Vertical Integration under Competition: Forward, Backward, or No Integration?

Yen-Ting Lin  
School of Business Administration, The University of San Diego, San Diego, CA 92110, linyt@sandiego.edu

Ali K. Parlaktürk, Jayashankar M. Swaminathan  
Kenan-Flagler Business School, The University of North Carolina at Chapel Hill, Chapel Hill, NC 27599, pturk@unc.edu, msj@unc.edu

We consider two competing supply chains, each consisting of a supplier, a manufacturer, and a retailer. The suppliers exert effort to improve product quality, and the retailers sell products competitively. Each manufacturer chooses one of the three strategies: forward integration, backward integration, or no vertical integration. We seek for a subgame perfect Nash equilibrium and study the resulting market structure. Moreover, we characterize the effect of vertical integration on profitability, product price, and quality in a competitive setting. Existing literature has shown that, when manufacturers consider only forward integration, they may choose not to vertically integrate in equilibrium. In contrast, we find that, when both forward and backward integration options are considered, disintegration cannot be an equilibrium outcome. In this case, both manufacturers either forward or backward integrate, and the degree of product perishability, cost of quality, and how much consumers value quality are critical for the chosen direction of integration. Furthermore, competition increases attractiveness of backward integration relative to forward integration. We show that, while integrating backward unilaterally is always beneficial, unilateral forward integration can harm a manufacturer’s profitability. Finally, vertical integration can result in a better quality product sold at a lower price.

Key words: Vertical integration, competition, supply chain, quality, pricing

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1. Introduction

Many manufacturers pursue vertical integration to gain competitive edge. Examples of vertical conglomerates who govern the entire supply chain are, however, less common. In practice, diversity in the direction of vertical integration occurs instead: some manufacturers choose to forward integrate retail operations, while others opt to backward integrate supply activities. This observation immediately leads to an important question: When should a manufacturer choose to forward integrate, backward integrate, or stay disintegrated? Furthermore, how do supply chain competition and other market and operational factors affect this decision? These questions motivate our paper.

Forward integration extends a manufacturer’s operational reach to product retailing, tightening
its grip on the demand side. For example, European fashion giant Zara and Los Angeles-based apparel retailer American Apparel manufacture products and sell them through their own retail channels. Tainan Enterprise, a Taiwan-based manufacturer, established its own brand, Tony Wear, in China in the late 1990’s (Ho 2002). Similarly, Pepsi purchased its bottlers for better control over the distribution of its growing product offerings (Collier 2009). Conversely, backward integration stretches a manufacturer’s operations toward the source of raw materials, strengthening its control on the supply side. For instance, steelmaker ArcelorMittal is moving deeper into the mining business to ensure stable material supply (Worthen et al. 2009); likewise, the Chinese apparel manufacturer Esquel, backward integrates supply functions by engaging in cotton farming (Peleg-Gillai 2007).

What is driving manufacturers to vertically integrate? Obviously, vertical integration eliminates the adverse effect of double marginalization. However, forward and backward integrations provide some additional benefits. Forward integration allows a manufacturer to better control its retail price, enabling it to respond to changes in demand more effectively. The value of this benefit increases with the degree of product perishability, the rate at which product popularity changes over time. Such a benefit of forward integration is quite significant for a fashion apparel manufacturer, for example, as it deals with quickly changing consumer trends. On the other hand, backward integration allows a manufacturer to seize a stronger control over its quality of material supply and, thereby, its quality of final products. This occurs because quality of raw materials is one of the important determinants of a final product’s quality. For instance, the quality of fabrics used in its production determines the texture quality of apparel.

Forward and backward integrations benefit firms in different ways, and each of these benefits is well understood in isolation. But we want to understand when the benefits of forward integration dominate those of backward integration, and vice versa. How these benefits interact with competitive dynamics in a supply chain is also not clear. These issues lead to a number of research questions. When do firms benefit from vertical integration in a competitive market? How does the choice between forward and backward integration depend on the competitor’s supply chain structure? What is the resulting equilibrium supply chain structure when firms competitively choose their integration strategies? Furthermore, how do product perishability, consumer sensitivity to quality (market factors) and cost of quality (an operational factor) affect answers to these questions?

To address these questions, we consider a model of two competing supply chains, each consisting of a supplier, a manufacturer, and a retailer. The supplier can exert effort to improve the quality of material it supplies to the manufacturer. The manufacturer then makes the product and sells it through its retailer exclusively. The product is sold in two periods, and its popularity, and
thereby the size of its potential market, decreases over time. Endogenizing quality investment and product perishability in our demand model allows us to capture the benefits of vertical integration besides eliminating double marginalization. Each manufacturer in our model chooses one of the three strategies: (1) forward integration, (2) backward integration, and (3) no integration. We seek a Subgame Perfect Nash Equilibrium (SPNE) in this model.

We find that not to vertically integrate at all cannot be an equilibrium outcome when manufacturers competitively choose their integration strategies. This result is in sharp contrast to the celebrated result of McGuire and Staelin (1983), in which disintegration, i.e., not to vertically integrate at all, can be an equilibrium outcome. The key difference in our model that leads to this contrast is that we consider a three-tier supply chain; the manufacturer can choose to either forward or backward integrate in our model, whereas prior studies allow only forward integration in a two-tier supply chain. In other words, we show that the result of prior studies (e.g., McGuire and Staelin 1983, Gupta and Loulou 1998, Trivedi 1998) is incomplete: not to vertically integrate at all can be an equilibrium strategy only when a backward integration option is not available.

In equilibrium, both manufacturers choose to vertically integrate in the same direction. The chosen direction depends on how benefits of more closely managing demand and supply side compare to each other. When demand (supply) side dynamics dominate, manufacturers choose forward (backward) integration. On the one hand, when the product is highly perishable, which implies a significantly lower second period demand, controlling retail price becomes more important. This in turn makes forward integration more attractive. On the other hand, when return from a quality investment is low due to either high cost of improving quality or low consumer sensitivity, an independent supplier would underinvest in quality too much. Therefore, backward integration, which allows directly controlling quality, becomes more attractive.

We find that the equilibrium exhibits a prisoner’s dilemma: both manufacturers choose to vertically integrate (either backward or forward) in equilibrium, and they both suffer. Furthermore, competition increases the attractiveness of backward integration relative to forward integration. Specifically, for the same demand and cost parameters, a manufacturer is more likely to choose backward integration in duopoly than in monopoly.

Moving from disintegration to backward integration always benefits a manufacturer regardless of its competitor’s integration strategy. In contrast, integrating forward unilaterally can hurt a manufacturer because it intensifies the retail competition, which in turn drops the retail price and hurts the manufacturer’s margin. Such a drop is less severe when the competing supply chain has
fewer intermediaries. Therefore, when its competitor vertically integrates, a manufacturer becomes more likely to choose forward over backward integration.

Controlling the retail end of the supply chain can enable improving brand perception. This improvement may increase consumer willingness to pay in some settings. Thus, we also study what happens when forward integration results in pricing advantage by reducing consumer price sensitivity. In this case, a forward-integrated manufacturer can make even the backward integration option unprofitable for the competing manufacturer.

Finally, we show that vertical integration (either forward or backward) results in a higher quality product sold at a lower retail price. Vertical integration lowers the retail price of a product because it reduces the number of intermediaries profiting from it. This benefit alleviates double marginalization and encourages more investment in quality improvement.

In the remainder of this paper, Section 2 presents our literature review. Section 3 describes our model. Section 4 characterizes the equilibrium decisions and discusses the value of vertical integration. Section 5 presents extensions to the base model. Finally, Section 6 offers our concluding remarks.

2. Related Literature

A number of papers in Operations Management and Marketing literature study the structure of competing supply chains. In their seminal work, McGuire and Staelin (1983) consider two competing supply chains, and they show that staying disintegrated can be an equilibrium for manufacturers. A number of papers have extended this result, confirming that vertically integrating with a downstream retailer may not be the profit-maximizing strategy for a manufacturer under various settings. For example, Moorthy (1988) finds that manufacturers may not vertically integrate even when competing products are complements. Coughlan (1985) extends the linear demand model of McGuire and Staelin (1983) to a more general demand model. Trivedi (1998) allows retailers to carry products of multiple competing manufacturers. In Gupta and Loulou (1998), manufacturers can invest in research and development to reduce unit production cost. In Gupta (2008), a manufacturer’s cost reduction investment also helps its competitor reduce its cost due to involuntary technology spillovers. In Liu and Tyagi (2011), retailers determine their competitive product position on a Hotelling line. Wu et al. (2007) allow for demand uncertainty. They show that whether demand variability affects the equilibrium supply chain structure depends on the form of the demand probability distribution function: when it takes a certain form, higher variability favors vertical integration.
All of these papers show that McGuire and Staelin (1983)’s disintegration result continues to hold in various scenarios; in contrast we show that the disintegration result does not hold in our model. The key difference that leads to this contrast is that these papers, including McGuire and Staelin (1983), consider two-tier supply chains and they allow only forward integration, whereas our model considers a three-tier supply chain allowing both forward and backward integrations.

Corbett and Karmarkar (2001) show that when two competing supply chains both adopt vertical integration, they both suffer a drop in their profitability. However, Corbett and Karmarkar (2001) do not study whether firms would choose vertical integration in equilibrium; in contrast, we study the vertical integration strategies manufacturers choose in equilibrium. Cho (2011) extends Corbett and Karmarkar (2001) by studying horizontal merger among firms within the same tier of a supply chain, whereas we focus on vertical integration. Cho (2011) finds that retail price is less likely to drop when the merger occurs in downstream rather than upstream of a supply chain. Pun and Heese (2010) consider two competing manufacturers who sell two complementary products through retailers, and they find that manufacturers can benefit from staying disintegrated as this mitigates their competition. Boyaci and Gallego (2004) consider two supply chains who compete on fill-rate assuming an exogenously determined retail price. In contrast firms compete on price and quality in our model. They identify a prisoner’s dilemma: both supply chains prefer having coordination among its manufacturer and retailer even though both achieve a lower profit.

There is a stream of papers addressing when a firm should outsource production or keep it in-house, which is essentially a form of backward integration. Cachon and Harker (2002) and Gilbert et al. (2006) show that competing firms may benefit from outsourcing production to a common supplier as this dampens the price competition. In our model, each manufacturer has a distinct supplier instead of a common one. Van Mieghem (1999) and Kostamis and Duenyas (2009) allow firms to continue producing in-house while outsourcing a part of their production.

Our paper differs from the aforementioned papers in three critical aspects. First, while we study a three-tier supply chain, they all consider vertical integration in two-tier supply chains, and therefore they cannot address the choice between forward and backward integration. Second, our paper differs in that the product quality is endogenously determined. Finally, there are two periods which allow capturing the product perishability. These elements enable us to capture quality and pricing benefits of vertical integration, and they are critical to our research questions and their results; dropping any of them leads to trivial outcomes.

Another relevant line of papers study the channel conflict that arises when a manufacturer competes with its retailer by directly selling to consumers, which is a form of forward integration.
These papers explicitly model the consumer choice between channels. They consider attributes like product availability, delivery lead time, and relative inconvenience of each channel (Chen et al. 2008) and consumer cost for searching online or traveling (Cattani et al. 2006, Guo and Liu 2008) in addition to prices. In these papers, the manufacturer decides whether to add a direct channel to compete with its independent retailer; the structures of the channels, however, are fixed. In contrast, in our paper, there are already two competing channels, and each one determines its structure by deciding whether to integrate vertically. Furthermore, these papers study competition within the supply chain of the same product produced by the same manufacturer, that is, competing products are identical. In contrast, competing supply chains independently determine the quality of their products in our model.

Our work also relates to papers that study joint quality and pricing decisions in a vertically disintegrated supply chain. Kaya and Özer (2009) find that vertical disintegration by outsourcing production to a contract manufacturer whereby the contract manufacturer decides product quality results in a lower quality product similar to our results. However, they find that vertical disintegration results in a lower price contrary to our findings. This contrast is due to difference in the sequence of events. In Kaya and Özer (2009), the downstream firm dictates the wholesale price by offering a two-part tariff contract before the upstream firm chooses the product quality, whereas in our paper the upstream firm dictates the wholesale price after choosing product quality, which leads to a higher wholesale price. Thus, in Kaya and Özer (2009), the downstream firm can affect the upstream firm’s quality choice through its contract terms, and not knowing the upstream firm’s cost or inability to contract on quality hurts the efficiency of its contract. In contrast, in our model the downstream firm can affect the product quality only by vertically integrating with the upstream firm. Furthermore, when considering vertical integration, firms need to take into account the competitor’s reaction. Our focus is on quantifying the impact of competition on vertical integration, whereas their focus is on quantifying the effect of quality risk due to asymmetric quality cost information and non-contractable quality in vertical disintegration. Similarly, Gilbert and Cvsa (2003) and Xu (2009) also consider joint quality and pricing decisions in a disintegrated supply chain, but they too do not consider competition and their focus is different. In Heese (2010), a retailer sells its own store-brand product in addition to the competing product of a national manufacturer, where both the retailer and manufacturer choose the quality of their products.

Finally, vertical integration can eliminate double-marginalization and achieve coordination, which is a very important issue for supply chains. There is quite a rich literature on this subject and Cachon (2003) and Kaya and Özer (2012) provide excellent reviews of this literature.
3. Model

We consider two competing supply chains \((i = 1, 2)\) selling products to a consumer market over two periods \((t = 1, 2)\). In the following, we introduce our consumer choice model, firm decisions and cost structure, and vertical integration choices. Table 1 summarizes the parameters and decision variables of our model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>(t)</td>
<td>Time period</td>
</tr>
<tr>
<td>(k)</td>
<td>Size of consumer population in the second period</td>
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<tr>
<td>(m)</td>
<td>Consumer reservation value</td>
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<tr>
<td>(\alpha)</td>
<td>Consumer quality sensitivity</td>
</tr>
<tr>
<td>(d)</td>
<td>Consumer disutility per unit deviation from the ideal product</td>
</tr>
<tr>
<td>(\psi_{i,t})</td>
<td>Distance between product (i) and a consumer’s ideal product in period (t)</td>
</tr>
<tr>
<td>(\theta_i)</td>
<td>Quality of product (i)</td>
</tr>
<tr>
<td>(r_i)</td>
<td>Raw material price charged by supplier (i)</td>
</tr>
<tr>
<td>(w_i)</td>
<td>Wholesale price charged by manufacturer (i)</td>
</tr>
<tr>
<td>(p_{i,t})</td>
<td>Retail price of product (i) in period (t)</td>
</tr>
<tr>
<td>(Q_{i,t})</td>
<td>Sales quantity of product (i) in period (t)</td>
</tr>
<tr>
<td>(c)</td>
<td>Cost coefficient for quality improvement</td>
</tr>
<tr>
<td>(N, F, B)</td>
<td>No integration, forward integration and backward integration respectively</td>
</tr>
<tr>
<td>(I_i)</td>
<td>Manufacturer (i)’s vertical integration strategy</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(\frac{cd}{2})</td>
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Following Salop’s (1979) spatial differentiation model, we assume consumers are utility maximizers and they are uniformly distributed along a circle at \(\frac{1}{2}\) units of density in each period.\(^1\) Each consumer is identified by a point on the circle which represents her ideal product. The two competing products are located at the two ends of the diameter.\(^2\) Each customer stays in the market only for one period, and a new batch of customers arrive in the next period. If buying a product yields nonnegative utility, the customer purchases one unit of the product that gives her the highest utility. Here

\[
U(\theta_i, p_{i,t}, \psi_{i,t}) = m + \alpha \theta_i - p_{i,t} - d\psi_{i,t}
\]

shows the utility from purchasing product \(i\) (i.e., the product of supply chain \(i\)) in period \(t\), where \(m\) shows consumer reservation value, \(\theta_i\) represents the quality of product \(i\), and \(p_{i,t}\) is the retail price of product \(i\) in period \(t\). Here, \(\alpha\) captures consumer sensitivity to product quality. We fix the

\(^1\) Density serves as a scaling factor; changing it does not alter our insights.

\(^2\) It is well known in equilibrium, symmetric duopoly firms choose locations at each end of the diameter of a circular market (c.f., Salop 1979). Thus, our circular market is identical to using Hotelling’s (1929) model with duopolists located on each end of the Hotelling line.
consumer price sensitivity, i.e., the coefficient to $p_{i,t}$ in (1), to 1. However, we relax this assumption and present additional insights in Section 5.1. Finally, $\psi_{i,t}$ is the shortest distance between product $i$ and the consumer’s ideal product as Figure 1.b illustrates, and a consumer incurs disutility $d > 0$ per unit of distance.

**Figure 1 Consumer Demand Model**

The product popularity decreases over time. Specifically, the mass of potential customers, i.e., market size, in period 1 is normalized to 1, but it shrinks to $k < 1$ in period 2. Thus, in $t = 2$, there are fewer consumers and they are distributed on a smaller circle as seen in Figure 1.a. In other words, differentiation between the products gets smaller in period 2. This approach assumes that customers who buy a product in period 2 do not have strong preferences between the products (relative to period 1 customers), and the price becomes a relatively more important factor.

Here, $k$ measures the demand perishability: A smaller $k$ indicates faster decrease in the product popularity over time, thus the demand perishability is proportional to $1 - k$. The demand perishability plays a critical role in the manufacturers’ forward integration decisions. Specifically, a smaller $k$ corresponding to highly perishable demand makes directly controlling the price through forward integration more valuable as it allows responding to changes in demand better.\(^3\)

Each supply chain $i$ consists of a supplier ($S_i$), a manufacturer ($M_i$) and a retailer ($R_i$), and all firms are risk neutral profit maximizers. A supplier provides raw materials to its downstream manufacturer at a unit material price $r_i$, which is determined by the supplier. Supplier $i$ invests in material quality which in turn determines the product quality $\theta_i$. Manufacturer $i$ produces each product $i$ with one unit of raw material and sells it to retailer $i$ at its chosen unit wholesale price $w_i$. Finally, retailer $i$ determines the retail price $p_{i,t}$ for product $i$, in each period $t$, and sells it in the consumer market.

\(^3\) Alternatively, the market sizes in periods 1 and 2 can be $1 - k$ and $k$ respectively with $k < 1/2$, so the total market size over two periods is independent of $k$. All of our key insights continue to hold in this alternative model.
The material price $r_i$ and wholesale price $w_i$ do not change across periods because firms often sign relatively long term contracts with their suppliers; long term contracts are common in B2B settings in many industries such as textile, computer hardware, telecommunication, mining and energy (Saigol 2012, AT&T 2012, Hachman 2012). In contrast, we allow the retail price $p_i$ to be adjusted from period to period, reflecting the fact that a retailer has more flexibility in pricing.

Supplier $i$ incurs a fixed cost $c\theta_i^2$ for achieving quality level $\theta_i$, where $c$ determines how expensive it is to improve quality. This assumes quality improvement is achieved through process improvement. Indeed, it is not uncommon to see quality improvement examples as fixed cost investments in practice. For example, Esquel, a major Chinese apparel manufacturer, provides its cotton suppliers training in process improvement techniques, such as seed selection and impurities elimination, to improve cotton quality (Peleg-Gillai 2007). Similarly, advances in spinning and knitting technologies improve the production process, allowing yarns to produce fabrics with superior quality (Bainbridge 2009). In addition, to keep our focus on the impact of vertical integration, we assume firms do not incur variable costs for production and retailing, and they have sufficient capacity to fulfill any demand.

As it will be evident, the fact that quality is endogenously determined plays a critical role in the manufacturers’ backward integration decisions in our model. Backward integration enables a manufacturer to better coordinate quality choice in its supply chain by directly controlling quality decision, but the manufacturer needs to absorb the quality investment cost. In fact, Appendix B shows that when the quality investment cost disappears, backward integration is always preferred. Furthermore, vertical (both forward and backward) integrations create an additional dynamic due to endogenous quality: reduction in double marginalization incentivizes further investment in quality, and this in turn can lead to an additional benefit from vertical integration.

While a number of papers including ours treat quality improvement as a fixed cost investment (e.g., Bonanno 1986, Demirhan et al. 2007, Bhaskaran and Krishnan 2009, Kaya and Özer 2009), there are others assuming quality improvement accompanies an increase in marginal production cost. Both examples are prevalent in practice, and our model considers a setting in which quality is improved by a one-time investment characterized by a fixed cost.

As Figure 2 indicates, we envision three integration strategies for each manufacturer: no integration ($N$), forward integration ($F$) and backward integration ($B$). When a manufacturer does not integrate, its material supply and product retail are accomplished through other independent firms.

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4 We also examine the situation where manufacturers can set a different wholesale price for each period in Section 5.2.
In that case, the manufacturer has control only over the wholesale price it charges to the retailer. When a manufacturer forward integrates, it sells the product through its own company stores and controls the retail price $p_i$. Finally, when a manufacturer backward integrates, it performs supply operations in-house and controls the quality level $\theta_i$.

When a manufacturer is indifferent between the integration options, we break the tie in favor of no integration over integration, and forward over backward integration. This refinement does not affect our insights. We follow this convention for ease of exposition, otherwise multiple equilibria exist at the boundary points of equilibrium regions. We use $I_i \in \{N, F, B\}$ to denote manufacturer $i$’s integration strategy, and $I_1 I_2$ to denote different scenarios of supply chain structures in the industry.

Let $Q_{i,t}$ be the sales quantity of product $i$ in period $t$. Then the profit functions for the retailer $\pi^N_{R_i}$, manufacturer $\pi^N_{M_i}$, and supplier $\pi^N_{S_i}$ in a disintegrated supply chain $i$ are:

\[
\pi^N_{R_i} = \sum_{t=1}^{2} (p_{i,t} - w_i) Q_{i,t}, \tag{2}
\]
\[
\pi^N_{M_i} = (w_i - r_i) \sum_{t=1}^{2} Q_{i,t}, \tag{3}
\]
\[
\pi^N_{S_i} = r_i \sum_{t=1}^{2} Q_{i,t} - c \theta_i^2. \tag{4}
\]

When manufacturer $i$ forward integrates, it sets the retail price itself. In this case, the profit function for manufacturer $i$ becomes:

\[
\pi^F_{M_i} = \sum_{t=1}^{2} (p_{i,t} - r_i) Q_{i,t}. \tag{5}
\]

On the other hand, backward integration allows manufacturer $i$ to dictate its quality level. This yields the following profit function for manufacturer $i$:

\[
\pi^B_{M_i} = w_i \sum_{t=1}^{2} Q_{i,t} - c \theta_i^2. \tag{6}
\]

To facilitate our discussion, we define $x^+ = max(x, 0)$ and

\[
\gamma = \frac{cd}{\alpha^2}. \tag{7}
\]
Essentially, $\gamma$ is a measure of relative return from quality investment, and a high $\gamma$ due to either high cost of quality $c$ or low consumer sensitivity to quality $\alpha$ indicates low return from a quality investment. Likewise, a high $d$, which yields a high $\gamma$, makes quality differentiation relatively less important compared to the spatial differentiation.

Furthermore, we make the following two parametric assumptions to keep our focus on the interesting scenarios and avoid trivial cases. These assumptions are only for ease of exposition; relaxing them, while adds additional equilibrium regions, does not change our insights.

A1. $m > d(\frac{3(5+4k)}{2} - \frac{1+k}{6\gamma})$, otherwise the market is not covered, and firms do not compete, forming local monopolies.

A2. $\frac{m}{\gamma}(11 - \frac{1}{k}) > 1$, otherwise in an asymmetric competition, a disintegrated supply chain competing against a vertically integrated supply chain cannot sell any products in period 2.

Assumption A2 avoids this trivial case by ensuring that the market size in period 2 is sufficiently large (i.e., large $k$) and the relative return from quality investment is not too high (i.e., large $\gamma$) so the advantaged vertically integrated supply chain cannot drive its disadvantaged competitor out of market in period 2.

The sequence of decisions is as follows: First, each manufacturer chooses its vertical integration strategy which determines the supply chain structure $I_1 I_2$ of the industry. Then, firms who control the material supply (a supplier or a backward-integrated manufacturer) competitively determine their quality levels. These firms then set the unit price they charge to their downstream customers. Thereafter, a manufacturer sets its wholesale price if it does not vertically integrate. Finally, the selling season begins, and firms that sell products to consumers (a retailer or a forward-integrated manufacturer) set their retail prices for each period and demand is realized. We seek a Subgame Perfect Nash Equilibrium (SPNE) of this game. When there are multiple equilibria, we eliminate the ones that are Pareto dominated by another equilibrium in the sense of resulting in lower payoffs for all parties. Specifically, there are multiple equilibria in only one scenario and it is explicitly stated in our discussion of equilibrium results. This approach is for ease of exposition and it does not affect our key findings.

Our sequence of events is consistent with the notion that quality decisions are made before pricing decisions as they need more advanced planning (Banker et al. 1998, Desai 2001, Chambers et al. 2006, Xu 2009). Furthermore, it is common to assume that upstream firms move before downstream firms in papers studying vertical integration and the structure of supply chains (McGuire and Staelin 1983, Gupta and Loulou 1998, Corbett and Karmarkar 2001, Wu et al. 2007). We follow this convention so our results can be contrasted with these papers.
4. Equilibrium and Vertical Integration

First, we describe how we derive the equilibrium. We then demonstrate the monopoly benchmark. Subsequently, we present price-quality equilibrium for two competing supply chains for all possible supply chain structures \( I_1 I_2 \). By comparing our results across different supply chain structures, we quantify the value of forward and backward integration, which also allows characterizing the vertical integration equilibrium. Proofs for all of our analytical results are provided in the online appendix.

4.1. Characterization of Equilibrium

When none of the manufacturers vertically integrate \((NN)\), a SPNE satisfies the following for \( i = 1, 2 \) and \( t = 1, 2 \):

\[
\begin{align*}
\theta^*_i &= \arg \max_{\theta_i} \sum_{t=1}^{2} Q_{i,t}(\theta, p^*(r^*, \theta)) - c\theta_i^2, \\
r^*_i &= \arg \max_{r_i} \sum_{t=1}^{2} Q_{i,t}(\theta, p^*(r, \theta)) - c\theta_i^2, \\
w^*_i &= \arg \max_{w_i} (w_i - r_i) \sum_{t=1}^{2} Q_{i,t}(\theta, p^*(w)), \\
p^*_i &= \arg \max_{p_{i,1}, p_{i,2}} \sum_{t=1}^{2} (p_{i,t} - w_i) Q_{i,t}(\theta, p).
\end{align*}
\]

Problems (8) to (11) formulate the optimization problems for the suppliers, manufacturers, and retailers. Essentially, problems (8) and (9) state each supplier chooses the quality and material price to maximize its profit. Problem (10) indicates each manufacturer sets the wholesale price to maximize its profit. Finally, Problem (11) states each retailer maximizes its profit by setting the retail price.

When manufacturer \( i \) vertically integrates, it no longer solves problem (10) and a new problem arises: when manufacturer \( i \) forward integrates, it sets retail prices and solves problem (11) with \( w_i \) replaced by \( r_i \). Alternatively, when manufacturer \( i \) backward integrates, it determines its quality \( \theta_i \), solving problems (8) and (9) with \( r_i \) replaced by \( w_i \). We derive the equilibrium by applying backward induction, essentially solving problems (8) to (11) in reverse order.

Let us first consider a monopolist supply chain as a benchmark; contrasting this benchmark with our main model allows us to extract the impact of competition. When the monopolist supply chain serves the entire market, vertical integration can not increase its demand (it has no impact on the quality and retail price as well). Thus, we confine our monopoly benchmark to regions in which the manufacturer is not limited by the market size in both periods, which is ensured by
Together with assumption A1, this means that the market is sufficiently big for a single firm so there is potential for increasing sales, but sufficiently small for two firms so they compete.

**Proposition 1.** Consider a monopolist supply chain.

(i) There exists a unique SPNE, and the equilibrium decisions are described in Appendix A.

(ii) The manufacturer and the entire supply chain always achieve higher profits when the manufacturer moves from disintegration (N) to forward (F) or backward (B) integration.

Vertical integration eliminates double-marginalization. It can improve sales only when the market is not already covered. In this case, it always increases profitability of the manufacturer and the entire supply chain.

Now we return our focus to duopolist supply chains. In this case, firms need to take both its competitor’s and downstream partner’s actions into account when making decisions.

**Lemma 1.** Consider two competing supply chains. There is a unique SPNE of price-quality competition in all supply chain scenarios \( I_1, I_2, I_i \in \{N, F, B\} \). Equilibrium product quality \( \theta_i \), retail price \( p_{i,t} \) and total sales quantity \( Q_i \) for each scenario are as follows.

(i) When none of the manufacturers vertically integrates, \( NN \):

\[
\theta_1 = \theta_2 = \frac{(1+k)\alpha}{6c}, \quad p_{1,1} = p_{2,1} = d(7+6k), \quad p_{1,2} = p_{2,2} = d(6+7k), \quad Q_1 = Q_2 = \frac{1+k}{2}.
\]

(ii) When only manufacturer 1 forward or backward integrates, \( FN \) or \( BN \):

\[
\theta_1 = \frac{(1+k)(63\gamma-2)\alpha}{12c(27\gamma-1)}, \quad p_{1,1} = \frac{d(9\gamma(55+43k)-4(4+3k))}{4(27\gamma-1)}, \quad p_{1,2} = \frac{d(9\gamma(43+55k)-4(3+4k))}{4(27\gamma-1)}, \quad Q_1 = \frac{(1+k)(63\gamma-2)}{4(27\gamma-1)};
\]

\[
\theta_2 = \frac{(1+k)(45\gamma-2)\alpha}{12c(27\gamma-1)}, \quad p_{2,1} = \frac{d(18\gamma(14+11k)-(11+9k))}{2(27\gamma-1)}, \quad p_{2,2} = \frac{d(18\gamma(11+14k)-(9+11k))}{2(27\gamma-1)}, \quad Q_2 = \frac{(1+k)(45\gamma-2)}{4(27\gamma-1)}.
\]

(iii) When both manufacturers forward or backward integrate, \( FF, BF, FB \) or \( BB \):

\[
\theta_1 = \theta_2 = \frac{(1+k)\alpha}{6c}, \quad p_{1,1} = p_{2,1} = \frac{d(5+3k)}{2}, \quad p_{1,2} = p_{2,2} = \frac{d(3+5k)}{2}, \quad Q_1 = Q_2 = \frac{1+k}{2}.
\]

When only manufacturer 1 is vertically integrated, Lemma 1.(ii) states that \( FN \) and \( BN \) scenarios share the same equilibrium quality, price and sales quantity outcomes. This is because the sequence of events and decisions are identical in these scenarios. Manufacturer 1, however, achieves different profits. In the \( BN \) scenario, it collects the profit of a supplier in a two-tier supply chain, whereas it collects the profit of a retailer in the \( FN \) scenario. Likewise, when both manufacturers are vertically integrated as in Lemma 1.(iii), \( FF, BF, FB \) and \( BB \) scenarios have the same equilibrium quality, price and sales quantity outcomes. Consistent with the sequence of events, it

\[\text{5} \text{Recall assumption A1 in our duopoly model assumes that two firms cover the market so they compete with each other rather than forming local monopolies. However, vertical integration can help a manufacturer increase its demand in both periods as one firm does not cover the entire market by itself since assumption A2 ensures the other firm has a positive market share.}\]
is straightforward to show that the supplier’s margin is higher than the manufacturer’s, which is in turn higher than the retailer’s in a disintegrated supply chain (this is also true in monopoly benchmark).

### 4.2. Vertical Integration Equilibrium

Having characterized price-quality equilibrium, we now turn our focus to the choice of vertical integration strategies and their impact on profitability. Specifically, in the following, Lemmas 2 and 3 show the value of forward and backward integration in a competitive market, and then Proposition 2 describes the chosen vertical integration strategies in equilibrium.

Let \( \Pi_{M_1}^{1I_2} \) be the equilibrium profit of manufacturer 1 when manufacturer \( i, i = 1, 2, \) chooses strategy \( I_i \). Recall that vertical integration always benefits a monopolist manufacturer (Proposition 1). In contrast, the next proposition shows that vertical integration can actually harm profitability in a competitive setting.

**Lemma 2.**

(i.a) \( \Pi_{M_1}^{1FN} \geq \Pi_{M_1}^{1NN} \) if and only if \( k \leq \delta_1 \).

(ii) \( \Pi_{M_1}^{1F2} \geq \Pi_{M_1}^{1N2} \) for \( I_2 \in \{F, B\} \) if and only if \( k \leq \delta_2 \).

(iii) \( \Pi_{M_1}^{1B2} > \Pi_{M_1}^{1N2} \) for \( I_2 \in \{F, N, B\} \).

Here, \( \delta_1 = 1 - \frac{2(8 - 396\gamma + 4779\gamma^2)^+ - 4(27\gamma - 1)\sqrt{(8 - 396\gamma + 4779\gamma^2)^+}}{4 - 180\gamma + 1863\gamma^2} \) and \( \delta_2 = 1 - \frac{2(8 - 324\gamma + 3159\gamma^2)^+ - 4(27\gamma - 1)\sqrt{(8 - 324\gamma + 3159\gamma^2)^+}}{4 - 108\gamma + 243\gamma^2} \).

In addition to enabling direct control on quality and price, vertical integration reduces double marginalization in a supply chain. Therefore, one may expect it to improve profitability. However, in a competitive market, vertical integration intensifies the price competition, which could negate its benefits. The manufacturer feels the competitive effect more strongly when it forward integrates as it moves closer to price competition. Therefore, while backward integration always benefits a manufacturer as part (ii) illustrates, forward integration can be detrimental as shown in parts (i.a) and (i.b). The benefit of forward integration depends on the level of demand perishability measured by \( k \). Specifically, a small \( k \) indicates a bigger drop in the market size (i.e., the mass of potential customers) in period 2, which makes the ability to directly control pricing by forward integration more valuable. Thus, when \( k \) is sufficiently small, the benefit of forward integration dominates.

The previous lemma discusses the value of vertical integration when the manufacturer moves from disintegration to vertical integration. The next proposition examines the choice between forward and backward integration.
Lemma 3.

(i) $\Pi_{M_1}^{BN} > \Pi_{M_1}^{FN}$.

(ii) $\Pi_{M_1}^{FI} - \Pi_{M_1}^{BI} > \Pi_{M_1}^{FN} - \Pi_{M_1}^{BN}$ for $I_2 \in \{F, B\}$.

(iii) $\Pi_{M_1}^{FI} \geq \Pi_{M_1}^{BI}$ for $I_2 \in \{F, B\}$ if and only if $k \leq \delta_3$.

Here $\delta_3 = 1 - \frac{2(18\gamma - 1)^{\tau} - 6\sqrt{\gamma(18\gamma - 1)^{\tau}}}{9\gamma - 1}$.

Because forward integration intensifies the impact of competition more than backward integration for the manufacturer, backward integration always dominates forward integration against a disintegrated competitor as in part (i). However, when the competing supply chain is vertically integrated, its price reaction is weaker due to fewer firms adjusting their margins. Thus, a vertically integrated competitor improves the relative attractiveness of forward integration as shown in part (ii). Indeed, when the competitor is vertically integrated, forward integration can dominate backward integration as part (iii) demonstrates.

The result in part (iii) depends on how benefits of controlling price and quality compare to each other. Recall that a high $\gamma$, which is defined in (7), indicates a lower return on quality investment. Moreover $\delta_3$ is decreasing in $\gamma$, thus a lower $\delta_3$ means a lower return from quality investment. Therefore, when $\delta_3$ is low, an independent supplier would underinvest in quality too much. Doing so increases the attractiveness of backward integration for the manufacturer as it allows directly controlling quality.

In contrast, when $\delta_3$ is high, quality investment becomes attractive. The suppliers overinvest in quality due to competition, thus the manufacturers do not need to control quality. Furthermore, overinvestment in quality hurts the suppliers’ profitability which gives the manufacturers another reason to stay away from backward integration. On the other hand, a low $k$ indicates a bigger intertemporal drop in market size, and this in turn makes controlling price through forward integration desirable. The overall effect depends on how $k$ compares to $\delta_3$ as shown in part (iii). To sum up, Lemma 3 shows that the choice between forward and backward integration critically depends on the structure of the competing supply chain, return on quality investment, and the degree of demand perishability.

Having characterized each manufacturer’s best response integration strategy in Lemmas 2 and 3, now, we are ready to state integration strategies $I_1^*I_2^*$ the manufacturers choose in equilibrium.

---

6 In line with this finding, when quality investment cost disappears, i.e., quality is exogenously fixed, backward integration is always preferred as seen in Appendix B.

7 As a robustness check, we have verified that this finding continues to hold when total market size is independent of $k$, specifically when period 1 and 2 market sizes are $1 - k$ and $k$ respectively with $k < 1/2$. 
**Proposition 2.**

(i) When manufacturers can choose among no integration (N) and backward integration (B), \( I_1^* I_2^* = BB \).

(ii) When manufacturers can choose among no integration (N) and forward integration (F), then

\[
I_1^* I_2^* = \begin{cases} 
FF & \text{for } k < \delta_1, \\
NN & \text{otherwise}.
\end{cases}
\]

(iii) When manufacturers can choose among no integration (N), both forward (F) and backward (B) integration, then

\[
I_1^* I_2^* = \begin{cases} 
FF & \text{for } k \leq \delta_3, \\
BB & \text{otherwise},
\end{cases}
\]

where \( \delta_1, \delta_2 \) and \( \delta_3 \) are given in Lemmas 2 and 3.

The proposition shows that symmetric manufacturers make symmetric choices. However, Section 5.4 shows that the cost asymmetry can lead to asymmetric equilibrium outcomes. Parts (i) and (ii) provide benchmarks in which only one of the integration options is available. Their results immediately follow from Lemmas 2 and 3. Specifically, backward integration is always desirable whereas a smaller \( k \) indicating higher degree of demand perishability makes forward integration desirable.

Part (iii) states the equilibrium for our main model. It shows that staying disintegrated cannot be an equilibrium outcome when the manufacturers consider both forward and backward integration options. This is in sharp contrast to the celebrated result that disintegration can be an equilibrium outcome (e.g., McGuire and Staelin 1983, Gupta and Loulou 1998, Trivedi 1998). The key difference is that in those papers the manufacturers can choose among no integration and forward integration (backward integration option is not available) as in part (ii). Not surprisingly, their findings are consistent with our part (ii) benchmark.

Part (iii) demonstrates both manufacturers choose to vertically integrate in the same direction in equilibrium. Consistent with Lemma 3, the chosen direction depends on how benefits of more closely controlling demand and supply side compare to each other. When \( k \) is small indicating higher degree of demand perishability, there is bigger need for controlling price, whereas when \( \delta_3 \) is small indicating low return from quality investment, the need for controlling quality prevails.

\( ^8 \) Note that both FF and NN can be equilibria for \( \delta_1 \leq k < \delta_2 \) but only NN survives because it is Pareto dominant in the sense that both manufacturers achieve a higher pay-off with NN.
Note that when \( k = 1 \), demand is no longer perishable and both periods become identical, thus the products are sold at the same price in each period. In this case, the model becomes similar to a single-period-model, yielding the same equilibrium structure as in Proposition 2, but quality investment becomes the only dynamic determining the equilibrium outcomes.\(^9\)

The next proposition reveals that the equilibrium in part (iii) of Proposition 2 results in a prisoner’s dilemma.

**Proposition 3.** \[ \Pi_{NN}^{M_1} > \Pi_{FF}^{M_1} \text{ and } \Pi_{NN}^{M_1} > \Pi_{BB}^{M_1}. \]

In equilibrium, both manufacturers vertically integrate, and they both suffer due to competing with a stronger competitor. Observe that when backward integration option is not available (i.e., in the setting of McGuire and Staelin 1983), the equilibrium does not necessarily lead to a prisoner’s dilemma as no integration can be an equilibrium outcome.

Proposition 2 characterizes the equilibrium under competition. It is worthwhile to contrast its results with the monopoly benchmark (Proposition 1) to see how competition affects the chosen vertical integration strategy.

**Corollary 1.** *The manufacturers are more likely to choose backward integration under duopoly competition compared to the monopoly benchmark.*

Forward integration increases the intensity of price competition more than backward integration. A monopolist does not need to worry about the price reaction of the competitor and more freely integrates forward whereas the competitor’s response is an issue for a duopolist. Therefore, a manufacturer is more likely to choose backward integration in duopoly than in monopoly.

So far we have showed that vertical integration can hurt a manufacturer. An immediate question becomes how it affects the entire supply chain. The next proposition addresses this question. Let \( \Pi_{C_1}^{I_1I_2} \) be the total equilibrium profit achieved by supply chain 1 when manufacturer \( i, i = 1, 2 \), chooses strategy \( I_i \).

**Proposition 4.**

(i) \[ \Pi_{C_1}^{FI_2} < \Pi_{C_1}^{NI_2} \text{ and } \Pi_{C_1}^{BI_2} < \Pi_{C_1}^{NI_2} \text{ for any } I_2. \]

(ii) \[ \Pi_{C_1}^{I_1I_2} \leq \Pi_{C_1}^{NN}. \]

\(^9\)To make the model exactly identical to a single-period-model, quality cost needs to be scaled to \( \hat{c} = 2c \) and profits need to be divided by two to account for the fact that quality investment is recovered over two periods instead of one.
Part (i) shows that both forward and backward integrations decrease profitability of the supply chain. Thus, even when the manufacturer benefits from vertical integration (as shown in Lemmas 2 and 3), this happens at the expense of its supply chain partners. Note that the adverse effect of vertical integration is due to the competitor’s reaction because in monopoly benchmark vertical integration always improves a supply chain’s total profit.

Proposition 3 indicates that the equilibrium results in a prisoner’s dilemma for manufacturers. Part (ii) of Proposition 4 shows both supply chains suffer in equilibrium as well. The equality in this case occurs when none of the manufacturers vertically integrate (i.e., $I_1 I_2 = NN$); otherwise supply chain profits decrease strictly when equilibrium involves vertical integration.

4.3. Impact on Product Quality, Sales Quantity and Retail Price

The previous section has focused on the impact of vertical integration on profitability. Here, we discuss the impact on sales quantity, product quality and retail price.

**Proposition 5.** For $t = 1, 2$,

(i) $Q_{11}^{I_1 I_2} > Q_{11}^{NI_2}$, $\theta_{11}^{I_1 I_2} > \theta_{11}^{NI_2}$ and $p_{11}^{I_1 I_2} < p_{11}^{NI_2}$, for $I_1 \in \{F, B\}$ and any $I_2 \in \{F, N, B\}$.

(ii) $Q_{22}^{I_1 I_2} < Q_{22}^{NI_2}$, $\theta_{22}^{I_1 I_2} < \theta_{22}^{NI_2}$ and $p_{22}^{I_1 I_2} < p_{22}^{NI_2}$, for $I_1 \in \{F, B\}$ and any $I_2 \in \{F, N, B\}$.

(iii) $Q_{i i}^{I_1 I_2} = Q_{11}^{NN}$, $\theta_{i i}^{I_1 I_2} = \theta_{11}^{NN}$, and $p_{i i}^{I_1 I_2} < p_{i i}^{NN}$, for $I_i \in \{F, B\}$ and $i = 1, 2$.

Interestingly, Proposition 5.(i) shows manufacturer 1’s vertical integration (both forward and backward) results in a better quality product sold at a lower retail price in both periods. Because vertical integration alleviates double marginalization, it encourages more quality investment for product 1, which can potentially increase the retail price. On the other hand, vertical integration removes intermediaries who add their margins to the retail price, and this leads to an opposite force which can lower the retail price. Similarly, vertical integration forces the competitor to drop its prices. This reaction intensifies the price competition which can also lead to a lower retail price. As a result, the latter forces dominate, and the quality of product 1 improves while its retail price drops. Part (ii) shows that the competitor decreases its quality investment because it now faces a stronger opponent (one with less double marginalization inefficiency).

Finally, part (iii) compares what happens when both of the manufacturers vertically integrate to when none of them do so. The retail prices are lower when they vertically integrate due to more intense competition. Furthermore, because no supply chain has the advantage, they equally split the market and choose the same product quality in both scenarios.
5. Extensions

In this section we consider various extensions of our model. Section 5.1 allows forward integration to alter consumer price sensitivity; Section 5.2 allows manufacturers to modify the wholesale price in period 2; Section 5.3 studies retailers’ quality contribution; finally, Section 5.4 allows suppliers to differ in their quality improvement costs.

5.1. Forward Integration and Price Sensitivity

So far we have assumed vertical integration does not alter the consumer price sensitivity. In this section, we relax this assumption and consider what happens when forward integration reduces the consumer price sensitivity. Intuitively, direct contact with consumers through company stores can generate better brand perception which can increase consumers’ willingness to pay. Indeed, it is common to see higher retail prices in company stores than in general retailers, and some examples are provided in Table 2.¹⁰

5.1.1. Symmetric Price Sensitivity

Suppose manufacturers enjoy equal reduction in the consumer price sensitivity when they forward integrate. Specifically, let \( \beta^I_i \) be the consumer price sensitivity to product \( i \) when manufacturer \( i \) chooses strategy \( I, I \in \{F, N, B\} \). Then the base model described in Section 3 entails \( \beta^I_i = 1, I \in \{F, N, B\}, i = 1, 2 \), as (1) shows. In this section, we relax that assumption, allowing \( \beta^F_i = \beta^F < 1 \) while keeping \( \beta^I_i = 1, I \in \{N, B\}, i = 1, 2 \). Furthermore, to account for \( \beta^F < 1 \), we generalize assumption A2 as

\[
\frac{\gamma_2 \beta^F}{3 + \beta^F} (1 - \frac{1}{k}) > 1 - \frac{\beta^F - 1}{k(3 + \beta^F)}
\]

which reduces to the original A2 when \( \beta^F = 1 \). We assume that the price sensitivity advantage of forward integration is not extreme, specifically \( \beta^F > \frac{1 + 3k}{1 - k + 3 + 4k} \), to avoid the trivial case in which a backward-integrated supply chain competing against a forward-integrated supply chain cannot sell any products in a period. We present the resulting equilibrium quality, price and sales quantity for all possible supply chain configurations in Appendix C.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Product</th>
<th>Company Store</th>
<th>Private Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>Price</td>
<td>Retailer</td>
<td>Price</td>
</tr>
<tr>
<td>Columbia Steep Slop Parka Men's Ski Jacket</td>
<td>$181.9</td>
<td>REI</td>
<td>$139.93</td>
</tr>
<tr>
<td>North Face Denali Thermal Women's Jacket</td>
<td>$199</td>
<td>Dick's Sporting Goods</td>
<td>$178.99</td>
</tr>
<tr>
<td>Nike Dri-Fit UV Men's Stripe Golf Polo</td>
<td>$70</td>
<td>Dick's Sporting Goods</td>
<td>$65</td>
</tr>
<tr>
<td>Apple iPod Nano 5th Gen 8GB</td>
<td>$149</td>
<td>Walmart</td>
<td>$133.99</td>
</tr>
<tr>
<td>Sony DSC-T90 Digital Camera</td>
<td>$279</td>
<td>Walmart</td>
<td>$248</td>
</tr>
</tbody>
</table>

¹⁰The prices are collected on February 11, 2010 from both physical and online stores.
While our discussion in the following will focus on new findings due to relaxing $\beta^F = 1$, we want to point that most of the key results of the base model continue to hold. Specifically, Proposition 6 shows vertical integration can be detrimental to a manufacturer, which is consistent with Lemma 2. Proposition 6 also characterizes the conditions that make forward integration more attractive than backward integration similar to Lemma 3. Proposition 8 shows vertical integration can improve product quality while reducing the retail price as in Proposition 5. However, allowing $\beta^F < 1$ complicates the analysis; characterizing equilibrium vertical integration strategies as in Proposition 2 becomes intractable. This difficulty arises because the profit expressions involve high order polynomials, and characterizing equilibrium regions requires comparing multiple high order polynomials. Even though we cannot fully characterize equilibrium integration strategies, Proposition 7 confirms that no integration still cannot be an equilibrium integration strategy when $\beta^F < 1$.

First, we present new findings regarding the value of vertical integration to a manufacturer.

**Proposition 6.**

(i) $\Pi^{FI}_{M_1} > \Pi^{NI}_{M_1}$ if and only if $\beta^F < \tau^I_{1}$ for $I_2 \in \{F, N, B\}$.

(ii) $\Pi^{FI}_{M_1} > \Pi^{BI}_{M_1}$ if and only if $\beta^F < \tau^I_{2}$ for $I_2 \in \{F, N, B\}$.

(iii) $\Pi^{BF}_{M_1} < \Pi^{NF}_{M_1}$ if and only if $\beta^F < \tau_3$; $\Pi^{BI}_{M_1} > \Pi^{NI}_{M_1}$ for $I_2 \in \{N, B\}$.

The thresholds $\tau^I_{1}$, $\tau^I_{2}$ and $\tau_3$ are characterized in the proof in online Appendix E.

When forward integration improves a manufacturer’s pricing advantage, manufacturers are more likely to adopt this strategy. Consequently, forward integration can be beneficial and it can be favored over backward integration when the pricing advantage is significant as in Proposition 6.(i) and (ii).

Proposition 6.(iii) demonstrates an additional insight: the potential benefit of backward integration can be nullified by the competitor’s pricing advantage. This is in direct contrast to our base model where backward integration is always beneficial. When manufacturer 2’s pricing advantage from forward integration is significant, it relies on this advantage and reacts to manufacturer 1’s backward integration by increasing the quality of its product. This in turn decreases manufacturer 1’s return from quality investment when it backward integrates.

Next, we formally state that staying disintegrated cannot be an equilibrium outcome when manufacturers competitively determine their vertical integration strategies.

**Proposition 7.** When $\beta^F < 1$ and manufacturers choose among no integration (N), forward (F), and backward (B) integration, then NN cannot be an equilibrium.
This occurs because backward integration does not affect the consumer price sensitivity. Thus, when a manufacturer chooses to stay disintegrated (N), its competitor can always improve its profitability by backward integration as Lemma 2 shows. Finally, we examine the effect of vertical integration on product quality and sales quantity.

**Proposition 8.** Let $I_1, I_2 \in \{F, N, B\}$:

(i.a) For $(I_1, I_2) = (B, F)$: $\theta_1^{I_1I_2} < \theta_1^{NI_2}$, $Q_1^{I_1I_2} < Q_1^{NI_2}$, $\theta_2^{I_1I_2} > \theta_2^{NI_2}$, $Q_2^{I_1I_2} > Q_2^{NI_2}$ if and only if $\beta^F < \frac{4}{2 + 2\gamma^F}$.

(i.b) For $(I_1, I_2) \neq (B, F)$: $\theta_1^{I_1I_2} \geq \theta_1^{NI_2}$, $Q_1^{I_1I_2} \geq Q_1^{NI_2}$, $\theta_2^{I_1I_2} \leq \theta_2^{NI_2}$, $Q_2^{I_1I_2} \leq Q_2^{NI_2}$.

(ii.a) $p_1^{FI_2} > p_1^{NI_2}$ if and only if $\beta^F < \sigma^{I_2}$.

(ii.b) $p_1^{BI_2} < p_1^{NI_2}$; $p_2^{I_1I_2} \leq p_2^{NI_2}$ for $I_1 \neq B$.

The threshold $\sigma^{I_2}$ is stated in the proof in online Appendix E and the inequalities in (i.b) and (ii.b) hold strictly when $I_1 \neq N$.

When facing a competing supply chain that is already forward integrated, Proposition 8.(i.a) shows that the quality of product 1 gets lower when manufacturer 1 backward integrates, whereas in the base model we find that vertical integration always increases product quality. This is because the competing supply chain relies on its pricing advantage, reacting to manufacturer 1’s backward integration by increasing quality of its product. Consequently, manufacturer 1’s return from quality improvement gets lower, forcing it to lower its product quality. Additionally, contrary to our earlier result where vertical integration always lowers retail price, case (ii.a) shows that forward integration can increase the retail price when the pricing advantage is significant, because consumers become very price insensitive. Thus, proposition 8 (i.b) and (ii.a) suggest that forward integration can increase both product quality and retail price when it has a significant pricing advantage, i.e., $\beta^F < \sigma^{I_2}$.

### 5.1.2. Asymmetric Price Sensitivity

In the previous section, the manufacturers have identical price sensitivity when they forward integrate, i.e., $\beta_1^F = \beta_2^F$. Now, we relax this assumption, allowing for $\beta_1^F \neq \beta_2^F$. Suppose $\beta_1^F < \beta_2^F \leq 1$ without loss of generality.

Similar to Section 5.1.1, we assume the price sensitivity advantage is not extreme, specifically $\frac{(1+3k)^{\beta_2^F} - (1-k)^{\beta_2^F}}{\beta_1^F} < 54\gamma k$, to avoid the trivial cases. Note that when $\beta_2^F = 1$, this assumption becomes identical to that of Section 5.1.1. The resulting equilibrium quality, price and sales quantity for each scenario are summarized in Appendix C. Here, we focus on results concerning competition between two forward integrated manufacturers with asymmetric price sensitivities, which is the most interesting scenario.
Proposition 9. (i) $Q_{1F}^N < Q_{1F}^F$, $\theta_{1F}^N < \theta_{1F}^F$, $Q_{2F}^N > Q_{2F}^F$, and $\theta_{2F}^N > \theta_{2F}^F$.

(ii) There exists $\xi_i^0$ and $\xi_i^Q$, $i = 1, 2$, such that

(a) $Q_{1F}^N < Q_{1F}^F$ if and only if $c < \xi_1^Q$, and $Q_{1F}^N < Q_{1F}^F$ if and only if $c < \xi_1^0$.

(b) $Q_{2F}^N > Q_{2F}^F$ if and only if $c < \xi_2^Q$, and $Q_{2F}^N > Q_{2F}^F$ if and only if $c < \xi_2^0$.

(iii) $\xi_i^0$ and $\xi_i^Q$ are stated explicitly in the proof of the Proposition and they increase in $\beta_2^F - \beta_1^F$.

When forward integration gives manufacturer 1 the pricing advantage ($\beta_1^F < \beta_2^F$), its product quality and sales quantity always increase while those of the competitor decrease as shown in part (i). What happens when the competitor forward integrates is more subtle. When manufacturer 2 forward integrates, the competing supply chain 1 relying on its pricing advantage can respond aggressively by increasing its quality and force supply chain 2 to lower its quality. Part (ii) shows that when quality cost $c$ is sufficiently low, supply chain 1 indeed follows this hostile strategy. However, when quality cost $c$ is sufficiently high, it cannot afford overinvesting in quality. In this case, supply chain 1 accommodates manufacturer 2’s forward integration by reducing its quality and quantity, allowing manufacturer 2 to take advantage of forward integration to have higher quality and sales quantity. Finally, part (iii) demonstrates that the competitor is more likely to respond aggressively to manufacturer 2’s forward integration when the competitor has a bigger pricing advantage.

5.2. Dynamic Wholesale Pricing

In the base model, manufacturers set the same wholesale price for both periods. In this section, we examine what happens if manufacturers can choose their wholesale prices dynamically in each period. Assumptions A1 and A2 described in Section 3 are replaced by $m > d(\frac{3(2+k)}{2} - \frac{1+k}{6\gamma})$ and $\frac{5\gamma}{9}(45 - \frac{9}{k}) + \frac{1}{9k} > 1$ respectively in this extension.

Next proposition shows that our key results continue to hold. Recall that $I_i^1 I_i^2$ denotes manufacturers’ equilibrium strategy.

Proposition 10. Suppose manufacturers set wholesale prices dynamically for each period and manufacturers choose among no ($N$), forward ($F$) and backward ($B$) integration.

(i) The equilibrium integration strategy is

\[ I_i^1 I_i^2 = \begin{cases} 
FF & \text{for } k \leq 1 - \frac{8(18\gamma - 1)^+ - 12\sqrt{7(18\gamma - 1)^+}}{63\gamma^2}, \\
BB & \text{otherwise.}
\end{cases} \]

(ii) Manufacturers encounter prisoner’s dilemma; i.e., $\pi_i^{I_i^1 I_i^2} < \pi_i^{NN}$. 

Proposition 10 shows that the key results of the base model continue to hold. That is, staying disintegrated is never an equilibrium outcome when manufacturers consider both forward and backward integrations. Manufacturers prefer forward integration when the product is highly perishable and backward integration when the return from quality investment is low (the threshold in part (i) decreases in $\gamma$). Furthermore, the equilibrium results in a prisoner’s dilemma. In the next corollary, we examine how manufacturers’ dynamic pricing ability affects their integration choices.

**COROLLARY 2.** *Manufacturers are more likely to backward integrate when they gain dynamic pricing ability.*

When manufacturers cannot set their wholesale prices dynamically, forward integration has the added benefit of enabling pricing flexibility. But when they already have the dynamic pricing ability, this benefit is not as valuable, making forward integration less attractive.

### 5.3. Retailers’ Quality Contribution

In the base model, product quality is determined solely by the supplier. In some settings, retailers can also affect the perceived product quality significantly, for example, by their store design and employee training. We now consider what happens if retailers can also contribute to the product quality. Thus, the consumer utility in (1) is appended as follows with an additional term $b\theta_{ir}$ which is controlled by the retailer.

$$U_{i,t} = m + \alpha \theta_i + b \theta_{ir} - p_{i,t} - d_{i}.$$  \hspace{1cm} (12)

Here, $\theta_{ir}$ is the quality level chosen by the retailer in supply chain $i$, and $b \geq 0$ shows the consumer sensitivity to retailer quality. To avoid introducing any bias, we assume the supplier and the retailer have equal quality cost coefficients. Specifically, the retailer incurs quality cost $c\theta_{ir}^2$. Similar to our base model, quality decisions are made before pricing decisions, and suppliers precede manufacturers and retailers. Specifically, a supplier’s quality decision is followed by that of the retailer, which is followed by the pricing decisions.

Assumptions A1 and A2 of the base model are replaced with $m > \frac{(b/\alpha)^2}{547(729^2 - 2(b/\alpha)^2)} + \frac{243(9^2 (5 + 4k) - (1 + k))}{2(729^2 - 2(b/\alpha)^2)} + \frac{941(11 - \frac{1}{k})}{72} > 1 - (\frac{b}{\alpha})^2(\frac{1}{72} (\frac{1}{k} - 23) + \frac{1}{972})(\frac{b}{\alpha})^2$ respectively. Note these assumptions and our results in this section reduce to those of the base model when $b = 0$. The following proposition shows that our key results continue to hold when retailers contribute to quality.

**PROPOSITION 11.** *Suppose both supplier and retailer control quality as in (12) and manufacturers choose among no (N), forward (F) and backward (B) integration.*

(i)
vertical integration under competition: forward, backward, or no integration?

\[ I_1^* I_2^* = \begin{cases} FF & \text{if } k \leq 1 - \frac{2\eta^+ + 18(81 \gamma - 2(\beta / \alpha)^2) \sqrt{\gamma \eta^+}}{4(\beta / \alpha)^2 - 19683(\beta / \alpha)^2 \gamma^2 + 59049 \gamma^2 (9 \gamma - 1)}, \\ BB & \text{otherwise}, \end{cases} \]

with \( \eta = 4(\beta / \alpha)^6 + 324(\beta / \alpha)^4 \gamma - 45927(\beta / \alpha)^2 \gamma^2 + 59049 \gamma^2 (18 \gamma - 1) \).

(ii) Manufacturers encounter prisoner’s dilemma; i.e., \( \pi_i^{I_1^* I_2^*} \leq \pi_i^{NN} \).

No integration still cannot be an equilibrium outcome when manufacturers consider both forward and backward integrations. Furthermore, the equilibrium results in a prisoner’s dilemma. Moreover, same as in our base model (see Proposition 2), the threshold in part (i) increases in \( \alpha \) and decreases in \( \gamma \) and \( \delta \). Therefore our corresponding insights continue to hold.

Finally, the threshold in part (i) increases in the consumer sensitivity to retailer quality \( b \), indicating that the manufacturers prefer forward integration when the retail quality has a significant impact on the overall perceived product quality. Intuitively, when \( b \) is high, the retailer chooses a high level of quality investment. Because of the resulting high quality product, the retailer tends to sell to a smaller segment, which in turn forces the manufacturers to drop its price to maintain its sales quantity. Forward integration eliminates this problem for the manufacturer.

5.4. Asymmetric Quality Cost

Here we consider what happens when the two supply chains differ in their efficiency of quality improvement. Specifically, \( c_i \) shows the quality improvement cost coefficient of supply chain \( i \).

Suppose \( c_1 > c_2 \) without loss of generality. Let \( c_1 = c \) and \( c_2 = vc_1 \), where \( v \in (0, 1) \). Assumptions A1 and A2 are replaced with \( m > d \left( \frac{1458 \gamma^4 (5 + 4k)v - 9 \gamma (19 + 17k)(1 + v) + 4(1 + k)}{12 \gamma (81 \gamma v - (1 + v))} \right) \) and \( \frac{9 \gamma}{3 + v} (11 - \frac{1}{k}) > 1 + \frac{1 - v}{k(3 + v)} \) respectively, which reduce back to A1 and A2 when \( v = 1 \). Furthermore, we assume that the cost advantage is not extremely high, i.e., \( v > \frac{1 + 3k}{1 - k + 34 \gamma k} \) so that the high cost supply chain can also sell its product in both periods.

**Proposition 12.** Suppose suppliers have asymmetric quality costs such that \( c_2 = vc_1, v \in (0, 1) \) and manufacturers choose among no (N), forward (F) and backward (B) integration.

(i) \( NN \) cannot be an equilibrium.

(ii) \( I_1^* I_2^* = NB \) for \( v \leq \delta_4 \) and \( k \geq min(\delta_6, \delta_7) \).

(iii) \( I_1^* I_2^* = FB \) for \( \delta_5 < k < min(\delta_6, \delta_7) \), \( \delta_4 \) to \( \delta_7 \) are stated in the addendum in Appendix D.

Proposition 12 shows even with asymmetric costs, both firms staying disintegrated cannot be an equilibrium. While Proposition 12 does not state all possible equilibrium outcomes (which are stated in Appendix D), it demonstrates the asymmetry in the quality costs can lead to asymmetry in
equilibrium outcomes. Part (ii) illustrates manufacturer 1 can stay disintegrated while manufacturer 2 integrating backward when manufacturer 2 has a sufficiently strong cost advantage. In this case, manufacturer 1 does not want to increase the intensity of competition due to the competitor’s cost advantage. Finally, part (iii) shows that with the cost asymmetry, manufacturers can choose to integrate in the opposite directions in contrast to our symmetric base model.

6. Concluding Remarks

In this paper we examine equilibrium vertical integration strategies under supply chain competition. To this end, we analyze two competing three-tier supply chains, each with a supplier, a manufacturer and a retailer. Each manufacturer considers three vertical integration strategies: forward, backward and no integration. Forward integration enables a manufacturer to better manage the demand side by directly controlling the retail price, whereas backward integration allows it to better manage the supply side by directly controlling the quality investment.

When competing manufacturers consider solely moving from disintegration to forward integration, a celebrated result in prior studies is that manufacturers may strategically choose to stay disintegrated (e.g., McGuire and Staelin 1983, Gupta and Loulou 1998). Interestingly, we demonstrate this result does not continue to hold when backward integration is also an option for manufacturers. When they have both forward and backward integration options, both manufacturers choose to vertically integrate (either forward or backward) even though this results in a prisoner’s dilemma situation.

We explain how the direction of vertical integration manufacturers choose in equilibrium depends on supply and demand elements. Specifically, when the dynamics of demand perishability dominate, forward integration becomes more attractive. In contrast when the dynamics of product quality dominate, backward integration is preferred. Our equilibrium results characterize this trade-off explicitly.

We also quantify the value of unilaterally implementing vertical integration and how this relates to the structure of the competing supply chain. A vertically integrated competitor compared to a disintegrated competitor improves attractiveness of forward integration relative to backward integration. In addition, backward integration always increases profitability regardless of the structure of the competing supply chain. In contrast, forward integration can be detrimental both when the competitor is vertically integrated and when it is disintegrated.

Examples from practice suggest that forward integration can reduce the consumer price sensitivity. In this case, forward integration can make backward integration unattractive for the competitor.
Basically, with pricing advantage a forward-integrated manufacturer undermines the return from quality investment for its competitor, disallowing the competitor to take advantage of backward integration. While we provide several examples that show forward integration may enable manufacturers to set higher prices in their company stores, there could be opposite examples whereby forward integration may increase the consumer price sensitivity. In those examples, we expect forward integration to be less attractive for manufacturers and it would be less likely an equilibrium outcome.

While manufacturers with symmetric costs never stay disintegrated in equilibrium, the manufacturer with the cost disadvantage can stay disintegrated when there is cost asymmetry in our model. Furthermore, organizational, cultural and geographical barriers, which are omitted in our model, can also stop manufacturers from pursuing vertical integration. These factors are also omitted in McGuire and Staelin (1983), Gupta and Loulou (1998), Trivedi (1998) and they argue that firms choose to stay disintegrated to avoid intensifying competition. Our results show that competition alone cannot explain why firms may choose to stay disintegrated when backward integration is an option in addition to forward integration.

Our model has limitations. We assume that each retailer sells a single product variant. However, in practice most retailers will be selling a product line, thus we expect product proliferation to increase the intensity of price competition. Because forward integration intensifies retail price competition more than backward integration, we expect forward integration to be less valuable than our model predicts. Furthermore, while each manufacturer has a single distinct supplier in our model, they may have multiple suppliers and share some of these suppliers with their competitors. In that case, there will be an additional incentive for backward integration due to the added benefit of controlling the competitor’s supply. This can be a fruitful direction for future research. More broadly, it would be worthwhile to study how the value of vertical integration depends on various supply chain configurations.

Forward and backward integration may have additional benefits beyond what is captured in our model. For example, forward integration may enable a manufacturer to collect more information about customer demand. Moreover, in our model, upstream firms move before downstream firms and make a take-it-or-leave-it offer. This is a common assumption in papers studying vertical integration decisions (McGuire and Staelin 1983, Gray et al. 2009, Pun and Heese 2010, Liu and Tyagi 2011). However, it would be worthwhile for future research to study how vertical integration choices depend on the relative bargaining power of the retailer and supplier within the supply chain using a bargaining framework. Finally, our analysis is restricted to competition between two
supply chains, and an obvious extension would examine the competition between many supply chains. Because retail competition would be fiercer due to more competitors, we expect increasing the number of competitors to deteriorate attractiveness of forward integration relative to backward integration.

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Appendix A: Monopoly Benchmark
The unique SPNE product quality \( \theta \), retail price \( p_t \), \( t = 1, 2 \), and sales quantity \( Q \) for a monopolist supply chain are as follows assuming it does not serve the entire market in both periods.

(i) When the manufacturer does not vertically integrate:
\[
\theta = \frac{m}{6c - \alpha \gamma}, \quad p_1 = p_2 = \frac{7cdm}{6c - \alpha \gamma}, \quad Q = \frac{2cm}{6c - \alpha \gamma}.
\]

(ii) When the manufacturer forward or backward integrates:
\[
\theta = \frac{m}{4cd - \alpha \gamma}, \quad p_1 = p_2 = \frac{3cdm}{4cd - \alpha \gamma}, \quad Q = \frac{2cm}{4cd - \alpha \gamma}.
\]

In equilibrium, the monopolist manufacturer chooses to forward integrate when \( \gamma < \frac{1}{2} \) and it chooses to backward integrate otherwise.

Appendix B: Exogenous Quality Benchmark
Let \( \theta \) be the quality level for both products which is exogenously determined. When the quality is determined exogenously, assumption A1 is replaced by \( m > \frac{3d(5+4k)}{2} - \alpha \theta \) which helps avoiding the trivial case where firms form local monopolies in the NN scenario.

Proposition 13. When quality is exogenously determined, manufacturers always backward integrate when they choose among no (N), forward (F) and backward (B) integration.

Appendix C: Forward Integration Decreasing Price Sensitivity

Lemma 4. Suppose \( \beta^F_1 < 1, \ i = 1, 2 \), the unique SPNE product quality \( \theta_i \), retail price \( p_{i,t} \) and sales quantity \( Q_i \) for each scenario \( I_1I_2, I_i \in \{N, F, B\} \), are as follows.

(i) When none of the manufacturers vertically integrates, NN:
\[
\theta_1 = \theta_2 = \frac{(1+k\ln)}{6c}, \quad p_{1,1} = p_{2,1} = d(7+6k), \quad p_{1,2} = p_{2,2} = d(6+7k), \quad Q_1 = Q_2 = \frac{1+k}{2}.
\]

(ii) When only manufacturer 1 forward integrates, FN:
\[
\theta_1 = (63\gamma - 2)A, \quad p_{1,1} = B_1, \quad p_{1,2} = B_1 - \frac{d(1-k)}{\beta_1}, \quad Q_1 = (63\gamma - 2)C;
\]
\[
\theta_2 = (45\gamma - 2)A, \quad p_{2,1} = B_2, \quad p_{2,2} = B_2 - d(1-k), \quad Q_2 = (45\gamma - 2)C.
\]

(iii) When only manufacturer 1 backward integrates, BN:
\[
\theta_1 = (63\gamma - 2)D, \quad p_{1,1} = E_1, \quad p_{1,2} = E_1 - d(1-k), \quad Q_1 = (63\gamma - 2)F;
\]
\[
\theta_2 = (45\gamma - 2)D, \quad p_{2,1} = E_1 + E_2, \quad p_{2,2} = E_1 + E_2 - d(1-k), \quad Q_2 = (45\gamma - 2)F.
\]

(iv) When both manufacturers vertically integrate, FF, BF, FB or BB:
$$
\theta_1 = (27\gamma \beta_2 - 2)G, p_{1,1} = H_1, p_{1,2} = H_1 - \frac{d(1-k)}{\beta_1}, Q_1 = \beta_1 (27\gamma \beta_2 - 2)J
$$

$$
\theta_2 = (27\gamma \beta_1 - 2)G, p_{2,1} = H_2, p_{2,2} = H_2 - \frac{d(1-k)}{\beta_2}, Q_2 = \beta_2 (27\gamma \beta_1 - 2)J
$$

where

$$
\gamma = \frac{cd}{\alpha^2}, A = \frac{(1+k)\alpha}{6c((54\gamma - 1)\beta_1 - 1)}, B_1 = \frac{d(k-1-15\beta_1 - 13k\beta_2 + \gamma \beta_1 (495 + 387\alpha))}{2\beta_1((54\gamma - 1)\beta_1 - 1)}, B_2 = \frac{d((504\gamma - 1)\beta_1 - 21 + k(396\beta_1 + \beta_1 - 19) - 21)}{2((54\gamma - 1)\beta_1 - 1)}.
$$

$$
C = \frac{(1+k)\alpha}{2(54\gamma - 1)\beta_1 - 2}, D = \frac{(1+k)\alpha}{12c(27\gamma - 17)}, E_1 = \frac{d(495\gamma + 3k(129\gamma - 4) - 16)}{4(27\gamma - 1)}, E_2 = \frac{3d(1+k)(3\gamma - 2)}{4(27\gamma - 1)}, F = \frac{(1+k)}{4(27\gamma - 1)},
$$

$$
G = \frac{\alpha(1+k)}{6c(27\gamma \beta_2 - \beta_1 - \beta_2)}, H_1 = \frac{d(1,35 + 81k) \beta_1 \beta_1 \beta_1 (k-1) \beta_1 - 1)}{2\beta_1(27\gamma \beta_2 - \beta_1 - \beta_2)}.
$$

$$
\beta_1 = \begin{cases} 
1 & \text{if } I_i = B \\
\beta^c < 1 & \text{if } I_i = F
\end{cases}
$$

Appendix D: Addendum to Proposition 12

(iv) $I_1 I_2 = FF$ for $k \leq \delta_5$.

(v) $I_1 I_2 = BB$ for $v > \delta_4$ and $k \geq \min(\delta_6, \delta_7)$.

$\delta_4$ is the largest root of $v$ such that

$$
(27\gamma - 1)(1 + v - 54\gamma v)^2(27\gamma v - 2)^2 + 27\gamma (45\gamma v - 2)^2(1 + v - 27\gamma v)^2 = 0
$$

$\delta_5$ is the smaller root of $k$ such that

$$
\left[ 6561(1 + 6k + k^2)\gamma^3 v^2 - 243\gamma^2 v(1 + 6v + 10k(1 + 2v) + k^2(1 + 6v)) \right.
\left. + 9\gamma(10v - 1 + 7v^2 + k(10v - 1 + 7v^2) + 2k(1 + 14v + 9v^2)) - 4(1 + k)^2 v \right] = 0
$$

$\delta_6$ is the smaller root of $k$ such that

$$
\left[ 4(1 + k)^2 - 6561(1 + 6k + k^2)\gamma^3 v^2 + 243\gamma^2 v(1 + 6v + 10k(1 + 2v)) + k^2(1 + 6v) + 9\gamma(v^2 - k^2(7 + 10v - v^2) - 2k(9 + 14v + v^2) - 7 - 10v) \right] = 0
$$

$\delta_7$ is the smaller root of $k$ such that

$$
\left[ (1 + v)^2(-7 + 2v + v^2 - k^2(7 - 2v - v^2) - 2k(9 + 2v + v^2)) - 177147\gamma^4 v^4 \\
+ 150k + k^2) + 13122\gamma^3 v(7 + v + k^2(7 + v) + k(86 + 74v)) \\
- 243\gamma^2 v(43 + 38v - 17v^2 + k^2(43 + 38v - 17v^2) + 2k(121 + 194v) + 61v^2) - 54\gamma v(1 + v)(-9 + 2v + 3v^2 - k^2(9 + 2v - 3v^2) - 2k(15 + 14v + 3v^2)) \right] = 0
$$

References

AT&T. 2012. TNCI signs long-term wholesale contract with AT&T for voice, data and IP networking solutions. AT&T news room.


