma EC-2, it is straightforward to show that quantity commitment results in the same equilibrium outcome as price commitment (stated in Proposition 7).

## G.3. Proofs of Commitment Results

*Proof of Lemma EC-1* We can define the total loss due to inter-temporal cannibalization $\Psi(\bar{\theta}, p_{2L}, p_{2H})$ similar to (13-14). The firm then solves the following problem

$$\begin{aligned} \max_{p_{2H}, p_{2L}} \quad & [(p_{2H} - c)(\bar{\theta} - \theta_{2H}) + (p_{2L} - \gamma c)(\theta_{2H} - \theta_{2L}) - \Psi(\bar{\theta}, p_{2L}, p_{2H})] \\ \text{st.} \quad & \bar{\theta} \geq \theta_{2H} \geq \theta_{2L} \geq 0, \\ & p_{2H}, p_{2L} \geq 0. \end{aligned}$$

Note that $\theta_H$ and $\theta_L$ are defined in (EC-1). $p_{2L} = \frac{1}{2}(\delta\beta + \gamma c)$, and $p_{2H} = \frac{1}{2}(\delta + c)$ solves the above problem. It is straightforward to compute the resulting $\Psi$ and $\Pi_2$ and the Lemma follows. □

*Proof of Lemma EC-2* The optimal $Q_L, Q_H$ should satisfy, $Q_H \geq 1 - \theta_1$ and $Q_L \geq \theta_1 - \bar{\theta}$, otherwise quantities cannot meet period 1 demand. The firm always sell its committed quantities otherwise it could commit to smaller quantities and reduce costs. Thus, period 2 prices are set to clear the remaining inventory. Specifically,

$$p_{2L} = \delta\beta(1 - Q_L - Q_H) \text{ and } p_{2H} = \delta\beta(1 - \beta Q_L - Q_H) - \delta(1 - \beta)(\theta_1 - \bar{\theta}). \tag{EC-3}$$

We can define the total loss due to inter-temporal cannibalization $\Psi(\bar{\theta}, Q_L, Q_H)$ as in (13-14) plugging in (EC-3) for $p_{2L}$ and $p_{2H}$. The firm then solves the following problem similar to (15).

$$\begin{aligned} \max_{Q_H, Q_L} \quad & [(Q_H - (1 - \theta_1))\,[\delta\beta(1 - \beta Q_L - Q_H) - \delta(1 - \beta)(\theta_1 - \bar{\theta})] \\ & \quad + (Q_L - (\theta_1 - \bar{\theta}))[\delta\beta(1 - Q_L - Q_H)] - \Psi(\bar{\theta}, Q_L, Q_H)] \\ \text{st.} \quad & Q_H \geq 1 - \theta_1, Q_L \geq \theta_1 - \bar{\theta}. \end{aligned}$$

The solution of this problem gives the results stated in the Lemma. □

*Proof of Proposition 7* The firm solves (15) in which $\Psi^{pc}$ and $\Pi_2^{pc}$ are now given by Lemma EC-1. The rest of the proof follows the same steps as those of the proof of Proposition 1. Equilibrium prices are as follows: $p_{1L} = \frac{1}{2}(\beta + \gamma c)$, $p_{2L} = \frac{1}{2}(\delta\beta + \gamma c)$, $p_{1H} = \frac{1}{2}(1 + c)$, $p_{2H} = \frac{1}{2}(\delta + c)$. Furthermore, $\bar{\theta} = \min(\frac{c\gamma}{2\beta}, \frac{c}{2}) + \frac{1}{2}$. □

*Proof of Corollary 2* We obtain the profit of single-product benchmarks with quantity and price commitments $\Pi^{qc,1}$ and $\Pi^{pc,1}$ by setting $\gamma = \beta = 1$ in $\Pi^{qc}$ and $\Pi^{pc}$. Comparisons of $\Pi^{qc,1}$ vs. $\Pi^{qc}$ and $\Pi^{pc,1}$ vs. $\Pi^{pc}$ yield the result of the Corollary. □

*Proof of Proposition 8* We show the result for $\Pi^{pch}$, the analysis is similar for $\Pi^{qch}$. We will show that $\Pi^{pch} = \Pi$ for $c > f_3$. Note that $\Pi^1 > \Pi$ only if $c > f_3$, thus the result follows. In the following, we show that the firm never sells product H in period 2 when it can commit to future prices of product H, hence $\Pi^{pch} = \Pi$ for $c > f_3$. We use backward induction. The optimal period 2 price for product L, $p_{2L}$, for given $\bar{\theta}$ and committed $p_{2H}$ is $p_{2L} = \beta p_{2H} - \frac{(\beta-\gamma)c}{2}$. One can define the total loss due to inter-temporal cannibalization $\Psi(\bar{\theta}, p_{2H})$ similar to (13-14). Suppose the firm aims to induce customers $[\bar{\theta}, 1]$ to buy in period 1, following our discussion in Section 7.3 optimal commitment price $p_{2H}$ maximizes $\Pi_2 - \Psi$. We find that $\arg\max_{p_{2H}}[\Pi_2(\bar{\theta}, p_{2H}) - \Psi(\bar{\theta}, p_{2H})] = \frac{c+\delta}{2}$. Furthermore product H is offered in period 2 only if

$$\bar{\theta} \geq \frac{1}{2} + \frac{(1-\gamma)c}{2(1-\beta)\delta}. \tag{EC-4}$$

The firm solves

$$\max_{0 \leq \bar{\theta} \leq \theta_1 \leq 1} [(1-\theta_1)(\theta_1 - c) + (\theta_1 - \bar{\theta})(\beta\bar{\theta} - \gamma c) - \beta(1-\theta_1)(\theta_1 - \bar{\theta}) - \Psi^{pch}(\bar{\theta}) + \Pi_2^{pch}(\bar{\theta})]. \tag{EC-5}$$

No solution of (EC-5) satisfies (EC-4), hence product H is never offered in period 2. □

## Appendix H: Loss due to Forward-Looking Strategic Behavior

When the firm ignores forward-looking strategic behavior in its pricing and product selection, the analysis is as follows. The firm chooses period 1 prices as in Proposition 6 assuming myopic consumers. However, consumers act strategically taking into account their conjectured pay-offs from buying a product in period 2. The marginal consumer $\bar{\theta}$ is then given by (11). Finally, in period 2, there is no room for strategic waiting as it is the last period and its equilibrium is characterized by Lemma 1.

When the firm ignores forward-looking strategic behavior only in its product selection (but not in its pricing), the analysis is as follows. Because the firm ignores forward-looking behavior in its product selection, it always offers both products H and L when $\beta > \gamma$ and only product L when $\beta \leq \gamma$ as shown in Table 1. Proposition 1 describes the equilibrium when the firm's product portfolio includes both products H and L. Proposition 5 states the equilibrium when the firm's portfolio includes only one of products H or L.