A Theory of Pyramidal Ownership and Family Business Groups*

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Abstract

We provide a rationale for pyramidal ownership (the control of a firm through a chain of ownership relations) that departs from the traditional argument that pyramids arise to separate cash flow from voting rights. With a pyramidal structure a family uses a firm it already controls to set up a new firm. This allows the family to access the entire stock of retained earnings of the firm it controls, and to share the security benefits of the new firm with minority shareholders of the original firm. Therefore, pyramids are more attractive when internal funds are important due to the poorly functioning capital markets, and when the security benefits of the new firm are low; conditions that hold in an environment with poor investor protection. Because our model departs from the traditional argument for pyramids as a device to separate cash flow from voting rights, it can differentiate between pyramids and dual-class shares even in situations in which the same deviation from one share-one vote can be achieved with either method. Unlike the traditional argument, our model is consistent with recent empirical evidence that some pyramidal firms are associated with small deviations between ownership and control. We also analyze the creation of family business groups (a collection of multiple firms under the control of a single family). Business groups also flourish when external markets are poorly developed because, in such cases, internal resources from the existing firms provide the family with a financing advantage vis-a-vis other competing entrepreneurs. Thus, the model predicts that in countries with poor investor protection family business groups should be more common, and that they are more likely to be organized as pyramids. Other predictions of the model are consistent with systematic and anecdotal evidence on pyramidal business groups.

Key words: pyramids, business groups, family firms, investor protection, ownership structure, dual-class shares.

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1 Introduction

Many firms in the world have a controlling shareholder, usually a family or the State (La Porta, Lopez-de-Silanes and Shleifer, 1999). In several countries, single individuals or families control a large number of firms; an organization typically referred to as a *family business group*.[1] The controlling family often organizes the ownership of the group member firms in a *pyramidal structure*.[2] In such a structure the family achieves control of the constituent firms by a chain of ownership relations: the family directly controls a firm, which in turn controls another firm, which might itself control other firms, and so forth.

Despite the ubiquity of pyramidal business groups, there is surprisingly no formal theory that explains their existence. There are, however, some informal arguments. The traditional one is that a pyramid allows a family to achieve control of a firm with a small cash flow stake.[3] For instance, a family that directly owns 50% of a firm, which in turn owns 50% of a different firm, achieves control of the latter firm with an *ultimate* cash flow stake of only 25%. Securing control through such arrangements is beneficial for the family when private benefits of control are large. Because this view suggests that pyramids are created to separate cash flow from voting rights, it predicts that pyramidal firms should always be associated with a substantial separation between ownership and control. In fact, there are a number of examples in the literature in which firms in pyramidal groups are characterized by considerable separation between ownership and control (see for example Claessens, Djankov and Lang, 2000).

Nevertheless, a more detailed examination of the available data on the characteristics of pyramidal ownership structures reveals some facts that cannot be adequately explained by the traditional view. For example, the finding that pyramidal firms are associated with large deviations from one

[1] The term business group is somtimes used in the literature to refer to other types of corporate groupings such as those in which the member firms are tied together by common ethnicity of the owners, interlocking directorates, school ties, etc. An example is the Japanese keiretsu, an organization in which individual managers have considerable autonomy in their firms but coordinate their activities through the President Council and a common Main Bank (Hoshi and Kashyap, 2001). Another example is the horizontal financial-industrial groups in Russia, 'which are more properly industry alliances' (Perotti and Gelfer, 2001, p. 1604). To avoid confusion, we use the term *family* business groups to refer to groups in which member firms are controlled by the same family, such as the groups in Western Europe, Latin America, and East Asia.

[2] See, among others, Claessens, Djankov, and Lang (2000) for the evidence on East Asia, Faccio and Lang (2002) and Barca and Becht (2001) for Western Europe, Khanna (2000) for emerging markets, and Morck, Strangeland and Yeung (2000) for Canada.

[3] This argument goes back at least to the beginning of the 20th century. Berle and Means (1932) and Graham and Dodd (1934) use this argument to explain the creation of pyramids in the U.S. in the early 20th century.

share-one vote is not universal. There are many cases in which the separation achieved is minimal and does not seem to warrant the use of a pyramid (see for example Franks and Mayer, 2001, and section 7 for a discussion of this and other evidence).

Moreover, even the cases in which pyramids *do* seem to separate cash flow from voting rights are not entirely explained by the traditional view. The reason is that pyramids are not the only way to achieve this separation. For example, the family can achieve any degree of separation by directly owning the firm and issuing shares with no voting rights. In such a case, why would a family choose to control a firm through a pyramid instead of issuing dual-class shares? Yet, despite this apparent equivalence, the empirical evidence indicates that pyramids are much more common throughout the world than dual-class shares (La Porta, Lopez-de-Silanes and Shleifer, 1999). This does not appear to be caused by restrictions to the use of dual-class shares. Although these restrictions set an effective upper bound to the deviation from one share one vote that can be achieved with dual-class shares, many pyramidal firms have deviations that fall *below* this permitted upper bound (Bianchi, Bianco and Enriques, 2001). All this evidence suggests that considerations other than separation of cash flow from voting rights motivate the creation of pyramidal business groups.

In this paper we present a model that provides a rationale for the existence of pyramids that does not rely on separation of cash flow from voting rights. The model is consistent with the finding that pyramids arise even in situations in which the family can use dual-class shares to facilitate control. The model can also explain why firms controlled through pyramids sometimes have substantial deviations between ownership and control, while other times the separation is minor. The theory addresses both the ownership structure of business groups, that is, why is it that groups are organized as pyramids as opposed to a structure in which group firms are owned directly by the controlling family, and the existence of the group itself, that is, why is it that a single family controls multiple independent firms. We show that the implications of the model are consistent with anecdotal and empirical evidence regarding the characteristics of pyramidal business groups.

The model has two key ingredients. The first one is the assumption of limited investor protection. If investor protection is poor, the family extracts private benefits from the firms it controls at the expense of minority shareholders. The second ingredient is the assumption that business groups are created over time, that is, the family initially sets up a firm and, at some point in the

future, the opportunity to set up another firm arises.

When this opportunity arises, the family must decide on the ownership structure of the business group. In a *pyramidal* structure the new firm is owned by all the shareholders of the original firm. As a result, the family shares the security benefits of the new firm with non-family shareholders of the existing firm, but it has access to the entire stock of retained earnings (cash) of the original firm.[4] We consider an alternative ownership structure in which the family controls the new firm by directly holding its shares. We refer to this direct ownership structure as a *horizontal structure*. Under this structure, non-family shareholders of the existing firm have no rights to the cash flows of the new firm, and thus the family captures the entire security benefits of the new firm. However, the family has access only to its share of the retained earnings of the original firm.[5]

The level of investor protection plays a crucial role in the choice of structure. Poor investor protection leads to high diversion of cash flows, which makes the pyramidal structure more attractive for two reasons. First, diversion increases the family's private benefits of control, at the expense of a reduction in security benefits.[6] Because in a pyramidal structure the family shares the security benefits with non-family shareholders, while in the horizontal structure it keeps them entirely, high diversion increases the family's payoff under the pyramidal structure relative to the payoff under the horizontal structure (payoff advantage). Second, high diversion makes it more difficult to finance the new firm with external investors as they anticipate the level of diversion and discount the terms at which they are willing to provide finance. Thus, the family's ability to use the entire stock of retained earnings of existing group firms when it chooses the pyramid becomes more valuable (financing advantage).

In addition to the level of investor protection, certain firm characteristics influence the choice of structure. In particular, we show that firms with high investment requirements and/or low profitability are more likely to be set up in pyramids. The argument is similar to that described in the previous paragraph. Because of their characteristics, these types of firms generate lower security benefits for investors. Thus, the family achieves a higher payoff and, at the same time,

[4] Security benefits represent the fraction of the firm's returns that is not diverted by the family and thus accrues to all shareholders. The remaining part (the diverted value) represents a private benefit of control for the family.

[5] Graham and Dodd (1934) argue that the ability to use the resources of an already established firm to set up or acquire new firms was one of the reasons for the existence of pyramids in the U.S. in the early 1900's (see p. 564).

[6] There is a large empirical literature providing evidence that private benefits of control are larger in poor investor protection countries. See Zingales (1994), Nenova (1999) and Dyck and Zingales (2004).

finds it easier to finance these firms when it uses a pyramid to set them up.

In sum, in our model pyramids are chosen by the family because of the payoff and financing advantages they provide when firms are expected to yield low security benefits relative to the required investments. This rationale for pyramids is clearly different from that proposed by the traditional view. In particular, pyramids can be optimal even if the opportunities for separating cash flow and votes with dual-class shares are not exhausted. To clearly make this point in the model, we assume that there are no legal restrictions to the use of dual-class shares, implying that any deviation from one share-one vote generated with the use of pyramids can also be achieved by directly holding shares in the firm (horizontal structure) and issuing dual-class shares. Under this assumption, the traditional argument predicts an equivalence between these two mechanisms. The fact that, even in such an environment, our model predicts that pyramid might be strictly preferred helps explain one of the puzzles raised above, namely, that the prevalence of pyramids does not seem to be explained only by restrictions to the use of dual-class shares.

The analysis above assumes that the family is the only party that can set up and control the new firm. That is, it assumes the existence of a business group. However, we also analyze the conditions under which the business group *itself* appears, that is, the conditions that allow the family to control the new firm. As it turns out, these conditions are very similar to those that are conducive to the creation of pyramids. A firm is more likely to be added to a business group when its security benefits are low relative to the required investments (for example, due to poor investor protection). In such cases, it is difficult for an outside, less wealthy entrepreneur to finance the required investment in the external market. As a result, families that already own successful firms might be the only ones with the financial resources to set up the new firm, regardless of whether they are the most efficient owners. The model thus predicts that pyramidal business groups should be very prevalent in poor investor protection countries, because both groups and pyramids are more likely to exist in such countries. This implication appears to be consistent with available empirical and anecdotal evidence (e.g., La Porta, Lopez-de-Silanes and Shleifer, 1999).

We also show that observed ultimate ownership is lower and equilibrium diversion is higher in firms that are controlled through pyramids. This result is driven by a selection effect. Firms with low security benefits relative to their investments require that the family sell more shares to set them up. As a result, the family's ultimate stake in these firms is low and diversion high.

But as explained above, when diversion from the new firm is expected to be high, this firm is likely to be set up in a pyramidal structure. Thus, firms with low security benefits relative to their investment requirements are associated with lower ownership concentration, high diversion and end up in pyramidal structures. Similarly, firms with high security benefits relative to their investment requirements are associated with high ultimate ownership concentration, low diversion and are more likely to be set up in horizontal structures, or even outside business groups. This prediction is consistent with evidence that shows significant expropriation of investors in firms that belong to pyramidal structures (Bertrand, Mehta and Mullanaithan, 2002, and Johnson et al., 2000). Notice, however, that in our model it is not the case that the pyramidal structure itself increases diversion. Rather, it is the expectation of high levels of diversion that makes the pyramidal structure an optimal choice for the controlling family.

Despite the fact that pyramidal firms are associated with lower ultimate ownership relative to firms controlled directly by the family, our model does not necessarily require –as the traditional argument does– that the ultimate ownership concentration in a pyramidal firm be small in an absolute sense.[7] In fact, our model is consistent with families holding either large or small ultimate ownership stakes in pyramidal firms, leading to either minor or substantial separation of cash flow from voting rights. Thus our model can explain why in some pyramidal firms –but not in all– deviations from one-share-one-vote appear to be minor. The model also suggests conditions under which we should observe a large separation between ownership and control in pyramidal firms. In particular, we show that in countries with poor investor protection it is more likely that pyramidal firms will be associated with large concentrations of cash flow rights in the hands of the family. The intuition is that in such countries, even firms in which the family holds large cash flow stakes will be associated with low security benefits, and thus will be optimally owned through pyramids. These observations help explain another of the puzzles raised above, namely that pyramidal structures with small deviation between cash flow and votes seem to be common.

Finally, we consider a few extensions of the basic model that address additional questions raised by the theory. First, we consider the optimal contracting problem at the time in which the first firm in the business group is set up. The fact that pyramids are only created when the security

[7] The selection argument above only suggests that families should hold smaller ownership stakes in firms that they control through pyramids, *relative* to firms that they own directly. This is not incompatible with high observed ownership stakes in pyramidal firms, in an *absolute* sense.

benefits of new firms that are added to the group are low raises the question of whether pyramids are ex-ante optimal for the family. We show that, because of the financing advantage of pyramids, the family might not benefit from ruling them out, even if it is contractually possible to do so. Second, we analyze whether it is optimal for the family to set up new firms as legally independent entities or as divisions inside existing firms. This question is important because if new firms are set up as divisions the resulting structure does not necessarily match the usual definition of a pyramid. We show that, as long as there is variation in the level of investor protection across firms in the same group, the family is very likely to set up the new firm as a partial subsidiary. We also consider additional extensions that verify the robustness of the results to some particular assumptions regarding the diversion technology and the characteristics of project's payoffs. We find that the results are unaffected by such considerations.

Besides the papers mentioned above, there is a large literature on business groups that is related to our paper. Some authors explain the presence of business groups as an efficient organizational form that adds value to member firms. Leff (1978) and, more recently Khanna and Palepu (1997, 1999), argue that business groups substitute for missing markets (e.g. labor and financial markets).[8] Another potential benefit of groups is that they are better positioned to lobby the governments for favors (Pagano and Volpin, 2001). Other benefits of groups include the possibility to "prop up" (inject money) failing firms (Friedman, Johnson, and Mitton, 2003) and that a group's deep pockets serve a strategic role in product market competition (Cestone and Fumagalli, 2004). None of these arguments considers the ownership structure (e.g., pyramids, horizontal structure, etc.) of the business group.

To explain the ownership structure of groups, the literature has relied on a different set of arguments. As we discussed above, the conventional wisdom for the existence of pyramidal ownership is that the pyramid is a device that allows the family to separate cash flow from voting rights. The question still remains as to why a pyramid is the best mechanism to achieve this separation. The same observation can be made regarding the models in Gomes (2000), who shows that separation of cash flow and voting rights might have reputation benefits, and Bebchuk (1999), who argues that an initial owner might want to separate cash flow and voting rights to prevent potential raiders from seizing valuable control. Regulatory or tax considerations might also help explain the exis-

[8] See also Aoki (1984), Ghatak and Kali (2001), and Kim (2004) for related arguments.

tence of pyramids. Indeed, taxes on inter-company dividends do seem to affect the incidence of pyramidal structures (Morck, 2003). Others have suggested that pyramidal structures can be used as an elusive tool to hide the identity of the ultimate owner from either the market or the state (Bianchi, Bianco, and Enriques, 2001).

Our paper is organized as follows. We start our analysis in section 2 by considering a version of the model in which the family already owns a given firm and has to decide on the structure to use (pyramidal or horizontal) to set up a new firm. We use this framework in section 3 to characterize the conditions that lead to the choice of each structure by the family. In section 4 we analyze the conditions that give rise to a business group, that is, the conditions under which the family who already owns a firm can set up the new firm (as opposed to ownership by an outside entrepreneur). In sections 3 and 4 we assume that diversion entails no costs. This assumption makes diversion insensitive to the firm's ownership structure, and simplifies the analysis considerably. In section 5 we relax this assumption, and derive implications regarding variations in diversion and ownership concentration in different structures. Section 6 considers a few additional questions and extensions, including the analysis of ex-ante contracting between the family and outside shareholders, some variations in the diversion technology and the question of whether new projects that are taken by the pyramid should be organized as stand-alone firms or divisions. Our theory generates a number of empirical implications, which we discuss in detail in section 7 together with the relevant empirical literature. Section 8 concludes the paper.

2 Pyramidal and horizontal structures

In this section we present a framework to analyze pyramidal business groups. The model has three dates. At date 0, a family sets up a firm (firm A), keeping a fraction α of its shares. At date 1, firm A generates cash flows of c, and the opportunity to set up another firm (firm B) arises. Firm B requires an investment i at date 1 and generates a revenue r at date 2, with $r > i$. We also assume for now that the family is the only possible owner of firm B. In section 4 we analyze the effects of competition from an alternative owner.

At date 1, the family chooses the optimal ownership structure for firm B (horizontal or pyramidal). In a pyramidal structure, the family sets up firm B as a partial subsidiary of firm A and thus can use the cash c in firm A to set up firm B. In an horizontal structure, the family itself –and

independently from firm A– sets up firm B. In this case, the family has access only to its personal wealth of αc. In either structure, the family sells shares of firm B to raise additional funds.

We assume that there are no legal restrictions to the use of dual-class shares. This assumption ensures that the family always retains complete control of firm B, irrespective of the structure it chooses and its ultimate ownership.

Control allows the family to divert cash from firm B into its pockets. We assume that when the family diverts dr of the cash flows, it pays a cost (one can think of this as waste involved in the diversion process) of $c(d,k)r$, where k is the level of investor protection. We discuss the particular functional form of $c(\cdot,\cdot)$ in sections 3 and 5.

One implicit assumption in this formulation is that diversion opportunities are the same regardless of the structure the family chooses. The reason for this assumption is that, because the family retains the same degree of control in both structures, the set of feasible actions the family can take and hence the diversion *opportunities* should be the same. Of course, as we will see below, *actual* diversion will be affected by the incentives that the family faces in each structure. The other implicit assumptions about diversion opportunities are not crucial to the results. For example, diversion occurs from firm B directly into the family's pockets instead of from firm B to firm A. Also, there is no direct diversion from firm A. In section 6, we discuss these assumptions and show that our results are robust.

Finally, we assume that the market interest rate is zero and that the family maximizes its date 2 payoff. We start by solving the model from date 1 and take the family stake in firm A, α, as given. In section 6.1, we endogenize α by solving the model from date 0. We state the family's problem for each of the two structures.

2.1 Horizontal structure

The family has personal wealth of αc. To set up firm B at date 1, the family contributes R_I^H (the subscript I stands for "internal" funds) of these funds and raises R_E^H from the external market by selling $1-\beta^H$ shares of firm B. The family's payoff at date 2 can be written as

$$\alpha c - R_I^H + \beta^H \left(R_I^H + R_E^H - i + (1-d)r\right) + (d - c(d,k))r. \tag{1}$$

At date 2, the family chooses the level of diversion that maximizes the above expression. Thus,

$d = \arg\max_d \beta^H(1-d) + d - c(d,k)$. This expression defines $d(\beta^H, k)$.

Because investors break even in equilibrium, we can write $R_E^H = (1-\beta^H)(R_I^H + R_E^H - i + (1-d)r)$. Solving this equation for R_E^H, plugging this value into Equation (1), and letting $NPV \equiv r - i - c(d,k)r$ be the NPV of firm B net of diversion costs, we obtain the payoff of the family as of date 1:

$$U^H = \alpha c + NPV. \tag{2}$$

This expression is the family's payoff conditional on firm B being set up. The family is able to set up firm B whenever $R_I^H + R_E^H \geq i$, which by replacing the value for R_E^H leads to

$$R_I^H + (1-\beta^H)(1-d)r \geq i. \tag{3}$$

We let $R^H \equiv R_I^H + (1-\beta^H)(1-d)r$. Conditional on setting up firm B, the family's date 1 problem is:

$$\begin{aligned} &\max_{R_I^H \in [0,\alpha c], \beta^H \in [0,1]} U^H \\ &\text{subject to } R^H \geq i \\ &\text{and to } d = d(\beta^H, k) \end{aligned} \tag{4}$$

2.2 Pyramidal structure

Firm A has retained earnings of c, out of which it contributes R_I^P to the set up cost of firm B. In addition, it raises R_E^P from the external market by selling $1 - \beta^P$ shares of firm B. The family's payoff at date 2 is given by

$$\alpha[c - R_I^P + \beta^P(R_I^P + R_E^P - i + (1-d)r)] + (d - c(d,k))r, \tag{5}$$

where $R_I^P + R_E^P - i + (1-d)r$ are the security benefits of firm B at date 2.

At date 2, the family chooses the level of diversion that maximizes the above expression. Thus, $d = \arg\max_d \alpha\beta^P(1-d) + d - c(d,k)$. As can be seen by comparing this expression with the corresponding one in the horizontal case, in both structures diversion depends on the same way on ultimate ownership (β^H in the horizontal structure and $\alpha\beta^P$ in the pyramidal). Therefore, diversion in the pyramidal case is given by $d(\alpha\beta^P, k)$, with this function being the same as the one defined in the previous section. In section 5, it will be more convenient to think about the family

as choosing its ultimate ownership concentration in firm B rather than the direct ownership. Thus, for future reference we let $\omega^H \equiv \beta^H$ and $\omega^P \equiv \alpha\beta^P$.

Moving back to date 1, we write $R_E^P = (1-\beta^P)(R_I^P + R_E^P - i + (1-d)r)$. Solving for R_E and plugging this expression into Equation (5), we get the family's payoff as of date 1:

$$U^P = \alpha c + NPV - (1-\alpha)[(1-d)r - i]. \tag{6}$$

The payoff differences between the horizontal and the pyramidal structures can be seen by comparing Equations (2) and (6). In the horizontal structure the family sets up firm B and, because new investors of firm B get the market return, the family ends up capturing the entire NPV of the project. In the pyramidal structure, firm A sets up firm B and so the NPV is shared between the family and non-family shareholders of firm A. However, the NPV is not distributed in proportion to the stakes in firm A because the family –but not the other shareholders of firm A– receives the diverted amount. Only the non-diverted NPV ($(1-d)r-i$) is divided in proportion to the stakes in firm A. That is, non-family shareholders of firm A get $(1-\alpha)[(1-d)r-i]$ and the family receives the rest.

For the family to be able to set up firm B, it must be that $R_I^P + R_E^P \geq i$. Replacing the value of R_E^P leads to

$$R_I^P + (1-\beta^P)(1-d)r \geq i \tag{7}$$

Letting $R^P \equiv R_I^P + (1-\beta^P)(1-d)r$, the family's problem conditional on setting up firm B is

$$\begin{aligned} &\max_{R_I^P \in [0,c], \beta^P \in [0,1]} U^P \\ &\text{subject to } R^P \geq i \\ &\text{and to } d = d(\alpha\beta_P, k) \end{aligned} \tag{8}$$

3 Choice of structure, investor protection and firm characteristics

There are two parts to the family's problem. First, the family finds the optimal ownership concentration for each of the two possible structures (problems in Equations (4) and (8)). Next, it chooses the structure that provides the higher payoff.

To provide the intuition for each of the two steps, we first consider a very simple cost of diversion function that guarantees that, in equilibrium, the cost of diversion is always zero. We show that this

simplifying assumption implies that the family's payoff is independent of ownership concentration. This allows us to abstract from the effects of ownership concentration and isolate the choice of structure effects.

In section 5 we allow diversion to be costly, using a similar framework to that in Burkart, Gromb, and Panunzi (1998), and Shleifer and Wolfenzon (2002). With this assumption, diversion, the cost of diversion, and consequently the family's payoff depend on ultimate ownership concentration. As we show in section 5, this new assumption allows us to derive additional implications regarding the optimal ownership concentration and equilibrium levels of diversion, but it does not change the substance of the implications of the model of this section, so we start our analysis with this simpler model.

We assume that diversion entails no cost and that the level of investor protection limits the amount of diversion that can take place (similar formulations of the diversion technology can be found in Pagano and Roell, 1998, and in Burkart and Panunzi, 2002). In other words, we assume that:

$$c(d,k) = \begin{cases} 0 & \text{if } d \leq \overline{d}(k) \\ +\infty & \text{otherwise} \end{cases}, \tag{9}$$

with $\frac{\partial \overline{d}}{\partial k} < 0$.

Because diversion up to $\overline{d}$ is costless, the family sets $d = \overline{d}$, regardless of the structure it uses. Using Equations (2) and (6) , we get

$$\begin{aligned} U^H &= \alpha c + NPV, \text{ and} \\ U^P &= \alpha c + NPV - (1-\alpha)[(1-\overline{d})r - i], \end{aligned} \tag{10}$$

where $NPV = r - i$. These payoffs, however, are conditional on the project being taken. Because payoffs are not affected by ownership concentration, the family is indifferent among all ownership concentration levels that allow it to raise the necessary funds. Therefore, without loss of generality, we assume that the family chooses the ownership concentration that allows it to raise the most funds.

In the case of the horizontal structure, we define:

$$\overline{R}^H \equiv \max_{R_I^H \in [0,\alpha c], \beta^H} R^H = \alpha c + (1-\overline{d})r. \tag{11}$$

The horizontal structure is feasible whenever $\overline{R}^H \geq i$. In this simplified model, because diversion does not depend on ownership concentration, the family maximizes the funds raised by fully dispersed ownership in firm B. This is not a general result. As we will see in section 5, with costly diversion the family always tries to keep ownership concentration as high as possible.

Similarly, for the pyramidal case we define:

$$\overline{R}^P \equiv \max_{R_I^P \in [0,c], \beta^P} R^P = c + (1 - \overline{d})r. \tag{12}$$

The pyramidal structure is feasible whenever $\overline{R}^P \geq i$. In this case, firm A contributes all of its retained earnings, c, and fully disperses ownership in firm B.

The following result fully characterizes the choice of structure in this version of the model.

Result 1 *If the non-diverted NPV of firm B, $(1 - \overline{d})r - i$, is positive, the family always chooses the horizontal structure. If the non-diverted NPV of firm B is negative and the pyramid is feasible ($\overline{R}^P > i$), the family chooses the pyramid. Otherwise ($(1 - \overline{d})r < i$ and $\overline{R}^P < i$), firm B is not set up by the family.*

The proof of this result, as well as all other proofs, is in the appendix. When the non-diverted NPV is positive, firm B can be financed in either structure because the contribution of external investors, $(1 - \overline{d})r$, is sufficient to pay the investment cost, i. In terms of payoffs, however, the family prefers the horizontal structure. If the family sets up the pyramid, it shares this positive non-diverted NPV with the non-family shareholders of firm A, whereas if it chooses the horizontal structure it gets to keep the entire amount. Therefore, in this case, the horizontal structure is chosen.

When the non-diverted NPV is negative, firm B is not always feasible because the maximum amount external investors contribute is less than the set up costs. Firm B is feasible only when the internal resources are sufficiently high. In addition, when the non-diverted NPV is negative, the family prefers the pyramid because this structure allows it to share this negative value with the other shareholders of firm A. Therefore, in this region, the family chooses the pyramidal structure whenever it is feasible.[9]

[9] There is never a case in which the family prefers the pyramid but the only feasible structure is the horizontal, because the pyramidal structure is feasible whenever the horizontal structure is ($\overline{R}_H < \overline{R}_P$).

Result 2 *Assume that $\overline{R}^P \geq i$, such that firm B is feasible under the pyramidal structure. Given this condition, firm B is less likely to be owned through a pyramid when*

- *Firm B generates higher revenues*
- *Firm B requires a smaller investment*
- *Investor protection increases*

This result follows from the fact that the non-diverted NPV is higher and so more likely to be positive when profitability increases, investment decreases or investor protection is stronger. Because the non-diverted NPV is more likely to be positive, the family is more likely to use a horizontal structure both because its payoff is higher, and because it becomes easier to finance the project.[10]

This simple model identifies a rationale for pyramids that is unrelated to considerations about control of voting rights. Our assumption that there are no legal restrictions to the use of dual-class shares implies that the family can use either structure to achieve control, regardless of how small a cash flow stake it wants to hold. In this framework, any argument for the existence of pyramids that relies on separation of ownership and control cannot make predictions as to which structure the family should use. Because in our model pyramids are not used to separate ownership from control, but rather to allow the family to maximize its internal sources of financing and to share the security benefits of new firms, they can be optimal in this environment. That is, in our model, pyramids are not equivalent to direct ownership with the (potential) use of dual-class shares, even when there are no legal restrictions to the use of dual-class shares.

4 Business groups

We define a business group as an organization in which a family owns and controls more than one firm. In the last section we assumed that the family is the only party with the ability to set up firm B. This effectively means that we assumed the existence of a business group. In this section we investigate the conditions under which a business groups arises.[11]

[10] We condition on firm B being feasible under the pyramidal structure because, empirically, only the set of projects that are feasible under the least restrictive conditions will be observable.

[11] This section is related to the model of family firms in Burkart, Panunzi and Shleifer (2003). The main differences are that they consider only one firm and thus do not model family business groups, while we do not consider the

We introduce the possibility that, at date 1, there is an alternative owner for firm B (whom we call the entrepreneur). The set up cost of firm B for the entrepreneur is also i. The entrepreneur might be a better or a worse manager than the family, a possibility that we capture by assuming that under his control revenues of firm B are $(1+t)r$. The parameter t can be positive or negative, and is a measure of the productivity differential between the family and the entrepreneur. We also assume that the entrepreneur has no personal wealth. Thus, if the parameter $t > 0$, the only advantage of the family is its higher financing capacity due to the accumulation of internal funds in the existing firms it owns (that is, the cash c of firm A).

For simplicity, we assume that the market in question only allows for one firm.[12] Thus, if $t < 0$ the family will be the natural owner of the firm because it has both a technological and a financing advantage. If $t > 0$, the entrepreneur is the most productive owner but might not own the firm because of the family's wealth advantage. We capture this possibility by assuming that, if the entrepreneur can raise sufficient funds, he will be the only one to enter the market because of his higher productivity. If he cannot raise the necessary funds, then the family sets up firm B using any of the two structures described in the last section.[13] Given this assumption, we can prove the following result.

Result 3 *Business groups are less likely to arise when*

- *The entrepreneur's productivity differential, t, is positive and large*
- *Firm B generates higher revenues*
- *Firm B requires a smaller investment*
- *Investor protection is higher*

possibility that the family might hire the entrepreneur as a professional manager and dilute the family's ownership, a situation that they argue should be more common in countries with good investor protection.

[12] Presumably, the family and the entrepreneur would engage in some form of competition for the market, which might involve a phase when both enter and attempt to capture the market. Our assumption that only one can enter can be seen as a reduced form of a competition game under which one of the firms must eventually prevail.

[13] The assumption that a more productive entrepreneur owns the firm whenever he can finance it is a bit extreme. A situation could arise in which the more wealthy family manages to drive out an entrepreneur that is only marginally viable, for example by using its financing clout to lower the output price. However, such a possibility would not lead to results that are qualitatively different than the ones we describe below, since it would still be the case that the entrepreneur would become the most natural owner if its productivity differential and/or firm B's security benefits are large enough. See also the proof of result 3.

If $t > 0$, the comparative advantage of the family is that they have accumulated wealth, and thus do not need to rely as much on external capital markets. As investor protection improves, the comparative advantage of the family eventually disappears and the entrepreneur is able to set up his firm. The entrepreneur is also more likely to raise the necessary funds to set up firm B when firm B's NPV is large, which happens when r and t are high, and i is low.

Notice that the conditions that are conducive to the formation of business groups are also conducive to the formation of pyramids (compare Results 2 and 3). In fact, in this simple model we can prove the following result.

Result 4 *Business groups that arise because of the family's financing advantage, that is, when* $t > 0$*, are always organized as pyramids. If* $t < 0$ *it is possible that business groups are organized horizontally.*

If $t > 0$, competition from the entrepreneur eliminates the region of the parameter space in which a horizontal structure arises. Thus, in our model, there is an endogenously derived equivalence between business groups that arise due to financing reasons and pyramids. The intuition for this result is that horizontal structures only appear when the non-diverted NPV of firm B is positive, because in such cases the family does not want to share the positive NPV of firm B with the existing shareholders of firm A. However, under such conditions entrepreneurial finance is possible, because the fraction of the profits of firm B that can be pledged to outside investors, $(1-\overline{d})(1+t)r$, is bigger than the investment i. Thus, the situations in which an horizontal structure is optimal are precisely the situations in which the business group loses its financing advantage over the entrepreneur. This also means that horizontal groups can only arise because of technological reasons, that is, when $t < 0$. Finally, notice that a corollary of result 4 is that conditional on the business group arising, a pyramid is more likely to appear when the family is *not* the most efficient owner of firm B.

It is worth discussing in more detail what is novel regarding the results in this section. The idea that business groups are more likely to arise in countries with poor investor protection because external financing is more limited is not new. This idea is related to the arguments in Leff (1978) and Khanna and Palepu (1997, 1999), mentioned in the introduction. However, these authors have not considered the optimal choice of ownership structure in a business group. Result 4 suggests that, if business groups are created to substitute for financial markets that are curtailed by poor

investor protection, they *should also be organized as pyramids*. In section 7 we discuss in greater detail the empirical implications of this result.

5 Ultimate ownership and diversion

The simple framework we have used so far generates several results about the conditions under which business groups appear and the type of structures they use. However, because we assumed that diversion is independent of ownership concentration, the family can fully dilute ownership without any implications for value. Thus the previous model is not well suited to address the question of concentration of cash flow rights in pyramidal firms. Furthermore, because diversion is the same irrespective of the organizational form, the model does not have predictions for the relationship between the pyramidal organizational form and diversion.

In this section, we endogenize diversion and allow for an optimal ownership concentration level of firm B. To this end, we assume that diversion is costly. In particular, we assume that $c(0,k) = 0$, $c_d > 0$, $c_{dd} > 0$, and $c_{dk} > 0$. These assumptions imply that a high degree of investor protection (high k) corresponds to a high cost of diversion.

5.1 Optimal ownership concentration in each structure

We start solving the model at date 2. Recall that, in both structures, $d(\omega, k)$ is defined by $d = \arg\max_d \omega(1-d) + d - c(d,k)$, where ω is the family's ultimate ownership concentration in firm B. Assuming an interior solution, $d(\omega, k)$ satisfies the first order condition of this problem: $c_d(d(\omega,k),k) = 1 - \omega$.

It follows from the properties of $c(\cdot,\cdot)$ that diversion is decreasing in ownership concentration ($d_\omega < 0$) and in the level of investor protection ($d_k < 0$). Also recall that $NPV = r - i - c(d,k)r$ is the NPV generated by firm B net of diversion costs. Note that $\frac{\partial NPV}{\partial \omega} > 0$ because higher ultimate ownership concentration reduces diversion and hence the total cost of diversion.

Moving back to date 1, the family solves the problem in Equation (4) and (8). We obtain the following result.

Result 5 *In both structures the family maximizes its ownership concentration in firm B. For this reason, the internal resources contributed to firm B are set to the maximum possible ($R_I^H = \alpha c$ and $R_I^P = c$). Also, the ultimate ownership concentration is set at the highest value that is consistent*

with the financing requirement. That is, for the horizontal structure, if $\alpha c \geq i$*, then* $\omega^H = 1$*, and if* $\alpha c < i$*,* ω^H *is the maximum value that satisfies*

$$R^H(\omega^H) = i. \tag{13}$$

For the pyramidal structure, if $c \geq i$*, then* $\omega^P = \alpha$ *(i.e.,* $\beta^P = 1$*), and if* $c < i$*,* ω^P *is the maximum value that satisfies*

$$R^P(\omega^P) = i. \tag{14}$$

In the horizontal structure the cost of diversion falls back on the family who gets the entire NPV of the project. Thus the family has an incentive to minimize diversion. To commit to a low level of diversion, the family maximizes its ownership concentration (recall that $d_\omega < 0$).

In the pyramidal structure, it is not a priori clear that the family wants to minimize diversion. The reason is that reducing diversion has two different effects on the family's payoff (see Equation (6)). First, it reduces the cost of diversion and hence increases the NPV of the firm B. This is a positive effect on the family's payoff. However, lower diversion also means that the family has to share a greater fraction of the NPV with existing shareholders (the term $(1-\alpha)[(1-d)r - i]$ goes up).

However, we show that it is always the case that the family wants to reduce diversion. Recall that the family bases its diversion decision on its ex-post stake in firm B, $\omega^P = \alpha\beta^P$. Actual diversion is then $d(\alpha\beta^P, k)$. Nevertheless, from the viewpoint of date 1, the family gets a fraction α of the non-diverted revenue (this is because diversion is priced in). That is, from the viewpoint of date 1, optimal diversion is $d(\alpha, k)$. Because the ex-post stake is lower, the family always diverts too much from the perspective of date 1 and hence benefits from reduction in the diversion level.

5.2 Comparison of the pyramidal and horizontal structures

We can now use Result 5 to find the optimal ownership levels for each structure. Then we compare the family's payoff with each structure at the optimal ownership level to determine the optimal structure and its corresponding ultimate ownership.

We concentrate on the most interesting cases in which the family needs to rely on the external market to finance firm B (i.e., $c < i$). By Result 5, we find the optimal ownership concentration

levels, ω^H and ω^P, by replacing $R_I^H = \alpha c$ and $R_I^P = c$ into the Equations (13) and (14), respectively, to obtain

$$\alpha c + (1 - \omega^H)(1 - d(\omega^H, k))r = i, \text{ and} \tag{15}$$

$$\alpha c + (1 - \omega^P)\left[m\left(\frac{\alpha c}{\omega^P}\right) + (1 - m)(1 - d(\omega^P, k))r\right] = i, \tag{16}$$

where $m \equiv \frac{\omega^P}{1-\omega^P}(\frac{1}{\alpha} - 1)$.

Equations (15) and (16) have been rearranged to highlight the distinction between a pyramidal and an horizontal structure. In an horizontal structure, there are only two types of shareholders of firm B: the family and the new shareholders. The family keeps a fraction ω^H of firm B and contributes all of its wealth, αc. New shareholders buy a fraction $1 - \omega^H$ of the firm and pay $(1 - \omega^H)(1 - d(\omega^H, k))r$, where $(1 - d(\omega^H, k))r$ is the market price per share. In a pyramidal structure, there are three types of shareholders: the family, non-family shareholders of firm A and new shareholders. The family retains a fraction ω^P of firm B and contributes αc. The remaining fraction, $1 - \omega^P$, is distributed between non-family shareholders of firm A and new shareholders of firm B.[14] The term in brackets in Equation (16) reflects the average price of these shares in which the variable m measures the weight of each of these two types of shareholders. New shareholders pay the market price of firm B, $(1 - d(\omega^P, k))r$, while non-family shareholders of firm A pay an implied price of $\alpha c/\omega^P$. The latter price follows because non-family shareholders of firm A contribute their share of retained earnings $(1 - \alpha)c$ for a fraction $(1 - \alpha)\beta^P$ of firm B. Thus, the implied price is $(1 - \alpha)c/[(1 - \alpha)\beta^P] = \alpha c/\omega^P$.

As Equations (15) and (16) indicate, even though the family has access to the entire stock of earnings of firm A when it chooses the pyramidal structure, this does not necessarily translate into a financing advantage. The reason is that, in the pyramidal structure, non-family shareholders receive shares, which could have been sold to the market if the horizontal structure had been used instead. The pyramidal structure has a financing advantage if and only if the implied price paid by non-family shareholders of firm A is higher than the market price of firm B.

To compare the payoff under the two structures, we need to know the relation between ω^H and ω^P and the value of the non-diverted NPV (see Equations (2) and (6)). We establish these facts

[14] Note that non-family shareholders of firm A indirectly own $(1-\alpha)\beta^P = (1-\omega^P)m$ of firm B and new shareholders hold $1 - \beta^P = (1 - \omega^P)(1 - m)$ of firm B.

for all parameter values and indicate the region of the parameter space over which each structure is chosen in Result 6. Before we do that, we explain in detail what happens in the model as one particular parameter varies (the investment level). This allows us to explain the main results graphically using Figure 1.

5.2.1 The relation between the ultimate ownership concentration levels ω^H and ω^P

In Figure 1 we plot the values of R^H and R^P as a function of the family's ultimate ownership in firm B, ω.[15] At $\omega = \overline{\omega}$, the family raises the same amount of funds with either structure (we call this amount $\overline{i}$) because the implied price paid by non-family shareholders of firm A equals the market price paid by new shareholders of firm B. For higher ownership concentration levels, $\omega > \overline{\omega}$, the market price increases (because there is less diversion in firm B), and the implied price declines (because non-family shareholders receive more shares but contribute the same amount). Thus, for $\omega > \overline{\omega}$, the market price is higher than the implied price. When this is the case, the family raises more funds with the horizontal structure because, with this structure, the number of shares sold to the market is larger. This explain why Figure 1 shows R^H above R^P for all $\omega > \overline{\omega}$. Conversely, for lower ownership concentration levels, $\omega < \overline{\omega}$, the market price of firm B is lower than the implied price. In this case, the family raises more funds with the pyramidal structure. Figure 1 shows R^P above R^H for $\omega < \overline{\omega}$.

Figure 1 also shows the optimal ownership concentration levels, ω^H and ω^P for two investment levels i_1 and i_2 with $i_1 < \overline{i} < i_2$.[16] For the low investment level, i_1, the ultimate ownership in the pyramidal structure is lower than that in the horizontal structure ($\omega_1^P < \omega_1^H$). The reason is that the ultimate ownership concentration consistent with a low investment cost is quite high (few shares need to be sold). As a result, the market price of shares of firm B is higher that the implied price. Therefore, if the family chooses this structure, it has to sell fewer shares to finance the investment. This allows the family to keep a higher ownership concentration in the horizontal structure. Conversely, at the high investment cost, i_2, $\omega_2^P > \omega_2^H$. The reason is similar. When the family needs to finance a high investment level, it has to sell a large fraction of the shares. The

[15] Figure 1 only shows the relevant part of the curves. Both R^H and R^P might have increasing and decreasing regions. However, the solution is never in an increasing region. The reason is that a slightly higher onwership concentration would increase NPV (recall that $\frac{\partial NPV}{\partial \omega} > 0$) and at the same time allow the family to raise more funds. Thus, the solution has to be in a decreasing region.

[16] Because, for $s = H$ or P, ω^s solves $R^s(\omega^s) = i$, the optimal ω is the value on the horizontal axis that corresponds to the intersection of R^s and an horizontal line at i.

resulting ownership concentration is low and hence the market price falls below the implied price paid by non-family shareholders of firm A. In this case, the pyramid has the financing advantage and leads to a higher ownership concentration.

5.2.2 The non-diverted NPV at the optimal ownership concentration

It turns out that the non-diverted NPV is negative for both structures when $i > \overline{i}$ and it is positive for both structures when $i < \overline{i}$. As we explained above, when $i > \overline{i}$, the implied price paid by non-family shareholders is higher than the market price of firm B. This implies that non-family shareholders get a lower return on their investment than market participants. Because non-family shareholders of firm A get the non-diverted NPV of firm B and market participants get a zero NPV, it follows that the non-diverted NPV is negative. A similar argument holds for the other case.

5.2.3 The choice of structure

The final step is to compare the maximum payoff with each structure (Equations (2) and (6)) at the optimal ownership concentration. The key result is that, if $i < \overline{i}$, the horizontal structure is chosen, and if $i > \overline{i}$, the pyramidal structure is chosen. When $i < \overline{i}$, the ultimate ownership in the horizontal structure is higher. A more concentrated ownership leads to a higher NPV (recall that $\frac{\partial NPV}{\partial \omega} > 0$). In addition, the non-diverted NPV in this region is positive and therefore it is better to capture this value entirely (horizontal structure) than to share it with the non-family shareholders of firm A (pyramidal structure). Conversely, when $i > \overline{i}$, the ultimate ownership in the pyramidal structure is higher and the non-diverted NPV of firm B is negative. Thus, in this region, the pyramid is the best option for the family.

We summarize the above discussion in the following Result.

Result 6 *Let $(\overline{\alpha}, \overline{c}, \overline{r}, \overline{i}, \overline{k})$ be parameters such that $\omega^H = \omega^P = \overline{\omega}$. Take parameters $(\overline{\alpha}, \overline{c}, \overline{r}, i, \overline{k})$. For $i > \overline{i}$, the ultimate ownership in a pyramid is higher ($\omega^P(i) > \omega^H(i)$), the non-diverted NPV of firm B is negative under both structures and, if feasible, the pyramidal structure is chosen (identical results hold when $r < \overline{r}$ and when $k < \overline{k}$). For $i < \overline{i}$, the ultimate ownership in a pyramid is lower ($\omega^P(i) < \omega^H(i)$), the non-diverted NPV of firm B is positive under both structures and the horizontal structure is always chosen (identical results hold when $r > \overline{r}$ and when $k > \overline{k}$).*

As the above discussion illustrates, the model with endogenous ownership produces essentially the same results regarding pyramidal and horizontal structures that we described in section 5. Pyramids are chosen only when the existing shareholders of firm A lose with the addition of firm B. Otherwise, the family prefers to set up a horizontal structure. Furthermore, pyramids are more likely to be chosen when investor protection and firm B's NPV are low.

As Result 6 indicates, the relative size of the family's ultimate cash flow stake in the pyramidal and in the horizontal structure depends on the parameter values. For some parameter values we have that $\omega^P > \omega^H$, and for others, $\omega^P < \omega^H$. However, result 6 cannot be taken to the data because it compares the hypothetical values of ownership concentration that would arise if each structure were to be chosen. It does not compare the observed (or effectively chosen) ownership concentration levels as the parameters vary. The following result establishes that relation.

Result 7 *Suppose that different structures are the result of variation in r, i or k (one by one). The ultimate ownership concentration level* observed *in any pyramidal structure is lower than the ultimate ownership observed in any horizontal structure. It follows directly that diversion from firm B is higher in pyramids.*

This is a direct implication of the discussion leading to Result 6. Figure 2 reproduces Figure 1 and indicates the range of ownership concentration levels that are associated with each structure. The pyramidal structure is chosen when $i > \overline{i}$. These investment levels map into ownership concentrations that satisfy $\omega^P < \overline{\omega}$. Also, the horizontal structure is chosen whenever $i < \overline{i}$. The associated ownership concentration levels are $\omega^H > \overline{\omega}$.

The point that ultimate ownership in a pyramid is low has been made informally by many authors. The traditional argument is that the chain of control mechanically reduces ultimate ownership. As Result 6 indicates, however, if one compares the ultimate ownership a firm would have under a pyramid and under a horizontal structure, it is not obvious that ultimate ownership would be lower in a pyramidal structure. One key feature that the traditional argument ignores is that because of the presence of retained earnings, the pyramid might have a financing advantage, and consequently, the family might need to sell fewer shares when using this structure to finance the new firm. Our argument in Result 7 is different. The pyramid is set up when the investment required is high, revenues are low and/or investor protection is low. But in these situations more

shares need to be sold to finance the set up costs of the new firm. This explains the fact that ultimate ownership is lower in a pyramidal structure than in a horizontal structure.

A similar argument holds for diversion. The traditional view is that diversion is higher in a pyramidal structure because the chain of control reduces the family's ultimate ownership. This argument ignores the fact that families are interested in reducing diversion because its cost falls back to them (see Result 5). But if families take into account that the cost of diversion falls back to them, why do they ever choose a structure that facilitates diversion? Our model is consistent with the empirical observation that there is more diversion in pyramids, while at the same time families try to minimize diversion. In our model expropriation is higher in firms that end up in a pyramidal structure. However, in these cases, diversion would have been even higher (because of a lower ultimate ownership concentration) if the horizontal structure had been chosen instead.

Result 7 shows that the observed ultimate ownership in pyramids is lower than in horizontal structures. It does not imply that ownership concentration should be low in a pyramid in an *absolute* sense. In fact, the threshold of ultimate ownership at which the family switches to the pyramidal structure, $\overline{\omega}$, can be quite high. If, for instance, this threshold is strictly above 50%, some pyramidal firms will have ultimate ownership concentration around 50%. Clearly, for these firms, the family could have achieved the same degree of control by simply holding shares directly. Yet, the family chooses the pyramids because of its financing and payoff advantages described above. Depending on the parameter values, our model can also predict very small ownership concentration in pyramidal firms. Thus, unlike the traditional argument for pyramids, our model accommodates both high and low ownership concentration in pyramidal firms. We see this as a strength of the model because the empirical evidence indicates that both cases are common. Moreover, we can analyze which characteristics make it more likely that a pyramid will be associated with large ultimate ownership concentration.

Result 8 *The threshold of ultimate ownership below which the pyramid is chosen, $\overline{\omega}$, is decreasing in investor protection.*

Recall that, at the threshold $\overline{\omega}$, the non-diverted NPV of firm B is zero. As investor protection deteriorates, ownership concentration must increase to keep the diversion level and the NPV constant. The implication of this result is that, in poor investor protection countries, it is more likely

that families hold a large ultimate cash flow stakes in pyramidal firms.

6 Extensions

In this section we provide a number of new results and also confirm the robustness of our main results to some specific assumptions made above. As our benchmark model, we use the model with costless diversion of section 3. The exception is section 6.3, in which we need the model with costly diversion to be able to characterize optimal ownership stakes.

6.1 Ex-ante optimality of pyramids

In our model whenever the pyramidal structure is chosen, shareholders of firm A realize a negative return because they share the negative non-diverted NPV with the family. This raises the question of whether shareholders will agree to buy into firm A at date 0 if they anticipate that a pyramid might be formed in the future. Even though it is possible that shareholders do not anticipate such an event, in this section we analyze a model in which shareholders can rationally foresee the future formation of the pyramid.[17]

To analyze this question, we extend the model with exogenous diversion of section 3 to date 0. We assume that, at date 0, firm A needs an investment of i_A and generates revenues of $r_A > i_A$ at date 1. Similarly, firm B requires an investment of i_B at date 1 and generates a revenue of r_B at date 2. For simplicity, we assume that there is no diversion of the cash flows of firm A, but the analysis can be easily extended to a case in which there is diversion (see section 6.3). We also assume that the family has no wealth at date 0, and we do not consider competition by the entrepreneur, neither at date 0, nor at date 1.[18]

Suppose first that the family cannot commit at date 0 not to set up the pyramid at date 1. In this case, the following result shows that pyramids arise under certain conditions.

[17] Aganin and Volpin (2004) document the case of the Pesenti group. The first firm in the group, Italcementi, was established in 1865. The creation of the Pesenti pyramid happened when the family started to acquire firms in 1945. The possibility that shareholders in 1865 foresaw the creation of a pyramid 80 years later, is possible, but unlikely.

[18] If there is competition at date 0, the family will own firm A if it is the most efficient owner, or if the family is sufficiently more wealthy than the entrepreneur. On the other hand, if the entrepreneur owns firm A, the entrepreneur "becomes" the family in date 1, from the perspective of our model. Competition at date 1 from a talented entrepreneur has the same effect it had in section 3: it eliminates horizontal structures if the pledgeable income of firm B is larger than the required investment.

Result 9 *Suppose that* $r_A + (1-\overline{d})r_B > i_A + i_B$ *and that* $(1-\overline{d})r_B < i_B$*. In this case, the family sets up firm A at date 0 and uses a pyramid to set up firm B at date 1. Shareholders of firm A break even from the perspective of date 0.*

Intuitively, the first firm that the family sets up must be profitable enough in order to compensate initial shareholders for the future expropriation associated with pyramids. If this condition holds, the group's shares can be priced low enough such that initial shareholders break even and the family can raise enough to finance firm A.

Under the assumption that the family cannot commit at date 0 to use a particular structure in the future, result 9 shows that the pyramid might appear irrespective of whether it is *ex-ante* optimal for the controlling family. Result 9 does not rule out the possibility that the family might benefit from a mechanism (such as a contract or a charter provision) that allows it to commit not to form pyramids. From the perspective of date 0, the family bears all the costs of future expropriation associated with pyramids.[19] Thus, its expected payoff might be higher if such a contractual commitment is possible. Importantly, however, our model also suggests that the family may not want to rule out pyramids by contract *even when* it can do so. We argue that there are cases in which the only way the family can set up both firms A and B is by allowing pyramidal structures. Ruling out pyramids might eliminate the possibility of setting up firm B. Since firm B is positive NPV, this is inefficient from an ex-ante perspective.

This type of situation arises if there is uncertainty regarding the cash flow produced by firm A. Suppose that the revenue generated by firm A is $r_A = \overline{r}_A - \Delta$ with probability $\frac{1}{2}$, and $r_A = \overline{r}_A + \Delta$ with probability $\frac{1}{2}$. The following result holds:

Result 10 *Suppose that the following conditions hold:*

$$\left(1 - \frac{i_A}{\overline{r}_A}\right)\Delta + \overline{r}_A - i_A + (1-\overline{d})r_B - i_B < 0, \tag{17}$$

and

$$\overline{r}_A - i_A + \frac{1}{2}\left[(1-\overline{d})r_B - i_B\right] > 0 \tag{18}$$

[19] In fact, in the version of the model we analyze in this section, pyramids do not have higher *deadweight* costs relative to horizontal structures. This is because diversion of cash flows from firm B is assumed to costless. Thus, conditional on the family being able to finance both firms, the family's ex-ante payoff is identical under pyramidal or horizontal structures. We can show, however, that in the model with endogenous diversion (section 5), ruling out pyramids ex-ante will generally decrease ex-post diversion and associated deadweight costs (see also Wolfenzon, 1999).

In this case, it is not optimal for the family to rule out pyramids at date 0.

Under condition in Equation (17), the family cannot set up firm B at date 1, even in the high cash flow state. In this situation it might be efficient for the family to allow pyramids to be formed at date 1 in the high cash flow state, because pyramids relax the date 1 financing constraint by increasing the cash available for investments. The problem with the pyramid is that because shareholders of firm A expect future expropriation, allowing pyramids to be formed tightens the date 0 financing constraint. The condition in Equation (18) is required for the date 0 constraint to be met if the pyramid is formed only in the high cash flow state.[20]

If the two conditions of Result 10 hold, ruling out pyramids prevents the family from setting up firm B. Clearly, when investor protection is perfect (i.e., $\overline{d} = 0$), these conditions do not hold. In this case, firm B can be set up even when pyramids are ruled out. A necessary condition for these conditions to hold is then that investor protection be imperfect.

So far we assumed that firm B can only be set up at date 1. The next result endogenizes the timing of this decision in the context of the current extension.

Result 11 *Suppose the family has access to both projects at date 0. Under the conditions of result 10, the optimal investment policy is to set up firm A first, and then set up firm B in a pyramid if the cash flows of firm A are high.*

Result 10 shows that pyramids can only have ex-ante benefits if the sum of the pledgeable incomes of firms A and B is lower than the sum of the required investments. This follows from the first condition in result 10, because $\overline{r}_A - i_A + (1-d)r_B - i_B < -\left(1 - \frac{i_A}{\overline{r}_A}\right)\Delta < 0$. In this case, the family cannot set up both firms with probability one. Furthermore, as we explained above, the pyramid will be set up when cash flows are high. Thus, pyramids are created following good performance of the existing firms in the group.

6.2 Should firm B be a separate firm or a division?

In the model of sections 3, 4, and 5 we define a pyramid as the structure that results when firm A sets up firm B. However, in addition to this feature, the definition of a pyramid requires firm B

[20] We show in the appendix that if conditions 17 and 18 hold, the family can never raise enough funds to set up firm A at date 0, and set up the pyramid in both states at date 1, because in this case shareholders of firm A cannot break-even.

not to be wholly-owned by firm A. Otherwise, one could think of firm B simply as a project of firm A. In this section we focus on the cases in which the family chooses the pyramidal structure and analyze the optimal financing of firm B. This financing can take the form of sale of shares of firm A, of firm B, or a combination of the two. We show that in a large number of cases it is optimal for the family to sell shares of firm B directly to the market, i.e., it is optimal for firm A to retain less than 100% of the cash flow rights of firm B. In these cases, it is clear that the resulting structure is a pyramid.

To analyze this question we need a model in which the optimal ownership concentration is well-defined. For that reason we use the model with costly diversion of section 5. In addition, we augment that model by allowing for the possibility of diversion from firm A. Because we model diversion from both firms A and B, we can analyze the trade-off between selling shares of these firms to raise funds.

In the previous section we assumed that the parameter k in the cost of diversion $c(d, k)$ was common to all firms in a country. The underlying assumption was that they are all subject to the same laws, regulations, and other institutions that protect outside investors. However, even though there is an important common component in investor protection, there is within-country variation in the degree of protection firms offer to their outside shareholders (Durnev and Kim (2004)). The reason for this firm-specific variation can be purely technological: some industries are inherently more obscure than others. It is also the case that firms can take actions to modify the degree of protection they provide. For example, firms can hire more or less outside directors, have their books audited by a reputable accounting firm, etc. Accordingly, in this section we assume that there is within-country variation in the degree of investor protection offered by firms by letting k_A be potentially different from k_B, where k_A and k_B are the parameters in the cost of diversion function for firm A and B, respectively. However, we maintain the assumption that there is a significant country component. In fact, Doidge, Karolyi, and Stulz (2004) find that a significant fraction of the variation in individual firm corporate governance measures is driven by country characteristics.

As in section 5, we consider a model that starts at date 1. At this date the family has a stake α_1 in firm A and and the possibility of setting up firm B appears. Instead of assuming that firm A generates cash flows at date 1, we assume that it only generates a cash flow, r_A, at date 2. Of course, the cash flow r_A that occurs at date 2 can be 'converted' into a date-1 cash flow by selling

claims against it. However, because we introduce costly diversion from firm A, this conversion will entail deadweight costs. The goal of the family is to set the ownership structure of firm A and firm B so as to raise the necessary funds in the most cost-efficient way.

We show in the Appendix that, whenever $k_B > k_A$, it is optimal to sell shares of firm B directly to the market. The reason is that the family prefers to sell shares of the firm with a higher diversion cost because a higher diversion cost helps the family commit to low levels of diversion and thus to an overall lower deadweight cost. Since firm A holds a stake in firm B, the family could sell rights to the cash flows of firm B indirectly by selling shares of firm A. However, when $k_B > k_A$, it is more efficient to sell shares of firm B while minimizing the number of shares of firm A that the family sells. To achieve this goal, the family needs to sells shares of firm B directly.

Notice that even under the more precise definition of a pyramid used in this section (e.g., the structure that results when firm A sets up firm B and sells shares of firm B directly to the market), we still get the result that pyramids are more common in countries with poor investor protection. The reason is that, as shown in sections 3 and 5, whether firm B is set up by firm A or directly by the family is determined by the *absolute* value of k_B. If there is at least some country component in the determination of k (Doidge, Karolyi, and Stulz (2004)) then it is more likely that firm B will be set up by firm A in poor investor protection countries. However, this result does not guarantee that there will be more pyramidal firms in these countries. If, for some reason, firm A is also more likely to set up firm B as a wholly-owned subsidiary in countries with poor investor protection, then our result will break down. However, in this section we have shown that, conditional of firm A setting up firm B, whether shares of firm B are sold directly to the market (pyramid) or not (wholly-owned subsidiary) depends only on the *relative* magnitudes of k_A and k_B. Because there is no reason to expect the probability of $k_B > k_A$ to vary systematically with investor protection, the proportion of pyramids to wholly-owned subsidiaries should not vary systematically across countries. Thus, the result that there should be more pyramids in poor investor protection countries is robust.

6.3 Other forms of diversion

In the benchmark model, we assume that cash flows are diverted from firm B directly into the family's pockets. While this assumption seems appropriate for some forms of diversion (e.g., personal loans from the firm that are never repaid, above market salary, etc.), it does not capture intra-firm

diversion (e.g., transfer pricing between two firms at non-market prices or financial transactions between firms at non-market prices). This latter form of diversion has been shown to occur in business groups (Bertrand, Mehta and Mullanaithan, 2002). In addition, the benchmark model abstracts from direct diversion from firm A.

In the following two section we extend the simple model of section 3 to allow for intra-firm diversion and diversion from firm A and show that our results hold. In particular, we show that when the non-diverted NPV of firm B is positive, the family chooses the horizontal structure. Otherwise, it chooses the pyramid, if it is feasible. That is, even with these different specifications of the diversion technology, our central result in section 3, Result 1, holds exactly. We provide a brief description of the modified model and the intuition for why Result 1 holds. In the Appendix we provide proofs of the results.

6.3.1 Intra-firm diversion

To analyze the effect of intra-firm diversion, we modify the model of section 3 by making the assumption that the cash flows of firm B can only be diverted from firm B to firm A, but not directly to the family. We maintain the assumption that diversion possibilities are the same across the two structures. That is, at date 2, the family chooses the fraction d of firm's B revenues that it diverts to firm A. As in section 3, the cost of diversion is zero up to a fraction $\overline{d}$, and is infinite above that level.[21]

The simple intuition for why we obtain Result 1 is as follows. In the pyramidal structure, the family shares both the diverted amount and the non-diverted NPV with the non-family shareholders of firm A, whereas in the horizontal structure, the family shares only the diverted amount and keeps the entire non-diverted amount for itself. Thus, as in the model of section 3, the choice of structure affects only the fraction of the *non-diverted* payoff that the family captures. Whenever the non-diverted payoff is positive, the family's payoff is higher under the horizontal structure. In such a case, the horizontal structure is always feasible and thus it is chosen. Conversely, when the non-diverted NPV is negative the family prefers the pyramidal structure and hence chooses this

[21] Naturally, diversion opportunities can be such that funds can be transferred in the other direction, from firm A to firm B. Note, however, that this type of diversion never occurs in a pyramidal structure since the cash flow stake of the family is always higher in firm A than in firm B. Also note that diversion from firm A to firm B in an horizontal structure is identical to direct diversion from firm A: funds are transferred out of firm A and the family captures the entire diverted amount even if it ends up in firm B, because new shareholders of firm B price in the expected inflow of funds when they buy firm B's shares.

structure whenever it is feasible.

6.3.2 Diversion from firm A

In this section we consider the possibility of diversion from firm A directly into the family's pockets. In particular, we assume that, at date 1, the family can divert a fraction d_A of the cash, c, generated by firm A. We maintain the assumption that, at date 2, the family can divert a fraction d_B of the revenue of firm B to its pockets. As in section 3, we assume that the cost of diversion is zero up to a given level ($\overline{d}_A$ and $\overline{d}_B$ for diversion from firms A and B, respectively), and infinite above that level.

As opposed to the model of section 3, in this version of the model, there are two sources internal funds at date 1: the amount diverted from firm A, and the post-diversion cash left in firm A. We maintain the assumption that the family is free to contribute any amount of the after-diversion cash to the set up of firm B. That is, in the horizontal structure $R_I^H \leq \alpha(1-d_A)c$, and in the pyramidal structure $R_I^P \leq (1-d_A)c$. However, we assume that the family can only invest the diverted amount, $d_A c$, in firm B at market prices.[22]

We get Result 1 for an identical reason to that in section 3 and section 6.3.1. The family engages in diversion from firm A in both structures and the only difference in payoff lies in the distribution of the non-diverted NPV of firm B.[23] The family shares this value when it chooses the pyramidal structure and keeps it entirely when it chooses the horizontal structure.

6.4 Can firm B have a negative NPV?

In the model, we assume that firm B is always a positive NPV investment once we take into account both the security benefits and the private benefits that accrue to the family. While the family will never want to undertake a negative NPV investment in a horizontal structure, it might want to do so in a pyramidal structure because in this case it shares the negative value consequences with the shareholders of A. In fact, we can show that if expected diversion is high enough the family might benefit from setting up firm B at date 1, even when firm B has a negative total NPV.

Nevertheless, the assumption that there are no negative NPV investments is less restrictive than

[22] Allowing the family to use the diverted amount from firm A to invest in firm B at better-than-market prices would constitute another form of diversion from firm B. We assume that this is already included in d_B.

[23] We are oversimplifying here for ease of explanation. As we show in the proof, diversion from firm A is not the same across structures. In fact, we show that $d_A^P \geq d_A^H$, which is another advantage of the pyramidal structure.

it seems. First of all, the model already suggests that the types of firms that tend to be owned through pyramids are negative NPV for the non-family shareholders. In this sense, the only new result that we would obtain if the total NPV of firm B were negative is that from the perspective of date 0, the family would always like to commit not to set up firm B. Second, even if the family can undertake such investments, they might not happen in equilibrium. Notice that the minimum diversion level that makes it worthwhile for the family to set up a negative-NPV firm B is higher than the level that makes the family use the pyramid in the model of sections 3 and 5. Thus, a negative-NPV firm B is harder to finance. Moreover, and perhaps more importantly, the family always prefers a positive to a negative NPV project. Thus, the family will prefer to use its wealth first on positive NPV projects, and only then on negative NPV ones. This makes it even more likely that there will be limited wealth available to invest in negative NPV projects.[24]

7 Empirical Implications

Our theory generates a number of empirical implications, which we list and discuss here. We also mention empirical and anecdotal evidence that is consistent with our theory, and suggest some additional implications that can be tested in the future.

1. **It is possible to observe pyramids in which the controlling family has high cash flow stakes in member firms, and thus the separation between ownership and control is not large**

The traditional argument for pyramids considers them simply as a device to separate cash flow from voting rights. Consistent with the traditional view, there are a number of examples in the literature in which a family has achieved substantial deviation from one share-one vote through the use of pyramids (see the examples presented in La Porta, Lopez-de-Silanes and Shleifer, 1999, and Claessens, Djankov and Lang, 2000). However, there are also many other cases in which the separation achieved is small and does not warrant the use of a pyramid. For example, Franks and Mayer (2001) find in their sample of German firms that, in 69% of the firms controlled through

[24] Furthermore, the model in section 6.1 suggests that if shareholders anticipate the future creation of pyramids, they tend to appear when the cash flows produced by firm A are high. Presumably, these are also situations in which the probability that firm B turns out to be a positive NPV project is high, because both the good performance of firm A and the value of firm B might be driven by a common component. This is an additional reason why our focus on positive NPV projects seems warranted.

pyramids, the controlling shareholder could have achieved the same level of control by simply holding shares directly in the firm. The authors conclude that, in Germany, pyramids are *not* used as a device to achieve control.[25] In a study of ownership and control of Chilean firms, Lefort and Walker (1999) find that the controlling shareholder owns more cash flow than necessary to achieve control. They compute the ultimate cash flow ownership of the controlling shareholder in all the members of a pyramidal group and find this 'integrated' ownership to be on average 57%. Thus, the separation of ownership and control achieved through pyramids is minimal. Attig, Fischer and Gadhoum (2003) find that, in Canada, the cash flow stake of the controlling shareholder in a pyramid is, on average, 31.78% while the controlling stake is only a bit higher, 41.68%. Faccio and Lang (2002) report that both dual-class shares and pyramids are commonly used in Western European countries. However, they find large deviations between ownership and control in only a few of the Western European countries they analyze. Demirag and Serter (2003) report similar findings for Turkey, where cash flow and voting rights appear to be closely aligned despite the widespread prevalence of pyramidal structures. Finally, Valadares and Leal (2001) draw a similar picture for Brazil, where according to the authors pyramids do not appear to be a mechanism to deviate from one-share-one-vote.

Our model is consistent with these cases. Even though we show that the ultimate ownership concentration in pyramids is lower than that in horizontal structures, it can still be the case that the ultimate ownership in a pyramid is high in an absolute sense (see our discussion following Result 7). In fact, depending on the parameter values, pyramidal firms can have either low or high ultimate ownership concentration (and consequently large or small separation of ownership and control). This is consistent with the evidence that in some pyramids a significant separation is achieved, while in others there is virtually no separation of ownership and control. In addition, we show in result 8 that pyramids with minor separation from ownership and control are more likely to arise in countries with poor investor protection. Because pyramids are more common in countries with

[25] Franks and Mayer define 25%, 50% and 75% as critical control levels and argue that voting power between any of these critical levels provide the same degree of control. They show that in 69% of their sample of pyramidal firms, the cash flow and control rights do not straddle a control treshold. To see that, when this is the case, the pyramid is not used to separate ownership and control, consider the following example. A family's ultimate cash flow rights in a firm that belong to a pyramid are 55% and his voting rights are 70%. If the same controlling party held 55% of the shares directly in the firm, he would have 55% of the votes (assuming one-share-one-vote). Because 55% and 70% are between the same two critical levels, direct holdings in the firm and the pyramid provide the controller with the same degree of control.

poor investor protection, this result helps explain why it is possible to observe pyramidal firms with small separation between cash flow and voting rights.

2. **The family might strictly prefer to create a pyramid, even when restrictions to the issuance of dual-class shares are not binding**

Because we identify a rationale for pyramids over and above the separation of cash flow from voting rights, our model can distinguish between pyramids and direct ownership with dual-class shares even if there are no legal restrictions to the use of dual-class shares. Therefore, according to the model, it should not be surprising to find that pyramids arise even when families have not exhausted the possibility of issuing dual-class shares.

We do observe pyramids in situations in which the family could have achieved the same separation with dual-class shares alone. For example, in Italy, Bianchi, Bianco, and Enriques (2001) measure the ultimate ownership in each firm that belongs to a pyramid, compute the number of units of capital that the controlling shareholder controls with one unit of his own capital, and average this ratio for all the firms in a pyramid. As a benchmark, consider a family who holds directly 50% of the cash flows and votes in a firm. In this case the ratio is 2. The family can increase this ratio because Italian law allows the issuance of 50% of the firm's capital in non-voting shares (savings shares or *azioni di risparmio*). If the family uses the maximum fraction of dual-class shares and retains 50% of the voting shares (i.e., 25% of the total capital), it can achieve a ratio of 4. Bianchi, Bianco and Enriques find that, while some pyramids allow the controlling shareholder to control a large amount of capital (e.g., the ratio for the De Benedetti group is 10.33 and that for the Agnelli group is 8.86), the ratio for other groups is below 4, and sometimes even below 2 (e.g., for the Berlusconi group, it is 3.66, and for the Pirelli group it is only 1.95). Finally, Brazil is another country in which dual-class shares can be issued. The evidence in Valadares and Leal (2001) suggests that pyramids are common despite the fact that many Brazilian firms do not exhaust the possibility of issuing shares with superior voting rights.

3. **Family business groups should be more prevalent in countries with poor investor protection**

In the model, families have a financing advantage over potentially more efficient, but less wealthy entrepreneurs, because families can utilize the funds of the firms they already control. In low

investor protection countries, this financing advantage is more important because it is more difficult to raise external finance (see result 3). Thus, the fraction of the corporate sector that ends up in business groups should be higher in such countries.

This implication is in the spirit of the arguments in Khanna and Palepu (1997, 1999), who argue that business groups arise in countries with underdeveloped markets. We believe there is no systematic evidence on this implication, but there is some scattered and anecdotal evidence. For example, Claessens, Fan and Lang (2002) show that the incidence of business groups is high in developing Asian countries, where more than 50% of the firms in their sample are affiliated with business groups. Faccio and Lang (2002) report similar results for Europe.

4. **Family business groups are more likely to be organized as pyramids, especially in countries with poor investor protection**

In our model, the conditions that are conducive to the appearance of business groups are also conducive to the choice of pyramids over horizontal structures. Families choose the pyramidal structure if the security benefits associated with the new firm are low (high investment, low revenues and poor investor protection). However, it is precisely in these cases that an outside, talented entrepreneur cannot finance this new venture in the external capital market. As a result, the business group is created and the pyramid is used.

As we discuss in section 4, if there are other reasons for firm B to be set up in a business group (for example, if the family is also the most efficient owner of firm B), then the business group may be organized horizontally. However, in this case implication 3 also breaks down because the underlying rationale for the existence of the business group does not necessarily correlate with investor protection. A more general way to state the implication of our model is thus that the *types* of business groups that appear because of poor investor protection (in our model those that appear because of financial market underdevelopment) tend to be organized as pyramids. Thus, while business groups are likely to have pyramidal structures in countries with poor investor protection, they might be organized horizontally in other countries with higher investor protection.

La Porta, Lopez-de-Silanes and Shleifer (1999) show evidence that pyramids are very common in countries with poor investor protection. Another piece of evidence that business groups are typically organized as pyramids is that researchers have treated these two terms as synonymous

when analyzing the role of family control in developing countries.[26]

5. **When a new firm is added to a pyramidal structure, the existing non-family shareholders of the pyramid realize a negative return**

Results 1 and 6 show that pyramids can only be chosen by the family when the pledgeable income of firm B is lower than the required investment, which implies that existing shareholders of firm A realize an ex-post loss when the pyramid is formed. As we show in section 6.1, this does not necessarily mean that shareholders also lose in an "ex-ante" sense because the business group's shares might be priced to reflect future expropriation.

Although the empirical literature has not tested this hypothesis directly, there is some evidence that M&A activity in firms owned by a controlling shareholder and in business groups is associated with expropriation of minority shareholders. Bae, Kang, and Kim (2002) find that Korean chaebols use M&A transactions between member firms to expropriate shareholders of the bidder firm and benefit the controlling family. Bigelli and Mengoli (2001) show evidence that around the time of the announcement of acquisitions by Italian firms (not only business groups), voting shares perform significantly better than non-voting shares, indicating that acquisitions are associated with large private benefits of control. On the other hand, Buysschaert, Deloof and Jegers (2003), who uses a sample of Belgian firms, Holmen and Knopf (2003), in Sweden, and Faccio and Stolin (2004), in a sample with several European countries, do not find evidence that M&A activity is associated with expropriation.

6. **Diversion is higher in firms placed in a pyramid, than in firms controlled directly by the family**

We show that the observed ultimate ownership is lower in pyramidal firms, and thus diversion is higher (result 7). It is important to emphasize that our model does not predict that, for given parameter values, ultimate ownership is lower in a pyramid. Even though this might seem obvious because, in a pyramid, the chain of control reduces ultimate ownership, it is not always the case. The reason is that there is an opposing effect when the pyramid is chosen: the family has access to more internal funds and so it can finance the investment by selling fewer shares of the new

[26] See for example the definition of a business group in Claessens, Fan and Lang (2002): "A group can be described as a corporate organization where a number of firms are linked through stock-pyramids and cross-ownership".

firm. Our result is that the *observed* ultimate ownership in a pyramid is lower. We show that the family chooses pyramids when the investment required is high, or when the revenues of the firm and investor protection are low. In these situations the family needs to sell more shares to finance the firm. As a result, ultimate ownership is lower and diversion higher than in horizontal structures. Thus, even though in our model families do not choose pyramids to facilitate diversion, pyramidal business groups are nonetheless associated with high levels of diversion because of this selection effect. This prediction is consistent with empirical findings that pyramids are associated with high expropriation (Bertrand, Mehta, and Mullanaithan 2002).

7. **Firm value and firm performance will tend to be lower in firms that are owned through pyramids, than in unaffiliated firms or horizontal structures**

Our model predicts that projects of lower profitability will be undertaken inside pyramids (results 2 and 6). Thus, even if the pyramid does not have a direct negative effect on performance, one should observe a negative relationship between measures of firm value such as Tobin's Q and pyramidal membership because of a selection effect. There is evidence that firms in business groups organized as pyramids have lower Tobin's Q than stand-alone firms and firms organized in horizontal groups (Claessens et al. 2002, Volpin 2002) and that this undervaluation is greater if the controlling shareholder has lower ultimate ownership (Holmen and Hogfeldt, 2004). There is also evidence that the separation of ownership and control is detrimental to performance (Claessens et al 2002, Lemmon and Lins 2003, Lins 2003, Mitton 2002, and Joh 2003).[27] Finally, Attig, Fischer and Gadhoum (2003) show that low Tobin's Q predicts membership in a pyramidal group. This last result is particularly consistent with the idea that pyramids undertake lower profitability projects.

8. **Firms in pyramids are larger, or they are more likely to belong to capital intensive industries**

As the required investment increases, results 2 and 6 suggest that firms are more likely to belong to pyramidal business groups. Attig, Fischer and Gadhoum (2003) find evidence consistent with this implication, using Canadian data. Claessens, Fan and Lang (2002) also find that in East Asia group firms tend to be larger than unaffiliated firms. Bianchi, Bianco and Enriques (2001) find

[27] There is also a literature that examines the relationship between valuation and firm membership in business groups, without distinguishing between pyramids and other types of groups. See Khanna and Rivkin (1999), Khanna and Palepu (2000), Fisman and Khanna (2000) and Claessens, Fan, and Lang (2002).

similar evidence for Italy.

9. **Pyramids tend to be created dynamically, following good performance of existing family firms**

The timing of the model is exogenously specified in most of the analysis. However, result 11 partially endogenizes the timing. It shows that pyramids will not be set up at a single point in time, even when the family has access to both firms at date 0. Thus, our model predicts that pyramids evolve over time, as a function of the performance of the existing firms in the pyramid. This is consistent with the claim of Khanna and Palepu (2000) that one of the most important role of groups is to set up new firms in which the family and the member firms acquire equity stakes (p. 869). Aganin and Volpin (2004) describe the evolution of the Pesenti group in Italy, and show that this group was created by adding new subsidiaries to the firms the Pesenti family already owned. One of their conclusions is that, in Italy, business groups expand through acquisitions when they are big and have significant cash resources. Claessens, Fan and Lang (2002) find that firms with the highest separation of votes and ownership (those at the bottom of the pyramid) are younger than those with less separation (those at the top).

8 Concluding Remarks

In this paper we propose a theory of pyramidal business groups. The theory explains why families use a pyramidal structure to achieve control of several firms in a business group, as opposed to holding shares directly in these firms (horizontal structure). We show that pyramids have both a payoff and a financing advantage over horizontal structures when the amount of diversion is expected to be high (e.g., because investor protection is poor). We also show that the cases in which the pyramidal structure is optimal for the family are also cases in which the business group itself is more likely to appear. Thus, our theory provides a rationale for why pyramidal business groups are a relatively common organizational structure in many countries of the world.

Our argument departs from the traditional story that pyramids are a device to separate cash flow from voting rights. Because of this feature, our model can generate cases in which pyramidal firms have only minor deviations from one share-one vote. It can also explain why pyramids arise even if the family is free to deviate from one share-one vote with the use of dual-class shares, i.e., it explains

why pyramids are different from the use of dual-class shares. Our theory can help understand recent empirical evidence –which is inconsistent with the traditional view of pyramids– that some pyramidal firms are associated with small separation between ownership and control. While some predictions of the model are consistent with existing empirical findings, other predictions are only backed by anecdotal evidence. Future empirical work could test these implications in a more systematic way to help us better understand these complex organizations.

In terms of the theory, we believe that a full understanding of the structure of business groups requires an answer to three different questions: 1) why are multiple assets in the hands of a single family, 2) why are these assets grouped into legally independent firms, and 3) what determines the choice of ownership structure of these firms (e.g., pyramidal, horizontal, or more complex structures). In this paper we deal with these three questions by using a single imperfection: poor investor protection. For this reason, in some cases we provide only partial answers. This is specially true on the issue of the boundaries of the firm. Rather than establishing the definitive theory of pyramidal groups, we believe the main contribution of the current paper is to provide a promising line of argument that matches well with both logic and the data. We hope that our paper will motivate future theoretical work to fill the gaps that we have left unanswered.

Another important issue that could be explored by future theoretical work regards the normative implications of the existence of business groups. We have argued that pyramidal business groups can be efficient for the family, but this is not enough to establish efficiency from the perspective of social welfare. Previous authors have argued that family business groups can have deleterious effects on overall economic efficiency because they foster an inefficient allocation of corporate control through family inheritances (Morck, Strangeland and Yeung, 2000), and because they might hamper the development of external capital markets (Almeida and Wolfenzon, 2004). In addition, even though we assume that the level of investor protection is exogenous, this assumption might be unwarranted. In the model, wealthy families benefit from poor investor protection since it acts as a barrier to entry for new entrepreneurs. Thus, these families have incentives to lobby for regulations that impede financial and economic development (Rajan and Zingales, 2003a, 2003b). A model that blends these ideas into our current framework might generate interesting normative implications regarding pyramidal business groups.

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Appendix

Proof of Result 1

When $(1-\overline{d})r > i$, it holds that $U^H > U^P$ and $\overline{R}^H \geq i$. Therefore, the horizontal structure is chosen.

When $(1-\overline{d})r < i$, it holds that $U^H < U^P$. In this case, we cannot guarantee that the pyramidal structure is always feasible. However, because $\overline{R}^P \geq \overline{R}^H$, the pyramidal structure is feasible whenever the horizontal structure is. As a result, the family chooses the pyramidal structure whenever it is feasible (i.e., it is never the case that the pyramid is preferred but only the horizontal is feasible).■

Proof of Result 2

The condition $(1-\overline{d})r < i$ is more likely to hold when r is low, i is high and investor protection is low (i.e., $\overline{d}$ is high). ■

Proof of Result 3

If $t < 0$, the entrepreneur never owns the firm. If $t > 0$, whenever the entrepreneur can finance the required investment i, he will set up firm B and thus business groups will not appear. Thus, the condition required for business groups not to appear is that the income that the entrepreneur can pledge to outside investors is enough to finance the investment:

$$\overline{R}^E = (1-\overline{d})(1+t)r \geq i \tag{19}$$

Clearly, if r is high, t is high and/or i is low, Equation (19) is more likely to hold. Furthermore, an increase in investor protection k decreases diversion $\overline{d}$ and facilitates entrepreneurial finance.

As we claim in footnote 13, notice that if the family can drive out entrepreneurs who are just marginally viable (that is, if $\overline{R}^E - i$ is positive but small), then r and t would have to be higher and/or i and $\overline{d}$ would have to be lower to generate entrepreneurial ownership. Such a possibility would thus lead to a result that is qualitatively identical to result 3.■

Proof of Result 4

By result 1, a horizontal structure can only arise when $(1-\overline{d})r \geq i$. However, this condition implies that when $t > 0$, $\overline{R}^E = (1-\overline{d})(1+t)r \geq i$, and thus the entrepreneur can finance the project. Thus, the business group does not appear. If $t < 0$, the family always owns firm B, and the horizontal group appears under the same conditions characterized in result 1.■

Proof of Result 5

We just need to show that, in both structures, the family benefits by committing to a low level of diversion. For the horizontal structure, we have $\frac{\partial U^H}{\partial d} = -c_d < 0$. Thus the family gains by reducing diversion.

For the pyramidal case, the optimal diversion level from the perspective of date 1 solves $\frac{\partial U^P}{\partial d} = 0$ or $1-\alpha = c_d(d,k)$, which using the definition of $d(\cdot,\cdot)$ can be expressed as $d(\alpha,k)$. However, diversion is decided at date 2, when the family ultimate ownership is $\alpha\beta^P$. Thus, actual diversion is given by $d(\alpha\beta^P,k)$. Since $\frac{\partial^2 U^P}{\partial d^2} = -c_{dd}(d,k)r < 0$, the closer $d(\alpha\beta^P,k)$ is to $d(\alpha,k)$, the higher is U^P. Because $d(\alpha\beta^P,k) > d(\alpha,k)$, the family gains by reducing diversion.■

Proof of Result 6

We first show that for the parameter values $(\overline{\alpha},\overline{c},\overline{r},\overline{i},\overline{k})$ it follows that $(1-d(\overline{\omega},\overline{k})) - \overline{i} = 0$ and $U^P = U^H$. By Equations (15) and (16),

$$R^H(\overline{\omega}) = R^P(\overline{\omega})$$

$$\overline{\alpha c} + (1-\overline{\omega})(1-d(\overline{\omega},\overline{k}))\overline{r} = \overline{c} + \left(1 - \frac{\overline{\omega}}{\overline{\alpha}}\right)(1-d(\overline{\omega},\overline{k}))\overline{r}$$

$$(1-d(\overline{\omega},\overline{k}))\overline{r} = \frac{\overline{\alpha c}}{\overline{\omega}}.$$

Thus, as explained in the text, the market price, $(1-d(\overline{\omega},\overline{k}))\overline{r}$, and the implied price, $\frac{\overline{\alpha c}}{\overline{\omega}}$, are the same. Plugging the last equality into Equation (15) leads to $(1-d(\overline{\omega},\overline{k}))\overline{r} = \overline{i}$. Now, $U^P = \overline{\alpha c} + NPV - (1-\overline{\alpha})[(1-d(\overline{\omega},\overline{k}))\overline{r} - \overline{i}] = \overline{\alpha c} + NPV = U^H$.

Now, we prove another intermediate result. Let $i_1 > i_2 > c$, then, it must be that $\omega^H(i_1) < \omega^H(i_2)$ and $\omega^P(i_1) < \omega^P(i_2)$.[28] We prove this result only for the horizontal structure (the pyramidal case is identical). Suppose towards

[28] This intermediate result is simply saying that to finance a larger investment level, the family needs to sell more shares. The reason why the proof is not trivial is that we cannot gurantee that the functions R^H and R^P are always decreasing with ω. In fact, there are regions in which these functions are increasing.

a contradiction that $i_1 > i_2 > c$ and $\omega^H(i_1) \geq \omega^H(i_2)$. First, because $R^H(1) = \alpha c < c < i_2 < i_1 = R^H(\omega^H(i_1))$, then by the Intermediate Value Theorem (R_H is continuous), there exists a $\widehat{\omega} \in (\omega^H(i_1), 1)$ such that $R^H(\widehat{\omega}) = i_2$. Now, since by assumption $\omega^H(i_1) \geq \omega^H(i_2)$, it must be that $\widehat{\omega} > \omega^H(i_2)$. But this a contradiction because $\omega^H(i_2)$ is defined as the highest ω such that $R^H(\omega) = i_2$.

We now prove the result for i. Proofs for r and k are identical. We consider the parameter $(\overline{\alpha}, \overline{c}, \overline{r}, i, \overline{k})$ with $i > \overline{i}$.Recall that we are considering only investment levels strictly above c,that is $\overline{i} > c$.

By the intermediate result shown above, $\omega^H(i) < \omega^H(\overline{i}) = \overline{\omega}$ and $\omega^P(i) < \omega^P(\overline{i}) = \overline{\omega}$. Also, because $(1 - d(\omega, k))r - i$ is increasing in ω and decreasing in i, and $(1-d(\overline{\omega}, \overline{k}))\overline{r} - \overline{i} = 0$, it must be that $(1-d(\omega^H(i), \overline{k}))\overline{r} - i < 0$ and $(1 - d(\omega^P(i), \overline{k}))\overline{r} - i < 0$.

Next, we show that $\omega^P(i) > \omega^H(i)$. We showed above that $(1-d(\omega^H(i), \overline{k}))\overline{r} - i < 0$. Replacing $i = R^H(\omega^H(i)) = \alpha c + (1-\omega^H(i))(1-d(\omega^H(i), \overline{k}))\overline{r}$ into this last inequality and rearranging leads to $\frac{\overline{\alpha c}}{\omega^H(i)} > (1-d(\omega^H(i), \overline{k}))\overline{r}$. Now, evaluating R^P at $\omega^H(i)$:

$$\begin{aligned} R^P(\omega^H) &= \overline{\alpha c} + \left(\frac{\omega^H}{\overline{\alpha}} - \omega^H\right)\left(\frac{\overline{\alpha c}}{\omega^H}\right) + \left(1 - \frac{\omega^H}{\overline{\alpha}}\right)(1 - d(\omega^H, \overline{k}))\overline{r} \\ &> \overline{\alpha c} + \left(\frac{\omega^H}{\overline{\alpha}} - \omega^H\right)(1 - d(\omega^H, \overline{k}))\overline{r} + \left(1 - \frac{\omega^H}{\overline{\alpha}}\right)(1 - d(\omega^H, \overline{k}))\overline{r} \\ &= \overline{\alpha c} + (1 - \omega^H)(1 - d(\omega^H, \overline{k}))\overline{r} = R^H(\omega^H) = i, \end{aligned}$$

where the inequality follows from $\frac{\overline{\alpha c}}{\omega^H(i)} > (1-d(\omega^H(i), \overline{k}))\overline{r}$. Since $R^P(\omega^H(i)) > i > c = R^P(\alpha)$, by the Intermediate Value Theorem, there must be a $\widehat{\omega} \in (\omega^H(i), \alpha)$ such that $R^P(\widehat{\omega}) = i$. Because $\omega^P(i)$ is defined as the highest ω such that $R^P(\omega) = i$, it must be that $\omega^P(i) \geq \widehat{\omega}$, and consequently $\omega^P(i) > \omega^H(i)$.

Finally, we compare utilities under both structures

$$U^P = \overline{\alpha c} + NPV(\omega^P) - (1 - \overline{\alpha})[(1 - d(\omega^P, \overline{k}))\overline{r} - \overline{i}] > \overline{\alpha c} + NPV(\omega^H) = U^H$$

The inequality follows because 1) $NPV(\omega^P) > NPV(\omega^H)$ since $\omega^P > \omega^H$ and $NPV(\omega)$ is increasing, and 2) $(1 - d(\omega^P, \overline{k}))\overline{r} - i < 0$.■

Proof of Result 7

Fix a parameter vector $(\overline{\alpha}, \overline{c}, \overline{r}, \overline{i}, \overline{k})$ such that, for these parameters, $\omega^H = \omega^P = \overline{\omega}$. Suppose that the different structures are chosen due to variation in i (an identical argument can be made with the other parameters). We know from Result 6 that the pyramidal (horizontal) structure is chosen for $i > \overline{i}$ ($i < \overline{i}$) and that $\omega^P(i > \overline{i}) < \overline{\omega}$ and $\omega^H(i < \overline{i}) > \overline{\omega}$. That is, all pyramids we observe have ultimate ownership below $\overline{\omega}$ and all horizontal structures have ultimate ownership above $\overline{\omega}$.■

Proof of Result 8

Recall that $\overline{\omega}$ is the ultimate ownership concentration at which both the pyramidal and the horizontal structure raise the same amount i. That is, it is defined by $R^H(\overline{\omega}) = i$ and $R^P(\overline{\omega}) = i$. We can re-write this system as

$$\overline{\omega}(1 - d(\overline{\omega}, k))r = \alpha c$$

and

$$(1 - d(\overline{\omega}, k))r = i.$$

Because the system has two equations, for it to hold after a change in k, at least two parameters need to change. We consider the effect on $\overline{\omega}$ and i. Differentiating the first equation with respect to k leads to $r\left[\overline{\omega}_k(1-d) - \overline{\omega}d_\omega\overline{\omega}_k - \overline{\omega}d_k\right] = 0$ or $\overline{\omega}_k = \overline{\omega}d_k/(1 - d - \overline{\omega}d_\omega) < 0$ because $1 - d \geq 0$, $d_\omega < 0$ and $d_k < 0$. The solution to i can be found from the second equation.■

■

Proof of Result 9

In order to finance firm A, the family sells a fraction $(1 - \alpha)$ of this firm and raises R. Because $(1 - \overline{d})r_B < i_B$, at date 1 the family sets up firm B in a pyramid. Thus, firm A does not pay a dividend at date 1, but rather invests the cash $c = R - i_A + r_A$ of firm A to set up firm B. To raise additional finance at date 1, firm A sells a stake of $(1 - \beta_P)$ of firm B to the market. We assume (wlog) that firm A raises just enough cash to set up firm B, that is, $R - i_A + r_A + (1 - \beta_P)(1 - \overline{d})r_B = i_B$. At date 2, firm A receives dividends of $\beta_P(1 - d)r_B$, which by the previous equation equals $R - i_A + r_A + (1 - \overline{d})r_B - i_B$. Because investors in firm A break even, we have that $R = (1 - \alpha)[R - i_A + r_A + (1 - \overline{d})r_B - i_B]$, or $R = \frac{1-\alpha}{\alpha}[r_A - i_A + (1 - \overline{d})r_B - i_B]$. Note that as long as

$r_A - i_A + (1-\overline{d})r_B - i_B > 0$, the family can raise any amount of money at date 0. In particular, the family can raise enough to fund firm A, and to make the pyramid feasible at date 1.■

Proof of Result 10

Consider first the case in which pyramids are ruled out by contract. At date 0, the family sells a fraction $1-\alpha$ of firm A to raise the set up cost, i_A (this is wlog – i.e. can show there is no benefit in raising more). Then $(1-\alpha)\overline{r}_A = i_A$. Let $\alpha^* = 1 - \frac{i_A}{\overline{r}_A}$ denote the stake that the family retains in firm A.

The cash that the family holds at date 1 is $\alpha^* r_A$ and so setting firm B in a horizontal structure is feasible if and only if: $\alpha^* r_A + (1-\overline{d})r_B > i_B$. In the low state this inequality becomes:

$$-\alpha^*\Delta + \overline{r}_A - i_A + (1-\overline{d})r_B > i_B$$

This inequality never holds since, by assumption, $\overline{r}_A - i_A + (1-\overline{d})r_B < i_B$. In the high cash flow state the horizontal structure is feasible when

$$\alpha^*\Delta + \overline{r}_A - i_A + (1-\overline{d})r_B > i_B. \tag{20}$$

Consider now the case in which pyramidal structures are not ruled out. In this case, the horizontal structure never arises since $(1-\overline{d})r_B < i_B$. At date 0, the family sells a fraction $1-\alpha$ of firm A and raises R. Suppose that investors expect the family to set up firm B in a pyramid only when the cash flows of firm A are high (we will show below that it will never be an equilibrium to expect that the family sets up the pyramid when cash flows are low). In case of a low cash flow, the family pays the cash in firm A, $c = R - i_A + \overline{r}_A - \Delta$, as dividends and does not set the pyramid. In the case of a high cash flow, the family uses all the cash in firm A to set up firm B in a pyramid. The family sells $1-\beta_P$ shares of firm B to raise additional cash to set up firm B. We assume (wlog) that the family raises just enough cash to set up firm B, that is:

$$R - i_A + \overline{r}_A + \Delta + (1-\beta_P)(1-\overline{d})r_B = i_B$$

At date 2, firm A receives dividends of $\beta_P(1-\overline{d})r_B$, which by the last equation equal $R - i_A + \overline{r}_A + \Delta + (1-\overline{d})r_B - i_B$. We now consider the relation between R and α. Because R must equal the expected cash flows to date 0 investors, we have:

$$R = (1-\alpha)\left[\frac{1}{2}\left(R - i_A + \overline{r}_A - \Delta\right) + \frac{1}{2}[R - i_A + \overline{r}_A + \Delta + (1-\overline{d})r_B - i_B]\right]$$

or

$$R = \frac{1-\alpha}{\alpha}\left[\overline{r}_A - i_A + \frac{1}{2}[(1-\overline{d})r_B - i_B]\right]. \tag{21}$$

To sustain the equilibrium, R needs to be large enough so that the pyramid is feasible only when cash flows are high. That is, we need:

$$R - i_A + \overline{r}_A + \Delta + (1-\overline{d})r_B \geq i_B,$$

and

$$R - i_A + \overline{r}_A - \Delta + (1-\overline{d})r_B < i_B.$$

Note that, as long as

$$\overline{r}_A - i_A + \frac{1}{2}[(1-\overline{d})r_B - i_B] > 0 \tag{22}$$

R can be set to any positive value by an appropriate choice of α (see Equation (21)).

Furthermore, notice that the only possible equilibrium when Equation (22) holds is the one we consider in which shareholders expect the family to set up the pyramid only when cash flows are high. It might seem that R can be set sufficiently high so as to finance the pyramid in all states. However, this is not an equilibrium because if R is sufficiently high to make the pyramid feasible in both states, investors anticipate that the family will always set up firm B in a pyramid, and the expression for R changes to $R = \frac{1-\alpha}{\alpha}\left[\overline{r}_A - i_A + (1-\overline{d})r_B - i_B\right]$. But the right hand side is always negative so this is not an equilibrium. We can also see from this explanation why it is not possible to have a pyramid only when the cash flows are low. If the pyramid is feasible in the low cash flow state, it will also be feasible in the high cash flow state and the family will not be able to raise any money.

Finally, notice that when Equation (22) holds, but Equation (20) does not, ruling out pyramids eliminates the possibility of setting up firm B, whereas not ruling them out at least allows the family to set up firm B in the high cash flow state. There is a region of the parameter space where it is possible to have both Equation (22) holding but not Equation (20). This region is defined by:

$$\alpha^*\Delta + \overline{r}_A - i_A + (1-\overline{d})r_B < i_B < 2(\overline{r}_A - i_A) + (1-\overline{d})r_B.$$

Because $2(\overline{r}_A - i_A)$ can be greater than $\alpha^*\Delta + \overline{r}_A - i_A$, this inequality is possible.■

Proof of Result 11

If the pyramid is set up at date 0, the maximum pledgeable income of firms A and B is $\overline{r}_A - i_A + (1-d)r_B - i_B < \left(1 - \frac{i_A}{\overline{r}_A}\right)\Delta + \overline{r}_A - i_A + (1-d)r_B - i_B < 0$, where the last inequality follow from condition 17. Thus the family cannot set up the pyramid at date 0 even if firm B is available at that date.■

Proof of the result of section 6.2 that, when $k_A < k_B$, the family sets $\beta^P < 1$ when setting a pyramid

To raise funds, firm A sells a fraction $1-\beta_2$ of firm B and, in addition, issues new shares of its own. Letting α_2 be the family's final stake in firm A, the amount of funds raised is given by:

$$R_P = \left(1 - \frac{\alpha_2}{\alpha_1}\right)\left((1-d_A)r_A + \beta_2(1-d_B)r_B\right) + (1-\beta_2)(1-d_B)r_B. \tag{23}$$

The first term is the amount collected by selling shares of firm A and the second term is the amount raised by selling shares of firm B. Note that because firm A keeps a fraction β_2 of firm A, the family sells a fraction of firm B indirectly through the sale of shares in firm A.

The family's payoff at date t_2 is given by:

$$U^P = \alpha_2\left((1-d_A)r_A + \beta_2(1-d_B)r_B\right) + \left(d_A - \frac{k_A d_A^2}{2}\right)r_A + \left(d_B - \frac{k_B d_B^2}{2}\right)r_B, \tag{24}$$

where the first term is its security benefits and the last two terms are the diverted amount from each firm net of the cost of diversion.

At date t_2 the family chooses d_A and d_B to maximize its payoff. Using the first order conditions (and assuming an interior solution) we get $d_A = (1-\alpha_2)/k_A$ and $d_B = (1-\alpha_2\beta_2)/k_B$.

From the viewpoint of date t_1, the goal of the family is to maximize U^P subject to $R_P \geq i_B$ and to the expressions for d_A and d_B. As we have shown before, the fact that diversion is costly implies that it does not pay for the family to raise more funds than the strictly necessary to set up firm B. Thus, at the solution $R_P = i_B$. Replacing this equality in the family's payoff leads to:

$$U^P = \alpha_1\left((1-d_A)r_A + (1-d_B)r_B - i_B\right) + \left(d_A - \frac{k_A d_A^2}{2}\right)r_A + \left(d_B - \frac{k_B d_B^2}{2}\right)r_B. \tag{25}$$

The problem of the family is to chose α_2 and β_2 to maximize its payoff (Equation (25)) subject to raising enough funds to set up firm B ($R_P \geq i_B$) and to the expressions $d_A = (1-\alpha_2)/k_A$ and $d_B = (1-\alpha_2\beta_2)/k_B$.

The family's problem is to choose α_2 and β_2 so as to maximize its payoff. It will be convenient to divide this problem in two steps. First, we fix β_2 and find the optimal α_2 and the maximum attainable payoff for the given value of β_2.We let $\alpha_2(\beta_2)$ and $U^P(\beta_2)$ be the optimal value of α_2 and the maximum attainable payoff, respectively, as a function of β_2. This first maximization problem can be written as

$$U^P(\beta_2) = \max_{\alpha_2} \alpha_1\left((1-d_A)r_A + (1-d_B)r_B - i_B\right) + \left(d_A - \frac{k_A d_A^2}{2}\right)r_A + \left(d_B - \frac{k_B d_B^2}{2}\right)r_B \tag{26}$$

subject to

$$\begin{aligned} &R_P \geq i_B, \\ &d_A = (1-\alpha_2)/k_A, \text{ and} \\ &d_B = (1-\alpha_2\beta_2)/k_B, \end{aligned} \tag{27}$$

where the objective function comes from Equation (25) and R_P is given in Equation (23). The second step is simply to maximize $U^P(\beta_2)$ over β_2.

Because the family sells shares to raise funds, the solution has $\alpha_2 \leq \alpha_1$ and $\beta_2 \leq 1$.It can be shown (by using a similar proof as that of Result 5) that increasing α_2 towards α_1 and β_2 towards 1 raises the family payoff. An implication of this fact is that the family does not raise more capital than it needs, that is, at the solution $R_P = i_B$, which using the expression for R_P in Equation (23) can be written as:

$$\left(1 - \frac{\alpha_2}{\alpha_1}\right)\left((1-d_A)r_A + \beta_2(1-d_B)r_B\right) + (1-\beta_2)(1-d_B)r_B = i_B, \tag{28}$$

where $d_A = (1-\alpha_2)/k_A$ and $d_B = (1-\alpha_2\beta_2)/k_B$. This equation implicitly defines α_2 as a function of β_2, i.e., it defines $\alpha_2(\beta_2)$. The expression for $U^P(\beta_2)$ can be found by plugging $\alpha_2(\beta_2)$ into the objective function.

The optimal value of β_2 is the one that maximizes $U^P(\beta_2)$. We are only interested in showing that $\beta_2 < 1$. It will be sufficient to show that $U^{P\prime}(\beta_2)\mid_{\beta_2=1} < 0$.

Differentiating the objective function in Equation (26) and recognizing that α_2 is a function of β_2 and that $d_A = (1-\alpha_2)/k_A$ and $d_B = (1-\alpha_2\beta_2)/k_B$ we get:

$$U^{P\prime}(\beta_2) = \frac{-\left(k_A\, r_B\, {\alpha_2}^2\, \beta_2\right) + \alpha_1\, \alpha_2'\, \left(k_B\, r_A + k_A\, r_B\, \beta_2\right) + \alpha_2\, \left(k_A\, r_B\, \alpha_1 - \left(k_B\, r_A + k_A\, r_B\, {\beta_2}^2\right)\, \alpha_2'\right)}{k_A\, k_B}, \tag{29}$$

where we use α_2 instead of $\alpha_2(\beta_2)$ and α_2' instead of $\alpha_2'(\beta_2)$ to lighten notation. Next, we obtain $\alpha_2'(\beta_2)$ by completely differentiating Equation (28) with respect to β_2 and rearranging:

$$\alpha_2'(\beta_2) = \frac{k_A\, r_B\, \alpha_2\, \left(1 - k_B + \alpha_1 - 2\, \alpha_2\, \beta_2\right)}{-\left(k_A\, r_B\, \left(1+\alpha_1\right)\, \beta_2\right) + k_B\, \left(r_A\, \left(-1 + k_A - \alpha_1\right) + k_A\, r_B\, \beta_2\right) + 2\, \alpha_2\, \left(k_B\, r_A + k_A\, r_B\, {\beta_2}^2\right)}. \tag{30}$$

Finally, we replace $\alpha_2'(\beta_2)$ into Equation (29) and evaluate the expression at $\beta_2 = 1$ to obtain

$$U^{P\prime}(\beta_2)\mid_{\beta_2=1} = \frac{\left(k_A - k_B\right)\, r_A\, r_B\, \left(\alpha_1 - \alpha_2\right)\, \alpha_2}{-\,k_B\, r_A\, \left(1 + \alpha_1 - k_A\right) - k_A\, r_B\, \left(1 + \alpha_1 - k_B\right) + 2\, \left(k_B\, r_A + k_A\, r_B\right)\, \alpha_2}. \tag{31}$$

We show that this expression is negative. Note that the numerator is negative. The reason is that $k_B > k_A$ and $\alpha_1 > \alpha_2(1)$. It is always the case that $\alpha_1 \geq \alpha_2(\beta_2)$ because the family sells some non-negative amount of shares of firm A. However, when $\beta_2 = 1$, the inequality is strict. If not, then R_P would be 0 which is not possible because α_2 and β_2 should be such that $R_P = i_B > 0$.

We now show that the denominator is positive. We do this by deriving a condition that $\alpha_2(1)$ must satisfy. We go back to the problem in Equation (26) and solve it for $\beta_2 = 1$. We let $\widehat{R}_P(\alpha_2)$ be the expression for the amount raised (given in Equation (23)) as a function of α_2 when $\beta_2 = 1$:

$$\widehat{R}_P = \left(1 - \frac{\alpha_2}{\alpha_1}\right)\left((1-d_A)r_A + (1-d_B)r_B\right)). \tag{32}$$

By replacing the expressions for $d_A = (1-\alpha_2)/k_A$ and $d_B = (1-\alpha_2\beta_2)/k_B = (1-\alpha_2)/k_B$ into the above expression and differentiating two times with respect to α_2, we obtain $\frac{\partial^2 \widehat{R}_P}{\partial \alpha_2^2} = \frac{-2\left(\frac{r_A}{k_A} + \frac{r_B}{k_B}\right)}{\alpha_1} < 0$. This implies that $\widehat{R}_P$ first increases and then decreases with α_2. We let $\overline{\alpha}$ be the value of α_2 at which $\widehat{R}_P$ achieves its maximum. The value of $\overline{\alpha}$ can be found by solving $\frac{\partial \widehat{R}_P}{\partial \alpha_2} = 0$ and is equal to

$$\overline{\alpha} = \frac{1}{2}\frac{k_B r_A(1+\alpha_1-k_A) + k_A r_B(1+\alpha_1-k_B)}{k_B r_A + k_A r_B}.$$

Note that $\alpha_2(1) \geq \overline{\alpha}$. Suppose not, i.e., $\alpha_2(1) < \overline{\alpha}$. By increasing α_2 to $\overline{\alpha}$ two things happen. One is that the amount raised goes up ($R_P^{'}(\alpha_2) > 0$ for $\alpha_2 < \overline{\alpha}$) and also the payoff of the family goes up (we explained above that the payoff of the family increases as α_2 increases towards α_1). Thus, it must be that $\alpha_2(1) \geq \overline{\alpha}$. Finally, the numerator in Equation (31) satisfies

$$\begin{aligned}
&-k_B r_A\left(1+\alpha_1-k_A\right) - k_A r_B\left(1+\alpha_1-k_B\right) + 2\left(k_B r_A + k_A r_B\right)\alpha_2 \\
&\geq -k_B r_A\left(1+\alpha_1-k_A\right) - k_A r_B\left(1+\alpha_1-k_B\right) + 2\left(k_B r_A + k_A r_B\right)\overline{\alpha} \\
&= 0,
\end{aligned}$$

where the inequality follows because $\alpha_2(1) \geq \overline{\alpha}$.

In sum, we show that the numerator in Equation (31) is negative and the denominator positive. Therefore $U^{P\prime}(\beta_2)\mid_{\beta_2=1} < 0$,which implies that the optimal value of β_2 satisfies $\beta_2 < 1$.■

Proof that Result 1 holds in the extension of Section 6.3.1

We show that when the family can commit to a level of diversion in the horizontal structure such that $(1-d)r > i$, it always chooses the horizontal structure. Otherwise, it chooses the pyramidal structure if this structure is feasible. This is an identical result to that in section 3.

Let d be the fraction of firm B that the family diverts *to firm A*. As in section 3, we assume that there is no cost of diversion, but that diversion can only take values in $[0, \overline{d}]$. Also, as in section 3, we consider a model in which firm A is already set up and we take the family's stake in this firm, α, as exogenous.

Modifying the family's payoff at date 2 (see Equation (1)) to reflect diversion from firm B to firm A, the family solves the problem,

$$\max_{d^H \in [0,\overline{d}]} \alpha c - R_I^H + \beta^H\left(R_I^H + R_E^H - i + (1-d^H)r\right) + \alpha d^H r,$$

where the term $\alpha d^H r$ is the share of the diverted amount that the family receives due to its stake α in firm A. The solution to this problem is

$$d^H = \begin{cases} 0 & \text{if } \alpha < \beta^H \\ \overline{d} & \text{if } \alpha \geq \beta^H \end{cases}.$$

Noting that $R_E^H = (1-\beta^H)\left(R_I^H + R_E^H - i + (1-d^H)r\right)$, solving for R_E^H, and replacing its value into the family's payoff, we obtain the family's payoff as of date 1:

$$U^H = \alpha c + (1-d^H)r - i + \alpha d^H r. \tag{33}$$

In the pyramidal case, modifying the family's payoff at date 2 (see Equation (5)) to reflect diversion from firm B to firm A, the family solves the problem:

$$\max_{d^P \in [0,\overline{d}]} \alpha[c - R_I^P + \beta^P(R_I^P + R_E^P - i + (1-d^P)r) + d^P r].$$

The optimal solution is $d^P = \overline{d}$. Noting that $R_E^P = (1-\beta^P)(R_I^P + R_E^P - i + (1-d^P)r)$, solving for R_E^P and plugging the value into the family's payoffs, we obtain the family's payoff as of date 1:

$$U^P = \alpha[c + r - i] \tag{34}$$

We turn now to the choice of structure. First, we show that, when $(1-d^H)r > i$ (or $d^H < 1 - \frac{i}{r}$) then $U^H > U^P$, and that when $(1-d^H)r < i$ then $U^P > U^H$. This results follows directly from 1) $\frac{\partial(U^H - U^P)}{\partial d} < 0$, and 2) $(U^H - U^P)_{d^H = 1-\frac{i}{r}} = 0$.

Thus,when 1) $\overline{d} < 1 - \frac{i}{r}$, or 2) the family can raise the necessary funds and still set $\beta^H > \alpha$, the family chooses the horizontal structure. The reason is that in both these cases $d^H < 1 - \frac{i}{r}$ so that $U^H > U^P$. It is also the case that the horizontal structure is feasible. In case 1) it follows because the family can completely dilute its ownership in firm B and raise $(1-\overline{d})r > i$. In case 2), it follows by assumption.

In all other cases, that is, when $\overline{d} > 1 - \frac{i}{r}$ and the family cannot raise i and still set $\beta^H > \alpha$, then $d^H = \overline{d} > 1 - \frac{i}{r}$ and so $U^P > U^H$. To prove that the pyramidal structure is chosen whenever it is feasible, the only thing left to prove is that, in this case, it is impossible to have the horizontal structure feasible but not the pyramidal structure. This result follows because the the maximum the family can raise in the horizontal structure is $\alpha c + (1-\overline{d})r$ which is lower than the maximum it can raise with the pyramidal structure, $c + (1-\overline{d})r$. ■

Proof that Result 1 holds in the extension of Section 6.3.2

We change the model of section 3 to allow diversion from firm A at date 1. Specifically, we let d_A be the fraction of firm A's cash flow, c, that the family diverts into its pockets. We continue to allow for the possibility of diversion from firm B at date 2, which we now denote by d_B. Also, we assume that the cost of diversion from firm A (firm B) is zero until diversion reaches $\overline{d}_A$ ($\overline{d}_B$). Beyond this level, the cost of diversion in infinite. The family can use the diverted amount $d_A c$ to invest in firm B at date 1. We assume that this investment can only be done at market prices.

We show a result identical to result 1. The family chooses the horizontal structure whenever $(1-\overline{d}_B)r - i < 0$ and it chooses the pyramid when $(1-\overline{d}_B)r - i > 0$ and the pyramid is feasible.

In the horizontal structure, the date 2 payoff of the family is $\alpha(1-d_A^H)c - R_I^H + d_A^H c + d_B^H r + \beta^H(R_E^H + R_I^P - i + (1-d_B^H))$ for $d_B^H \leq \overline{d}_B$ and $-\infty$ for $d_B^H > \overline{d}_B$. Clearly, the family sets $d_B^H = \overline{d}_B$. Moving back to date 1, the family decides the level of diversion from firm A, d_A^H, the level of internal funds contributed to firm B, R_I^H, and the fraction of shares to sell from firm B, β^H. The amount of funds contributed to the firm, R_I^H, must satisfy $R_I^H \leq \alpha(1-d_A^H)c$. The family can also buy shares of firm B with the diverted amount, $d_A^H c$, but, because this transaction takes place at market prices, this source of funds shows up in the amount collected from the external market, R_E^H. This last term is given by $R_E^H = (1-\beta^H)(R_I^H + R_E^H - i + (1-\overline{d}_B)r)$. Solving for R_E^H and plugging it into the feasibility constraint $R_I^H + R_E^H \geq i$, leads to $R_I^H + (1-\beta^H)(1-\overline{d}_B)r \geq i$. Plugging R_E^H into the family's payoffs leads to the family's problem at date 1:

$$\begin{aligned} &\max_{d_A^H, R_I^H, \beta^H} \alpha(1-d_A^H)c + d_A^H c + r - i \\ &\text{subject to } R_I^H + (1-\beta^H)(1-\overline{d}_B)r \geq i \\ &\text{and to } R_I^H \in [0, \alpha(1-d_A^H)c] \end{aligned}$$

There is no loss in setting $\beta^H = 1$ and $R_I^H = \alpha(1-d_A^H)c$ since doing so does not affect the objective function and relaxes the feasibility constraint. Also, the objective function is increasing in d_A^H, so d_A^H is set to the maximum possible. That is, $d_A^H = \overline{d}_A$ whenever $\alpha(1-\overline{d}_A)c + (1-\overline{d}_B)r \geq i$ and to $d_A^H = 1 - \frac{i-(1-\overline{d}_B)r}{\alpha c}$ otherwise.[29]

[29] Of course, we are talking about the cases in which firm B is feasible. In the horizontal structure, this occurs when $\alpha c + (1-\overline{d}_B)r \geq i$.

In the pyramidal structure, the date 2 payoff of the family is $\alpha[(1-d_A^P)c - R_I^P + \beta^P(R_I^P + R_E^P - i + (1-d_B^P)r)] + d_A^P c + d_B^P r$ for $d_B^P \leq \overline{d}_B$ and $-\infty$ for $d_B^P > \overline{d}_B$. Clearly, the family sets $d_B^P = \overline{d}_B$. Moving back to date 1, the amount contributed by firm A, R_I^P, must satisfy $R_I^P \leq (1-d_A^P)c$. Again, any amount contributed by the family from the diverted funds $d_A^P c$ is invested at market prices and hence shows up in the term R_E^P, where $R_E^P = (1-\beta^P)(R_I^P + R_E^P - i + (1-\overline{d}_B)r)$. Solving for R_E^P and plugging it into the feasibility constraint $R_I^P + R_E^P \geq i$, leads to $R_I^P + (1-\beta^P)(1-\overline{d}_B)r \geq i$. Also, plugging the value of R_E^P into the family's payoff leads to the family's problem at date 1:

$$\begin{aligned}
&\max_{d_A^P, R_I^P, \beta^P} \alpha(1-d_A^P)c + d_A^P c + \alpha[(1-\overline{d}_B)r - i] + \overline{d}_B r \\
&\text{subject to } R_I^P + (1-\beta^P)(1-\overline{d}_B)r \geq i \\
&\text{and to } R_I^P \in [0, (1-d_A^P)c]
\end{aligned}$$

Again, there is no loss in setting β^P and R_I^P to their maximum values, 1 and $(1-d_A^P)c$, respectively. The objective function is increasing in d_A^P so the family sets it to the highest possible value. That is, $d_A^P = \overline{d}_A$ whenever $(1-\overline{d}_A)c + (1-\overline{d}_B)r \geq i$ and $d_A^P = 1 - \frac{i-(1-\overline{d}_B)r}{c}$ otherwise.

Note that when,$(1-\overline{d}_B)r - i \geq 0$, it is the case that both $\alpha(1-\overline{d}_A)c + (1-\overline{d}_B)r \geq i$ and $(1-\overline{d}_A)c + (1-\overline{d}_B)r \geq i$ and so the family sets d_A to $\overline{d}_A$ in both structures. It is also the case that both structures are feasible and that the payoff under the horizontal structure is higher. Thus, in this case, the horizontal structure is always chosen. This result parallels result 1 in section 3.

When $(1-\overline{d}_B)r - i < 0$, the pyramidal structure is feasible whenever the horizontal structure is. First note that, $d_A^P \geq d_A^H$. Next, we show that the family prefers the pyramidal structure

$$\begin{aligned}
U^H &= \alpha(1-d_A^H)c + d_A^H c + r - i \\
&\leq \alpha(1-d_A^P)c + d_A^P c + r - i \\
&\leq \alpha(1-d_A^P)c + d_A^P c + \alpha[(1-\overline{d}_B)r - i] + \overline{d}_B r = U^P,
\end{aligned}$$

where the first inequality follows from $d_A^P \geq d_A^H$ and the second inequality follows from $(1-\overline{d}_B)r - i < 0$. Finally, we show that whenever the horizontal structure is feasible so it the pyramidal structure. This follows because the maximum the family raises with the horizontal structure, $\alpha c + (1-\overline{d}_B)r$, which is lower than the maximum the family raises with the pyramidal structure, $c + (1-\overline{d}_B)r$.■

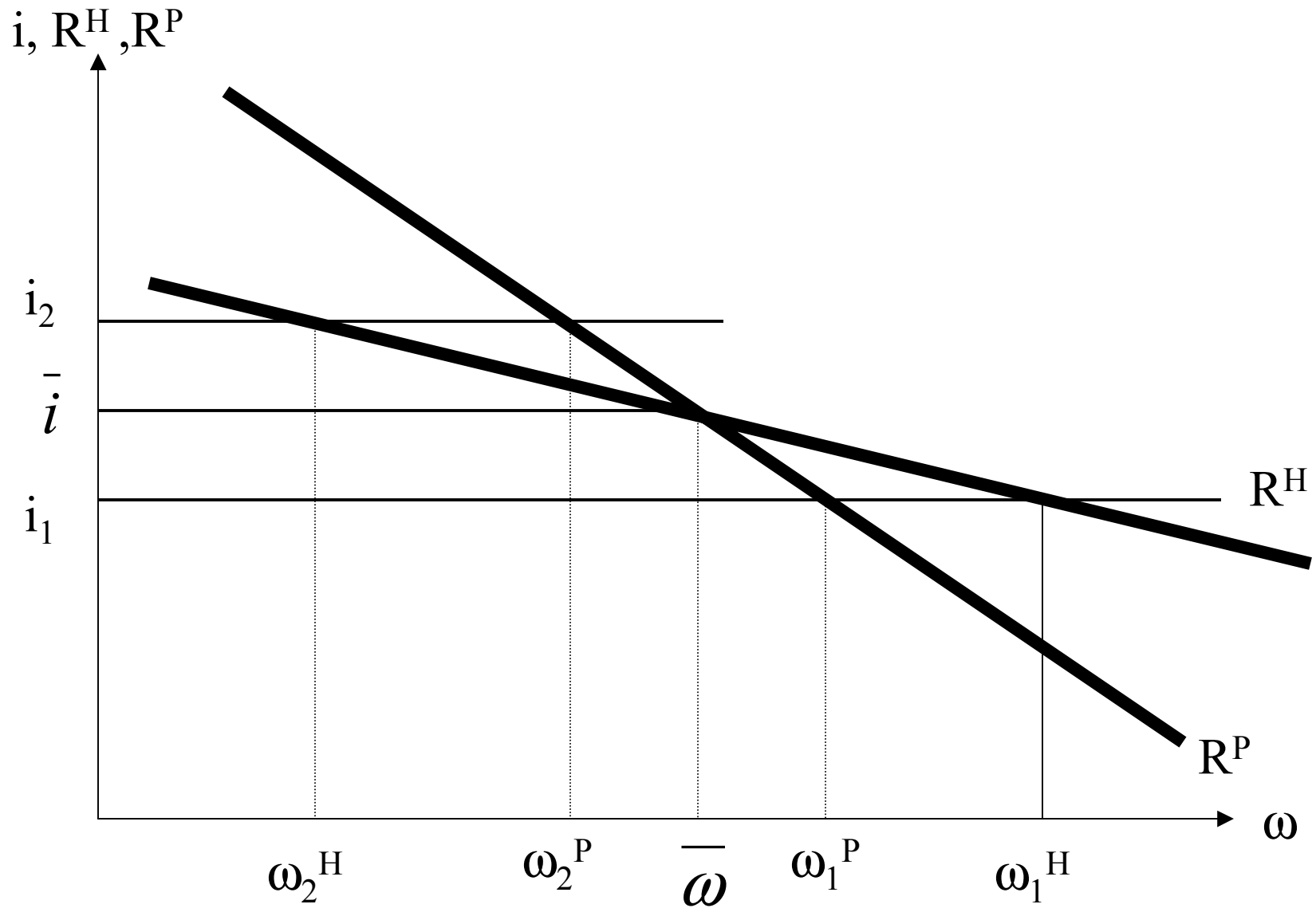

Figure 1: Determination of the ultimate ownership concentration as a function of the required investment i.

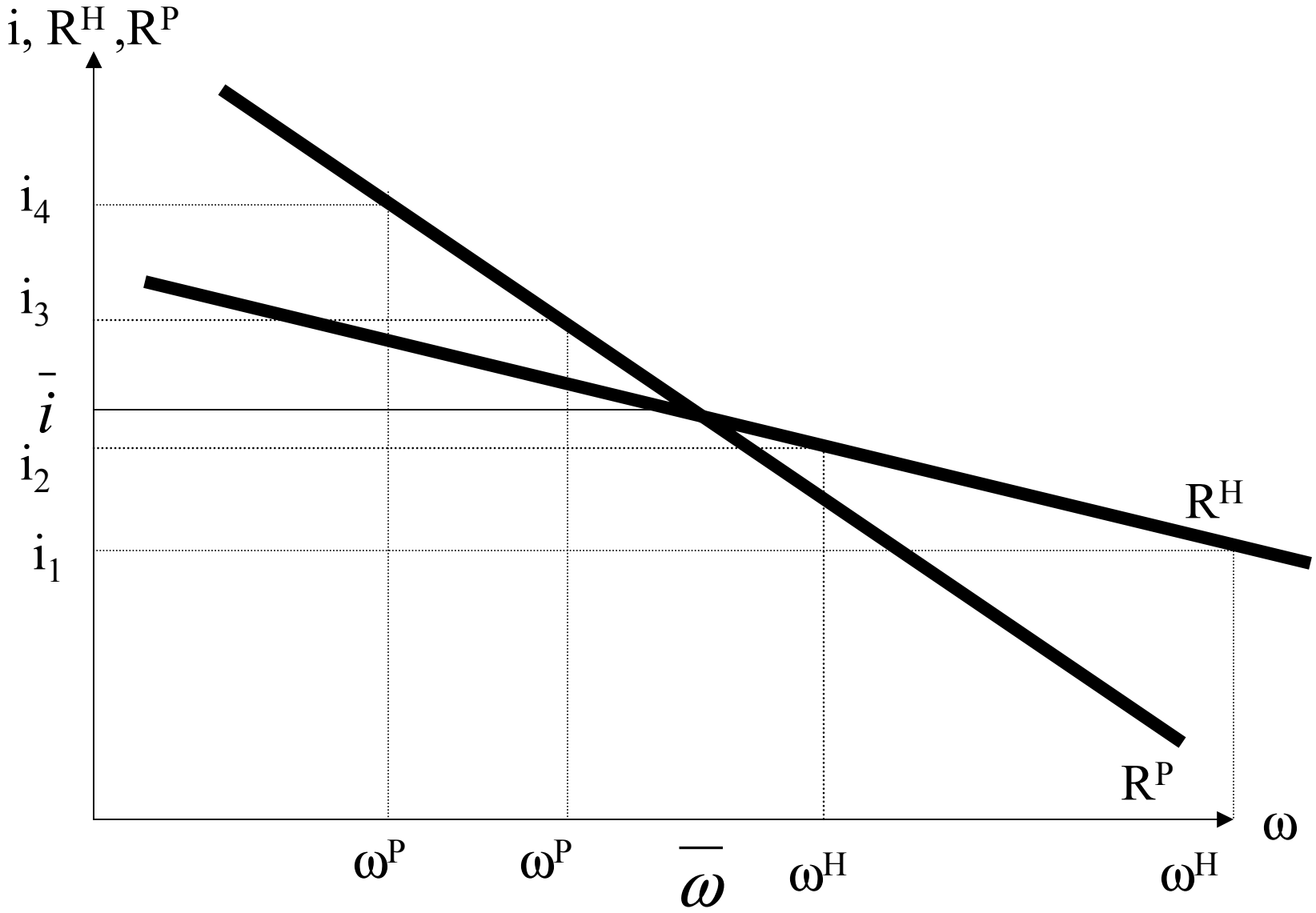

Figure 2: Observed ultimate ownership levels in the horizontal and in the pyramidal structures