Online Supplement to Competitive Quality Choice and Remanufacturing

December 7, 2012

In this online Appendix, we study monopoly no-remanufacturing, monopoly remanufacturing and exogenous benchmark models in Appendix A, provide CS and SS results for our extensions in Appendix B and provide the proofs of the analytical results in Appendix C in the order their corresponding results appear in the paper.

A. Benchmarks

A.1. Monopoly No-Remanufacturing (NR) Benchmark

The *monopoly no-remanufacturing benchmark* considers a monopolist OEM who only sells the new product. The OEM decides on the quality and quantity of its product by solving the following problem

$$\max_{q_n,s} \ \pi_{OEM}(q_n,s)=[s(1-q_n)-\beta s^2]q_n$$
$$s.t \ \ q_n,s\geq 0$$

Firstly, notice that, for $s\geq\frac{1}{\beta}$, the profit function is negative. Hence, the optimum quality satisfies $s<\frac{1}{\beta}$. $\frac{\partial^2\pi_{OEM}}{\partial q_n^2}=-2s<0$. Hence, it is concave in q_n and the optimum is $q_n^*(s)=\frac{1}{2}(1-s\beta)$. If we plug this into the profit function, we have $\pi_{OEM}(s)=\frac{1}{4}s(-1+s\beta)^2$. This function is unimodal for $s<\frac{1}{\beta}$ and has its maximum at $s^*=\frac{1}{3\beta}$. Hence, $q_n^*=\frac{1}{3}$ and $\pi_{OEM}^*=\frac{1}{27\beta}$. From the optimal quality and the new product quantity, it can be found that consumer surplus is $\frac{1}{54\beta}$ and the social surplus is $\frac{1}{18\beta}$ for the no-remanufacturing benchmark. $\square$

A.2. Monopoly Remanufacturing Benchmark

The *monopoly remanufacturing benchmark* considers a monopolist OEM who may sell both the new product and the remanufactured products. The OEM decides on the quality of the new products and the quantity of new and remanufactured products by solving the following problem

$$\max_{q_n,q_r,s} \ \pi_{OEM}(q_n,q_r,s)=[s(1-q_n-\delta q_r)-\beta s^2]q_n+[\delta s(1-q_n-q_r)-\alpha\beta s^2]q_r$$
$$s.t \ \ q_n,s\geq 0$$
$$q_n\geq q_r\geq 0$$

We first optimize for q_n and q_r. In this case, the Hessian of π_{OEM} is $\begin{pmatrix}-2s & -2s\delta\\ -2s\delta & -2s\delta\end{pmatrix}$. Hence, it is jointly concave in q_n and q_r. From the first order conditions, it is straightforward to show that the interior solution is $q_r = \frac{\beta s(\delta-\alpha)}{2(1-\delta)\delta}$ and $q_n = \frac{\beta s(\alpha-1)+1-\delta}{2(1-\delta)}$. It can be seen that, $q_r \leq 0$ if and only if $\alpha \geq \delta$; therefore, the OEM does not remanufacture for $\frac{\alpha}{\delta} \geq 1$ and remanufactures otherwise. If it does not remanufacture, all the decisions are same as in the no-remanufacturing benchmark. From $0 < q_r < q_n$, this case applies if $s < \frac{\delta(-1+\delta)}{\beta(\alpha-2\delta+\alpha\delta)} \triangleq s_0$. Similarly, if the core constraint binds, $q_n = q_r = \frac{1+\delta-\beta s(1+\alpha)}{2(1+3\delta)}$ and this case applies if $s \geq s_0$. It is easy to see that in equilibrium $s < \frac{1+\delta}{\beta(1+\alpha)}$. Now we can optimize for quality. For $\frac{\alpha}{\delta} < 1$, the profit, as a function of s ($\pi_{OEM}(s)$), is a piecewise function and changes characteristic at s_0. $\pi_{OEM}(s)$ is continuous at s_0. It can be shown that for $s \geq s_0$, π_{OEM} is a unimodal function and has only one maximizer at $s = \frac{1+\delta}{3(1+\alpha)\beta}$. Similarly, π_{OEM} is either unimodal or an increasing function for $s < s_0$. If it is unimodal, maximizer is $s = -\frac{-2\delta+2\delta^2+\sqrt{(-1+\delta)\delta\left(3\alpha^2-6\alpha\delta+\delta(-1+4\delta)\right)}}{3\beta\left(\alpha^2+\delta-2\alpha\delta\right)} \triangleq s_1$. Using these, it can be shown that if $0 < \frac{\alpha}{\delta} \leq \frac{1-5\delta}{2\delta^2-5\delta-1}$, the core constraint binds and $s^* = \frac{1+\delta}{3(1+\alpha)\beta}$, $q_n^* = q_r^* = \frac{1+\delta}{3(1+3\delta)}$. Note that, in this case optimum quality is higher than the NR benchmark. On the other hand if $\frac{1-5\delta}{2\delta^2-5\delta-1} < \frac{\alpha}{\delta} < 1$, the core constraint does not bind and the optimum solution is $s^* = s_1$, $q_r^* = \frac{\beta s^*(\delta-\alpha)}{2(1-\delta)\delta}$ and $q_n^* = \frac{\beta s^*(\alpha-1)+1-\delta}{2(1-\delta)}$. Similar to the previous case, it can be shown that optimal quality is higher than the NR benchmark.

For this model, by some algebra it can be shown that $CS = \pi_{OEM}/2$. Hence, if the profit increases, CS and SS increase as well and vice-versa. Notice that, under remanufacturing, profit cannot be lower than the no-remanufacturing case. Hence, CS and SS is more than or equal to no-remanufacturing.

Following proposition states the effect of OEM remanufacturing on environment by comparing it to the NR benchmark.

PROPOSITION 8. *The following compares environmental impact of the monopoly remanufacturing benchmark to the NR benchmark.*

- *When the OEM does not remanufacture, the environmental impact is the same as the NR benchmark level.*
- *When the OEM remanufactures but does not remanufacture all available cores, the environmental impact is lower than the NR benchmark level if and only if* $\frac{e}{E} < \frac{(-1+3\alpha-2\delta)\delta+\sqrt{(-1+\delta)\delta\left(3\alpha^2-6\alpha\delta+\delta(-1+4\delta)\right)}}{3(\alpha-\delta)} \triangleq r^m$.
- *When the OEM remanufactures all available cores, the environmental impact is lower than the NR benchmark level if and only if* $\frac{e}{E} < \frac{2\delta}{1+\delta}$.

When the OEM remanufactures maximum $\frac{e}{E}$ *ratios below which remanufacturing improves environmental impact stated in this Proposition is always higher than that of the base model stated in Proposition 5.*

Region	Condition	q_n^*	q_r^*
$R1^{exo}$	$\frac{\alpha}{\delta} \geq \frac{1+\beta s_f}{2\beta s_f}$	$\frac{1-\beta s_f}{2}$	0
$R3^{exo}$	$\frac{\left(3\beta s_f-1+\delta\right)}{(2+\delta)\beta s_f} < \frac{\alpha}{\delta} < \frac{1+\beta s_f}{2\beta s_f}$	$\frac{2-\delta+\beta s_f(\alpha-2)}{4-\delta}$	$\frac{\delta+\beta s_f(\delta-2\alpha)}{\delta(4-\delta)}$
$R4^{exo}$	$0 < \frac{\alpha}{\delta} \leq \frac{\left(3\beta s_f-1+\delta\right)}{(2+\delta)\beta s_f}$	$\frac{1-\beta s_f}{(2+\delta)}$	$\frac{1-\beta s_f}{(2+\delta)}$

Table 5 **Equilibrium when product quality is exogenously given**

A.3. Exogenous Quality Benchmark

In the *exogenous quality benchmark*, the OEM sells the new product and the IR sells the remanufactured product, but the quality level is fixed at s_f. In this case, the OEM's optimization problem is $\max_{q_n} \pi_{OEM}(q_n|s_f)$ and that of the IR is $\max_{q_r} \pi_{IR}(q_r|s_f)$ subject to the feasibility constraints. Table 5 describes the equilibrium of this benchmark. In the proof of proposition 1, we first solve the quantity game for a given quality level. Hence, proof of the exogenous quality benchmark is included in there. $\square$

B. Consumer and Social Welfare Results for Extensions to the Base Model

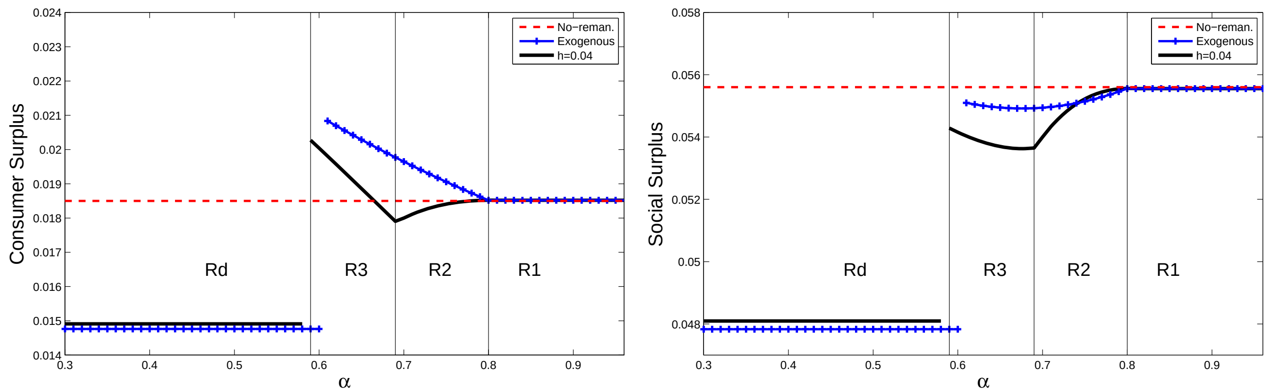
B.1. Preemptive Collection

In this section, we study CS and SS when the OEM can collect and dispose of the used cores to compete with the IR. Figure 8 is a representative illustration of the resulting CS and SS levels from our numerical study.

When the cost-to-value ratio $\frac{\alpha}{\delta}$ is high $(0.59 < \alpha < 1)$, the OEM does not preemptively collect cores and the equilibrium decisions are similar to those in the base model. Hence, same as in our base model, the IR's threat and actual entry can decrease the CS and the SS compared to NR benchmark.

When the cost-to-value ratio is low $(0 < \alpha \leq 0.59)$, the IR is a bigger threat and the OEM relies on preemptive collection as a competitive strategy. The OEM decreases its total new product quantity and collects all cores to deter the IRs entry. This strategy decreases the CS and SS significantly compared to NR benchmark. This result is also consistent with our base model where we show that entry deterrence reduces both CS and SS.

In the exogenous quality benchmark, new product quality is kept at the NR benchmark quality disregarding the OEM's quality response to the IR's threat as before. Figure 8 shows that when the OEM does not use preemptive collection strategy and the IR remanufactures (i.e, cost-to-value ratio is high), the CS and SS are lower than the exogenous quality benchmark as in the base model. However, when the OEM collects and disposes all available cores to deter the IR's entry (i.e, cost-to-value ratio is low), the CS and SS are higher than the exogenous benchmark with a small margin. The reason is as follows: In this case, the OEM mainly relies on collection of used cores to deter the IR's entry. Having additional lever quality allows the OEM to use collection strategy more efficiently in terms of the consumer surplus and firm's profits. Hence the CS and SS are higher than the exogenous quality benchmark.



(a) Consumer Surplus (b) Social Surplus

Figure 8 Comparison of Consumer and Social Surplus with NR and Exogenous Quality Benchmarks when the OEM can collect and dispose used cores ($\alpha = 0.4, \beta = 1$ and Exogenous quality $s_f = \frac{1}{3\beta}$**)**

B.2. Price Competition

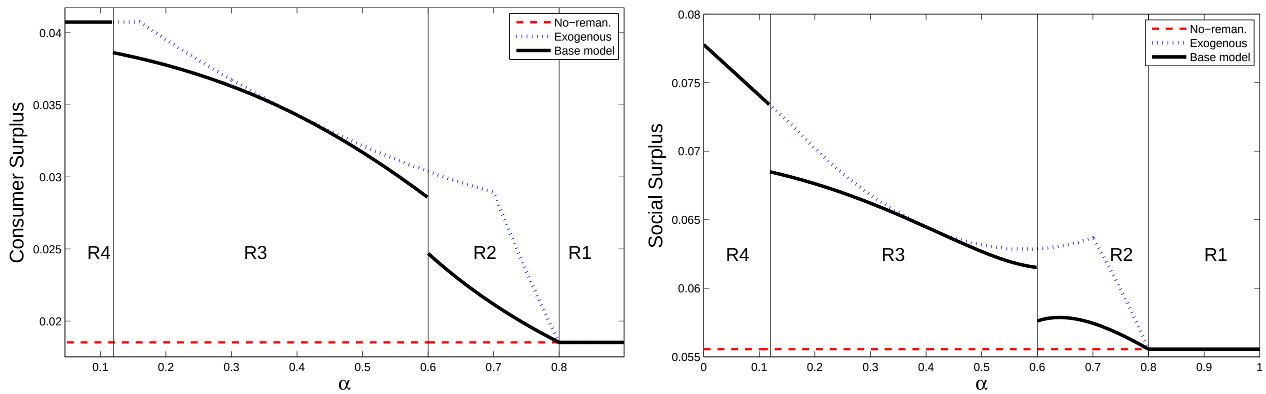
Figure 9 demonstrates our findings. It is well known that price competition leads to a more intense competition and a higher CS and SS than quantity competition (Singh and Vives 1994). Consistent with this fact, Figure 9 shows that CS and SS are higher than the NR benchmark when the OEM and the IR compete in prices. The Figure also illustrates that CS and SS are lower than the exogenous quality benchmark (with NR benchmark quality). Thus, similar to our base model, ignoring the OEM's quality decision leads to overestimating the benefits of remanufacturing for social welfare.

B.3. Alternative Remanufacturing Cost

In this section we study the CS and SS when the IR incurs an additional cost n independent of the product quality level.

Since the IR's total unit remanufacturing cost is now $\beta\alpha s^2 + n$, IR's competitive position depends not only on the ratio $\frac{\alpha}{\delta}$ (as in the base model) but also on the quality-independent cost component n. Specifically, the IR's competitive position is strong when $\frac{\alpha}{\delta}$ and n are simultaneously low.

In the base model we showed that when the IR's competitive position is strong enough, the CS and SS are always higher than the NR benchmark levels. Otherwise, the CS and SS are lower than



(a) Consumer Surplus (b) Social Surplus

Figure 9 Comparison of Consumer and Social Surplus in Price competition with NR and Exogenous Quality Benchmarks ($\delta = 0.4, \beta = 1$ and Exogenous quality $s_f = \frac{1}{3\beta}$**)**

the NR benchmark levels (see Propositions 3 and 4). Figures 10 and 11 illustrate that these results continue to hold in the presence of an additional quality-independent cost component n. In Figure 10, n is low ($n = 0.02$). On the same figure, when $\frac{\alpha}{\delta}$ is also low (Given $\delta = 0.4$, $0 < \alpha < 0.59$ for CS and $0 < \alpha < 0.34$ for SS), the IR is strong and the CS and the SS are above the NR benchmark levels. On the other hand when n is high ($n = 0.06$) as in Figure 11 or $\frac{\alpha}{\delta}$ is high (Given $\delta = 0.4$, $\alpha \geq 0.59$ for CS and $\alpha \geq 0.34$ for SS) as in Figure 10, the IR is weak and the CS and SS are always lower than the NR benchmark case.

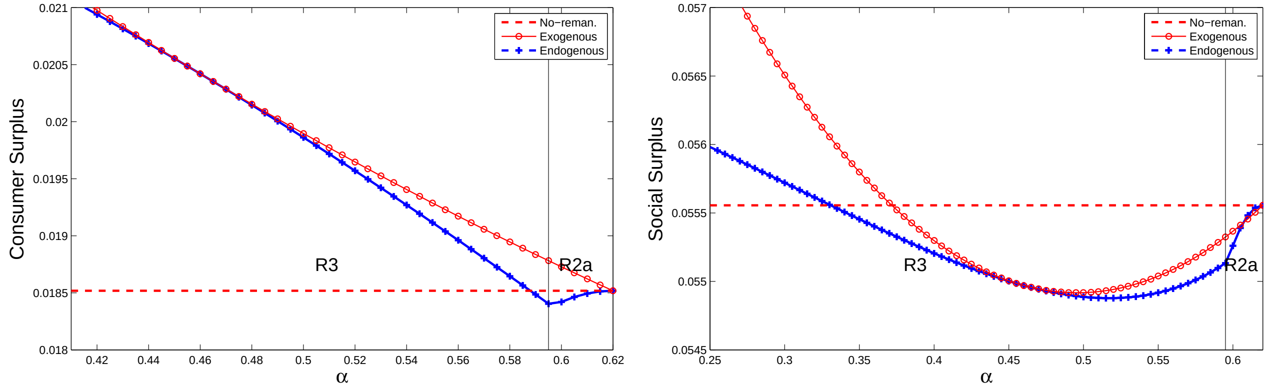
The Figures also illustrate that ignoring the OEM's quality decision may result in overestimating remanufacturing benefits, since the CS and the SS in exogenous benchmark is always higher than that of endogenous quality model when the IR remanufactures.

B.4. Independent Quality Gap

In this section we study CS and SS when the quality gap between the new and remanufactured product is independent of product quality.

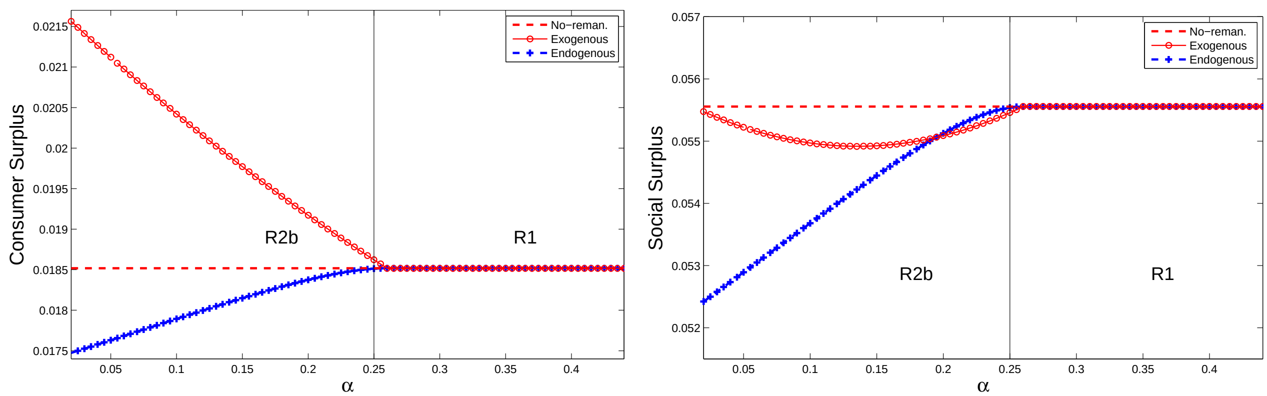
Figure 12 illustrates that all the insights we derived from the base model continue to hold for this extension. More specifically, the CS and SS can decrease compared to the NR benchmark when the OEM deters the IR's entry or when a weak IR (high ϕ) enters the market. To achieve a higher CS than the NR benchmark, the IR needs to be strong (small ϕ).

In the exogenous quality benchmark, the Figure shows that, as opposed to endogenous quality, independent of the IR's competitive position remanufacturing always increases CS. And SS in endogenous quality model is always lower than the exogenous quality benchmark when the IR remanufactures. These results are same as those derived in the base model.



(a) Consumer Surplus (b) Social Surplus

Figure 10 **Comparison of Consumer and Social Surplus with NR and Exogenous Quality Benchmarks in the presence of quality independent remanufacturing cost (**$\delta = 0.4, \beta = 1, n = 0.02$ and Exogenous quality $s_f = \frac{1}{3\beta}$ **)**



(a) Consumer Surplus (b) Social Surplus

Figure 11 **Comparison of Consumer and Social Surplus with NR and Exogenous Quality Benchmarks in the presence of quality independent remanufacturing cost (**$\delta = 0.4, \beta = 1, n = 0.06$ and Exogenous quality $s_f = \frac{1}{3\beta}$ **)**

C. Proofs

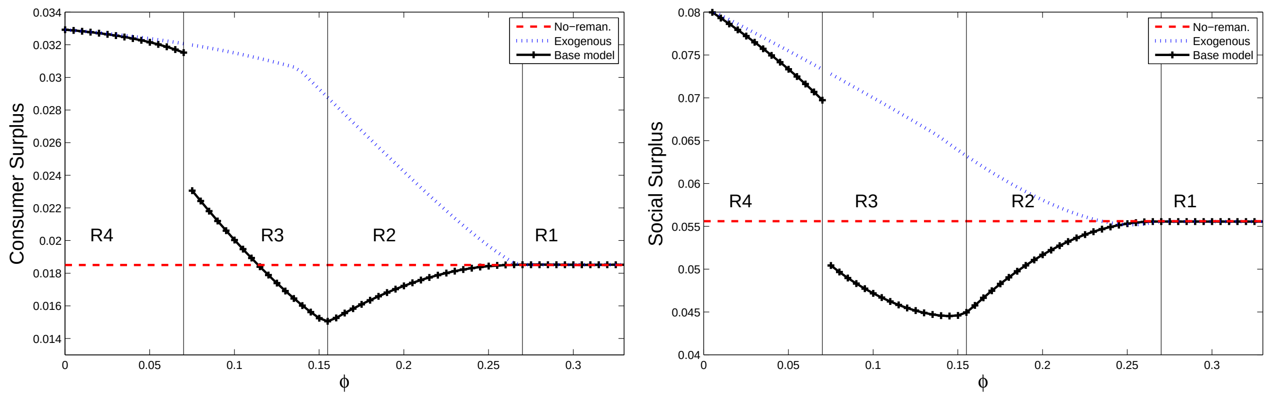
Proof of Proposition 1

Given s and q_r, $\frac{\partial^2 \pi_{OEM}}{\partial q_n^2} = -2s < 0$. Hence, it is concave in q_n and the optimum[1] is $q_n^*(s) = \frac{1}{2}(1 - \beta s - q_r \delta)$. This is positive if and only if $q_r < \frac{1-\beta s}{\delta}$ and $s < \frac{1}{\beta}$. Therefore, equilibrium quality satisfies $s < \frac{1}{\beta}$. For the IR, $\frac{\partial^2 \pi_{IR}}{\partial q_r^2} = -2\delta s < 0$ and interior solution is $q_r^i \triangleq \frac{-\beta \alpha s + \delta - q_n \delta}{2\delta}$. Thus, there can be three cases:

1. If $q_r^i \leq 0$, then $q_r^* = 0$.

In this case $q_n^* = \frac{1-\beta s}{2}$ and from $q_r^i = \frac{-\beta \alpha s + \delta - q_n^* \delta}{2\delta} \leq 0$ and $q_n^* > 0$, we have $\frac{\alpha}{\delta} \geq \frac{(1+\beta s)}{2\beta s}$ $(\equiv s \geq \frac{\delta}{\beta(2\alpha - \delta)})$

[1] It is straightforward to show that $q_n = 0$ can never be an equilibrium; therefore, we do not consider this case.



(a) Consumer Surplus (b) Social Surplus

Figure 12 Comparison of Consumer and Social Surplus with NR and Exogenous Quality Benchmarks when the perceptual quality gap is independent of new product quality ($\alpha = 0.4, \beta = 1$ and Exogenous quality $s_f = \frac{1}{3\beta}$**)**

and $\alpha > \delta$.

2. If $0 < q_r^i < q_n$, then $q_r^* = q_r^i$.

By solving $q_r = q_r^i$ and $q_n = \frac{1}{2}(1 - s\beta - q_r\delta)$, we obtain $q_n^* = \frac{2-\delta+\beta s(\alpha-2)}{4-\delta}$ and $q_r^* = \frac{\delta+s\beta(\delta-2\alpha)}{(4-\delta)\delta}$. From the condition $0 < q_r^* < q_n^*$, we have $\frac{\alpha}{\delta} > \frac{\delta(3\beta s-1+\delta)}{(2+\delta)\beta s}$ $(\equiv s < \frac{(-1+\delta)\delta}{\beta(2\alpha+\alpha\delta-3\delta)})$ for $\alpha < \delta$, and $\frac{\alpha}{\delta} < \frac{(1+\beta s)}{2\beta s}$ $(\equiv s < \frac{\delta}{\beta(2\alpha-\delta)})$ for $\alpha \geq \delta$.

3. If $q_n \leq q_r^i$, then $q_r^* = q_n$.

In this case $q_r^* = q_n^* = \frac{1-\beta s}{2+\delta}$ and from $q_r^i \geq q_n^* > 0$, we have $\frac{\alpha}{\delta} \leq \frac{\delta(3\beta s-1+\delta)}{(2+\delta)\beta s}(\equiv s \geq \frac{(-1+\delta)\delta}{\beta(2\alpha+\alpha\delta-3\delta)})$ and $\alpha < \delta$.

For the exogenous quality benchmark, by considering $\alpha \to \delta$, $\delta \to 0$ for $\alpha \geq \delta$ and $\delta \to 1$ for $\alpha < \delta$, it can be shown that all these three cases exist for any $s < \frac{1}{\beta}$.

Now, we proceed for the solution of the equilibrium quality. From the quantity game equilibrium, if $\alpha > \delta$ and $s < \frac{\delta}{\beta(2\alpha-\delta)}$, then $q_r^* = q_r^i$. On the other hand, if $\alpha > \delta$ and $s \geq \frac{\delta}{\beta(2\alpha-\delta)}$, then $q_r^* = 0$. This means that for $\alpha > \delta$, the profit function is a piecewise function and changes characteristic at $s_0 \triangleq \frac{\delta}{\beta(2\alpha-\delta)}$. We define $\pi_1 \triangleq \pi_{OEM}(s \geq s_0)$ and $\pi_2 \triangleq \pi_{OEM}(s < s_0)$. $\pi_{OEM}(s)$ is continuous at s_0, i.e $\pi_1(s_0) = \pi_2(s_0)$. π_2 can be written as $sq_{n_2}^2$ where $q_{n_2} \triangleq \frac{2-\delta+\beta s(\alpha-2)}{4-\delta}$. π_2 has one root at $s = 0$ and two roots at $s = \frac{2-\delta}{(2-\alpha)\beta}$ and it is positive between these roots. Since $\alpha > \delta$, $\frac{2-\delta}{(2-\alpha)\beta} > \frac{1}{\beta}$. $\frac{\partial \pi_2}{\partial s}$ has roots at $s = \frac{2-\delta}{(2-\alpha)\beta}$ and at $s = \frac{2-\delta}{3(2-\alpha)\beta}$. One of these roots $(s = \frac{2-\delta}{(2-\alpha)\beta})$ is same as the roots of π_2 and the other one satisfies $\frac{1}{\beta} > \frac{2-\delta}{3(2-\alpha)\beta} > 0$. Thus, for $s < \frac{1}{\beta}$, π_2 is unimodal and the maximizer is $s = \frac{2-\delta}{3(2-\alpha)\beta}$. Similarly, we can write $\pi_1 = s\left(\frac{1-\beta s}{2}\right)^2$ and show that this is unimodal for $s < \frac{1}{\beta}$ with a unique maximizer at $s = \frac{1}{3\beta}$. By checking the derivatives of π_1 and π_2 at the boundary s_0, we can determine where s^*, the maximizer of the profit function, is. For $\frac{\partial \pi_1}{\partial s}|_{s=s_0} \geq 0$ and $\frac{\partial \pi_2}{\partial s}|_{s=s_0} \geq 0$, the optimum s^* is in the region $s \geq s_0$ and it is $s^* = \frac{1}{3\beta}$. We can show that these inequalities are satisfied if and only if $\frac{\alpha}{\delta} \geq 2$. Therefore, for $\frac{\alpha}{\delta} \geq 2$, we have $s^* = \frac{1}{3\beta}$, $q_n^* = \frac{1}{3}$ and $q_r^* = 0$. Recall that, these are the no-remanufacturing benchmark optimum quality and quantities. For $\frac{\partial \pi_1}{\partial s}|_{s=s_0} < 0$ and $\frac{\partial \pi_2}{\partial s}|_{s=s_0} \geq 0$, the optimum s^* is at the boundary s_0. Similar to the previous case, inequalities are satisfied if and only if $\frac{8-\delta}{4+\delta} \leq \frac{\alpha}{\delta} < 2$. For this case we have $s^* = \frac{\delta}{\beta(2\alpha-\delta)}$, $q_n^* = \frac{2(2-\delta)}{3(4-\delta)}$ and $q_r^* = 0$. Hence, the OEM deters the IR's entry in this region. For $\frac{\partial \pi_1}{\partial s}|_{s=s_0} \leq 0$ and $\frac{\partial \pi_2}{\partial s}|_{s=s_0} < 0$, the optimum s^* is in the region $s < s_0$ and it is $\frac{2-\delta}{3(2-\alpha)\beta}$. The inequalities are satisfied if and only if $\frac{8-\delta}{4+\delta} > \frac{\alpha}{\delta} > 1$. For this case we have $s^* = \frac{2-\delta}{3(2-\alpha)\beta}$, $q_n^* = \frac{2(2-\delta)}{3(4-\delta)}$ and $q_r^* = \frac{(8-\delta)\delta-\alpha(4+\delta)}{3(2-\alpha)(4-\delta)\delta}$. Hence, the IR enters and collects a portion of the available cores. It is straightforward to show that $\frac{\partial \pi_1}{\partial s} > 0$ and $\frac{\partial \pi_2}{\partial s} < 0$ is infeasible.

For $\alpha = \delta$, from the quantity game equilibrium, only $q_n > 0$ and $q_n > q_r > 0$ applies. Hence, the equilibrium quality and the quantities are the same as the equilibrium outcome in region $\frac{8-\delta}{4+\delta} > \frac{\alpha}{\delta} > 1$.

For $\alpha < \delta$, from the equilibrium of the quantity game, $q_r^* > 0$ is always true. The core availability constraint may or may not bind depending on s. If $s < \frac{\delta(-1+\delta)}{\beta(2\alpha-3\delta+\alpha\delta)} \triangleq s_1$, then $0 < q_r^* < q_n^*$ and if $s \geq s_1$, then $q_r^* = q_n^*$ (see the quantity game equilibrium.). We define $\pi_3 \triangleq \pi_{OEM}(s \geq s_1)$ and $\pi_2 \triangleq \pi_{OEM}(s < s_1)$ (essentially this is the same function as π_2 defined for $\alpha > \delta$). $\pi_{OEM}(s)$ is continuous at $s = s_1$, i.e $\pi_2(s_1) = \pi_3(s_1)$. Before we look at how π_2 and π_3 behave at the boundary $s = s_1$, we first show that π_3 has only one maximizer in the region of interest, $s \in (0, \frac{1}{\beta})$. π_3 can be written as $s\left(\frac{1-\beta s}{2+\delta}\right)^2$, which has one root at $s = 0$ and two roots at $s = \frac{1}{\beta}$. Similar to π_1, for $s < \frac{1}{\beta}$, π_3 is unimodal and has one maximizer at $s = \frac{1}{3\beta}$. If $\frac{\partial \pi_2}{\partial s}|_{s=s_1} < 0$ and $\frac{\partial \pi_3}{\partial s}|_{s=s_1} \leq 0$, the optimum quality is in $s < s_1$ and it is $s^* = \frac{2-\delta}{3(2-\alpha)\beta}$. The inequalities hold if and only if $\frac{3\delta^2}{2+\delta} \leq \alpha < \delta$. In this case, the equilibrium quality and quantities are $s^* = \frac{2-\delta}{3(2-\alpha)\beta}$, $q_n^* = \frac{2(2-\delta)}{3(4-\delta)}$ and $q_r^* = \frac{(8-\delta)\delta-\alpha(4+\delta)}{3(2-\alpha)(4-\delta)\delta}$. It is easy to show that $\frac{\partial \pi_2}{\partial s}|_{s=s_1} >= 0$ and $\frac{\partial \pi_3}{\partial s}|_{s=s_1} < 0$ is infeasible. Therefore, s_1 can never be an optimum. If $\frac{\partial \pi_2}{\partial s}|_{s=s_1} \geq 0$ and $\frac{\partial \pi_3}{\partial s}|_{s=s_1} \geq 0$, the optimum quality is in $s > s_1$ and it is $s^* = \frac{1}{3\beta}$. Inequalities are satisfied if and only if $\alpha \leq \frac{3\delta^2}{4-3\delta+2\delta^2}$. In this case, $q_n^* = \frac{2}{3(2+\delta)} = q_r^*$. If $\frac{\partial \pi_2}{\partial s}|_{s=s_1} < 0$ and $\frac{\partial \pi_3}{\partial s}|_{s=s_1} > 0$, we need to compare the profit function's values at $s = \frac{1}{3\beta}$ and $s = \frac{2-\delta}{3(2-\alpha)\beta}$. Inequalities are satisfied if and only if $\frac{3\delta^2}{4-3\delta+2\delta^2} < \alpha < \frac{3\delta^2}{2+\delta}$. $\pi_2(s = \frac{2-\delta}{3\beta(2-\alpha)}) - \pi_3(s = \frac{1}{3\beta}) = -\frac{4\left(\alpha(-4+\delta)^2-\delta^2\left(18-8\delta-2\delta^2+\delta^3\right)\right)}{27(-2+\alpha)\beta(-4+\delta)^2(2+\delta)^2}$ and it is positive for this region if and only if $\frac{\delta(18-8\delta-2\delta^2+\delta^3)}{(4-\delta)^2} < \frac{\alpha}{\delta} < \frac{3\delta}{2+\delta}$. If we combine this case with the previous cases we can conclude that if $0 < \frac{\alpha}{\delta} \leq \frac{\delta(18-8\delta^2-2\delta^3+\delta^4)}{(4-\delta)^2}$, then $s^* = \frac{1}{3\beta}$, $q_n^* = q_r^* = \frac{2}{3\beta(2+\delta)}$. If $\frac{\delta(18-8\delta^2-2\delta^3+\delta^4)}{(4-\delta)^2} < \frac{\alpha}{\delta} < \frac{8-\delta}{4+\delta}$, then $s^* = \frac{2-\delta}{3(2-\alpha)\beta}$, $q_n^* = \frac{2(2-\delta)}{3(4-\delta)}$, $q_r^* = \frac{(8-\delta)\delta-\alpha(4+\delta)}{3(2-\alpha)(4-\delta)\delta}$. $\square$

Proof of Proposition 2

In $R3^{exo}$, where the IR enters and the core constraint does not bind, consumer surplus is $CS_3^{exo} \triangleq \frac{s_f\left(4+\delta-\delta^2\right)}{2(-4+\delta)^2} - \frac{s_f^2\beta\left(4-2\alpha(-2+\delta)-3\delta+\delta^2\right)}{(-4+\delta)^2} + \frac{s_f^3\beta^2\left(\alpha^2(4-3\delta)+(4-3\delta)\delta+2\alpha\delta^2\right)}{2(-4+\delta)^2\delta}$. If the OEM is monopoly without remanufacturing, consumer surplus is $CS_m^{exo} = \frac{s_f}{8} - \frac{s_f^2\beta}{4} + \frac{s_f^3\beta^2}{8}$. $\Delta = CS_3^{exo} - CS_m^{exo} = -\frac{s_f\delta(-12+5\delta)}{8(-4+\delta)^2} - \frac{s_f^3\beta^2\left(-8\alpha\delta^2+\delta^2(4+\delta)+4\alpha^2(-4+3\delta)\right)}{8(-4+\delta)^2\delta} - \frac{s_f^2\beta(-8\alpha(-2+\delta)+\delta(-4+3\delta))}{4(-4+\delta)^2}$. Δ has three roots for s_f: $\{0, \frac{\delta}{\beta(2\alpha-\delta)}, \frac{\delta(12-5\delta)}{\beta\left(8\alpha+4\delta-6\alpha\delta+\delta^2\right)}\}$. In Δ, coefficient of s_f^3 is positive for $\frac{\delta}{2} < \alpha < 1$, and $\frac{\delta(12-5\delta)}{\beta\left(8\alpha+4\delta-6\alpha\delta+\delta^2\right)} \geq \frac{\delta}{\beta(2\alpha-\delta)}$ for $\alpha \geq \delta$. Recall that for the exogenous quality model, the condition for $R3^{exo}$ (where the IR enters and the core constraint does not bind) is $s_f < \frac{\delta}{\beta(2\alpha-\delta)}$ if $\alpha \geq \delta$. Therefore if $\alpha \geq \delta$, $\Delta > 0$. If $\alpha < \delta$, the condition for $R3^{exo}$ is $s_f < \frac{(-1+\delta)\delta}{\beta(2\alpha-3\delta+\alpha\delta)}$. If $\frac{\delta}{2} < \alpha < \delta$, it is easy to show that $\frac{(-1+\delta)\delta}{\beta(2\alpha-3\delta+\alpha\delta)} < \frac{\delta(12-5\delta)}{\beta\left(8\alpha+4\delta-6\alpha\delta+\delta^2\right)} < \frac{\delta}{\beta(2\alpha-\delta)}$. Then if $\frac{\delta}{2} < \alpha < \delta$, $\Delta > 0$. If $0 < \alpha < \frac{\delta}{2}$, then $\frac{\delta(12-5\delta)}{\beta\left(8\alpha+4\delta-6\alpha\delta+\delta^2\right)} > \frac{(-1+\delta)\delta}{\beta(2\alpha-3\delta+\alpha\delta)} > 0 > \frac{\delta}{\beta(2\alpha-\delta)}$ and the coefficient of s_f^3 is negative, therefore $\Delta > 0$. Finally, for $\alpha = \frac{\delta}{2}$, Δ is a second order polynomial of s_f and roots are $\{0, \frac{12-5\delta}{2\beta(4-\delta)}\}$. For $\alpha = \frac{\delta}{2}$, $0 < \frac{(-1+\delta)\delta}{\beta(2\alpha-3\delta+\alpha\delta)} < \frac{12-5\delta}{2\beta(4-\delta)}$ and the coefficient of s_f^2 is negative; hence, $\Delta > 0$.

If the IR enters and the core constraint binds, $CS_4^{exo} \triangleq \frac{s_f(1+3\delta)}{2(2+\delta)^2} - \frac{s_f^2\beta(1+3\delta)}{(2+\delta)^2} + \frac{s_f^3\beta^2(1+3\delta)}{2(2+\delta)^2}$ and $CS_4^{exo} - CS_m^{exo} = \frac{s_f(-1+s_f\beta)^2(8-\delta)\delta}{8(2+\delta)^2}$ and this is always positive. □

Proof of Proposition 3

In $R2$, since the IR cannot enter, and the OEM acts like a monopoly without remanufacturing, consumer surplus is same as the NR benchmark.

Consumer surplus in $R2$ is $CS_2 \triangleq \frac{(\alpha-\delta)^2\delta}{2\beta(2\alpha-\delta)^3}$ and consumer surplus for the NR benchmark is $CS_m \triangleq \frac{1}{54\beta}$. $CS_2 - CS_m = -\frac{(8\alpha-7\delta)(\alpha-2\delta)^2}{54\beta(2\alpha-\delta)^3}$ and this is always negative for $\alpha > \delta$.

For $R3$, consumer surplus is $CS_3 \triangleq (-2+\delta)\left(-\frac{2(-2+\delta)\left(-4+6\alpha-6\delta+\delta^2\right)}{27(-2+\alpha)^2\beta(-4+\delta)^2} + \frac{((-8+\delta)\delta+\alpha(4+\delta))^2}{54(-2+\alpha)^3\beta(-4+\delta)^2\delta}\right)$. We show that CS_3 at $\alpha = \delta$ is greater than CS_m and CS_3 at $\alpha = \frac{(8-\delta)\delta}{4+\delta}$ is smaller than the CS_m. Then, we show that in $R3$, CS_3 is always decreasing in α which proves that there exists a critical α_c satisfying $\delta < \alpha_c < \frac{\delta(8-\delta)}{4+\delta}$ such that $CS_3 > CS_m$ if and only if $\alpha < \alpha_c$ in $R3$. $CS_3|_{\alpha=\delta} = \frac{(12-5\delta)\delta}{54\beta(-4+\delta)^2}$ and it is always positive. Similarly, $CS_3|_{\alpha=\frac{\delta(8-\delta)}{(4+\delta)}} = \frac{(12-5\delta)\delta^2}{54\beta(-4+\delta)^3}$ and it is always negative. $\frac{\partial CS_3}{\partial\alpha} = \frac{\alpha^2(-2+\delta)\left(-16-56\delta+23\delta^2\right)}{54(-2+\alpha)^4\beta(-4+\delta)^2\delta} + \frac{(-2+\delta)\left(192-400\delta+172\delta^2-19\delta^3\right)}{54(-2+\alpha)^4\beta(-4+\delta)^2} + \frac{2\alpha(-2+\delta)\left(-16+40\delta+19\delta^2-17\delta^3+2\delta^4\right)}{27(-2+\alpha)^4\beta(-4+\delta)^2\delta}$ and we want to show that this is negative. This expression is negative if and only if $q \triangleq \alpha^2\left(16+56\delta-23\delta^2\right) + \delta\left(-192+400\delta-172\delta^2+19\delta^3\right) - 4\alpha\left(-16+40\delta+19\delta^2-17\delta^3+2\delta^4\right) < 0$. q has two roots with respect α, i.e $\alpha_1 = \frac{x-\sqrt{y}}{z}$ and $\alpha_2 = \frac{x+\sqrt{y}}{z}$, where $x \triangleq -2\left(-16+40\delta+19\delta^2-17\delta^3+2\delta^4\right)$, $y \triangleq (-4+\delta)^2(-2+\delta)^2\left(16-8\delta+105\delta^2-80\delta^3+16\delta^4\right)$, $z \triangleq -16-56\delta+23\delta^2$. Since $z < 0$, $\alpha_2 < \alpha_1$. In $R3$, if the boundaries of α lies within α_1 and α_2, then $\frac{\partial CS_3}{\partial\alpha} < 0$. More specifically, if $\alpha_2 < \frac{\delta^2(18-8\delta-2\delta^2+\delta^3)}{(4-\delta)^2} < \alpha_1$ and $\alpha_2 < \frac{\delta(8-\delta)}{4+\delta} < \alpha_1$, then consumer surplus is decreasing in α in $R3$. By some tedious algebra, similar to the one in the proof of Proposition 4 (skipped here), it can be shown that this is indeed the case. Therefore, CS_3 is decreasing in α. Therefore, $CS_3 = CS_m$ has a unique solution in $R3$ and it is defined as α_c. Once we show that in $R4$, consumer surplus is always higher than the NR benchmark, this proves the existence of α_c satisfying $1 < \frac{\alpha}{\delta} < \frac{8-\delta}{4+\delta}$, and consumer surplus is higher than the NR benchmark if and only if $\alpha < \alpha_c$.

For $R4$, $CS_4 \triangleq \frac{2(1+3\delta)}{27\beta(2+\delta)^2}$ and $CS_4 - CS_m = \frac{(8-\delta)\delta}{54\beta(2+\delta)^2}$ which is always positive.

α_c is only a function of δ and δ is in the bounded region $(0,1)$; therefore, we can numerically verify that the derivative of α_c with respect to δ is always positive in $R3$.

The CS in $R1$ is same as the NR benchmark. If we exclude this region, we find that CS is strictly lower than the NR benchmark for $\frac{\alpha_c}{\delta} < \frac{\alpha}{\delta} < 2$. $\square$

Proof of Proposition 4

In $R1$, the IR cannot enter, and the OEM acts like a monopoly without remanufacturing. Hence, social surplus is same as NR benchmark.

In $R2$, social surplus is $SS_2 \triangleq \frac{3(\alpha-\delta)^2\delta}{2\beta(2\alpha-\delta)^3}$. The NR benchmark social surplus is $SS_m \triangleq \frac{1}{18\beta}$. $SS_2 - SS_m = \frac{(8\alpha-7\delta)(\alpha-2\delta)^2}{18\beta(2\alpha-\delta)^3}$ and it is easy to see that this is always positive for $\alpha > \delta$.

In $R3$, social surplus is $SS_3 \triangleq \frac{(-2+\delta)2\alpha\delta\left(-224+60\delta+3\delta^2-2\delta^3\right)}{54(-2+\alpha)^3\beta(-4+\delta)^2\delta} + \frac{(-2+\delta)\alpha^2\left(48+104\delta-53\delta^2+8\delta^3\right)}{54(-2+\alpha)^3\beta(-4+\delta)^2\delta} + \frac{(-2+\delta)\delta\left(192+128\delta-80\delta^2+11\delta^3\right)}{54(-2+\alpha)^3\beta(-4+\delta)^2\delta}$. We evaluate SS_3 at $\alpha=\delta$ and at $\alpha = \frac{\delta(8-\delta)}{4+\delta}$, and show that SS_3 is greater than SS_m at $\alpha=\delta$ and smaller than SS_m at $\alpha = \frac{\delta(8-\delta)}{4+\delta}$. Then, we show that SS_3 has a negative derivate in α, if $\delta > \frac{1}{2}$, and there exists a α' such that SS_m has a negative derivative if $\delta < \frac{1}{2}$ and $\alpha < \alpha'$ in $R3$, where $\frac{\delta(8-\delta)}{4+\delta} > \alpha' > \delta$. These imply that $SS_3 = SS_m$ has a unique solution for $\frac{\delta^2(18-8\delta-2\delta^2+\delta^3)}{(4-\delta)^2} < \alpha < \frac{\delta(8-\delta)}{4+\delta}$ (recall that these inequalities define $R3$) and are sufficient for the existence of an α_s value such that $SS_3 > SS_m$ if and only if $\alpha < \alpha_s$ in R3. $(SS_3 - SS_m)|_{\alpha=\frac{\delta(8-\delta)}{4+\delta}} = \frac{(12-5\delta)\delta^2}{18\beta(-4+\delta)^3}$ and this is negative. $(SS_3 - SS_m)|_{\alpha=\delta} = \frac{\delta(4+\delta)}{54\beta(-4+\delta)^2}$ and this is positive. $\frac{\partial SS_3}{\partial\alpha} = -\frac{(-2+\delta)\left(\alpha^2\left(48+104\delta-53\delta^2+8\delta^3\right)+\delta\left(-320+624\delta-228\delta^2+25\delta^3\right)\right.}{54(-2+\alpha)^4\beta(-4+\delta)^2\delta} + \frac{\alpha\left(192-480\delta+28\delta^2+44\delta^3-8\delta^4\right)}{54(-2+\alpha)^4\beta(-4+\delta)^2\delta}$ and denominator is always positive. Therefore we only need to consider the polynomial $p = \alpha^2\left(48+104\delta-53\delta^2+8\delta^3\right) + \delta\left(-320+624\delta-228\delta^2+25\delta^3\right) + \alpha\left(192-480\delta+28\delta^2+44\delta^3-8\delta^4\right)$. p is convex in α and has two roots α, i.e., $\alpha_1 = \frac{f-\sqrt{g}}{\eta}$ and $\alpha_2 = \frac{f+\sqrt{g}}{\eta}$ where $f = 2\left(-48+120\delta-7\delta^2-11\delta^3+2\delta^4\right)$, $g = \left(144-264\delta+481\delta^2-184\delta^3+16\delta^4\right)(-4+\delta)^2(-2+\delta)^2$ and $\eta = \left(48+104\delta-53\delta^2+8\delta^3\right)$. Since $\eta > 0$ it can be seen that $\alpha_1 < \alpha_2$. We want to show that $\alpha_1 < \frac{\delta^2\left(18-8\delta-2\delta^2+\delta^3\right)}{(4-\delta)^2}$. This can be simplified to showing $-20480 + 53760\delta - 50944\delta^2 + 9008\delta^3 + 13480\delta^4 - 5559\delta^5 - 1504\delta^6 + 1020\delta^7 - 32\delta^8 - 53\delta^9 + 8\delta^{10} < 0$. It can be further simplified to showing $20480 - 53760\delta + 50944\delta^2 - 16929\delta^3 > 0$. It is straightforward to show that this is true for $0 < \delta < 1$. In a similar way, it can be shown that $\alpha_2 > \frac{\delta^2\left(18-8\delta-2\delta^2+\delta^3\right)}{(4-\delta)^2}$. Now, we compare α_2 with $\frac{\delta(8-\delta)}{4+\delta}$. $\alpha_2 = \frac{f+\sqrt{g}}{\eta} > \frac{\delta(8-\delta)}{4+\delta}$ if and only if $(4+\delta)\sqrt{g} > (8\delta-\delta^2)\eta - f(4+\delta)$. Both sides are positive and this inequality is equivalent to $(-2+\delta)\delta(1-2\delta)\left(48+104\delta-53\delta^2+8\delta^3\right) > 0$. It can be easily seen that this is true if and only if $\delta > \frac{1}{2}$. Then, $\alpha_2 > \frac{\delta(8-\delta)}{4+\delta}$ if and only if $\delta > \frac{1}{2}$. So far, we showed that SS_3 is always decreasing in α in $R3$ if and only if $\delta > \frac{1}{2}$, or $\delta < \frac{1}{2}$ and $\alpha < \alpha_2$. Since $(SS_3 - SS_m)\big|_{\alpha=\frac{\delta(8-\delta)}{4+\delta}} < 0$ and $(SS_3 - SS_m)|_{\alpha=\delta} > 0$, $\alpha_2 > \delta$. This proves that if α increases from $\frac{\delta^2\left(18-8\delta-2\delta^2+\delta^3\right)}{(4-\delta)^2}$ to $\frac{\delta(8-\delta)}{4+\delta}$, it crosses SS_m only once. The crossing point is defined as α_s and it is the unique solution to the equation $SS_3 = SS_m$ in $R3$.

In $R4$, $SS_4 \triangleq \frac{2\left(3-2\alpha+5\delta-\alpha\delta+3\delta^2\right)}{27\beta(2+\delta)^2}$. $SS_4 - SS_m = \frac{-4\alpha(2+\delta)+\delta(8+9\delta)}{54\beta(2+\delta)^2}$ and it is easy to see that this expression is always positive for $\alpha < \delta$. Taken together with the previous results for $R1$, $R2$ and $R3$, this proves that SS is higher than the NR benchmark if and only if $\alpha < \alpha_s$, where α_s satisfies $1 < \frac{\alpha_s}{\delta} < \frac{8-\delta}{4+\delta}$.

Finally, α_s is only a function of δ and δ is confined to the region $(0,1)$. Hence, we numerically proved that α_s is increasing in δ.

In $R1$, SS is same as the NR benchmark. If we exclude this region, we find that SS is strictly smaller than the NR benchmark when $\frac{\alpha_s}{\delta} < \frac{\alpha}{\delta} < 2$ $\square$

Proof of Proposition 5

In $R1$, the IR cannot enter and the OEM acts like a monopoly without remanufacturing. Hence, the environmental impact is same as the NR benchmark and it is constant in α and δ.

In $R2$, new product quantity is lower than the monopoly without remanufacturing new product quantity. In addition to that, the IR cannot enter the market and cannot remanufacture. Hence, the environmental impact is lower than the NR benchmark. In $R2$, new product quantity increases in α and decreases in δ; therefore, the environmental impact also increases in α and decreases in δ.

In $R3$, the environmental impact is lower than the NR benchmark if and only if $Eq_{n_3} + eq_{r_3} < Eq_m$, where q_{n_3}, q_{r_3} are new and remanufactured product quantities in $R3$ and q_m is the monopoly without remanufacturing new product quantity (which is $\frac{1}{3}$). This can be rewritten as $\frac{e}{E} < \frac{1/3-q_{n_3}}{q_{r_3}} = \frac{(-2+\alpha)\delta^2}{(-8+\delta)\delta+\alpha(4+\delta)}$. Without loss of generality we can assume that $E = 1$, then $\frac{\partial q_{n_3}}{\partial \alpha} + e\frac{\partial q_{r_3}}{\partial \alpha} = \frac{e(-2+\delta)}{3(-2+\alpha)^2\delta} < 0$. Hence the environmental impact is decreasing in α for $R3$. $\frac{\partial^2 q_{n_3}}{\partial \delta^2} + e\frac{\partial^2 q_{r_3}}{\partial \delta^2} = \frac{-2\left(-4(-2+\alpha)\delta^3+e\left(-4\delta^3+\alpha\left(64-48\delta+12\delta^2+\delta^3\right)\right)\right)}{3(-2+\alpha)(-4+\delta)^3\delta^3}$ and denominator of this is always positive. We only need to consider the numerator, $r \triangleq 4(-2+\alpha)\delta^3 - e\left(-4\delta^3 + \alpha\left(64 - 48\delta + 12\delta^2 + \delta^3\right)\right)$. $\frac{\partial r}{\partial e} = 4\delta^3 - \alpha\left(64 - 48\delta + 12\delta^2 + \delta^3\right)$ and this is positive if and only if $\alpha < \frac{4\delta^3}{64-48\delta+12\delta^2+\delta^3}$. By some algebra, it can be shown that $\frac{4\delta^3}{64-48\delta+12\delta^2+\delta^3} < \frac{\delta^2\left(18-8\delta-2\delta^2+\delta^3\right)}{(4-\delta)^2}$; therefore, in $R3$, $\frac{\partial r}{\partial e}$ can never be positive. Hence r is decreasing in e. Since the minimum value of e is 0 and r is decreasing in e, maximum value of the r is $r_{e=0} = 8(-2+\alpha)\delta^3 < 0$. Since the maximum value of the r is negative, the environmental impact is a concave function of δ.

Region	s^*	p_n^*	p_r^*	q_n^*	q_r^*
$R1^p$	$\frac{1}{3\beta}$	$\frac{2}{9\beta}$	$\frac{\alpha}{9\beta}$	$\frac{1}{3}$	0
$R2^p$	$\frac{2\delta}{3\alpha\beta}$	$\frac{4\delta}{9\alpha\beta}$	$\frac{4\delta^2}{9\alpha\beta}$	$\frac{1}{3}$	0
$R3^p$	$\frac{2(1-\delta)}{3\beta(2-\alpha-\delta)}$	$\frac{4(1-\delta)^2(8-2\alpha-3\delta)}{9\beta(4-\delta)(2-\alpha-\delta)^2}$	$\frac{2(1-\delta)^2(\delta(8-3\delta)+\alpha(4-3\delta))}{9\beta(4-\delta)(2-\alpha-\delta)^2}$	$\frac{4}{3(4-\delta)}$	$\frac{(8-3\delta)\delta-\alpha(4+\delta)}{3(4-\delta)\delta(2-\alpha-\delta)}$
$R4^p$	$\frac{1}{3\beta}$	$\frac{2-\delta}{9\beta}$	$\frac{\delta}{9\beta}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 6 Price Competition Equilibrium product quality, new and remanufactured product prices and quantities

In $R4$, the environmental impact is lower than the NR benchmark if and only if $\frac{e}{E} < \frac{1/3-q_{n_4}}{q_{r_4}}$ where q_{n_4} and q_{r_4} are new and remanufactured product quantities in $R4$. This is equivalent to $\frac{e}{E} < \frac{\delta}{2}$. In $R4$, new and remanufactured product quantity is only a function of δ. Hence, the environmental impact is constant in α. In this region, if δ increases, new and remanufactured quantities decreases. Therefore, the environmental impact is decreasing in δ. □

Proof of Proposition 6

The equilibrium quality, new and remanufactured product prices and quantities are provided in Table 6.

In the price competition game, using the utility functions for the remanufactured and the new product, it is straightforward to show that if $q_n > 0$ and $q_r > 0$, $q_n = 1 - \frac{p_n-p_r}{s(1-\delta)}$ and $q_r = \frac{p_n-p_r}{s(1-\delta)} - \frac{p_r}{\delta s}$. Using these, profit functions can be written as $\pi_{OEM}(p_n,p_r) = (p_n - \beta s^2)(1 - \frac{p_n-p_r}{s(1-\delta)})$and $\pi_{IR}(p_n,p_r) = (p_r - \beta\alpha s^2)(\frac{p_n-p_r}{s(1-\delta)} - \frac{p_r}{\delta s})$ for the differentiated market. If $q_r = 0$, then $q_n = 1 - \frac{p_n}{s}$, and the OEM's profit function can be written as usual. Given s, it can be shown that OEM's profit function is piecewise concave in p_n and the IR's profit function is concave in p_r. Following the similar steps as in the proof of proposition 1, we plug in the optimal prices as a function of quality. For $\alpha/\delta \le (>)1$, the OEM's profit function is a piecewise concave function of quality. This can be solved for the equilibrium quality as in the proof of proposition 1. □

Proof of Proposition 7

Similar to the base model, given s, profit functions of both the OEM and the IR are concave in production quantities and in the equilibrium $s < \frac{1}{\beta}$ is satisfied(see the proof of proposition 1). Using these we can show that for $\alpha \ge \delta$ there can be two cases and these are stated as follows:

C1. $q_r^* = 0$ and $q_n^* = \frac{1-\beta s}{2}$ if one of the followings is satisfied:

(a) $0 < n < -\frac{\delta^2}{-16\alpha\beta+8\beta\delta}$ and $0 < s \le -\frac{\delta}{2\beta(-2\alpha+\delta)} - \frac{1}{2}\sqrt{\frac{-16n\alpha\beta+8n\beta\delta+\delta^2}{\beta^2(2\alpha-\delta)^2}} \triangleq s_0$

(b) $0 < n < -\frac{\delta^2}{-16\alpha\beta+8\beta\delta}$ and $s_1 \triangleq -\frac{\delta}{2\beta(-2\alpha+\delta)} + \frac{1}{2}\sqrt{\frac{-16n\alpha\beta+8n\beta\delta+\delta^2}{\beta^2(2\alpha-\delta)^2}} \le s$

(c) $n \ge -\frac{\delta^2}{-16\alpha\beta+8\beta\delta}$

C2. The IR remanufactures but the core constraint does not bind and $q_r^* = \frac{2n+s(2s\alpha\beta-(1+s\beta)\delta)}{s(-4+\delta)\delta}$ and $q_n^* = \frac{-n+s(-2-s(-2+\alpha)\beta+\delta)}{s(-4+\delta)}$ if $n < -\frac{\delta^2}{-16\alpha\beta+8\beta\delta}$ and $s_0 < s < s_1$.

From above, the OEM's profit function is a piecewise function of s. For $C1$, define the profit function as π_1 and for $C2$ define the profit function as π_2. Unconstraint optimum for π_1 is $\frac{1}{3\beta}$ and for π_2 is $\frac{-2+\delta-\sqrt{4-24n\beta+12n\alpha\beta-4\delta+\delta^2}}{6(-2\beta+\alpha\beta)}$. Using a similar approach as in the proof of Proposition 1, one can show that only one of these unconditional optimums can exist at the same time and the profit function is unimodal in s. These lead us the following equlibrium regions and decisions:

1. If $\frac{\alpha}{\delta} \geq 2$, or $2 > \frac{\alpha}{\delta} > 1$ and $n \geq \frac{2\delta-\alpha}{9\beta}$, the IR cannot enter. Equilibrium decisions are as follows: $s^* = \frac{1}{3\beta}$, $q_n^* = \frac{1}{3}$, $q_r^* = 0$.

2. If $2 > \frac{\alpha}{\delta} \geq \frac{8-\delta}{4+\delta}$ and $\frac{2\delta-\alpha}{9\beta} > n$, or $\frac{8-\delta}{4+\delta} > \frac{\alpha}{\delta} \geq \frac{5}{4}$ and $\frac{2\delta-\alpha}{9\beta} > n \geq \frac{-16\alpha+32\delta+8\alpha\delta-28\delta^2+3\alpha\delta^2+3\delta^3}{144\beta-192\alpha\beta+64\alpha^2\beta+24\beta\delta-16\alpha\beta\delta+\beta\delta^2} \triangleq n_0$, the OEM deters IR's entry by increasing quality. The equilibrium decisions are $s^* = s_1$, $q_n^* = \frac{1-\beta s_1}{2}$, $q_r^* = 0$.

3. If $\frac{5}{4} > \frac{\alpha}{\delta} \geq 1$ and $\frac{2\delta-\alpha}{9\beta} > n \geq n_0$, the OEM deters the IR's by reducing quality. The equilibrium decisions are $s^* = s_0$, $q_n^* = \frac{1-\beta s_0}{2}$, $q_r^* = 0$.

4. If $\frac{8-\delta}{4+\delta} > \frac{\alpha}{\delta} \geq 1$ and $n_0 > n$, the IR enters the market but does not remanufacture all available cores. The equilibrium decisions are $s^* = \frac{-2+\delta-\sqrt{4-24n\beta+12n\alpha\beta-4\delta+\delta^2}}{6(-2\beta+\alpha\beta)}$, $q_n^* = \frac{-n+s^*(-2-s^*(-2+\alpha)\beta+\delta)}{s^*(-4+\delta)}$, $q_r^* = \frac{2n+s^*(2s^*\alpha\beta-(1+s^*\beta)\delta)}{s^*(-4+\delta)\delta}$.

□

Proof of Proposition 8

The monopoly remanufacturing benchmark achieves a lower environmental impact than the NR benchmark if and only if $eq_r + Eq_n < \frac{1}{3}E$. This leads to the $\frac{e}{E}$ thresholds stated in Proposition 8.

The binding region in the base model includes the binding region in the monopoly remanufacturing benchmark. Therefore, we need three types of comparison to show that maximum $\frac{e}{E}$ ratios in the monopoly remanufacturing benchmark are higher than that of the base model for $\frac{\alpha}{\delta} < 1$. These are as follows:

1. Compare thresholds when core constraint binds both in the monopoly remanufacturing benchmark and the base model.

2. Compare thresholds when core constraint does not bind in the monopoly remanufacturing benchmark and it binds in the base model.

3. Compare thresholds when core constraint does not bind both in the monopoly remanufacturing benchmark and the base model.

Then it can be easily shown that monopoly remanufacturing benchmark thresholds higher than the base model thresholds. □