Online Appendix

Appendix F: When the Retailers Can Decide Whether to Adopt QR

Here we describe what happens when the retailers can simultaneously determine whether to adopt QR. Let a and b be retailer 1 and 2's QR decision, a, $b = F$, S. Then there are three possible scenarios for equilibrium outcome: FF, FS and SS. A scenario is an equilibrium if none of the retailers is better off by deviating to anther decision (changing its decision from F to S or S to F). Using the SPNE derived in section 4, we compare the retailers' profits across scenarios, and obtain the following result:

PROPOSITION 15. *When the retailers choose whether to adopt QR simultaneously, the equilibrium choices* (a,b) *is*

$$(a,b) = \begin{cases} (S,S) & for\, v \leq v_1^F \\ (S,S)\, or\, (F,F) & for\, v_1^F < v \leq v_1^S \\ (F,F) & for\, v_1^S < v \end{cases}.$$

$v_1^S > v_1^F$, *and they are given in Table 2.*

Figure 4 describes the equilibrium region given in Proposition 15. As the figure shows, both of the retailers choose not to have QR ability when demand variability is too low, and both adopt QR when demand variability is too high. Nevertheless, both the SS and FF scenarios can be equilibria when the demand variability is moderate.

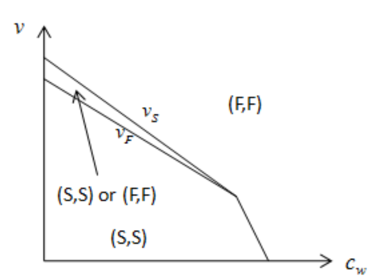

Figure 4 Retailers' Equilibrium QR Adoption Decisions

Appendix G: Proofs

Proof of Lemma 1

The retailers' expected profits are

$$\pi_1 = \mathbb{E}[(A - Q_1 - q_1 - Q_2)(Q_1 + q_1) - c_q q_1] - c_w Q_1,$$

$$\pi_2 = \mathbb{E}[(A - Q_1 - q_1 - Q_2)Q_2] - c_w Q_2,$$

where q_1 is given by (4). It can be shown $\frac{\partial^2 \pi_i}{\partial Q_i^2} < 0$. Let q^H and q^L denote the fast retailer's QR order quantities in a high and low market, respectively. Then solving the first order conditions $\frac{\partial \pi_i}{\partial Q_i} = 0$, for $i = 1, 2$, yields the following initial order quantities:

a. For $\bar{\theta}^{FS} \le c_q$: $Q_1 = Q_2 = \frac{m - c_w}{3}$, and $q^H = q^L = 0$.

b. For $\underline{\theta}^{FS} \le c_q < \bar{\theta}^{FS}$:

$$(Q_1, Q_2) = \begin{cases} (\frac{3m-5v-8c_w+5c_q}{10}, \frac{2(m-c_w)}{5}) & , \text{ for } c_w \le \alpha_1 \\ (0, \frac{3m-v-4c_w+c_q}{6}) & , \text{ for } \alpha_1 < c_w \le \alpha_2 \text{ , and } q^H > 0 \text{ while } q^L = 0 \\ (0,0) & , \text{ for } \alpha_2 < c_w \end{cases}$$

c. For $c_q < \underline{\theta}^{FS}$:

$$(Q_1, Q_2) = (0, \frac{m + c_q}{2} - c_w), \; q^H \ge 0 \text{ and } q^L \ge 0.$$

$\alpha_1 = \frac{3m-5v+5c_q}{8}$ and $\alpha_2 = \frac{3m-v+c_q}{4}$ correspond to the thresholds such that $Q_1 = 0$ and $Q_2 = 0$ for $\underline{\theta}^{FS} \le c_q < \bar{\theta}^{FS}$, while $\bar{\theta}^{FS}$ and $\underline{\theta}^{FS}$ correspond to the thresholds such that $q_1 = 0$ in a high market and $q_1 = 0$ in a low market. □

Proof of Proposition 1

The manufacturer solves the following problem to maximize its profit:

$$\max_{c_q} \mathbb{E}[\pi_M] = \mathbb{E}[(c_q - \delta)q_1] + c_w(Q_1 + Q_2),$$

where q_1 is given by (4) and Q_i, $i = 1, 2$, is characterized in Lemma 1. It can be shown that $\mathbb{E}[\pi_M]$ is a piecewise concave function: it is continuous and concave in c_q for $c_q > \underline{\theta}^{FS}$ and $c_q < \underline{\theta}^{FS}$ respectively, but is discontinuous at $c_q = \underline{\theta}^{FS}$, because in equilibrium $Q_1 = 0$ for $c_q \le \underline{\theta}^{FS}$ and $Q_1 > 0$ otherwise. In other words, the discontinuity is due to the fast retailer's change in behavior: it places an initial order only when the QR price is sufficiently high, but it does not place any initial order when the QR price is too low. Since $\mathbb{E}[\pi_M]$ is concave in c_q

for $c_q > \underline{\theta}^{FS}$, we obtain the optimal QR price by applying the first order conditions, and the threshold β^{FS} is given by the solution to $Q_1 = 0$ in this case. Furthermore, following Lemma 1, this optimal price is feasible only for $c_q < \bar{\theta}^{FS}$ which translates to $\delta < v$. Otherwise, demand uncertainty is too low and QR is never used. Similarly, for $c_q \leq \underline{\theta}^{FS}$ we derive the optimal QR price for this case by applying the first order conditions. Comparing the manufacturer's profit for $c_q > \underline{\theta}^{FS}$ and $c_q \leq \underline{\theta}^{FS}$ with the optimal QR price for each of these cases reveals the manufacturer is always better off by using the optimal c_q for $c_q > \underline{\theta}^{FS}$. That is, the manufacturer induces the fast retailer to place a QR order only in a high market. □

Proof of Lemma 2

Retailer i maximizes its expected profit

$$\pi_i = \mathbb{E}[(A - Q_i - q_i - Q_j - q_j)(Q_i + q_i) - c_w Q_i - c_q q_i],$$

where q_i and q_j are given by (5). It can be shown $\frac{\partial^2 \pi_i}{\partial Q_i^2} < 0$. Let q_i^H and q_i^L be retailer i's QR order quantities in a high and low market respectively. Then the equilibrium order quantities can be obtained by solving $\frac{\partial \pi_i}{\partial Q_i} = 0$, for $i = 1, 2$, leading to the following results:

(i) For $\bar{\theta}^{FF} \leq c_q$: $Q_1 = Q_2 = \frac{m - c_w}{3}$ and $q_i^H = q_i^L = 0$.

(ii) For $\underline{\theta}^{FF} \leq c_q < \bar{\theta}^{FF}$:

$$Q_1 = Q_2 = \begin{cases} \frac{m - v - 2c_w + c_q}{3} & , \text{ for } c_w < \frac{c_q + m - v}{2} \\ 0 & , \text{ otherwise} \end{cases}, \text{ and } q_i^H > 0 \text{ while } q_i^L = 0.$$

(iii) For $c_q < \bar{\theta}^{FF}$: $Q_1 = Q_2 = 0$, $q_i^H > 0$ and $q_i^L > 0$.

The threshold $\bar{\theta}^{FF}$ is derived from the condition $q^H = 0$ for the cases (i) and (ii), and $\underline{\theta}^{FF}$ is derived from the condition $q^L = 0$ for the cases (ii) and (iii). □

Proof of Proposition 2

The procedure of this proof essentially follows that of Proposition 1. The manufacturer solves the following problem to maximize its profit:

$$\max_{c_q} \mathbb{E}[\pi_M] = \mathbb{E}[(c_q - \delta)(q_1 + q_2) + c_w(Q_1 + Q_2)],$$

where q_1 is given by (5) and Q_i, $i = 1, 2$, is characterized in Lemma 2. It can be shown that $\mathbb{E}[\pi_M]$ is piecewise concave in c_q but discontinuous at $c_q = \underline{\theta}^{FF}$ because the wholesale price is sufficiently small and the retailers do not place any QR order for $c_q \leq \underline{\theta}^{FF}$. Since $\mathbb{E}[\pi_M]$ is concave in c_q for $c_q > \underline{\theta}^{FF}$, we solve $max_{c_q > \underline{\theta}^{FF}} \mathbb{E}[\pi_M]$ by applying the first order conditions, which leads to the optimal c_q in which $Q_1 = Q_2 \geq 0$, $q_i^H \geq 0$ and $q_i^L = 0$. Similarly, we obtain the optimal c_q for $c_q \leq \underline{\theta}^{FF}$ using the first order conditions, leading to another optimal c_q in which $Q_1 = Q_2 = 0$, $q_i^H > 0$ and $q_i^L > 0$. Finally, comparing the manufacturer's profits between these two cases reveals the boundary β^{FF}. $\square$

Proof of Proposition 3

Parts (i) and (ii) of this proposition are straightforward by showing that $\Pi_M^F \geq \Pi_M^S$ and $\Pi_R^F \geq \Pi_R^S$. In addition, combining (i) and (ii) leads to (iii) of this proposition. $\square$

Proof of Proposition 4

The results are straightforward from comparing the manufacturer's expected profitΠ_M across the scenarios, and v_M is derived by solving $\Pi_M^{FS} = \Pi_M^{FF}$. $\square$

Proofs of Propositions 5 and 6:

The results are derived by comparing each of the retailer's expected profit across the scenarios. $\square$

Proof of Proposition 7

The results are derived by comparing the expected total channel profit across the scenarios. $\square$

Proof of Lemma 3

The optimal wholesale price is given by the solution to the following problem

$$max_{c_w} \mathbb{E}[\pi_M] \tag{8}$$

for SS, FS and FF scenarios. It can be shown that $\mathbb{E}[\pi_M]$ is concave in c_w in each of these scenarios, and the optimal wholesale price $c_w = \frac{m}{2}$ can be derived by solving the first order conditions. $\square$

Proof of Proposition 8

First, we obtain the firms' expected profits using the wholesale price $c_w = \frac{m}{2}$ given in Lemma 3. Then part (i) of the proposition appears straightforward in comparing Π_M^{SS}, Π_M^{FS} and Π_M^{FF}. Similarly, parts (ii) and (iii) of the proposition are straightforward from comparing the expected profits of a retailer and the entire channel respectively across the scenarios. $\square$

Proof of Proposition 9

We provide the proof for the model $E1$. We first consider the FS scenario and next the FF scenario. In each scenario, following backward induction, we first derive the manufacturer's choice of c_q which is described by Lemma 5, followed by the retailers' equilibrium regular order decisions which are given by Lemma 6. The results for the model $E2$ can be derived following the same procedure, which yields the same results as $E1$.

In the last stage game in the FS scenario for the model $E1$, the fast retailer determines its QR order quantity as given in (4). Using this QR order quantity, in the second stage game, the manufacturer determines its QR price to maximize its expected profit $\mathbb{E}[\pi_M]$, which is piecewise concave in c_q. The manufacturer's optimization of QR price leads to the following pricing scheme:

LEMMA 5. *The optimal QR price for the manufacturer in the FS scenario for model $E1$ is given below:*

(1) For $0 \le Q_1 \le \sigma_1$: $c_q = \frac{m-Q_2+\delta}{2} - Q_1$, and the fast retailer places a QR order for both high and low market outcomes;

(2) For $\sigma_1 < Q_1 < \sigma_2$: $c_q = \frac{m-Q_2+v+\delta}{2} - Q_1$, and the fast retailer places a QR order only in a high market;

(3) For $\sigma_2 \le Q_1$: $c_q = \delta$, and the fast retailer does not place a QR order for any market outcome;

where $\sigma_1 = \frac{m-(1+\sqrt{2})v-\delta-Q_2}{2}$ and $\sigma_2 = \frac{m+v-\delta-Q_2}{2}$.

Next, in the first stage game, each of the retailers places an initial order to maximize its expected profit $\mathbb{E}[\pi_i]$, which is piecewise concave in Q_i as c_q is discontinuous on $Q_1 = \sigma_1$. Observe that the equilibrium initial order quantities must satisfy one of the cases stated in Lemma 5, and $\mathbb{E}[\pi_i]$ is concave in Q_i for each of the cases in that lemma. Therefore, we apply the first order conditions to derive the expressions for equilibrium order quantities (if it exists). Nevertheless, we need to verify that no retailer has incentive to deviate from these quantities so that they are equilibrium. This procedure leads to the following results:

LEMMA 6. *There exists a unique equilibrium for the FS scenario in model $E1$:*

(1) For $v \le \delta - c_w$, $Q_i = \frac{m-c_w}{3}$ for $i = 1, 2$. The fast retailer does not place a QR order for any market outcome.

(2) For $\delta - c_w < v$, $Q_1 = (\frac{7m-8c_w-v+\delta}{22})^+$ and $Q_2 = (\frac{4(7m-8c_w-v+\delta)}{77})^+$. The fast retailer places a QR order only in a high market.

proof: We derive cases (1) and (2) in this lemma as follows. Case (1) concerns an equilibrium in which $q_1 = 0$ in all market outcomes, corresponding to case (1) of Lemma 5, and solving the first order conditions yields $Q_i = (m - c_w)/3$ for $i = 1, 2$. Since $\mathbb{E}[\pi_i]$ is piecewise concave in Q_i, the first order condition only provides a necessary condition for an equilibrium; we also need to confirm that no retailer has incentive for unilateral deviation. For the quantities derived in this case, it suffices to ensure that the fast retailer has no incentive to place a QR order in a high market even when $c_q = \delta$, i.e.,

$$\frac{d\pi_1}{dq_1}|_{Q_1=Q_2=\frac{m-c_w}{3},q_1=0,A=m+v} \leq 0,$$

which implies $v \leq \delta - c_w$.

Case (2) concerns an equilibrium in which QR is used only in a high market, corresponding to case (2) of Lemma 5. Solving the first order condition yields (Q_1,Q_2)=$((\frac{7m-8c_w-v+\delta}{22})^+,(\frac{4(7m-8c_w-v+\delta)}{77})^+)$. Moreover, $q_1^L < 0$ implies $v > \delta - c_w$.

Now we have to ensure no retailer has incentive to deviate. For the fast retailer, deviation such that $Q_1 \geq \sigma_2$ is unattractive, because $\mathbb{E}[\pi_1]$ is concave in Q_1 for $Q_1 \geq \sigma_2$ and

$$\frac{d\mathbb{E}[\pi_1]}{dQ_1}|_{Q_1=\sigma_2,Q_2=\frac{4(7m-8c_w-v+\delta)}{77}} \leq 0.$$

Now consider retailer 1's deviation so that $Q_1 \leq \sigma_1$. Since $\mathbb{E}[\pi_1]$ is concave in Q_1 for $Q_1 \leq \sigma_1$,

$$\frac{d\mathbb{E}[\pi_1]}{dQ_1}|_{Q_1=\sigma_1 \geq 0},$$

and deviating to $Q_1 = \sigma_1$ is unattractive, we conclude that retailer 1 has no incentive to deviate to $Q_1 \leq \sigma_1$. Applying similar analysis reveals that the slow retailer has no incentive to deviate either.

Similar analysis can be applied to examine what happens when QR is used in both low and high markets, i.e., corresponding to case (3) of Lemma 5. This analysis reveals that $c_q = \frac{2c_w+\delta}{3}$ in equilibrium, implying that $c_q < c_w$ for $c_w > \delta$. Moreover, $q_1^L > 0$ implies $c_w > \frac{3v}{4} + \delta$, and therefore assuming $c_w \leq \delta$ eliminates an equilibrium in which QR is used in both of the market outcomes. □

Now consider the FF scenario. We apply the same procedure described above to derive the SPNE for this scenario. In the last stage game, the retailers determine their QR order quantities as given in (5). Next in the second stage game, the manufacturer determines c_q to maximize its expected profit $\mathbb{E}[\pi_M]$. Using the QR order quantities described in (5), the manufacturer's expected profit $\mathbb{E}[\pi_M]$ is again piecewise concave in c_q, and the manufacturer's optimization problem leads to the following result:

LEMMA 7. *The optimal QR price for the manufacturer in the FF scenario for model $E1$ is given below:*

(a) For $min(\sigma_4,\sigma_5,\sigma_7)\le Q_1\le min(\sigma_3,\sigma_6)$*:* $c_q=\frac{2m-3(Q_1+Q_2)+2(v+\delta)}{4}$*, which yields* $q_1^H>0$, $q_2^H>0$, $q_1^L=0$, $q_2^L=0$*;*

(b) For $min(\sigma_3,\sigma_{15})\le Q_1$ *and* $\frac{m-(1+\sqrt{2})v-\delta}{3}\le Q_2\le\frac{m+v-\delta}{3}$*:* $c_q=\frac{m-3Q_2+v+\delta}{2}$*, which yields* $q_1^H=0$, $q_2^H>0$, $q_1^L=0$, $q_2^L=0$*;*

(c) For $\sigma_8\le Q_1\le min(\sigma_5,\sigma_{10},\sigma_{11})$*:* $c_q=\frac{7m-12Q_1-9Q_2+v+7\delta}{14}$*, which yields* $q_1^H>0$, $q_2^H>0$, $q_1^L>0$, $q_2^L=0$*;*

(d) For $min(\sigma_9,\sigma_{10})\le Q_1\le min(\sigma_4,\sigma_{16})$*:* $c_q=\frac{m-2Q_1-Q_2+v+\delta}{2}$*, which yields* $q_1^H>0$, $q_2^H=0$, $q_1^L=0$, $q_2^L=0$*;*

(e) For $\sigma_{11}\le Q_1\le min(\sigma_7,\sigma_{12})$*:* $c_q=\frac{2m-3Q_1-3Q_2+2\delta}{4}$*, which yields* $q_1^H>0$, $q_2^H>0$, $q_1^L>0$, $q_2^L>0$*;*

(f) For $Q_1\le min(\sigma_8,\sigma_9)$*:* $c_q=\frac{m-2Q_1-Q_2+\delta}{2}$*, which yields* $q_1^H>0$, $q_2^H=0$, $q_1^L>0$, $q_2^L=0$*;*

(g) For $\sigma_{13}\le Q_1$ *and* $Q_2\le\frac{m-(1+\sqrt{2})v-\delta}{3}$*:* $c_q=\frac{m-3Q_2+\delta}{2}$*, which yields* $q_1^H=0$, $q_2^H>0$, $q_1^L=0$, $q_2^L>0$*;*

(h) For $min(\sigma_6,\sigma_{12})\le Q_1\le min(\sigma_{13},\sigma_{15})$*:* $c_q=\frac{3m-3Q_1-6Q_2+v+3\delta}{6}$*, which yields* $q_1^H>0$, $q_2^H>0$, $q_1^L=0$, $q_2^L>0$*;*

(i) For $\sigma_{16}\le Q_1$ *and* $Q_2\ge\frac{m+v-\delta}{3}$*:* $c_q=\delta$*, which yields* $q_1^H=0$, $q_2^H=0$, $q_1^L=0$, $q_2^L=0$*;*
where σ_3 *to* σ_{16} *are given in Table 3.*

$\sigma_3=(2m-3Q_2+2v-\sqrt{2}(m-3Q_2+v-\delta)-2\delta)/3$
$\sigma_4=Q_2-(m-3Q_2+v-\delta)/\sqrt{3}$
$\sigma_5=(14m-15Q_2-10v-\sqrt{7}(m-3Q_2+7v-\delta)-14\delta)/27$
$\sigma_6=Q_2+(4v+\sqrt{6}(-m+3Q_2+v+\delta))/3$
$\sigma_7=(2m-3Q_2-2(v+\sqrt{2}v+\delta))/3$
$\sigma_8=Q_2-v/2-\sqrt{7/6}(m-3Q_2+v-\delta)/2$
$\sigma_9=(m-Q_2-v-\sqrt{2}v-\delta)/2$
$\sigma_{10}=(21m-33Q_2-15v-\sqrt{21}(-m+3Q_2+5v+\delta)-21\delta)/30$
$\sigma_{11}=Q_2+(4v+\sqrt{14}(-m+3Q_2+v+\delta))/6$
$\sigma_{12}=(3m-3Q_2-v-\sqrt{3}(m-3Q_2+v-\delta)-3\delta)/6$
$\sigma_{13}=(3m-6Q_2+v+\sqrt{6}(-m+3Q_2+\delta)-3\delta)/3$
$\sigma_{14}=(m-(1+\sqrt{2})v-\delta)/3$
$\sigma_{15}=m+(-6Q_2+v-\sqrt{3}(m-3Q_2+v-\delta)-3\delta)/3$
$\sigma_{16}=(m-Q_2+v-\delta)/2$

Table 3 Threshold Values for c_q in the FF scenario of the model $E1$

Figure 5 depicts the regions described in Lemma 7 for $m=1$, $v=0.7$, $c_w=0.5$, $\delta=0.5$. In

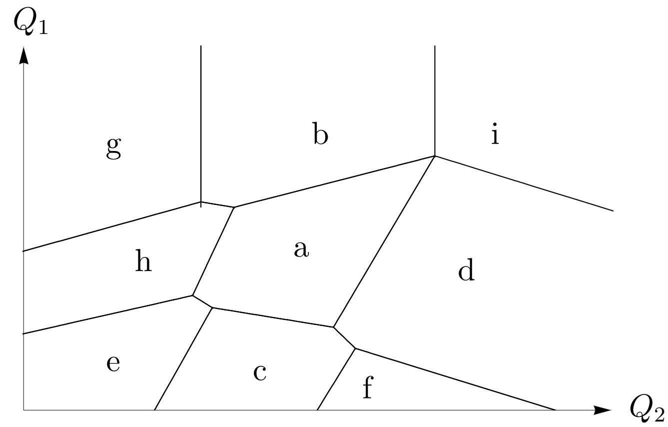

Figure 5 **The Regions Characterized in Lemma 7 for** $m=1$, $v=0.7$, $c_w=0.5$, $\delta=0.5$

Note: Some regions may not exist, depending on m, v, c_w and δ.

the first stage game, the retailers determine their initial order quantities to maximize their expected profits. Similar to the FS scenario, the manufacturer's chosen c_q described in Lemma 7 is discontinuous on some of the boundaries due to piecewise concavity of $\mathbb{E}[\pi_M]$. As a result, a retailer's expected profit $\mathbb{E}[\pi_i]$ is piecewise concave in Q_i, and discontinuity occurs on some of the boundaries given in Table 3. Nevertheless, $\mathbb{E}[\pi_i]$ is concave in each of the cases (a) to (i) described in Lemma 7. Since an equilibrium must satisfy one of these cases, we can apply the first order conditions to derive the order quantities for an equilibrium. Then we check for retailers' incentive for deviation to characterize an equilibrium. This process leads to the following symmetric result, i.e., $Q_1=Q_2$:

LEMMA 8. *There exists a unique equilibrium for the FF scenario in model $E1$ only for $v\le\epsilon_1$ and $v\ge\epsilon_2$, and there does not exist a pure-strategy equilibrium otherwise. The unique equilibrium is given below:*

(1) For $v\le\epsilon_1$, $Q_i=\frac{m-c_w}{3}$ for $i=1,2$. The retailers do not place a QR order for any market outcome.

(2) For $v\ge\epsilon_2$, $Q_i=(\frac{19m-24c_w-5v+5\delta}{60})^+$ for $i=1,2$. Each retailer places a QR order only in a high market,

where $\epsilon_1=\delta-c_w$ and $\epsilon_2=\frac{13m-168c_w+155\delta}{155}$.

proof: We derive cases (1) and (2) in this Lemma as follows. Case (1) concerns an equilibrium in which $q_i=0$ in all market outcomes, corresponding to case (i) of Lemma 7. Solving the first order conditions yields $Q_i=\frac{m-c_w}{3}$ for $i=1,2$. This quantity is an equilibrium only if $q_i^H\le 0$, which implies $v\le\delta-c_w$.

Case (2) concerns an equilibrium in which QR is used only in a high market, corresponding to case (a) of Lemma 7. Solving the first order condition yields $Q_i = (\frac{19m-24c_w-5v+5\delta}{60})^+$. This is an equilibrium only if no retailer has incentive to deviate, and it can be shown that deviation is attractive for $v < \frac{13m-168c_w+155\delta}{155}$. In that case, a retailer has incentive to deviate by purchasing more initially but not using QR at all.

Finally, applying the analysis described above reveals that there does not exist an equilibrium (asymmetric) corresponding to the other cases described in Lemma 7. Therefore cases (1) and (2) characterize the unique equilibrium for $v \le \delta - c_w$ and $v \ge \frac{13m-168c_w+155\delta}{155}$, and there is no pure-strategy equilibrium otherwise. $\square$

Note that there does not exist an equilibrium for the FF scenario for $\delta - c_w < v < \frac{13m-168c_w+155\delta}{155}$. This happens because the retailers' profit functions are piecewise concave in their regular order quantities, leading to multiple local maxima and hence the discontinuity of their best response functions. Finally, Proposition 9 for the model $E1$ proceeds by combining Lemmas 6 and 8. $\square$

Proof of Proposition 10

The proof of this proposition involves two parts: (1) obtaining the SPNE of each scenario, and (2) comparing profits across scenarios. We illustrate the derivation and the results of the first part; the latter part is straightforward after the first part is obtained.

Basically, the derivation of SPNE follows the steps shown in sections 4.2 and 4.3. The key difference is driven by the introduction of the QR capacity limit k, which results in additional cases to be analyzed in each stage game.

For the FS scenario, using the first order conditions we derive the fast retailer's QR order quantity:

$$q_1 = min((\frac{A-c_q-2Q_1-Q_2}{2})^+, k).$$

Next we proceed to solve for the retailers' equilibrium regular order quantities with this QR ordering policy. This yields a result similar to Lemma 1 with one additional case: For $v \ge k$ and $min(c_w+k-v, \frac{2c_w+k+2m-4v}{4}) < c_q \le min(c_w-k+v, \frac{2c_w-7k+2m+4v}{4})$, the fast retailer orders $q_1^H = k$ and $q_1^L = 0$. That is, when the demand variability is large enough and c_q is not overly high, the QR capacity is fully used in a high market. It can also be shown that in equilibrium $Q_2 > Q_1$, and $Q_1 > 0$ implies $c_w > m - \frac{5k}{3}$. Next we derive the manufacturer's optimal QR price for $Q_1 > 0$, which yields

$$c_q = \begin{cases} c_w + v & for\, v \leq \delta,\, and\; q_1^H = q_1^L = 0, \\ \frac{2c_w+v+\delta}{2} & for\, \delta < v < 2k+\delta,\, and\; 0 < q_1^H < k,\; q_1^L = 0, \\ c_w + v - k & for\, 2k+\delta \geq v,\, and\; q_1^H = k,\; q_1^L = 0. \end{cases}$$

For the FF scenario, first we solve for the retailers' equilibrium QR order quantities. Without loss of generality, we assume that $Q_1 \geq Q_2$. Recall that we assume that when the retailers' total QR order quantity exceeds the manufacturer's QR capacity, the manufacturer allocates its capacity evenly between the retailers. This complicates the analysis and the equilibrium is characterized in seven regions. Using this result, next we derive the retailers' equilibrium regular order quantities. This yields a result similar to Lemma 2 with one additional case: For $min(c_w + \frac{3k}{2} - v, m - v) < c_q \leq min(c_w + \frac{3k}{2} + v, m + v - 3k)$, the retailers order $q_i^H = k$ and $q_i^L = 0$. This case is relevant only for $v \geq \frac{3k}{2}$, and $Q_i > 0$ implies $c_w < m - \frac{3k}{2}$. Recall that $Q_1 > 0$ in the FS scenario requires that $c_w < m - \frac{5k}{3}$, and therefore $Q_i > 0$ for both of the FS and FF scenarios requires that $c_w < m - \frac{5k}{3}$. Knowing the retailers' ordering policies, finally we study the manufacturer's QR pricing decision for $c_w < m - \frac{5k}{3}$, which yields:

$$c_q = \begin{cases} c_w + v & for\, v \leq \delta,\, and\, q_i^H = q_i^L = 0, \\ \frac{2c_w+v+\delta}{2} & for\, \delta < v < \frac{3}{2}k+\delta,\, and\, 0 < q_i^H < k,\; q_i^L = 0, \\ c_w + v - \frac{3}{2}k & for\, \frac{3}{2}k+\delta \leq v,\, and\, q_i^H = k,\; q_i^L = 0. \end{cases}$$

The above results implies that the QR capacity is fully utilized in both FF and FS scenarios only for $v \geq max(2k+\delta, \frac{3k}{2}+\delta)$. Also note we assume $v < m$, and hence $m > max(2k+\delta, \frac{3k}{2}+\delta)$ is the necessary condition for QR to be fully used, which implies $k < \frac{m-\delta}{6}$. Finally, we obtain the firms' equilibrium profits with the above results, and comparing these profits across the scenarios yields the results described in this proposition. □

Proof of Proposition 11

The results are derived by comparing the manufacturer's expected profit across the scenarios. □

Proof of Propositions 12

The results are derived by comparing each of the retailer's expected profit across the scenarios. □

Proof of Proposition 13

The results are derived by comparing the channel's total expected profit across the scenarios. □

Proof of Lemma 4

In this case, the retailer's profit is given by

$$\pi_R = (A - Q - q)(Q + q) - c_q q - c_w Q,$$

where Q and q are the initial and QR order quantities respectively. It is straightforward that π_R is concave in q, and the retailer's optimal QR order quantity is given by

$$q = (\frac{A - c_q - 2Q}{2})^+.$$

Given the QR ordering policy, the retailer determines its initial order Q to maximize its expected profit $\mathbb{E}[\pi_R]$. Simple algebra reveals that $\frac{\partial \mathbb{E}[\pi_R]}{\partial Q} \leq 0$, and applying the first order condition yields the retailer's optimal initial order quantity as follows:

$$Q = \begin{cases} \frac{m-c_w}{2} & for\, c_w + v < c_q,\, and\, q^H = q^L = 0, \\ \frac{m-v-2c_w+c_q}{2} & for\, c_w < c_q \leq c_w + v,\, and\, q^H > 0\, q^L = 0, \\ 0 & for\, m - v < c_q \leq c_w,\, and\, q^H \geq 0\, q^L \geq 0, \\ \frac{m-v-c_q}{2} & for\, c_q \leq min(m - v, c_w),\, and\, q^H \geq 0\, q^L \geq 0. \end{cases}$$

Anticipating the retailer's initial and QR order quantities as described above, the manufacturer chooses its QR price, c_q, to maximize its expected profit

$$\mathbb{E}[\pi_M] = \mathbb{E}[(c_q - \delta)q] + c_w Q.$$

It can also be confirmed that $\mathbb{E}[\pi_M]$ is piecewise concave in c_q, and solving $\frac{\partial \mathbb{E}[\pi_M]}{\partial c_q} = 0$ leads to the results given in the proposition with

$$c_F = \begin{cases} \frac{m+\sqrt{(2m-\delta)\delta}}{2} & for\, v \leq min(\delta, \frac{m-\delta}{2}), \\ \frac{m+\sqrt{m^2-4mv+4v(v+\delta)}}{2} & for\, \frac{m-\delta}{2} < v \leq \delta, \\ \frac{m+\sqrt{m^2-4mv+5v^2+2v\delta+\delta^2}}{2} & for\, max(\delta, \frac{m-\delta}{2}) < v, \\ \frac{m+\sqrt{v^2+2mv-2v\delta}}{2} & for\, \delta < v \leq \frac{m-\delta}{2}. \end{cases}$$

□

Proof of Proposition 14 The result is straightforward by comparing retailer profit across different sequence of events. □

Proof of Proposition 15

The result is established by showing that no retailer has incentive to deviate from these decisions. □