Online Appendix

Appendix E: Proofs

Proof of Proposition 1

First we derive the equilibrium when the manufacturer does not vertically integrate. We drop the subscript i for notational simplicity. Following backward induction, first consider the retailer's pricing decision. Inverting the marginal customer's utility, the retailer's demand in period t, assuming the market is not fully covered, is $Q_t = (m - p_t + \alpha\theta)/d$, and its profit is $\pi_R = \sum_{t=1}^{2}(p_t - w)Q_t$. Then the first order condition shows that the optimal price is $p_1 = p_2 = (m + w + \alpha\theta)/2$ and the total sales quantity is $Q_1 + Q_2 = (m - w + \alpha\theta)/d$. The manufacturer maximizes its profit $\pi_M = (w - r)(Q_1 + Q_2)$, which gives the optimal wholesale price $w = (m + r + \alpha\theta)/2$. Finally, the supplier maximizes its profit $\pi_S = r(Q_1 + Q_2) - c\theta^2$ by first choosing its quality θ and then its price r. Similarly, the first order conditions lead to the optimal price $r = (m + \alpha\theta)/2$ and quality $\theta = \frac{m\alpha}{8cd-\alpha^2}$, and thus the equilibrium decisions described in this proposition. Here, we need $m < \frac{dk}{\gamma}(8\gamma - 1)$ to avoid full market coverage in period 2, i.e., every customer purchases the product. This condition is obtained by requiring $Q_2 < k$, that is, the sales quantity in period 2 does not exceed the market size. Note we assume $m < \frac{dk}{\gamma}(4\gamma - 1)$ which immediately implies $m < \frac{dk}{\gamma}(8\gamma - 1)$. Also, the uniqueness of this SPNE is straightforward because firm profits are concave in every decision variable.

Similarly, we derive the unique equilibrium decisions when the manufacturer backward or forward integrates. These two cases yield the same equilibrium decisions because the sequence of events are identical. Here, we need $m < \frac{dk}{\gamma}(4\gamma - 1)$ to avoid full market coverage in period 2, which is assumed in Section 4.1.

Using the SPNE decisions, we derive firm profits and the second result in the proposition is straightforward by comparing firm profits across integration scenarios. Specifically, let Π^I, $I \in \{N, F, B\}$, be the manufacturer's equilibrium profit. Then it can be shown that $\Pi^F - \Pi^N = \Pi^B - \Pi^N = \frac{2c^2dm^2(32c^2d^2-\alpha^4)(12(cd)^2+(6cd-\alpha^2)^2)}{(8cd-\alpha^2)^2(4cd-\alpha^2)^2} > 0.$ □

Proof of Lemma 1

We present the proof for NN scenario; the equilibrium decisions for other scenarios can be derived following the same procedure. Following backward induction, first we derive the retailers' equilibrium retail prices for each period assuming products compete. Retailers i's sales quantity $Q_{i,t}$ can be obtained by solving $U(\theta_i, p_{i,t}, Q_{i,t}) = U(\theta_j, p_{j,t}, \rho_t - Q_{i,t})$ for $j = 3 - i$, which yields

$$Q_{i,t} = \frac{\alpha(\theta_i - \theta_j) - p_{i,t} + p_{j,t} + d\rho_t}{2d}, \quad (13)$$

where $\rho_1 = 1$ and $\rho_2 = k$ are the market size in each period. Using the sales quantity given by (13), retailers determine their retail price $p_{i,t}$ competitively to maximize their profits. It is straightforward to show retailer profit in the NN scenario $\pi_{R_i}^{NN}$ is concave in $p_{i,1}$ and $p_{i,2}$. Since inventory is not carried over to the second period, we can solve the retailer pricing problem separately for each period using the first order conditions, and obtain the equilibrium price and sales quantity:

$$p^*_{i,t} = d\rho_t + \frac{\alpha(\theta_i - \theta_j) + 2w_i + w_j}{3}, \tag{14}$$

$$Q^*_{i,t} = \frac{\rho_t}{2} + \frac{\alpha(\theta_i - \theta j) + (w_j - w_i)}{6d}. \tag{15}$$

Next we solve for the manufacturers' game where they simultaneously choose a wholesale price w_i to maximize profit $\pi^{NN}_{M_i}$ given in (3). It is straightforward to show $\pi^{NN}_{M_i}$ is concave in w_i, and the equilibrium for the wholesale price game can be derived by solving $\partial \pi^{NN}_{M_i} / \partial w_i = 0$ simultaneously for $i = 1$, 2, which yields:

$$w^*_i = \frac{3d(1+k)}{2} + \frac{(2r_i + r_j) + \alpha(\theta_i - \theta_j)}{3}.$$

Next we solve for the suppliers' problem where each of them determines the material price r_i. Each supplier sets r_i to maximize its profit $\pi^{NN}_{S_i}$ given in (4). Again, it is straightforward that the profit function is concave in r_i. Therefore the equilibrium satisfies $\partial \pi^{NN}_{S_i} / \partial r_i = 0$ for $i = 1$, 2, which yields:

$$r^*_i = \frac{27d(1+k) + 2\alpha(\theta_i - \theta_j)}{6}.$$

Finally, we consider the supplier quality game. Each supplier determines its quality θ_i to maximize profit π^{NN}_{Si}. It can be shown that $\frac{\partial^2 \pi^{NN}_{S_i}}{\partial \theta_i^2} < 0 \Leftrightarrow c > \frac{\alpha^2}{81d}$, which holds under assumption A2. Thus, we solve for the suppliers' equilibrium quality decision following the first order conditions and obtain

$$\theta^*_i = \frac{(1+k)\alpha}{6c}.$$

Then the results in Lemma 1 follow plugging this equilibrium quality into each firm's equilibrium decisions. Plugging the equilibrium retail price and quality into the utility function of the marginal customer who is indifferent between the products, it follows that we need assumption A1, $m > d(\frac{3(5+4k)}{2} - \frac{1+k}{6\gamma})$, so that the marginal customer generates positive utility from the purchase.

We apply the same approach to derive equilibrium decisions for other scenarios. Moreover, the equilibrium demand for product 2 in FN and BN scenarios, $D^{FN}_{2,2}$ and $D^{BN}_{2,2}$ respectively, imply assumption A2:

$$D^{FN}_{2,2} = D^{BN}_{2,2} = \frac{9\gamma(11k-1) - 4k}{216\gamma - 8} > 0 \Leftrightarrow \frac{9\gamma}{4}(11 - \frac{1}{k}) > 1. \tag{16}$$

Finally, it can be shown that under assumptions A1 and A2 the profit functions for firms are concave in decision variables in every scenario and products compete. □

Proof of Lemma 2

The proof proceeds by comparing manufacturer profits across integration scenarios using the equilibrium quality and price given in Lemma 1. For part (i.a), it can be shown that $\Pi^{FN}_{M_1} - \Pi^{NN}_{M_1} = 0 \Leftrightarrow 36(5 + 34k + 5k^2)\gamma - 81(23 + 190k + 23k^2)\gamma^2 - 4(1 + 6k + k^2) = 0$ which has only one root $k = \delta_1$ that takes value in the parametric space defined by assumptions A1 and A2. Similarly, δ_2 is the only root for $\Pi^{FI_2}_{M_1} - \Pi^{NI_2}_{M_1} = 0$ that takes value under assumptions A1 and A2. Part (ii) proceeds by showing $\Pi^{BI_2}_{M_1} > \Pi^{NI_2}_{M_1}$ under parametric assumptions A1 and A2. □

Proof of Lemma 3

Parts (i) and (ii) proceed by showing that $\Pi_{M_1}^{BN} > \Pi_{M_1}^{FN}$ and $\Pi_{M_1}^{FI_2} - \Pi_{M_1}^{BI_2} > \Pi_{M_1}^{FN} - \Pi_{M_1}^{BN}$ under parametric assumptions A1 and A2. In part (iii), δ_3 is the only root for $\Pi_{M_1}^{FI_2} - \Pi_{M_1}^{BI_2} = 0$ that takes value under assumptions A1 and A2. □

Proof of Proposition 2

Part (i) follows from $\Pi_{M_1}^{BI_2} > \Pi_{M_1}^{NI_2}$ by Lemma 2. Part (ii) follows from $\Pi_{M_1}^{FF} > \Pi_{M_1}^{NF}$ for $k < \delta_2$ and $\Pi_{M_1}^{NN} > \Pi_{M_1}^{FN}$ for $k > \delta_1$ by Lemma 2. Moreover, it can be shown that $\delta_1 \leq \delta_2$ under assumptions A1 and A2 and Proposition 3 shows that NN Pareto dominates FF when both can be equilibrium outcomes.

The proof of part (iii) proceeds by showing the manufacturers have no incentive to deviate. First we show manufacturer 1 does not deviate from backward integration for $k > \delta_3$. This result is established by two facts: (1) Lemma 2 shows deviation from BB to NB is unattractive, and (2) Lemma 3 states manufacturer 1 does not deviate from BB to FB for $k > \delta_3$. Now we show manufacturer 1 does not deviate from forward integration for $k \leq \delta_3$. This result is established by the following two facts: (1) Lemma 3 shows manufacturer 1 does not deviate from FF to BF for $k \leq \delta_3$, and (2) it can be shown that $\delta_3 < \delta_2$ and therefore manufacturer 1 does not deviate from FF to NF for $k \leq \delta_3$. Similarly, it can be shown that manufacturer 2 has no incentive to deviate from the equilibrium strategy following the same procedure. □

Proof of Proposition 3

The proof is straightforward because $\Pi_{M_1}^{NN} - \Pi_{M_1}^{FF} = \frac{d(1+6k+k^2)}{4} > 0$ and $\Pi_{M_1}^{NN} - \Pi_{M_1}^{BB} = \frac{(1+k)^2\alpha^2}{36c} > 0$. □

Proof of Corollary 1

In a monopolist setting, using the equilibrium decisions described in Proposition 1, it can be shown that $\Pi_M^B > \Pi_M^F$ if and only if $\gamma > \frac{1}{2}$ for a monopolist manufacturer. In addition, Lemma 3 shows that a manufacturer backward integrates if and only if $\gamma > \frac{(1+k)^2}{9(1+6k+k^2)}$ under duopoly supply chains. It can be shown that $\frac{1}{2} > \frac{(1+k)^2}{9(1+6k+k^2)}$ for $0 \leq k \leq 1$, and therefore the manufacturer is more likely to backward integrate than forward integrate under supply chain competition. □

Proof of Proposition 4

First consider part (i) of this proposition. For $I_2 \in \{B, F\}$, we have $\Pi_{C_1}^{NI_2} - \Pi_{C_1}^{FI_2} = \Pi_{C_1}^{NI_2} - \Pi_{C_1}^{BI_2} = \frac{(1+k)^2(18\gamma-1)\alpha\gamma(477\gamma-20)}{16c(27\gamma-1)^2} > 0$. For $I_2 = N$, we have $\Pi_{C_1}^{NI_2} - \Pi_{C_1}^{FI_2} = \Pi_{C_1}^{NI_2} - \Pi_{C_1}^{BI_2} = \frac{(1+k)^2\alpha^2\gamma(20-927\gamma+10125\gamma^2)}{16c(27\gamma-1)^2} > 0$.

For part (ii), first note Proposition 2 states that $I_1^* I_2^* = NN$, FF or BB depending on the strategies that are considered. Then part (ii) follows because $\Pi_{C_1}^{NN} - \Pi_{C_1}^{FF} = \Pi_{C_1}^{NN} - \Pi_{C_1}^{BB} = \frac{9}{4}d(1+k)^2 > 0$. □

Proof of Proposition 5

The proof proceeds by comparing the equilibrium price and sales quantity given in Lemma 1 across scenarios in the parameter space specified by assumptions A1 and A2. □

Proof of Proposition 6

The proof proceeds by comparing manufacturer profits across scenarios using the equilibrium quality and price described in Lemma 4. In the following, we demonstrate the derivation of τ_1^N and the derivation of τ_1^F, τ_1^B, $\tau_2^{I_2}$ and τ_3 can be obtained following the same procedure. Here, we use β to denote β^F for ease of notation. It can be shown that $\Pi_{M_1}^{FN} - \Pi_{M_1}^{NN} = \frac{d}{4\upsilon_3}(\upsilon_2 - \upsilon_1\upsilon_3)$, where $\upsilon_1 = 3(1+k)^2$, $\upsilon_2 = 81c^2d^2(85+26k+85k^2)\beta^2 - 36cd\alpha^2\beta(3+10\beta+k(8\beta-6)+k^2(3+10\beta)) + \alpha^4(1+2\beta+5\beta^2+k^2(1+2\beta+5\beta^2)-k(2+4\beta-6\beta^2)$, $\upsilon_3 = \beta(54cd\beta - \alpha^2(1+\beta))^2$. Thus solving $\Pi_{M_1}^{FN} - \Pi_{M_1}^{NN} = 0$ is equivalent to solving $\upsilon_2 - \upsilon_1\upsilon_3 = 0$ which has only one real root for $0<\beta<1$. Then τ_1^N is given by this root because $\upsilon_2 - \upsilon_1\upsilon_3 < 0$ for $\beta = 1$ and $\upsilon_2 - \upsilon_1\upsilon_3 > 0$ for $\beta = 0$. □

Proof of Proposition 7

NN cannot be an equilibrium, because Lemma 2 (ii) states $\Pi_{M_1}^{BN} > \Pi_{M_1}^{NN}$, showing manufacturer 1 has incentive to deviate by choosing backward integration. □

Proof of Proposition 8

Let β denote β^F for ease of notation. The proof proceeds by comparing the equilibrium quality, pricing decisions and sales quantity given in Lemma 4. For part (ii.a), solving $p_{1,t}^{FI_2} - p_{1,t}^{NI_2} = 0$ reveals that $p_{1,t}^{FI_2} > p_{1,t}^{NI_2}$ if and only if $\beta < \sigma^{I_2}$, where σ^{I_2} is the larger root of $\chi^{I_2} = 0$, $I_2 \in \{F, N, B\}$, where

$$\chi^F = \begin{cases} \alpha^2(k-1-\beta-k\beta+14\beta^2+12k\beta^2) - 9cd\beta(84\beta-55+k(72\beta-43)), & for\, t=1 \\ \alpha^2(1-\beta+12\beta^2+k(14\beta^2-1-\beta)) - 9cd\beta(72\beta-43+k(84\beta-55)), & for\, t=2 \end{cases}$$

$$\chi^N = \begin{cases} 18cd\beta(28\beta-15+k(22\beta-9)) + \alpha^2(5-16\beta-\beta^2+k(3-16\beta+\beta^2)), & for\, t=1 \\ 18cd\beta(22\beta-9+k(28\beta-15)) + \alpha^2(3-16\beta+\beta^2+k(5-16\beta-\beta^2)), & for\, t=2 \end{cases}$$

$$\chi^B = \begin{cases} \alpha^4(1-k-2\beta-2k\beta-11\beta^2-9k\beta^2) + 243c^2d^2\beta(15-28\beta+k(9-22\beta)) \\ -9cd\alpha^2(3+14\beta-61\beta^2-k(3-8\beta+49\beta^2)), & for\, t=1 \\ \alpha^4(k(1-2\beta-11\beta^2)-1-2\beta-9\beta^2) - 243c^2d^2\beta(22\beta-9+k(28\beta-15)) \\ +9cd\alpha^2(3-8\beta+49\beta^2-k(3+14\beta-61\beta^2)), & for\, t=2 \end{cases}$$

□

Proof of Proposition 9

Let β_i denote β_i^F for ease of notation. First, we obtain firms' equilibrium decisions given in Appendix C following the procedure described in the proof of Lemma 1. Then the results in this proposition proceed by comparing equilibrium quality and sales quantity across scenarios, which leads to the following threshold values

$$\xi_1^\theta = \frac{\alpha^2(4\beta_2 + \beta_1(3\beta_2 - 5) + \sqrt{\beta_1^2(5-3\beta_2)^2 - 16\beta_1\beta_2 + 16\beta_2^2})}{54d\beta_1\beta_2},$$

$$\xi_2^\theta = \frac{\alpha^2(15 - 11\beta_2 - 2\beta_1 + \sqrt{81 - 66\beta_2 + \beta_2^2 - 60\beta_1 + 44\beta_1\beta_2 + 4\beta_1^2})}{54d(6 - 5\beta_2)\beta_1},$$

$$\xi_1^Q = \xi_2^Q = \xi_1^\theta.$$

Part (iii) of this proposition follows by replacing $\beta_1^F = \beta_2^F - \Delta$ and the fact that $\frac{d\xi_i^\theta}{d\Delta} > 0$ and $\frac{d\xi_i^Q}{d\Delta} > 0$. □

Proof of Proposition 10

In this extension, the manufacturers can set the wholesale price they charge to retailers dynamically in each period. First we derive the SPNE decisions under every integration scenario (as in Lemma 1 for the base model). Then we compare firm profits across scenarios to characterize the manufacturers' equilibrium integration strategy (as in Proposition 2 for the base model).

Next, we illustrate in the NN scenario how the derivation of equilibrium decisions differs from the base model. The derivation for SPNE decisions in the NN scenario is identical to the proof of Lemma 1 until (15). Next when considering the manufacturers' choice of wholesale prices, manufacturer i maximizes its profit $\pi_{M_i} = \sum_{t=1}^{2}(w_{i,t} - r_i)Q_{i,t}$ where $Q_{i,t}$ is given by (15).

It is straightforward to show π_{M_i} is concave in $w_{i,t}$, and the equilibrium for the wholesale price game can be derived by solving $\partial \pi_{M_i}/\partial w_{i,t} = 0$ simultaneously for $i = 1, 2$ and $t = 1, 2$, which yields:

$$w_{i,t}^* = \frac{9d\rho_t + 2r_i + r_j + \alpha(\theta_i - \theta_j)}{3}.$$

Given the manufacturers' response for the wholesale prices in each period, next we characterize the suppliers' choice of material prices. Each supplier sets its material price r_i to maximize its profit given in (4). Again, it is straightforward that the profit function is concave in r_i. Therefore the equilibrium satisfies $\partial \pi_{S_i}/\partial r_i = 0$ for $i = 1, 2$, which yields:

$$r_i^* = \frac{27d(1+k) + 2\alpha(\theta_i - \theta_j)}{6}.$$

Finally, we consider the suppliers' quality investment. Each supplier determines its quality θ_i to maximize profit π_{Si}. It can be shown that $\frac{\partial^2 \pi_{S_i}}{\partial \theta_i^2} < 0 \Leftrightarrow c > \frac{\alpha^2}{81d}$, which is satisfied under the revised assumption A2. Next, we solve for the suppliers' equilibrium quality decision following the first order conditions and obtain:

$$\theta_i^* = \frac{(1+k)\alpha}{6c}.$$

Then the SPNE price and sales quantity can be derived using this equilibrium quality. Plugging the equilibrium decisions into the utility function, it can be shown that the marginal customer who is indifferent between the products generates positive utility from the purchase when the revised assumption A1 holds.

Following the same procedure we derive the SPNE decisions in every possible vertical integration scenario. It can be shown that $D_{2,2} = \frac{45\gamma(5k-1)+(1-9k)}{432\gamma-16} > 0$ for the BN and FN scenarios leads to the revised assumption A2. Under the revised assumptions A1 and A2, the profit functions for firms are concave in their decisions variables and products compete.

Finally, we compare the manufacturers' profits across integration scenarios to characterize their equilibrium integration strategy. It can be shown that $\pi_{M_1}^{BN} > \pi_{M_1}^{NN}$ and $\pi_{M_2}^{BB} > \pi_{M_2}^{BN}$ and thus no integration at any manufacturer cannot be an equilibrium. The threshold value in part (i) can be derived by solving $\pi^{FI}_{M_1} = \pi^{BI}_{M_1}$ for $I \in \{F, B\}$. □

Proof of Corollary 2

The proof proceeds by showing $1 - \frac{8(18\gamma-1)^+ - 12\sqrt{\gamma(18\gamma-1)^+}}{63\gamma-4} \le \delta_3$ with equality holds for $k=1$. That is, with dynamic wholesale pricing BB region becomes larger in the parameter space. □

Proof of Proposition 11

This extension differs from the base model in that each retailer also chooses a quality investment level immediately after the suppliers determine their quality investments. Again, first we derive the SPNE decisions under every integration scenario (as in Lemma 1 for the base model). Then we compare profits across scenarios to characterize the manufacturers' equilibrium choice of integration strategy (as in Proposition 2 for the base model).

Next, we derive the SPNE decisions for the NN scenario in this extension. Following the proof of Lemma 1, we obtain supplier i's equilibrium material price:

$$r_i^* = \frac{27d(1+k) + 2(\alpha(\theta_i - \theta_j) + b(\theta_{ir} - \theta_{jr}))}{6}. \tag{17}$$

We next derive the retailers' choice of quality investment by plugging this price into retailer i's profit function:

$$\pi^N_{R_i} = \sum_{t=1}^{2} (p_{i,t} - w_i) Q_{i,t} - c\theta_i^2. \tag{18}$$

Here, we focus on $c > \frac{b^2}{729d}$ so that retailer profit is concave in θ_{ir}. We solve the first order conditions for both retailers which yields the following equilibrium retailer quality:

$$\theta^*_{ir} = \frac{b(27c(27d(1+k) + 2\alpha(\theta_i - \theta_j)) - 2b^2(1+k))}{54c(729cd - 2b^2)}. \tag{19}$$

Plugging this retailer quality into the suppliers' profit functions, it reveals that we need $c \ge \frac{4b^2 + 9\alpha^2 + 3\alpha\sqrt{8b^2+9\alpha^2}}{1458d}$ so that each supplier's profit function is concave in its quality choice θ_i. This requirement is satisfied under the revised assumption A2. We solve the first order conditions for both suppliers which yields the following equilibrium supplier quality:

$$\theta_i^* = \frac{243d(1+k)\alpha}{1458cd - 4b^2}. \tag{20}$$

The SPNE retailer quality, prices and sales quantity are then derived using this equilibrium supplier quality. Plugging the equilibrium prices and qualities into the utility function of the marginal customer in period 1 who is indifferent between the products, it follows that we need the revised assumption A1 stated in Section 5.3 so that the marginal customer generates positive utility from the purchase and products compete.

Following the same procedure, we derive the equilibrium outcome for other scenarios. It can be shown that $D_{2,2} = \frac{27b^2cd(1-23k) + 2b^4k + 486cd(9cd(11k-1) - 4k\alpha^2)}{4(b^4 - 324b^2cd + 972cd(27cd - \alpha^2))} > 0$ in the FN and BN scenarios implies the revised assumption A2. Under the revised assumptions A1 and A2, the profit functions for firms are concave in their decision variables and products compete.

Next we plug the equilibrium decisions into profit functions to derive the manufacturers' profits under every integration scenario. It can be shown that $\pi^{BN}_{M_1} > \pi^{NN}_{M_1}$ and $\pi^{BB}_{M_2} > \pi^{BN}_{M_2}$ and thus no integration at any manufacturer cannot be an equilibrium. In other words, the manufacturers choose to vertically integrate in equilibrium. Then the threshold value in part (i) can be derived by solving $\pi^{FI}_{M_1} = \pi^{BI}_{M_1}$ for $I \in \{F, B\}$. □

Proof of Proposition 12

In this extension supply chain 2 has a lower cost for quality improvement. First we derive the SPNE decisions under every integration scenario (as in Lemma 1 for the base model). Then we compare manufacturer profits across scenarios to characterize manufacturers' equilibrium choice of integration strategy (as in Proposition 2 for the base model).

For ease of notation, we drop the subscript 1 for c_1 in this proof. The derivation of SPNE decisions for each scenario essentially follows the same procedure described in the proof of Lemma 1. The only difference is that the cost for quality improvement in supply chain 2 becomes $vc\theta^2$, where $v \in (0,1)$, instead of $c\theta^2$ as in the base model. Once the SPNE decisions are derived, we plug them into profit functions to derive the manufacturers' equilibrium profits under every integration scenario. The result of this proposition is then derived by comparing manufacturer profits across integration scenarios and shown that no manufacturer can achieve higher profit by unilaterally deviating from the equilibrium strategy described in the proposition. Here, δ_4 is the solution to $\pi^{NB}_{M_1} = \pi^{BB}_{M_1}$, δ_5 is the solution to $\pi^{FB}_{M_1} = \pi^{FF}_{M_1}$, δ_6 is the solution to $\pi^{FB}_{M_1} = \pi^{BB}_{M_1}$, δ_7 is the solution to $\pi^{FB}_{M_1} = \pi^{NB}_{M_1}$. □

Proof of Proposition 13

Here we analyze the manufacturers' equilibrium vertical integration strategy when the quality improvement level for both products is exogenously determined at θ. First we derive the SPNE decisions under every integration scenario following the procedure described in the proof of Lemma 1. Using the equilibrium decisions in the NN scenario, it can be shown that the marginal customer who is indifferent between buying the product from either supply chain generates positive utility for $m > \frac{3d(5+4k)}{2} - \alpha\theta$. This condition also ensures that marginal customers in every integration scenario generate positive utility from their purchase. Then it can be shown that $\pi^{BN}_{M_1} > \pi^{NN}_{M_1} > \pi^{FN}_{M_1}$. Thus no integration cannot be an equilibrium outcome. Finally, the result for this proposition proceeds as $\pi^{BI}_{M_1} > \pi^{FI}_{M_1}$ for $I \in \{F, B\}$. □

Proof of Lemma 4

The derivation for the equilibrium decisions in this lemma follows the procedure described in the proof of Lemma 1 with $\beta^F_i < 1$. □