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Product Line Design with Component Commonality and Cost-Reduction Effort

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Market pressure for low prices paired with customer demand for high product variety presents a considerable dilemma for many manufacturers. Industry practice and research to date suggest that approaches based on component commonality can substantially lower the costs of proliferated product lines, but at the cost of reducing product differentiation and revenues. We analyze a stylized model of a manufacturer who designs a product line consisting of two products for sale to two market segments with different valuations of quality. The manufacturer determines the component quality levels, the amount of effort to reduce production costs, and whether to use common or different components for the two products.

Explicitly considering potential interdependencies between cost-reduction effort and quality decisions, we characterize environments where the optimal product line involving component commonality features products of higher quality and yields higher revenues. Counter to earlier research we show that it can be preferable to make those components common that, relative to their production cost, are attributed a higher importance by customers. Disregarding the interactions between commonality, production cost, quality, and effort decisions can lead manufacturers to offer product lines with excessive differentiation and inefficiently low quality.

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1. Introduction

Steadily growing customer demand for greater product variety at lower prices presents a considerable dilemma for manufacturers. Attempts to gain market share by matching products to increasingly heterogeneous consumer needs have serious negative implications on manufacturing efficiency due to increased product and process complexities and reduced scale economies. One popular remedy is component standardization or commonality that can mitigate the effects of product proliferation on product and process complexity (Swaminathan 2001). However, increased component commonality limits a firm's potential to extract price premiums through product differentiation.

Consider the platform concept in the automobile industry. By using platforms with identical components (engine, suspension, gearboxes, and so on) across different car types, Volkswagen (VW) saved hundreds of millions of dollars and gained the largest market share in Europe (Miller 1999). Popular belief is that the platform concept could translate into weaker demand for up-market brands, where profits are highest. Indeed, more and more manufacturers using common platforms face serious problems in sales revenue owing to undifferentiated models (Brown 2001). Customer perceptions of platform sharing can vary. Toyota's strategy to use the Camry platform for its luxury Lexus sedans has, partly due to a competitor's extensive marketing campaign, become a widely known example for a platform strategy with an extremely negative impact on customer perceptions. Marketed as an executive sedan, most "executives saw it for what it was: a thinly veiled Toyota Camry with a price premium" (Edmunds.com 2004). On the other hand, there are instances where customers are willing to pay a substantial price premium for products built on a common platform. For example, the Volkswagen Passat has benefited from its outstanding quality and impressive driving characteristics, which are partially due to the underlying Audi A4 platform (Mahoney 2000). At the same time, Audi has been steadily expanding market share, mitigating the fear of downward cannibalization.

In this paper we investigate what circumstances support component sharing as a profitable strategy and, more specifically, which components are the best candidates for commonality. The conventional paradigm is that, while enabling potentially substantial cost reductions, commonality generally reduces the attractiveness of a product line and, ceteris paribus, leads to lower revenues. Clearly there are many factors that determine whether or not an approach based on component commonality is successful. The vast majority of research on commonality in operations management has investigated the cost effects associated with scale economies, risk pooling effects, and reductions in product and process complexity but has mostly neglected the substantial impact that commonality can have on the sales performance of a product line (Baker et al. 1986, Lee 1996). Even when the benefits of commonality on sales are modeled, they focus on shorter lead times as a result of commonality-based strategies such as postponement (Swaminathan and Tayur 1998). See Swaminathan and Lee (2003) for a detailed literature review of the above streams of work. However, a manufacturer should also consider commonality decisions in determining how many products to offer, and how to position and price these products relative to each other and to competing products of other firms. Mussa and Rosen (1978) completed one of the earliest studies on the monopolist's product line design problem with self-selection constraints. In their model, customer types differ only in their willingness to pay for product quality, which is the sole dimension of (vertical) product differentiation. Moorthy (1984) considers the same problem with a finite number of customer segments and more general utility functions. Both papers emphasize the risk of cannibalization when self-selecting customers with a high willingness to pay for quality are attracted to cheaper products of lower quality. Our model is based on this classical framework and we explicitly capture the effects of component commonality on product differentiation and cannibalization.

Based on a recent review of research on product variety, Ramdas (2003) concludes that too many prescriptive models "focus on narrow tradeoffs within functional silos, ignoring important interdependencies across decisions" (p. 97). Some researchers have taken a more cross-functional approach focusing on the conflict between the implications of commonality on costs (operations perspective) and on reduced product differentiation and sales (marketing perspective). Kim and Chhajed (2000) explicitly study the trade-off between cost savings and losses due to reduced product differentiation resulting from component commonality. They assume premium and discount effects on two market segments with lower and higher valuation of quality, respectively. They find component commonality is profitable only when the target segments' valuations of quality are not too different or when the lower-valuation segment is substantial, such that the premium effect is more pronounced than the discount effect. Desai et al. (2001) ignore such direct valuation changes and focus on the trade-off between revenue losses resulting from reduced product differentiation and cost savings induced through design effort. In a model similar to Moorthy (1984), they consider a monopolist who offers two products to a market with two segments that have different valuations for quality, which is the only source of product differentiation. Each product consists of two components and the manufacturer decides if a common component is used in both products (common configuration) or if two distinct variants are used (unique configuration). For each component, the manufacturer further has the option to invest in design effort to reduce production costs. Desai et al. (2001) derive conditions for the overall profitability of commonality as well as an index to rank order components in terms of their attractiveness for commonality. Their results support the conventional paradigm, finding that commonality always leads to cost reductions and a less attractive product line with decreased revenues. This result is most clearly reflected in their proposition to make those components common that, relative to their production costs, are of little importance to the customer.

As in Desai et al. (2001) we do not capture direct valuation changes that could be induced when consumers have knowledge about a common component.

We believe that adding such exogenous valuation changes as in Kim and Chhajed (2000) to our model would complicate the analysis and distract from our focus on the impact of cost-reduction effort on the value of component commonality. Hence, while our model and research focus is similar to Desai et al. (2001), our approach fundamentally differs in how the relation between effort and production costs is modeled. Desai et al. (2001) use an additive model and thereby implicitly assume that the marginal cost of quality is independent of effort. We use a more general function that explicitly captures potential interactions. Our formulation allows us to generalize earlier results and to compare and contrast the results derived from different model specifications. We challenge the conventional paradigm that the loss of product differentiation under commonality always leads to less attractive product lines. In the presence of interactions between quality and effort decisions, we show that commonality might indeed lead to a more attractive product line and higher revenues. This result is contrary to the conventional paradigm: It proposes a strategy according to which components should be made common because they have a strong impact on customer valuation relative to their production costs. We find that settings with a pronounced lower class market segment and a relatively small difference between customer segments in terms of valuation of quality favor component sharing. Finally, we show that disregarding interdependencies between quality and effort decisions can lead manufacturers to offer excessively differentiated product lines with average quality far below the optimal level.

The remainder of the paper is structured as follows. In §2 we develop our model and discuss basic assumptions. Section 3 contains the analysis of our model. We discuss our key findings and conclude the paper in §4. Proofs are relegated to the appendix.

2. The Model

A manufacturer sells to a market of fixed size M with two customer segments that have different marginal valuations of quality. The segment with the higher valuation of quality θ represents a fraction $\beta \in (0, 1)$ of the total demand M, and the segment with the lower valuation $\alpha\theta$ (where $\alpha \in (0, 1)$) the remainder. This is a common modeling framework that has been used by many other researchers (e.g., Moorthy 1984, Kim and Chhajed 2000, Desai et al. 2001). Our focus is on internal design decisions of a single firm and we do not consider multi-firm competition to maintain analytical tractability.[1] Assuming rational customers who maximize their net-utility, i.e., the difference between utility gained through the purchase of a product (a customer's valuation for quality multiplied by the product's quality) and its price, we use the standard product positioning and pricing problem of a monopolist with second-degree price discrimination:

$$\max \Pi(P_H, P_L, Q_H, Q_L) = M[(1-\beta)(P_L - c(Q_L)) + \beta(P_H - c(Q_H))], \qquad (1)$$

subject to

$$\alpha\theta Q_L \geq P_L; \qquad (2)$$

$$\theta Q_H \geq P_H; \qquad (3)$$

$$\alpha\theta Q_L - P_L \geq \alpha\theta Q_H - P_H; \qquad (4)$$

$$\theta Q_H - P_H \geq \theta Q_L - P_L; \qquad (5)$$

$$Q_H \geq Q_L \geq 0, \qquad P_H \geq P_L \geq 0. \qquad (6)$$

Here P_L, P_H, Q_L, and Q_H denote the prices and quality levels of the two products offered to the different market segments, and $c(Q)$ is the cost of producing one product with quality Q. Constraints (2) and (3) guarantee that all customers in the two market segments purchase one of the two offered products (participation) and constraints (4) and (5) ensure that they choose the product addressed at their segment (self-selection). We assume it is optimal for the manufacturer to offer two different products to the two customer segments. This assumption implies that $\alpha > \beta$, because otherwise it would be better for the manufacturer to provide just one product for the

[1] Models of competition between vertically differentiated product lines (e.g., Stole 1995, Villas-Boas and Schmidt-Mohr 1999, Desai 2001) do not consider a firm's ability to exert effort, which is a key aspect of our work. In addition, in these models it is generally assumed that product lines are of identical length, an assumption that supports mathematical tractability, but that severely limits the analysis of a scenario where a firm may use component commonality to compete with another firm that uses uniquely designed components.

higher market segment in order to reduce cannibalization effects. Fixed costs will not alter any of our results and are omitted without loss of generality. At interior optima, only constraints (2) and (5) are binding (e.g., Moorthy and Png 1992) and profits are maximized by setting $P_L = \alpha\theta Q_L$ and $P_H = P_L + \theta(Q_H - Q_L)$. Hence, subject to $Q_H \geq Q_L \geq 0$, the above problem is equivalent to the following problem:

$$\max \Pi(Q_H, Q_L) = M[(1-\beta)(\alpha\theta Q_L - c(Q_L)) + \beta(\alpha\theta Q_L + \theta(Q_H - Q_L) - c(Q_H))]. \tag{7}$$

We assume each product is composed of J components (indexed with j) with importance weights $w_j > 0$ assigned by the customers. These importance weights represent the influence that each component has on the product quality as perceived by the customer. Fisher et al. (1999) discuss such categorization schemes and provide examples of their use in practice. As in Desai et al. (2001), we assume quality to be additive, such that the qualities of the two products Q_L and Q_H are the sums of their constituent components' quality levels weighted with the factors w_j.

Consistent with an extensive literature in economics and operations management (e.g., Mussa and Rosen 1978, Moorthy 1984, Garvin 1988, Kim and Chhajed 2000, Desai et al. 2001), we assume processing costs to be convex in quality. For mathematical tractability, we assume a quadratic function, such that the per-unit production cost of a component j with quality q is $c_j(q) = k_j q^2$, where $k_j > 0$ is a cost coefficient that reflects the differences in cost of producing quality across different component types. The total cost of a product is assumed to be the sum of its components' costs.

We consider a manufacturer who, besides determining quality levels, may exert effort to reduce production costs. Research and development (R&D) expenditures of major car manufacturers often explicitly aim at aggressive cost reduction (Gupta and Loulou 1998). Effort on R&D or innovation is just one example, where a manufacturer's investments can lead to production cost reductions. Different approaches have been used to model the effect of process innovation effort on unit production costs. Some researchers assume such effort leads to a certain absolute reduction in unit production cost (e.g., Bonanno and Haworth 1998, Gupta and Loulou 1998). Similar to the effect of effort in Desai et al. (2001), these models ignore potential interdependencies between decisions on effort and quality, which in this context is often associated with product innovation. However, Schmidt and Porteus (2000) discuss several examples of innovation and conclude that, in reality, "product innovation and process innovation are innately intertwined" (p. 325).

Such interaction effects are better captured in multiplicative models, where process innovation effort leads to a certain proportional reduction in unit production costs. For instance, Adner and Levinthal (2001) assume process innovation lowers unit production costs by a certain percentage, arguing that the resulting geometric cost decrease has an empirical basis in the learning curve literature. Supporting this connection, an empirical study by Hatch and Mowery (1998) in the semiconductor industry suggests that learning curve effects are highly moderated by deliberate engineering effort. Lambertini and Orsini (2000) and Bandyopadhyay and Acharyya (2004) explicitly consider how quality decisions affect unit production cost, and, in their models, process innovation effort decreases the marginal cost of quality, emphasizing the interdependency between quality and effort decisions.

Another context for interactions between quality and effort decisions is discussed in Mayer et al. (2004), who argue that supply or plant inspection effort should decrease the marginal cost of providing (reliability) quality, because they facilitate the supplier's learning and also encourage the supplier to ensure high quality to maintain business. Similarly, investments in new equipment with lower variable operating cost are yet another example for efforts toward production cost reductions. De Groote (1994) considers a manufacturer with the option of reducing the cost of product variety by investing in flexible technology with lower setup cost between batches; his analysis also emphasizes the mutually reinforcing effects between the investment and the product variety decision.

Hence, there is a wide literature that supports the notion that production costs and specifically the marginal cost of quality might be affected by deliberate effort that the manufacturer exerts in pursuit of cost reductions. In the following, we use a multiplicative model to make this externality explicit, and we take an additive model as proxy for cases where effort does not affect the marginal cost of quality. We consider a relatively general relation between effort and unit production costs, assuming that with an effort e the unit production cost of a component of quality q is $c_j(q, e) = a_j(e)k_jq^2 - b_j(e)$. This model includes both the general additive model (with $a_j(e) \equiv 1$) and the general multiplicative model (with $b_j(e) \equiv 0$) as special cases. Also note that the specification in Desai et al. (2001) is just a special instance of our model when $a_j(e) = 1$ and $b_j(e) = \sqrt{e}$. We assume $a'_j(e) < 0$, $b'_j(e) > 0$, $a''_j(e) > 0$, and $b''_j(e) < 0$, such that effort reduces cost with decreasing marginal effect. We further assume $a_j(0) = 1$ and $b_j(0) = 0$, so a zero effort does not cause any cost reductions. Whereas for the multiplicative model we can assure positive production costs by assuming $a_j(e) > 0$, there is no straightforward way to ensure this desirable property for the additive case.

It is easily verified that the problem is separable, such that the optimal quality and effort level for each component and the decision whether to make this component common is independent from decisions made for other components. This separability property, which is a consequence of the additive product quality and cost and the constant marginal valuation of quality by the consumers, allows us to evaluate the effects of commonality decisions on quality, investments in cost-reduction effort, and profitability for one component at a time. Hence, without loss of generality, we analyze the one-component problem and for notational convenience we drop the subscripts. Let q_b, q_p, and q_c denote the qualities of the components used uniquely in the lower-quality product (basic), uniquely in the higher-quality product (premium), or commonly in both products (common). Similarly, e_b, e_p, and e_c denote the cost-reduction efforts directed at these components. Substituting the assumed production cost and quality models into (7) and considering a single component, profits in the common (subscript C) and the unique (subscript U) configuration can be written as follows:

$$\Pi_C(q_c, e_c) = M\alpha\theta w q_c - M(a(e_c)kq_c^2 - b(e_c)) - e_c; \quad (8)$$

$$\begin{aligned}\Pi_U(q_b, q_p, e_b, e_p) &= M\theta w(\alpha q_b + \beta(q_p - q_b)) - M\beta(a(e_p)kq_p^2 - b(e_p)) \\ &\quad - M(1-\beta)(a(e_b)kq_b^2 - b(e_b)) - e_p - e_b. \quad (9)\end{aligned}$$

The manufacturer determines the optimal quality levels and cost-reduction efforts for each configuration and chooses the configuration that yields higher profits at the optimum. Note that comparing the profits for the two configurations directly reveals that if two different variants of the component are offered in the unique configuration, this product differentiation yields a premium that depends on the size of the higher-valuation segment (β) and the quality differential between the two components ($q_p - q_b$). Hence, revenues will always decrease in the common configuration, unless a component with sufficiently high quality is offered to compensate for the loss of differentiation. We use a prime (double prime) to denote the first (second) derivative with respect to e.

Lemma 1. *The profit functions $\Pi_C(q_c, e_c)$ and $\Pi_U(q_b, q_p, e_b, e_p)$ in* (8) *and* (9) *are strictly jointly concave in quality levels and efforts, if $a'(e) = 0$ or $a(e)a''(e) > 2a'(e)^2$.*

For the rest of the paper, the condition for joint concavity given in Lemma 1 is assumed to be satisfied. A notational index is given in Table 1.

3. Analysis

In this section, we derive necessary and sufficient conditions for the optimal quality levels and investments in cost-reduction effort. We investigate the effects of

Table 1 **Notation**

θ	Marginal valuation of quality of the premium segment
$\alpha\theta$	Marginal valuation of quality of the basic segment ($\alpha < 1$)
M	Total size of the market
β	Fraction of the market with high valuation of quality
w	Importance weight attributed to the component of interest
k	Production cost coefficient
ρ	Surrogate parameter that reflects the trade-off between revenue and cost drivers, $\rho = w^2/k$
$a(\cdot)$, $b(\cdot)$	Functions used to describe the impact of effort on the marginal production cost: $c(q, e) = a(e)kq^2 - b(e)$
q_b, q_p, q_c	Quality levels (decision variables)
e_b, e_p, e_c	Cost-reduction efforts (decision variables)
Π_U (Π_C)	Profits in the unique (common) configuration
$\Delta\Pi^*$ (ΔR^*)	Change in optimal profits (revenues) when a common component is used in place of two different variants

different parameters on design decisions and how these relations in turn are determined by the way in which effort affects production costs. Our main objective is to test the conventional wisdom that commonality enables cost savings but is always detrimental to revenues. For convenience and simplicity, we introduce a surrogate parameter $\rho = w^2/k$. This parameter explicitly captures the trade-off between a component's revenue impact (its relative importance to the customers) and its cost characteristics.

The conventional paradigm and recent research by Desai et al. (2001) suggest making those components common that are characterized by low values of ρ, i.e., parts that are costly to produce, but not regarded as differentiating or important by the customers. In §3.1, we analyze the additive model (i.e., we assume that $a(e) \equiv 1$) that disregards the interactions between cost-reduction efforts and quality levels. This model generalizes the model in Desai et al. (2001) and leads to similar managerial insights. However, in §3.2 we show that if effort affects quality-related costs, and if this interdependency is considered in evaluating commonality, the conventional paradigm does not necessarily hold. In a multiplicative model that captures such interactions (where we assume that $b(e) \equiv 0$), we demonstrate that commonality can lead to substantially higher product quality, increasing the overall attractiveness of the product line, as well as revenues, in spite of reduced product differentiation. To characterize environments that support this outcome, we analyze the effects of the valuation differential across segments (captured by α) and the relative size of the higher-valuation segment (captured by β) on the profitability of commonality as well as on how the profitability of commonality is affected by the trade-off between cost and revenue characteristics (as captured by the surrogate parameter ρ).

3.1. Effort Does Not Affect the Marginal Cost of Quality

In this subsection, we analyze a model where effort has no effect on quality-related costs. We model such a setting with a general additive model by assuming that $a(e) \equiv 1$ for all levels of effort e, i.e., $c(q, e) = kq^2 - b(e)$. Recall our assumptions $b(0) = 0$, $b'(e) > 0$, and $b''(e) < 0$, and note that the specific function $b(e) \equiv \sqrt{e}$ assumed in Desai et al. (2001) is a special instance of this model. As before, a prime denotes the first derivative with respect to e, and an asterisk denotes optimal values.

Proposition 1. *The optimal quality levels and cost-reduction efforts satisfy*

a. $q_b^* = \dfrac{w\theta(\alpha - \beta)}{2k(1-\beta)}$; $q_p^* = \dfrac{w\theta}{2k}$; $q_c^* = \dfrac{w\theta\alpha}{2k}$.

b. $b'(e_b^*) = \dfrac{1}{M(1-\beta)}$; $b'(e_p^*) = \dfrac{1}{M\beta}$; $b'(e_c^*) = \dfrac{1}{M}$.

c.

	q_b^*	q_p^*	q_c^*	e_b^*	e_p^*	e_c^*
$\partial/\partial\alpha$	$+$	0	$+$	0	0	0
$\partial/\partial\beta$	$-$	$+$	0	$-$	$+$	0

Proposition 1a shows that, in this model, the optimal component qualities do not depend on the magnitude of cost-reduction effort. Similarly, in Proposition 1b it can be seen that the components' quality levels do not affect the optimal level of such effort. Because in this model investments in effort have no effect on the cost of providing quality, the decisions regarding quality levels and the associated levels of effort are made independently. Proposition 1c demonstrates that the optimal effort depends on the production quantities of the corresponding parts (captured by the parameters β and M), but not on how the market segments differ in their valuation of quality (captured by α). This supports the conventional paradigm that the sole benefit of commonality lies in cost reductions associated with scale economies. Because quality and effort decisions are made independently, potential second-order revenue effects of cost-reduction effort are not captured.

Proposition 2. *The optimal quality levels and cost-reduction efforts satisfy*

a. $q_p^* > q_c^* > q_b^*$.

b. $e_p^* > e_b^* \Leftrightarrow \beta > 1/2$; $e_c^* > e_b^*$; $e_c^* > e_p^*$.

Proposition 2b demonstrates that relatively higher efforts are directed at those components with higher production quantity (captured by the parameter β), when neglecting the effects of cost reductions on quality decisions, and vice versa. Consequently, the effort directed at common components always exceeds effort directed at components that address a unique market segment. The relative distribution of effort between the components in the unique configuration merely depends on their relative production quantities and is independent of each component's relative importance in generating revenues. Hence, the potential revenue enhancing effect of commonality is not considered. Proposition 2a shows that in this model the optimal quality of a common component always lies between the optimal qualities of the components that would be used if two different variants were offered. Hence, under commonality, customers of the higher-valuation segment are offered a product of lower quality that is also less differentiated from the basic product. As a consequence, revenues from the higher-valuation segment always decrease under commonality. Revenues from the lower-valuation segment, however, increase, because this segment receives a better product and does not value product differentiation. We analyze the sum of these two effects in the following. To analyze the profitability of replacing unique components by a common component, we use the difference function $\Delta\Pi^* = \Pi_C(q_c^*, e_c^*) - \Pi_U(q_b^*, q_p^*, e_b^*, e_p^*)$. Again, we use $\rho = w^2/k$ to capture the trade-off between the relative importance of a component to the customers (i.e., its revenue effects) and its cost characteristics. Substitution of the optimal qualities from Proposition 2 gives the following:

$$\Delta\Pi^* = -\frac{M\theta^2\rho\beta(1-\alpha)^2}{2(1-\beta)} + \frac{M\theta^2\rho\beta(1-\alpha)^2}{4(1-\beta)} + M(b(e_c^*) - (1-\beta)b(e_b^*) - \beta b(e_p^*)) - e_c^* + e_b^* + e_p^*. \tag{10}$$

The function $\Delta\Pi^*$ represents the change in profits if a common component of optimal quality is used in place of two unique optimal components. The first term reflects the resulting change in revenues and we denote it with ΔR^*. The remaining terms represent the change in costs of production and effort. We note that in this model effort has no impact on the change in revenues and only trades off investments with the associated cost savings. In the following proposition, we show that commonality in this model always leads to lower revenues. We analyze how the profitability of commonality is affected by the component's importance relative to its cost characteristics (ρ), and how this relation and the general profitability of commonality depend on the differences across segments in terms of valuation of quality (α) and size (β).

Proposition 3.

a. $\Delta R^* < 0$.

b. $\dfrac{\partial\Delta\Pi^*}{\partial\rho} < 0$.

c. $\dfrac{\partial\Delta\Pi^*}{\partial\alpha} > 0$; $\dfrac{\partial}{\partial\alpha}\left(\dfrac{\partial\Delta\Pi^*}{\partial\rho}\right) > 0$.

d. $\dfrac{\partial\Delta\Pi^*}{\partial\beta} < 0 \iff b(e_b^*) - b(e_p^*) < \dfrac{\theta^2\rho(1-\alpha)^2}{4(1-\beta)^2}$; $\dfrac{\partial}{\partial\beta}\left(\dfrac{\partial\Delta\Pi^*}{\partial\rho}\right) < 0$.

If effort has no effect on the marginal cost of quality, the reduction of product differentiation under commonality always leads to lower revenues (Proposition 3a). Proposition 3b shows that the profitability of commonality decreases with the relative importance of a component compared to its cost characteristics (ρ). Because this model only captures benefits of commonality on the cost side, commonality clearly becomes a less attractive option for components that are relatively more valued by customers. Hence the ranking proposed by Desai et al. (2001) for the specific case $b(e) \equiv \sqrt{e}$ is valid for more general additive models. Proposition 3c shows that a small differential in quality valuations across market segments mitigates this negative revenue effect and increases the profitability of commonality. We have seen that revenues generated on the higher-valuation segment are always decreased under commonality, and Proposition 3d demonstrates that this effect is amplified by β. The impact of β on the overall profitability depends on the specific shape of $b(e)$, i.e., on the effectiveness of effort in achieving cost reductions. Whereas an increase in β almost always reduces the profitability of commonality, closer analysis of the condition given in Proposition 3d reveals that the inequality might be violated for large values of α and small values of β. Because profits in the common configuration are independent of β (cf. Equation (8)), this finding suggests that, as the premium segment gets larger, profits in the unique configuration are reduced. The reason for this surprising result is that for very small β and large α the two components offered in the unique configuration are of almost identical quality (cf. Proposition 4).

Whereas the lower-quality segment pays for the quality of the lower component, which is particularly high in this case, the higher-valuation segments pays for the quality differential (cf. Equation (9)). In general, increases in β lead to greater quality differentials and lower quality of the basic component, amplifying the role of the premium segment in generating revenues. However, in this particular setting with a negligible quality differential, revenues are largely generated by the lower-valuation segment, such that a reduction of its size might at first result in reduced profits. Finally, note that this occurs only in settings of this specific character and the insights reflect trade-offs internal to the unique configuration.

As a summary of this subsection, we find that for the model where effort does not affect the marginal cost of quality revenues are always decreased under commonality; this effect is strongest for components that, relative to their production costs, are valued highly by the customers. Cost savings represent the only benefit of commonality in this model and their magnitude solely depends on the quantities that are produced of the different components. As a consequence, this model supports the conventional suggestion to make those parts common that are costly to produce, but unimportant or invisible to the customer (cf. platform concept in the automobile industry). In general, the profitability of commonality is higher in environments with a substantial lower-valuation segment and a small quality differential between market segments.

3.2. Effort Affects the Marginal Cost of Quality

In this subsection, we analyze a special case of our model with $b(e) \equiv 0$ for all levels of effort e. In this model, effort leads to reductions in quality-related costs, and the interdependencies between effort and quality decisions are made explicit.

Proposition 4. *The optimal quality levels and cost-reduction efforts satisfy*

a. $q_b^* = \dfrac{w\theta(\alpha-\beta)}{2ka(e_b^*)(1-\beta)}$; $q_p^* = \dfrac{w\theta}{2ka(e_p^*)}$; $q_c^* = \dfrac{w\theta\alpha}{2ka(e_c^*)}$.

b. $-\dfrac{a(e_b^*)^2}{a'(e_b^*)} = \dfrac{M\rho\theta^2(\alpha-\beta)^2}{4(1-\beta)}$; $-\dfrac{a(e_p^*)^2}{a'(e_p^*)} = \dfrac{M\theta^2\rho\beta}{4}$; $-\dfrac{a(e_c^*)^2}{a'(e_c^*)} = \dfrac{M\theta^2\rho\alpha^2}{4}$.

c.

	q_b^*	q_p^*	q_c^*	e_b^*	e_p^*	e_c^*
$\partial/\partial\alpha$	+	0	+	+	0	+
$\partial/\partial\beta$	−	+	0	−	+	0

Proposition 4a shows that if effort affects quality-related production costs, the resulting interdependencies lead to higher levels of quality (recall that $a(e) \le 1$). In the previous model, effort had no effect on quality decisions and the optimal effort level was determined only by the production quantities. In this model, it also depends on the difference in valuations of quality across segments (α), and how investments in effort are distributed among the different components (Proposition 4c). This result reflects the fundamental difference between the two models in how scale economies associated with such effort under component commonality are evaluated. Whereas the previous model only considered the resulting cost savings, the model analyzed in this section takes second-order effects of cost-reduction effort on revenues explicitly into consideration. Even though the comparative statics for the optimal qualities are identical in sign across the two models, in this model the magnitude of these effects also depends on the specific effort decisions.

Proposition 5. *The optimal quality levels and cost-reduction efforts satisfy*

a. $q_c^* > q_b^*$; $q_p^* > q_b^*$; $a(e_c^*) < a(e_p^*)\alpha \Leftrightarrow q_c^* > q_p^*$.

b. $e_c^* > e_b^*$; $\alpha < \beta + \sqrt{\beta(1-\beta)} \Leftrightarrow e_p^* > e_b^*$; $\alpha < \sqrt{\beta} \Leftrightarrow e_p^* > e_c^*$.

Proposition 5a shows that if the interdependency between efforts and quality levels is captured, the manufacturer might find it optimal to provide a common component of quality that exceeds the quality of the component that would be offered to the higher-valuation segment, if two variants of the component were produced. The reason for this result lies in scale economies. Producing a common component augments the payoff of cost-reduction effort. Increases in effort reduce the cost of producing quality, providing an incentive to offer higher-quality components.

When the condition in Proposition 5a is satisfied, all customers are offered higher-quality products under commonality. Recall that in the previous model the quality of the optimal common component was always lower than the optimal quality level of the premium component in the unique configuration, such that the quality of the product offered to customers of the higher segment was always reduced under commonality. However, in this case an increase in quality might compensate these customers for the loss in product differentiation. Applying the results of Proposition 4, we see that designing a common component with such high quality is more likely for larger values of α and lower values of β, i.e., in environments with a substantial lower-valuation segment and a relatively small differential in quality valuations across segments.

Proposition 5b shows that now the relative ordering of components with respect to cost-reduction effort might also depend on the differentiation between segments in terms of valuing quality (captured by the parameter α). As before, compared to the basic component in the unique configuration, higher effort is directed at the common component, because it provides both higher quality and higher production quantity. However, the other comparisons depend on the relative sizes of the two market segments (β) and on how different their valuations of quality are (α). As in the previous model where quality levels were not considered in the effort decision, larger investments might be directed toward the lower-quality component. However, this outcome is less likely in this model, because $\beta + \sqrt{\beta(1-\beta)} > 1$ for all $\beta > 1/2$.

If the differential of quality valuations is sufficiently small compared to the size of the higher-valuation segment, it might be optimal to direct lower cost-reduction effort at the common component than at the premium component. Hence, this result does not support the finding of Desai et al. (2001) that net-savings due to effort are always increased in common configurations.

We again use the difference function $\Delta\Pi^*$ to evaluate the profitability of commonality and how it is affected by the trade-off between revenue and cost characteristics (captured by the parameter $\rho = w^2/k$). Substitution of the optimal quality levels gives the following:

$$\Delta\Pi^* = \frac{M\theta^2\rho}{2}\left(\frac{\alpha^2}{a(e_c^*)} - \frac{(\alpha-\beta)^2}{(1-\beta)a(e_b^*)} - \frac{\beta}{a(e_p^*)}\right) - \frac{M\theta^2\rho}{4}\left(\frac{\alpha^2}{a(e_c^*)} - \frac{(\alpha-\beta)^2}{(1-\beta)a(e_b^*)} - \frac{\beta}{a(e_p^*)}\right) - e_c^* + e_b^* + e_p^*. \tag{11}$$

As before, the first term of $\Delta\Pi^*$ reflects the change in revenues if a common component is used in place of two different variants; we denote this term with ΔR^*. The remaining terms represent the change in costs of production and effort. Comparing this expression to (10), we see that if effort affects the marginal quality cost, cost savings and revenue effects are no longer evaluated separately in the effort decision. In this model, cost reductions resulting from effort are the means to achieve higher-quality products and higher revenues. The results given in Proposition 6 (in the appendix) demonstrate that here commonality can indeed lead to increased revenues, and as a consequence the profitability of making a component common might increase in its importance relative to its cost impact (ρ). Whereas in the additive model the importance of a component relative to its cost characteristics always has a negative effect on the profitability of commonality, in this model the role of ρ depends on the specific functional properties of $a(e)$ and it is positive if and only if commonality brings about an increase in revenues. This is more probable in environments that favor higher investments directed at the common component (Proposition 6a). Proposition 4 suggests that such environments are likely to be characterized by a sufficiently large lower-valuation segment and a small differential in terms of quality valuations. Indeed we find that small valuation differentials across segments (α) increase the profitability of the common configuration compared with the unique configuration (Proposition 6b). This is intuitive, because with smaller quality-valuation differentials the benefit of differentiating between customer segments is reduced. As in the previous model, the impact of the relative size of the larger segment (β) on the profitability of commonality depends on the effectiveness of effort, but is negative in most cases (see Proposition 6c and discussion of Proposition 3). Hence, approaches based on commonality generally are more profitable in environments with a substantial lower-valuation segment and a small valuation differential. Kim and Chhajed (2000) derive similar results with a different model. An interesting result is the apparent parallelism between the comparative statics for the overall profitability of commonality and the impact of ρ, which demonstrates that a component's attractiveness for commonality is highly determined by its impact on revenues relative to production costs ($\rho = w^2/k$). This finding contradicts the conventional recommendation to make those components common that have little impact on revenues, but are costly to produce. In this model, where effort reduces quality-related production costs, the top candidates for commonality will often be those parts that matter to the customers and are important in differentiating products. Hence, considering cost savings as the sole or primary driver in commonality decisions might lead to excessively differentiated product lines with low-quality products.

4. Discussion

We develop a stylized model of a manufacturer who offers two products to a market with two segments that have different valuations for quality. These products consist of components that can be common or unique to both products. The manufacturer chooses the components' quality levels and decides if a component is used commonly in both products or if two distinct variants are used. For each component, the manufacturer further has the option to exert effort in order to reduce production costs. Assuming a general relation between such effort and production costs, we test the conventional paradigm that commonality leads to cost savings, whereas the loss of product differentiation always leads to less attractive product lines and reduced revenues.

Explicitly capturing interactions between quality and effort decisions, we show that this paradigm does not always hold. An optimally designed product line involving common components might be more attractive and yield higher revenues than a product line based on different variants. The intuition behind this finding is that commonality can induce scale economies that increase the effectiveness of cost-reduction effort. This leads to higher optimal levels of such effort and consequently to lower unit production costs. If these costs depend on quality, reductions of these costs support a higher optimal quality level, which eventually overcompensates customers of the premium market segment for the lost product differentiation and leads to increased revenues. Under certain circumstances, a manufacturer may find it profitable to proactively use this lever to substantially increase the average quality of the product line. These findings are contrary to the conventional paradigm and they contrast strongly with the results in Desai et al. (2001), because they suggest a strategy according to which at times those components should be made common that have a strong impact on customer valuation relative to their production costs.

Consider the platform concept in the automobile industry and the example discussed at the outset of this article, which raises the question why VW's customers do not seem to mind the platform sharing between the Passat and the Audi A4, whereas Toyota faces serious problems when using its Camry platform for the Lexus. Our results lead us to conjecture that one key difference between these two instances of platform sharing lies in the quality of the shared component. Both firms use the same platform for a basic and a premium brand, but the two models of the VW group supposedly share a high-quality platform, whereas Toyota uses its basic platform for both brands. The majority of Toyota's customers are interested in the reliable and reasonably priced Toyota brand, suggesting their environment to be supportive of a strategy based on component commonality (rather large α and small β). However, our analysis suggests that the optimal quality of a common component will always be higher than that designed for the basic product, and that at times it might even exceed the quality of the component designed for the premium product (Proposition 5). Using the Camry's basic quality platform in the premium Lexus sedan must make Lexus customers question whether the substantial price premium is justified. Our findings suggest that in this case the negative revenue implications of reduced product differentiation dominate the cost savings.

In the case of VW, one might expect that the quality of the premium Audi platform that is shared with the Passat might be closer to the optimal quality level for a common component. In addition, the characteristics of VW's customer base are at least as supportive of a platform-sharing strategy as those of Toyota's customer base. Unit sales of the VW brand are about four times those of the Audi brand (Volkswagen 2004), implying a premium market segment of relatively modest size (small β). VW also found that only 4% of Audi shoppers even consider VW, and vice versa, implying a relatively low risk of cannibalization between the brands (*The Toronto Star* 2002). Given this low degree of overlap between customers of the different brands, Volkswagen appears to be emphasizing the Passat's heritage from the sportive Audi brand—by introducing an eight-cylinder, 275-horsepower version. Priced at \$55,000, it is difficult to consider customers for this Passat to be price sensitive. This indicates a relatively small differential between the different customer segments in terms of quality valuation (large α), which we found to favor a platform strategy.

Strategies based on component commonality have been widely used by many manufacturers of assembled goods and consequently have also received substantial attention in the research community. However, although the application of such methods to physical product design seems straightforward, it appears to us that there might be some overlap with the problem of service design. For example, although it would be more cost efficient to offer just one type of fare (cf. commonality), most airlines offer different fares to segment their customers and extract price premiums. An important question is which dimensions should be used to differentiate between customer segments. Besides the amount of leg room and free in-flight food, fares are often set apart along less tangible dimensions, such as advance purchase requirements or cancellation policies. The trade-off between cost savings (e.g., more seats in a plane, fewer expenses for in-flight food, better planning due to rigid policies) and the risk of cannibalization (why should a customer buy the more expensive first class ticket?) is the same as in the problem we have studied, and the problem of selecting the dimensions to differentiate fares appears similar to the question of which components not to share across products. A direct application of our findings to this problem seems to find support in the observation that most of the upcoming low-cost (or no-frills) airlines offer much simpler fare schemes and often only one passenger class. With most of its customers being rather price sensitive (i.e., small β), it could indeed be optimal to follow a full-commonality strategy that minimizes costs by, in essence, offering only one product to the market.

However, we understand that there are more dimensions that need to be considered in service design research. It might be interesting to investigate the extent to which the results of the commonality literature apply to such problems. Studying the commonalities as well as the differences between these two applications could be a valuable venue for future research.

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Appendix

Proof of Lemma 1. The function is separable in the corresponding pairs of quality levels and efforts.

$$\frac{\partial^2\Pi}{\partial q_c^2} = -2kMa(e_c) < 0; \quad \frac{\partial^2\Pi}{\partial e_c^2} = -M(kq_c^2 a''(e_c) - b''(e_c)) < 0;$$

$$\frac{\partial^2\Pi}{\partial q_c^2}\frac{\partial^2\Pi}{\partial e_c^2} - \left(\frac{\partial^2\Pi}{\partial q_c \partial e_c}\right)^2 > 0$$

$$\Leftrightarrow a(e_c)a''(e_c) - 2a'(e_c)^2 > \frac{a(e_c)b''(e_c)}{kq_c^2}; \quad \text{and}$$

$$\frac{a(e_c)b''(e_c)}{kq_c^2} < 0.$$

$$\frac{\partial^2\Pi}{\partial q_p^2} = -2kM\beta a(e_p) < 0;$$

$$\frac{\partial^2\Pi}{\partial e_p^2} = -M\beta(kq_p^2 a''(e_p) - b''(e_p)) < 0;$$

$$\frac{\partial^2\Pi}{\partial q_p^2}\frac{\partial^2\Pi}{\partial e_p^2} - \left(\frac{\partial^2\Pi}{\partial q_p \partial e_p}\right)^2 > 0$$

$$\Leftrightarrow a(e_p)a''(e_p) - 2a'(e_p)^2 > \frac{a(e_p)b''(e_p)}{kq_p^2}; \quad \text{and}$$

$$\frac{a(e_p)b''(e_p)}{kq_p^2} < 0.$$

$$\frac{\partial^2\Pi}{\partial q_b^2} = -2kM(1-\beta)a(e_b) < 0;$$

$$\frac{\partial^2\Pi}{\partial e_b^2} = -M(1-\beta)(kq_b^2 a''(e_b) - b''(e_b)) < 0;$$

$$\frac{\partial^2\Pi}{\partial q_b^2}\frac{\partial^2\Pi}{\partial e_b^2} - \left(\frac{\partial^2\Pi}{\partial q_b \partial e_b}\right)^2 > 0$$

$$\Leftrightarrow a(e_b)a''(e_b) - 2a'(e_b)^2 > \frac{a(e_b)b''(e_b)}{kq_b^2}; \quad \text{and}$$

$$\frac{a(e_b)b''(e_b)}{kq_b^2} < 0.$$

The proof for $a'(e) = 0$ is straightforward (because $\partial^2\Pi/\partial q\partial e = 0$) and therefore omitted. □

Proof of Proposition 1. By Lemma 1, first-order conditions are necessary and sufficient. For given efforts

$$\frac{\partial\Pi}{\partial q_b} = 0 \Rightarrow q_b^* = \frac{w\theta(\alpha-\beta)}{2ka(e_b)(1-\beta)}, \tag{A1}$$

$$\frac{\partial\Pi}{\partial q_p} = 0 \Rightarrow q_p^* = \frac{w\theta}{2ka(e_p)}, \tag{A2}$$

$$\frac{\partial\Pi}{\partial q_c} = 0 \Rightarrow q_c^* = \frac{w\theta\alpha}{2ka(e_c)}. \tag{A3}$$

With $a(e) \equiv 1$, this proves Part 1a. After substituting these optimal quality levels into the profit functions (Equations (8) and (9)), we get the following necessary and sufficient first-order conditions for the optimal levels of effort.

$$\frac{\partial\Pi}{\partial e_b} = 0 \Rightarrow M(1-\beta)b'(e_b^*) - \frac{M(\alpha-\beta)^2\theta^2\rho}{4(1-\beta)}\frac{a'(e_b^*)}{a(e_b^*)^2} = 1, \tag{A4}$$

$$\frac{\partial\Pi}{\partial e_p} = 0 \Rightarrow M\beta b'(e_p^*) - \frac{M\beta\theta^2\rho}{4}\frac{a'(e_p^*)}{a(e_p^*)^2} = 1, \tag{A5}$$

$$\frac{\partial\Pi}{\partial e_c} = 0 \Rightarrow Mb'(e_c^*) - \frac{M\alpha^2\theta^2\rho}{4}\frac{a'(e_c^*)}{a(e_c^*)^2} = 1. \tag{A6}$$

If $a(e) \equiv 1$, (A4)–(A6) imply the conditions given in Part 1b. Part 1c:

$$\frac{\partial q_b^*}{\partial\beta} < 0 \Leftrightarrow \frac{-(1-\beta)+(\alpha-\beta)}{(1-\beta)^2} < 0 \Leftrightarrow \alpha < 1.$$

The other entries follow from inspection. □

Proof of Proposition 2. Part 2a:

$$q_p^* > q_c^* \Leftrightarrow \frac{w\theta}{2k} > \frac{w\theta\alpha}{2k} \Leftrightarrow 1 > \alpha \quad \text{and}$$

$$q_c^* > q_b^* \Leftrightarrow \frac{w\theta\alpha}{2k} > \frac{w\theta(\alpha-\beta)}{2k(1-\beta)} \Leftrightarrow \alpha > \frac{\alpha-\beta}{1-\beta} \Leftrightarrow \alpha < 1.$$

Part 2b: Recall that $b''(e) < 0$, such that $b'(e)$ is strictly monotonous decreasing in e. Hence $e_c^* > e_b^* \Leftrightarrow b'(e_c^*) < b'(e_b^*) \Leftrightarrow 1/M < 1/(M(1-\beta)) \Leftrightarrow \beta > 0$, $e_c^* > e_p^* \Leftrightarrow b'(e_c^*) < b'(e_p^*) \Leftrightarrow 1/M < 1/(M\beta) \Leftrightarrow \beta < 1$, and $e_p^* > e_b^* \Leftrightarrow b'(e_p^*) < b'(e_b^*) \Leftrightarrow 1/(M\beta) < 1/(M(1-\beta)) \Leftrightarrow \beta < 1/2$. □

Proof of Proposition 3. Part 3a:

$$\Delta R^* = -\frac{M\theta^2\rho\beta(1-\alpha)^2}{2(1-\beta)} < 0.$$

Part 3b:

$$\frac{\partial\Delta\Pi^*}{\partial\rho} = -\frac{M\theta^2\beta(1-\alpha)^2}{4(1-\beta)} < 0.$$

Part 3c:

$$\frac{\partial\Delta\Pi^*}{\partial\alpha} = \rho\frac{\partial^2\Delta\Pi^*}{\partial\rho\partial\alpha} \quad \text{and} \quad \frac{\partial^2\Delta\Pi^*}{\partial\rho\partial\alpha} > 0 \Leftrightarrow \frac{M\theta^2\beta(1-\alpha)}{2(1-\beta)} > 0.$$

Part 3d:

$$\frac{\partial^2\Delta\Pi^*}{\partial\rho\partial\beta} = -\frac{M\theta^2(1-\alpha)^2}{4}\left(\frac{1}{(1-\beta)^2}\right) < 0,$$

$$\frac{\partial\Delta\Pi^*}{\partial\beta} < 0 \Leftrightarrow \rho\frac{\partial^2\Delta\Pi^*}{\partial\rho\partial\beta} + M(b(e_b^*) - b(e_p^*)) < 0$$

$$\Leftrightarrow b(e_b^*) - b(e_p^*) < \frac{\theta^2\rho(1-\alpha)^2}{4(1-\beta)^2}. \quad \square$$

Proof of Proposition 4. Parts 4a and 4b follow from Equations (A1)–(A6) with $b(e) \equiv 0$. Part 4c: For the efforts, use the implicit function theorem with the optimality conditions in (A4)–(A6) for $b(e) \equiv 0$. With

$$h(e) = \left(\frac{\partial}{\partial e}\left(-\frac{a(e)^2}{a'(e)}\right)\right)^{-1} = \frac{a'(e)^2}{a(e)(a(e)a''(e) - 2a'(e)^2)},$$

we have

$$\frac{\partial e_b^*}{\partial\alpha} = \frac{M\rho\theta^2}{4}\left(\frac{2(\alpha-\beta)}{1-\beta}\right)h(e_b^*) > 0,$$

$$\frac{\partial e_b^*}{\partial\beta} = \frac{M\rho\theta^2}{4}\left(\frac{-2(\alpha-\beta)(1-\beta)+(\alpha-\beta)^2}{(1-\beta)^2}\right)h(e_b^*)$$

$$= -\frac{M\rho\theta^2}{4}\left(\frac{\alpha-\beta}{(1-\beta)^2}(2-\beta-\alpha)\right)h(e_b^*) < 0,$$

$$\frac{\partial e_p^*}{\partial\alpha} = 0, \quad \frac{\partial e_p^*}{\partial\beta} = \frac{M\rho\theta^2}{4}h(e_p^*) > 0,$$

$$\frac{\partial e_c^*}{\partial\alpha} = \frac{M\rho\theta^2}{4}2\alpha h(e_c^*) > 0, \quad \text{and} \quad \frac{\partial e_c^*}{\partial\beta} = 0.$$

For the quality levels, use the optimality conditions in (A1)–(A3).

$$\frac{\partial q_b^*}{\partial\alpha} > 0 \Leftrightarrow \frac{\partial}{\partial\alpha}\left(\frac{\alpha-\beta}{a(e_b^*)}\right) > 0$$

$$\Leftrightarrow \frac{a(e_b^*) - (\alpha-\beta)a'(e_b^*)(\partial e_b^*/\partial\alpha)}{a(e_b^*)^2} > 0,$$

because $a'(e_b^*) < 0$ and $\partial e_b^*/\partial\alpha > 0$.

$$\begin{aligned}
&\frac{\partial q_b^*}{\partial\beta} < 0\\
&\Leftrightarrow \frac{\partial}{\partial\beta}\left(\frac{\alpha-\beta}{a(e_b^*)(1-\beta)}\right) < 0\\
&\Leftrightarrow \frac{-a(e_b^*)(1-\beta)-(\alpha-\beta)(a'(e_b^*)(\partial e_b^*/\partial\beta)(1-\beta)-a(e_b^*))}{(a(e_b^*)(1-\beta))^2} < 0\\
&\Leftrightarrow \frac{(1-\alpha)}{(\alpha-\beta)(1-\beta)}a(e_b^*) > -a'(e_b^*)\frac{\partial e_b^*}{\partial\beta},
\end{aligned}$$

which is true because $a'(e_b^*) < 0$ and $\partial e_b^*/\partial\beta < 0$.

$$\frac{\partial q_p^*}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left(\frac{w\theta}{2ka(e_p^*)}\right) = -\frac{w\theta}{2k}\frac{a'(e_p^*)}{a(e_p^*)^2}\frac{\partial e_p^*}{\partial\alpha} = 0;$$

$$\frac{\partial q_p^*}{\partial\beta} > 0 \Leftrightarrow \frac{\partial}{\partial\beta}\left(\frac{1}{a(e_p^*)}\right) > 0 \Leftrightarrow -\frac{a'(e_p^*)}{a(e_p^*)^2}\frac{\partial e_p^*}{\partial\beta} > 0,$$

because $a'(e_p^*) < 0$ and $\partial e_p^*/\partial\beta > 0$.

$$\frac{\partial q_c^*}{\partial\alpha} > 0 \Leftrightarrow \frac{\partial}{\partial\alpha}\left(\frac{\alpha}{a(e_c^*)}\right) > 0 \Leftrightarrow \frac{a(e_c^*)-\alpha a'(e_c^*)(\partial e_c^*/\partial\alpha)}{a(e_c^*)} > 0,$$

because $a'(e_c^*) < 0$ and $\partial e_c^*/\partial\alpha > 0$.

$$\frac{\partial q_c^*}{\partial\beta} = \frac{\partial}{\partial\beta}\left(\frac{w\theta\alpha}{2ka(e_c^*)}\right) = -\frac{w\theta\alpha}{2k}\frac{a'(e_c^*)}{a(e_c^*)^2}\frac{\partial e_c^*}{\partial\beta} = 0. \quad \square$$

Proof of Proposition 5. Part 5a: From Part 5b, we know that $a(e_c^*) < a(e_b^*)$ and $a(e_p^*) < a(e_b^*)$. Hence

$$q_c^* > q_b^* \Leftrightarrow \frac{w\alpha\theta}{2ka(e_c^*)} > \frac{w\theta(\alpha-\beta)}{2ka(e_b^*)(1-\beta)}$$

is true if $\alpha > (\alpha-\beta)/(1-\beta) \Leftrightarrow \alpha < 1$, and

$$q_p^* > q_b^* \Leftrightarrow \frac{w\theta}{2ka(e_p^*)} > \frac{w\theta(\alpha-\beta)}{2ka(e_b^*)(1-\beta)}$$

is true if $1 > (\alpha-\beta)/(1-\beta) \Leftrightarrow \alpha < 1$. Finally, we have

$$q_c^* > q_p^* \Leftrightarrow \frac{w\alpha\theta}{2ka(e_c^*)} > \frac{w\theta}{2ka(e_p^*)} \Leftrightarrow a(e_c^*) < a(e_p^*)\alpha.$$

Part 5b: Because

$$\frac{\partial}{\partial e}\left(-\frac{a(e)^2}{a'(e)}\right) = \frac{a(e)(a(e)a''(e)-2a'(e)^2)}{a'(e)^2} > 0,$$

we can draw conclusions about the ordering of the optimal efforts by comparing the right-hand sides of the conditions given in Proposition 4. $e_c^* > e_b^* \Leftrightarrow \alpha^2 > (\alpha-\beta)^2/(1-\beta) \Leftrightarrow (\alpha-\alpha^2)+(\alpha-\beta) > 0$, which is true for all $\beta < \alpha < 1$. $e_p^* > e_b^* \Leftrightarrow \beta > (\alpha-\beta)^2/(1-\beta) \Leftrightarrow \alpha < \beta + \sqrt{\beta(1-\beta)}$. $e_p^* > e_c^* \Leftrightarrow \beta > \alpha^2 \Leftrightarrow \alpha < \sqrt{\beta}$. $\square$

Proposition 6.

a. $\Delta R^* > 0 \Leftrightarrow \dfrac{\partial\Delta\Pi^*}{\partial\rho} > 0 \Leftrightarrow \dfrac{a(e_c^*)}{a'(e_c^*)} < \dfrac{a(e_b^*)}{a'(e_b^*)} + \dfrac{a(e_p^*)}{a'(e_p^*)}$.

b. $\dfrac{\partial\Delta\Pi^*}{\partial\alpha} > 0$; $\quad \dfrac{\partial}{\partial\alpha}\left(\dfrac{\partial\Delta\Pi^*}{\partial\rho}\right) > 0$.

c. $\dfrac{\partial\Delta\Pi^*}{\partial\beta} > 0 \Leftrightarrow \dfrac{\partial}{\partial\beta}\left(\dfrac{\partial\Delta\Pi^*}{\partial\rho}\right) < 0 \Leftrightarrow \dfrac{a(e_b^*)}{a(e_p^*)} > \dfrac{(\alpha-\beta)(2-\alpha-\beta)}{(1-\beta)^2}$.

Proof of Proposition 6. Part 6a:

$$\Delta R^* > 0 \Leftrightarrow \frac{\partial\Delta\Pi^*}{\partial\rho} > 0 \Leftrightarrow \frac{\alpha^2}{a(e_c^*)} - \frac{(\alpha-\beta)^2}{(1-\beta)a(e_b^*)} - \frac{\beta}{a(e_p^*)} > 0$$

by inspection. From Equations (A4)–(A6),

$$a(e_b^*) = -\frac{a'(e_b^*)}{a(e_b^*)}\frac{M\rho\theta^2}{4}\frac{(\alpha-\beta)^2}{(1-\beta)}, \qquad a(e_p^*) = -\frac{a'(e_p^*)}{a(e_p^*)}\frac{M\theta^2\rho}{4}\beta,$$

and

$$a(e_c^*) = -\frac{a'(e_c^*)}{a(e_c^*)}\frac{M\theta^2\rho}{4}\alpha^2.$$

Substitution and simplification gives

$$\frac{M\theta^2}{4}\left(\frac{\alpha^2}{a(e_c^*)} - \frac{(\alpha-\beta)^2}{(1-\beta)a(e_b^*)} - \frac{\beta}{a(e_p^*)}\right) > 0 \Leftrightarrow \frac{a(e_c^*)}{a'(e_c^*)} < \frac{a(e_b^*)}{a'(e_b^*)} + \frac{a(e_p^*)}{a'(e_p^*)}.$$

Part 6b:

$$\frac{\partial\Delta\Pi^*}{\partial\alpha} = \rho\frac{\partial^2\Delta\Pi^*}{\partial\alpha\partial\rho} = \frac{M\theta^2\rho}{4}\left(\frac{2\alpha}{a(e_c^*)} - \frac{2(\alpha-\beta)}{(1-\beta)a(e_b^*)}\right).$$

We have $a(e_c^*) < a(e_b^*)$ from Proposition 5, such that

$$\frac{2\alpha}{a(e_c^*)} - \frac{2(\alpha-\beta)}{(1-\beta)a(e_b^*)} > 0, \quad \text{if } 2\alpha - \frac{2(\alpha-\beta)}{(1-\beta)} > 0 \Leftrightarrow \alpha < 1.$$

Part 6c:

$$\frac{\partial\Delta\Pi^*}{\partial\beta} = \rho\frac{\partial^2\Delta\Pi^*}{\partial\beta\partial\rho} < 0,$$

iff

$$\frac{M\theta^2\rho}{4}\left(\frac{2(\alpha-\beta)(1-\beta)-(\alpha-\beta)^2}{(1-\beta)^2a(e_b^*)} - \frac{1}{a(e_p^*)}\right) < 0 \Leftrightarrow \frac{a(e_b^*)}{a(e_p^*)} > \frac{2\alpha-2\beta+\beta^2-\alpha^2}{(1-\beta)^2} = \frac{(\alpha-\beta)(2-\alpha-\beta)}{(1-\beta)^2}. \quad \square$$

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