In this paper we study the optimal pricing strategies when a product is sold on two channels such as the Internet and a traditional channel. We assume a stylized deterministic demand model where the demand on a channel depends on prices, degree of substitution across channels and the overall market potential. We first study four prevalent pricing strategies which differ in the degree of autonomy for the Internet channel. For a monopoly, we provide theoretical bounds for these pricing strategies. We also analyze the duopoly case where an incumbent mixed retailer faces competition with a pure retailer and characterize price equilibria. Finally, through a computational study, we explore the behavior (price and profits) under different parameters and consumer preferences for the alternative channels.

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Keywords: Pricing; Channel; Demand; Bound

1. Introduction

The Internet has provided a new avenue to sell products to consumers. While it provides an opportunity to attract more customers and improve sales by attracting customers who would have otherwise not bought the product, it could also threaten existing channel relationships through possible cannibalization. In this paper, we study the pricing strategies for a retailer which sells a product through two channels. For example, a retailer such as Best Buy that was primarily selling products through the traditional channel, decides to sell products through the Internet channel as well. In such a setting, it is important for the firm to provide the right control and pricing across the two channels. The introduction of an Internet channel poses two important questions – the degree of independence of the Internet channel and the pricing of goods across the two channels. Retailers that have faced this issue in the last few years have taken different courses of action. For example, Best Buy, an electronics retailer, was quite slow to adopt the Internet channel. However, on adoption Best Buy decided to treat the customers of the online and physical stores in an identical manner [34]. On the other hand, Barnes and Noble, a book retailer, decided to create an independent unit for Internet operations that essentially competed with the traditional retail store.

Our motivation for this research came from multiple different sources. (1) A survey by Ernst and Young reported that nearly two-thirds of companies price products identically for their online and offline operations [16]. This enabled the firm to capture a share of the Internet friendly customers. The survey also indicated that the majority of customers expect to find lower prices online. This is further validated by the empirical findings of researchers. Brynjolfsson and Smith [7] indicate that prices of books and CD’s were 9–16% lower on the Internet than in traditional stores. Further, the prices charged by pure e-tailers and e-tailers with traditional channels may be different. Tang and Xing [32] find that the price charged by...
pure e-tailers for DVD titles is 14% lower than those charged by e-tailers with traditional channels. Ancarani and Shankar [1] develop hypotheses on how prices and price dispersion compare among pure-play Internet, bricks-and-mortar, and bricks-and-clicks retailers and test them through an empirical analysis. Their results, based on an analysis of 13,270 price quotes, show that when posted prices are considered, traditional retailers have the highest prices, followed by multichannel retailers, and pure-play e-tailers. (2) In a recent research, Cattani et al. [9] study four prevalent pricing strategies for a monopolist under varying degree of autonomy for the Internet channel and find through a detailed computational study that an identical pricing strategy may indeed be very close to the optimal. (3) We did our own survey of the pricing of a small subset of popular products. As shown in Table 1, the actual pricing strategy for these items shows a wide dispersion. For example, some products are priced identical by competitors (like Introduction to Probability Model and A Course in Game Theory) whereas for some others the price on the Internet channel offered by a firm that has both a traditional and Internet channel are higher than those offered by a pure Internet player (like World is Flat or Devinci Code). For products sold by two pure competing players, we find that prices could either be higher or lower (like Corning Set and Hamilton Beach Tea Maker). The above pieces of evidence motivated us to explore the possibility of theoretical bounds and optimal pricing in monopoly and duopoly environments with retailers having pure and mixed channels.

As opposed to the micro-level demand model considered in Cattani et al. [9], in this paper we use a stylized linear demand function that depends on price of the product in the channels as well as on the degree of substitutability across the two channels. First, we analyze a monopolistic retailer who opens a new Internet channel and consider four pricing strategies with different degrees of autonomy for the Internet channel. We theoretically show that under mild conditions web price to optimize the joint profit (while holding the traditional price) is at most 4% from the optimal. If the costs across the two channels are identical then this bound is at most 3% away from optimal and, the profit under the identical pricing is at most 4% away from optimal. Next, we consider the case where the Internet channel is introduced by a different firm, and show the existence of a unique Nash equilibrium for the cases where the incumbent has a pure traditional channel or a combination of Internet and traditional channel. We develop conditions under which internet price of the firm with both channels is greater or less than the price of the pure e-tailer. Through a limited computational study, we explore the behavior (price and profits) of the above models under different parameters and consumer preference for the alternative channels. Finally, we validate our results for a variant of the demand model that has also been well studied in literature. In the process we provide managerial insights on the following questions (1) how sub-optimal are some of the prevalent pricing policies? (2) What happens to optimal prices and profits when a new channel is introduced under monopoly and competition? (3) When do mixed channel firms price goods above the pure firm?

The rest of paper is organized as follows. In Section 2, we discuss relevant literature. In Section 3, we study the monopolistic setting under four different pricing strategies. In Section 4, we study duopoly with a new entrant. We study a variant of the demand model in Section 5 and conclude in Section 6.

### 2. Related literature and our contribution

In the last few years, researchers have begun to study issues related to electronic supply chains [31]. Cattani et al. [8] provide a comprehensive review of models which deal with coordination of traditional and Internet channels under monopoly. Tsay and Agarwal [32] provide a complete review of the same issues under competition in the multi-channels.

---

Table 1

<table>
<thead>
<tr>
<th>Category</th>
<th>Product</th>
<th>Amazon.com ($)</th>
<th>BarnesNoble.com ($)</th>
<th>BN local store ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOOK</td>
<td>The World is Flat</td>
<td>16</td>
<td>16.5</td>
<td>19.25</td>
</tr>
<tr>
<td></td>
<td>Devinci Code</td>
<td>14.5</td>
<td>14.97</td>
<td>17.47</td>
</tr>
<tr>
<td></td>
<td>Introduction to Probability Model</td>
<td>89.95</td>
<td>89.95</td>
<td>89.95</td>
</tr>
<tr>
<td></td>
<td>A Course in Game Theory</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>A Million Little Pieces</td>
<td>8.97</td>
<td>10.46</td>
<td>11.96</td>
</tr>
<tr>
<td>DVD</td>
<td>Why Harry Met Sally</td>
<td>11.21</td>
<td>11.45</td>
<td>14.99</td>
</tr>
<tr>
<td></td>
<td>The Matrix</td>
<td>14.97</td>
<td>17.98</td>
<td>19.99</td>
</tr>
<tr>
<td></td>
<td>The Sound of Music</td>
<td>18.89</td>
<td>21.58</td>
<td>26.99</td>
</tr>
<tr>
<td>HOME</td>
<td>Full Ultimate Memory Foam Mattress Topper</td>
<td>129.95</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td></td>
<td>King Ultimate Memory Foam Mattress Topper</td>
<td>189.95</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corning French White 12-pc Bake and Serve Set</td>
<td>39.99</td>
<td>49.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hamilton Beach Iced Tea Maker</td>
<td>24.99</td>
<td>19.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shaper Image Air Purifier Ionic Breeze 3.0</td>
<td>249.99</td>
<td>249.99</td>
<td></td>
</tr>
</tbody>
</table>
There is a volume of literature in marketing on channel management. Most of these models consider vertical competition and channel coordination. Choi [11] focuses on the intra- and inter-channel price competition and its implication on the channel structure and profits. Ingene and Parry [20] focus on the case of a single manufacturer selling an identical product to two competing retailers. They examine the implications of two simple wholesale pricing strategies: a linear quantity discount schedule and a two-part tariff, and show the circumstances where a manufacturer prefer different pricing strategies. Coughlan [12], Moorth [26], Coughlan and Wernerfelt [13], and Gupta and Loulou [19] study the possibilities where the manufacturers are better off using the intermediaries instead of vertically integrating. More recently, Cattani et al. [10] study price coordination issues when a manufacturer opens a direct internet channel. As opposed to the above models, our focus is on horizontal competition.

Horizontal competition has been studied in inventory theory by Parlar [28], Lippman and McCardle [23], and Bernstein and Federgruen [4]. Parlar [28], and Lippman and McCardle [24], study a pair of “newsvendor” firms who compete because their products are partially substitutable, while Bernstein and Federgruen [4] examine an oligopoly in which sales are awarded based on the competitors’ relative selling prices and fill rates. As opposed to the above papers that focus on a single channel our focus is on studying such competition in dual channels.

Some of the recent related work in the area of managing dual channels include Reinhardt and Levesque [29], Bernstein et al. [5], Lal and Sarvary [22], Zettelmeier [35], Balasubramanian [3], Druhl and Porteus [15], Cattani et al. [9] and Pan et al. [22].

The first four papers are focussed on the interaction effects across the two channels, so the primarily concern in those papers is how the introduction of a new channel (such as an internet channel) may provide an opportunity for customers to search for the product on either of the two channels and that eventually could lead to a purchase on the other channel. Our model does not take such interactions into account in that, we do not capture the cases where a customer could use the information about the product on the Internet channel to refine their decisions of purchase on the traditional channel or vice versa. We (and the last four papers) assume that the presence of another channel only acts as a substitute avenue for purchase.

Balasubramanian [3] models competition between direct marketers and conventional retailers. His model considers inelastic demand with customers uniformly distributed on a circle, retailers evenly spaced around the circle, and the direct marketer in the center. All retailers have fixed costs and customers of the direct channel incur a monetary disutility that may be of any value. Among other issues, his focus is on the role of information in multiple-channel markets and he shows that even with zero information costs, providing information to all consumers may not be optimal, especially when the product is not well adapted to the direct channel. In that case, high market coverage (from having a greater number of retailers) may depress profits. Druhl and Porteus [15] consider price competition between a traditional and an internet retailer. They adopt a utility based demand model and assume that only some proportion of the market has access to the Internet and is willing to buy online. They find the Nash and Stackelberg equilibrium prices and show that a pure Nash equilibrium may not exist in some cases. Cattani et al. [9] study coordination of pricing on Internet and traditional channels by modeling micro-level consumer behavior for demand generation. In their model, customers are at a random physical distance from traditional retailers, and at a random virtual distance from the direct marketer, independent of the physical distance. The market then is segmented according to the utility each customer attains from either the direct channel or the traditional channel. Customers are not excluded from a specific market; thus both markets have a chance to compete for all customers. Pan et al. [22] consider price competition between a traditional retailer and an e-tailer, and a dual channel retailer and Internet-only firm, based on the Hotelling model. They show that the price at a pure e-tailer is lower than that of a dual channel firm. However, the main assumption of the above paper is that they assume both the stores have the same marginal cost and, and the demand function is only differentiated by the transaction costs and prices. Further, when they compare the Nash equilibrium prices for the pure e-tailer and the brick-and-clicks retailer, they assume that the bricks-and-clicks retailer sets the same price at its two stores.

In contrast to the four papers above, one of the main aims in this paper is to provide theoretical insights on the near optimality of some of the prevalent pricing strategies (non-optimal). In particular, we use the demand model introduced by McGuire and Staelin [25] where the demand depends linearly on the prices on the different channels and the degree of substitutability across them. For the case when there are three possible channels for purchase (a mixed channel competing with a pure Internet channel), we utilize an extension similar to one developed by Granot and Sosic [18]. Further, we do not restrict ourselves to identical unit costs and prices, and characterize the optimal price and profits over a range of parameters. The main contributions of our work include – theoretical bounds on performance of sub-optimal yet prevalent pricing strategies; analysis of the implication of new alternative channel on prices and profits; and the effect and benefits of having a mixed channel under competition. It is to be noted that in the rest of the paper we will assume that the incumbent pure player is a traditional retailer and the new channel is an Internet channel but since the model is symmetric all the results should hold if other alternatives to an Internet channel are considered.


In this section we first introduce the demand model and then consider four different pricing strategies and finally provide bounds on performance.
3.1. Demand model

We first assume there is one firm that sells products only through the traditional channel. Let $q_T$ denote the demand of this firm when its price is at $p_T$, and let $c_T$ denote its cost, and $g_T$ denote its profit. We assume a linear downward sloping demand function. The demand and profit are given as below.

$$q_T = S(1 - \beta p_T); \quad g_T = q_T(p_T - c_T),$$

(1)

where $\beta$ and $S$ are positive real values. The constant $S$ is a scale factor equal to industry demand or market potential and $\beta$ is the price elasticity factor. In such a setting, the firm decides the optimal price $p_T^*$ by maximizing its profit. The optimal price $p_T^*$ and the optimal profit $g_T^*$ are given by

$$p_T^* = \frac{1 + c_T \beta}{2\beta}; \quad g_T^* = \frac{S(1 - c_T \beta)^2}{4\beta}.$$  

(2)

Now consider the case where the firm begins to sell products through another channel (like the Internet). For example, a firm such as Wal-Mart decides to open walmart.com. We utilize the demand function given by McGuire and Staelin [25] for two channels. Let $q_T$ denote the demand for the traditional channel, and $q_W$ denote the demand for the Internet channel. Then

$$q_T = uS\left(1 - \frac{\beta}{1 - \theta}p_T + \frac{\beta \theta}{1 - \theta}p_W\right); \quad q_W = (1 - u)S\left(1 + \frac{\beta \theta}{1 - \theta}p_T - \frac{\beta}{1 - \theta}p_W\right).$$

(3)

where $0 \leq u \leq 1$, $0 \leq \theta < 1$, and $\beta$ and $S$ are positive. All the proofs for this paper are in Appendix.

The parameters $u$ and $\theta$ capture two different aspects: the absolute difference in demand and the substitutability of the end products as reflected by the cross-elasticities. The parameter $u$ captures the market share. For example in the case of traditional and internet channel, if the population density is very high near a retail outlet, then $u$ (the market share for the traditional channel) is likely to be higher all other factors being constant. On the other hand, in a remote place with lower population density the market share $u$ of the traditional channel is expected to be lower. Similarly, if the population is dominated by families with high Internet literacy and dual career with high premium for time, then $1 - u$ (the market share for the Internet channel) can be expected to be higher. However, it is quite possible that a high population density such as a city may also have a higher internet literacy in which case their relative strengths will determine which channel will have more market share. The parameter $\theta$, in contrast, affects the substitutability of the product across the two channels in terms of changes in prices. More specifically, $\theta$ is the ratio of the rate of change of quantity with respect to the price on the other channel to the rate of change of quantity with respect to price on the own channel. In our setting, $\theta$ captures characteristics related to the type of product that is sold. For example, consumers prefer to buy clothes at retail stores, but are more amenable to buy computer or electronics product on the Internet. Therefore, the substitutability factor $\theta$ can be expected to be lower for clothes industry than for electronics. Purchase decisions are also affected by factors such as immediate availability and savings in traveling and purchase time. A combination of the $\theta$ and $u$ can capture these effects. As indicated earlier, this model does not capture the cross interactions in purchase decisions like an individual first searching the Internet and then making the purchase on the traditional channel. Despite the above shortcomings we believe this model captures several interesting dynamics related to managing dual channels and lends itself to theoretical analysis and characterization such as worst case bounds on prevalent (yet non-optimal) pricing policies.

In order to utilize the above model, it is necessary to impose additional inequality constraints on these parameters in order to guarantee that (1) prices exceed marginal costs, (2) quantities are nonnegative, and (3) industry demand does not increase with increase in prices for either product. McGuire and Staelin [25] derive the conditions given below to accomplish the same.

**Constraint 1:** quantities nonnegative: $c_T \beta \leq 1$ and $c_W \beta \leq 1$

**Constraint 2:** negative coefficients of prices in the quantities functions

$$\left\{ \theta \leq \frac{1 - u}{u} \quad \text{and} \quad \theta \leq \frac{u}{1 - u} \right\} \iff \left\{ \frac{\theta}{1 + \theta} \leq u \leq \frac{1}{1 + \theta} \right\}. \tag{4}$$

3.2. Alternative pricing strategies

Ernst and Young [12] reported that the need for organizational discipline might prompt retailers to consider setting up a separate online entity to manage itself. In particular they found that in the United States, 27% retailers fully integrated their traditional and Internet operations, 53% of Internet operations are semi-autonomous, and 20% are totally autonomous. They conclude that there are no clear best practices in place. Further, coordinating prices on traditional and Internet chan-
nals has been a challenging task for many firms. The natural inclination for many firms is to price products on the web at the same price as their traditional retailers [2, 24]. However, evidence exists of companies charging both higher and lower prices on their Internet channel [24, 30].

We consider four prevalent pricing strategies when a retailer adds an Internet channel to an existing traditional channel based on Cattani et al. [9]. These strategies differ in the degree of autonomy given to the Internet channel. In the first strategy, the retailer optimizes profit on the Internet channel by choosing an independent price for that channel without adjusting the price on the traditional channel. This strategy is suitable for retailers who have a very large traditional retail business and the new internet channel is very small in comparison. The logic behind such a strategy is that the firm may not want to make major changes on its traditional channel due to the presence of the Internet channel. In the second strategy, the retailer finds the price which optimizes the combined profit on traditional and Internet channels without adjusting the price on the traditional channel. In this case, the autonomy to the Internet channel is lower and they are somewhat forced to price the product for the benefit of the whole firm. In the third strategy, the retailer chooses an identical price across the two channels which optimizes the combined profits. Such a strategy has been followed by retailers such as Best Buy who want to provide a consistent seamless shopping experience for the consumer. Clearly, the autonomy while choosing prices on the Internet channel is further limited in the case. In the fourth strategy, the retailer chooses prices across the two channels which optimize the combined profits. Strategy one and strategy two minimize disruptions to the traditional channel, however they may be sub-optimal from the profit standpoint. Strategy three (a popular practice in industry) prices the product identically across the two channels, thereby minimizing cannibalization of demand due to differential pricing and maintaining the brand. Strategy four is the theoretical optimal solution however it affects pricing on both channels.

### 3.2.1. Pricing strategy I

The firm chooses the price on the traditional channel which maximizes its own profit in (1), and decides the optimal price for the Internet channel only by maximizing the profit from the Internet channel. The policy of profit maximization over the web channel without consideration of the traditional channel could arise if the web channel were managed separately for the Internet channel only by maximizing the profit from the Internet channel. The policy of profit maximization over the whole firm results in strategy 1.

The profit of the whole firm \( W \), and the optimal price for the Internet channel and the corresponding profit are given by:

\[
g_W = (1 - u)S \left( 1 + \frac{(1 + ct\beta)\theta}{2(1 - \theta)} - \frac{\beta}{(1 - \theta)} p_W \right) (p_W - c_W),
\]

\[
p_W^* = \frac{2(1 + c_w\beta) - \theta(1 - c_t\beta)}{4\beta}, \quad g_W^* = \frac{S(1 - u)(2(1 - c_w\beta) - \theta(1 - c_t\beta))^2}{16\beta(1 - \theta)}.
\]

Given the Internet channel optimal price, the profit for the retailer earned on the traditional channel is

\[
g_T^* = \frac{S u \left( (1 - c_t\beta)(2\theta(1 - c_w\beta) - (2 - \theta^2)(1 - c_t\beta)) \right)}{8\beta(1 - \theta)}.
\]

### 3.2.2. Pricing strategy II

The firm chooses the price on the traditional channel which maximizes its own profit in (1), and decides the optimal price for the Internet channel only by maximizing the profit from the Internet channel. The policy of profit maximization over the web channel without consideration of the traditional channel could arise if the web channel were managed separately from the traditional operations, as recommended by some experts (e.g., [6]). In this case, the objective profit function for the Internet channel is given by

\[
g_W = (1 - u)S \left( 1 + \frac{(1 + ct\beta)\theta}{2(1 - \theta)} - \frac{\beta}{(1 - \theta)} p_W \right) (p_W - c_W),
\]

\[
p_W = \frac{2(1 + c_w\beta)(1 - u) - \theta(1 - c_t\beta)(1 - 2u)}{4(1 - u)\beta}, \quad g_W^* = \frac{S(2(1 - c_w\beta)(1 - u) - \theta(1 - c_t\beta))(2(1 - c_w\beta)(1 - u) + \theta(1 - c_t\beta)(2u - 1))}{16(1 - u)\beta(1 - \theta)},
\]

\[
g_T^* = \frac{S u \left( (1 - c_t\beta)^2(1 - u) - 2\theta(1 - c_w\beta)(1 - u) - \theta^2(1 - c_t\beta)(1 - 2u) \right)}{8(1 - u)\beta(1 - \theta)}.
\]

**Lemma 1.** The profit of the whole firm \( g_W + g_T^* \) is concave with respect to price for the Internet channel \( (p_W) \).

The optimal price and profit for the firm on the traditional and Internet channel are computed as

\[
p_W^* = \frac{2(1 + c_w\beta)(1 - u) - \theta(1 - c_t\beta)(1 - 2u)}{4(1 - u)\beta},
\]

\[
g_W^* = \frac{S(2(1 - c_w\beta)(1 - u) - \theta(1 - c_t\beta))(2(1 - c_w\beta)(1 - u) + \theta(1 - c_t\beta)(2u - 1))}{16(1 - u)\beta(1 - \theta)},
\]

\[
g_T^* = \frac{S u \left( (1 - c_t\beta)^2(1 - u) - 2\theta(1 - c_w\beta)(1 - u) - \theta^2(1 - c_t\beta)(1 - 2u) \right)}{8(1 - u)\beta(1 - \theta)}.
\]
3.2.3. Pricing strategy III

In this case the retailer charges the same price in both channels. Although setting $p_T = p_W$ adds a constraint to the optimization problem (thereby potentially reducing profits), it has the benefit of avoiding channel conflict and perceived consumer inequality. In this case, the firm utilizes the same price ($p_T = p_W = p$) for the traditional and Internet channel which optimizes its total profit given as follows:

$$g_T + g_W = S(1 - p\beta)(p + c_W(-1 + u) - c_T u).$$  \hspace{1cm} (10)

**Lemma 2.** The total profit of the firm is concave with respect to $p_T = p_W = p$.

The identical optimal price for the tradition and Internet channels is

$$p^*_T = p^*_W = \frac{1 + c_W\beta + c_T u\beta - c_W u\beta}{2\beta}. \hspace{1cm} (11)$$

The corresponding profits of the two channels are given by

$$g^*_T = \frac{S(u + c_T(-2 + u)\beta - c_W(-1 + u)\beta)(1 + c_W(-1 + u)\beta - c_T u\beta)}{4\beta},$$

$$g^*_W = \frac{S(-1 + u)(1 + c_W(-1 + u)\beta - c_T u\beta)(-1 - c_T u\beta + c_W(1 + u)\beta)}{4\beta}.$$  

3.2.4. Pricing strategy IV

In this case, the firm maximizes the total profits without constraining the prices. The total profit for the firm is given as follows:

$$g_T + g_W = \frac{S}{1 + \theta} ((\beta p_W(1 - u) - (c_T - p_T)u(-1 + p_T\beta + \theta) + c_W(-1 + u)(-1 + p_W\beta + \theta - p_T\beta\theta) + p_W(-1 + u + \theta - u\theta - p_T\beta\theta + c_T u\theta)).$$  \hspace{1cm} (12)

**Lemma 3.** When $4u - 4u^2 - \theta^2 > 0$, the total profit of the firm is jointly concave with respect to $p_T$ and $p_W$.

The optimal prices for the traditional and Internet channels are given by

$$p^*_T = \frac{2u(1 + c_T\beta)(u - 1) - (1 - c_W\beta)\theta(2u - 1)(u - 1) + \theta^2(1 - u + uc_T\beta)}{\beta(-4u + 4u^2 + \theta^2)},$$

$$p^*_W = \frac{2u(1 + c_W\beta)(u - 1) + u\theta(1 - c_T\beta)(1 - 2u) + \theta^2(c_W\beta + u - uc_W\beta)}{\beta(-4u + 4u^2 + \theta^2)}.$$  \hspace{1cm} (13)

3.3. Prices and profits

**Proposition 1.** The Internet channel optimal price is less than the traditional channel optimal price ($p^*_W < p^*_T$) under the different pricing strategies as follows:

(i) Strategy I: $p^*_W < p^*_T$ when $c_T \geq c_W$ or $c_T < c_W$ and $c_W < \frac{\beta(2 - \theta)c_T\theta + \theta}{2\theta}$;

(ii) Strategy II: $p^*_W < p^*_T$ when $c_T \geq c_W$ and $u < 1/2$;

(iii) Strategy III: $p^*_W(III) < p^*_W(I, II)$ when $c_T \geq c_W$;

(iv) Strategy IV: $p^*_W < p^*_T$ when $c_T \geq c_W$ and $u < 1/2$.

3.3.1. Discussion

From the above proposition it is to be noted that the Internet channel price $p^*_W$ is less than the traditional price $p^*_T$ in most cases when $c_W < c_T$. For strategy I, $p^*_W < p^*_T$ even when $c_W > c_T$ as long as it not too large. Note that $\frac{\beta(2 - \theta)c_T\theta + \theta}{2\theta}$ is increasing in $\theta$ due to constraint 1 ($c_T\beta < 1$). Therefore, as $\theta$ increases, i.e., when consumers feel products sold on the Internet channel are more substitutable, $p^*_W$ is more likely to be less than $p^*_T$, even when $c_W \geq c_T$. For strategy II, the above result shows that it is worthwhile to price lower on Internet channel only if it has larger market share and sufficiently lower production cost in order to maximize the total profits for the firm. The above result for strategy III shows that if the cost of selling products on the traditional channel is higher than the cost of selling the product on the Internet channel, then the optimal equal price is still lower than the original traditional channel price. Thus, even in a monopolistic situation, consumers may benefit from the introduction of a new channel if the Internet channel is less expensive to operate than the traditional channel. In this case, the presence of an Internet channel helps in subsidizing the price on the traditional channel.
channel. For strategy IV it is worthwhile to price lower on the Internet channel if it has larger market share and sufficiently lower production cost in order to maximize the total profits for the firm. Next we provide theoretical bounds for the profit for strategies II and III as compared to the optimal solution under mild assumptions.

**Theorem 1.** The ratio of the total profit of the firm under pricing strategy II to that under pricing strategy IV is at least 96%.

**Theorem 2.** When the costs in the two channels are identical \((c_W = c_T)\), the ratio of the total profit of two channels under pricing strategy II to that under strategy IV is at least 97%.

**Theorem 3.** Under the assumption \(c_T = c_W = c\), the ratio of the total profit of two firms in pricing strategy III to that in pricing strategy IV is at least 96%.

### 3.3.2. Discussion

Theorems 1–3 provide some interesting insights that may be influencing prevalent pricing practices. As indicated earlier, many firms when they introduced their Internet channel, tended to give semi-autonomy to the Internet channel in that the Internet channel price was set assuming minimal changes to the traditional channel. Our results indicate that if a firm kept the same price on the traditional channel and optimized the firm profit with the Internet channel price (strategy II) or matched prices on traditional and Internet channels (strategy III) then in the worst case the profits of the firms are no more than 4% away from the maximum profits. This has an important implication for managers because changing prices on two channels is onerous and has large implications and influences on customers. Our results show that semi-autonomy to the Internet channel (via strategy II or III) maybe a very pragmatic non-optimal strategy for managers in practice. This may be one of the reasons as why to 53% of retailers (operating under monopoly and competition) in practice have adopted a semi-autonomous structure as indicated in the Ernst and Young report.

### 4. Two firms: Introduction of a new channel

In this section, we study the duopoly setting where the Internet channel is introduced by a different firm. We first consider the case where the incumbent has a pure traditional channel and then the case where the incumbent has both a traditional and Internet channel.

#### 4.1. Incumbent with a traditional channel

In the case where there is an incumbent with a traditional channel and an entrant with an Internet channel, this problem mathematically is identical to the problem studied by McGuire and Staelin [25] who show the existence of a Nash equilibrium in a different context.

**Lemma 4.** There exists a Nash equilibrium where optimal prices are given by

\[
p_{NT} = \frac{-2 - 2c_T\theta + \theta - c_W\theta^2 + \theta^2}{\beta(-4 + \theta^2)},
\]

\[
p_{NW} = \frac{-2 - 2c_W\theta + \theta - c_T\theta^2 + \theta^2}{\beta(-4 + \theta^2)}.
\]

In addition, we can also show the following result regarding concavity.

**Proposition 2.** The optimal prices of the two firms in the Nash equilibrium are concave decreasing functions with respect to \(\theta\), the substitutability factor between the two firms’ products.

The equilibrium prices are decreasing concavely in \(\theta\) because the channels become more substitutable as \(\theta\) increases. Therefore, the competition intensifies and prices get pushed down at a faster rate.

We now provide analytical insights on the effect of the new entrant on incumbent profits. In particular, we consider a case where the new entrant brings an additional set of customers to the market. Let \(S\) represent the whole market when both firms are present and \(S_1\) represent the market when only the first firm is present. Further let the market share of the incumbent be \(u\) after the new e-tailer being introduced into market. Then we assume that \(S_1 = Su\), i.e., the introduction of a new player does not affect retailers share.
Theorem 4.

(i) The ratio of the incumbent firm’s profit before and after the presence of new e-tailer is decreasing with \( c_T \) and increasing with \( c_W \).

(ii) The incumbent firm’s profit before the introduction of new e-tailer is greater than that after introduction when \( c_T < c_W \) and

\[
1 - c_W \beta < (1 - c_T \beta) \left( \frac{4 - 2\theta^2 - (4 - \theta^2)\sqrt{1 - \theta}}{2} \right).
\]

Counter to normal intuition, the above proposition shows that the competition from the new e-tailer can in fact be profitable for the incumbent traditional-only firm when the Internet channel costs are substantially higher than the traditional channel costs since the expression \( \left( \frac{4 - 2\theta^2 - (4 - \theta^2)\sqrt{1 - \theta}}{2} \right) \leq 1 \). On a closer look, we find that indeed the new Internet channel helps the incumbent because it brings along with it a new set of customers (since \( S_1 = S_0 \)) thereby increasing the potential market. However, once those customers are in the market they switch to the traditional channel because of channel choice and lower prices on the traditional channel (resulting from substantially lower cost). If the customers are assumed to be completely loyal to the Internet channel then one would not observe this result.

4.2. Incumbent with a mixed channel

In this section, we study the case where the incumbent firm has both traditional and Internet channels and faces competition from a new Internet firm. The demand functions obtained are natural extensions of the demand functions used earlier. The incumbent firm sells products through two channels, the traditional and Internet channels. The new entrant sells products only through its own Internet channel. We consider the three channels among the two firms as three distinctive players. Assume that there is a central decision maker for the two channels in the incumbent firm which aims to maximize its total profit. There is a separate decision maker for the new entrant firm which aims to maximize its own profit. Let 1, 2 denote the Internet and traditional channel of the incumbent firm, respectively. Let 3 denote the Internet channel of the new entrant. In such a setting, the deterministic and linear demand functions, introduced by Granot and Sosic [18], are given by

\[
q_i = u_i S \left[ 1 - \beta \frac{1}{(1 - \theta_i)(1 - \theta_k)} p_i + \beta \frac{\theta_0 (2 - \theta_i)}{2(1 - \theta_i)(1 - \theta_k)} p_j + \beta \frac{\theta_0 (2 - \theta_j)}{2(1 - \theta_i)(1 - \theta_k)} p_k \right]
\]

with \( i, j, k \in \{1, 2, 3\} \) such that \( i \neq j \neq k \). \( 0 \leq u_i, u_2, u_3 \leq 1 \), \( u_1 + u_2 + u_3 = 1 \), \( \beta > 0 \), \( S > 0 \), and \( 0 \leq \theta_i < 1 \). Here let \( S \) represent a scale factor corresponding to the industry demand when the prices of all products are set to zero, while the \( u_i \)'s and \( \theta_i \)'s represent two aspects of product differentiation – the absolute difference in demands and the substitutability of two products, respectively. When \( \theta_i = 0 \), product \( i \) and \( j \) have independent demands; product substitutability increases with \( \theta_i \), and they become highly substitutable as \( \theta_i \to 1 \).

It is necessary to impose additional inequality constraints on these parameters in order to guarantee (1) prices exceed marginal costs; (ii) quantities are nonnegative; (iii) industry demand must not increase with increases in prices.

Constraint 1: quantities nonnegative:

\[
2(1 - \theta_j)(1 - \theta_k) - 2\beta p_i + \beta \theta_0 (2 - \theta_i) p_j + \beta \theta_0 (2 - \theta_j) p_k \geq 0
\]

when \( i, j, k \in \{1, 2, 3\}, i \neq j \neq k \).

Constraint 2: negative coefficients of prices in the quantities functions

\[
\begin{align*}
&u_1 (2 - (-1 + \theta_{12})\theta_{13}(-2 + \theta_{23}) - 2\theta_{23}) + (u_2 (2 + \theta_{12} - \theta_3) + \theta_{13} - \theta_{12}\theta_3)(-2 + \theta_{23}) > 0, \\
&(-2 + \theta_{13})(u_1 (2 - \theta_{23}) + \theta_{23} - \theta_{12}\theta_3) \\
&+ u_2 (2 + 2\theta_{13} - 2\theta_{12}\theta_3 + \theta_{13}(-2 + (-1 + \theta_{12})\theta_{23})) > 0, \\
&2 - 2\theta_{13} + u_3 (-2 + 2\theta_{13} - \theta_{23} + \theta_{12}(2 + \theta_{13} - \theta_{13}\theta_{23}) \\
&+ u_2 (-2 + 2(-1 + \theta_{13})\theta_{23} + \theta_{12}(2 + \theta_{23} - \theta_{13}\theta_{23})) > 0.
\end{align*}
\]

We must guarantee that the whole profit function of the incumbent firm is jointly concave with respect to \( p_1 \) and \( p_2 \) in order to analyze the Nash equilibrium.
Constraint 3: condition to preserve concavity

\[-\theta_{12}^2(u_2(-1 + \theta_{13})(-2 + \theta_{23}) + u_1(-2 + \theta_{13})(-1 + \theta_{23}))^2 + 16u_1u_2(-1 + \theta_{13})(-1 + \theta_{23}) > 0.\]  

(19)

In the following passages, we will provide theoretical insights on how changes in costs and substitutability factors affect optimal pricing and profits. We will consider symmetric cases where most parameters are identical and asymmetric cases where firms may have differential costs, substitutability and markets shares across channels.

4.2.1. Effect of cost and substitutability on prices

Under the above conditions (constraints 1–3), we consider the game beginning with both the incumbent and entrant firms simultaneously choosing prices, where \(p_{N1}, p_{N2}\) are the prices of the two channels of the incumbent firm, and \(p_{N3}\) is the price of the Internet-only entrant firm. We can show that there exists a unique Nash equilibrium in this game under the above assumptions. In this section, we first characterize the equilibrium prices for some special settings where we can prove a definite ordering of prices based on cost and substitutability. Then we use numerical results to demonstrate their effect under general settings.

Identical substitutability factors: Let \(\theta_{12} = \theta_{13} = \theta_{23} = \theta\) in this case.

**Proposition 3.** If the costs of the products from the two firms are identical (\(c_1 = c_2 = c_3 = c\)) and the substitutability factor across the three players are identical (\(\theta_{12} = \theta_{23} = \theta_{13} = 0\)), the Nash equilibrium price for the Internet channel for the incumbent firm is always greater than the price for the Internet-only new entrant firm (\(p_{N3} < p_{N1}\)).

**Proposition 4.** If we assume that \(c_1 = c_2 = c, \theta_{12} = \theta_{13} = \theta_{23} = \theta, u_1 = u_2 = u\), there exists a \(u = \frac{\beta(2 - 2\theta + \beta^2)(2\theta - 1 + \theta)^2}{\beta(4 - 6\theta + 3\theta^2)}\) such that if \(c_3 > c_2\), then the Nash equilibrium price of the Internet-only entrant firm is greater than that of the incumbent firm \((p_{N3} > p_{N1})\).

Under identical substitutability across the three channels, it appears that unless the cost for the entrant is significantly high (greater than \(c_3\)), the incumbent is likely to have higher prices on the Internet channel. This may provide some theoretical validation as to why Tang and Xing [32] empirically found that the price charged by pure e-tailers for DVD titles is 14% lower than those charged by e-tailers with traditional channels. Although it is important to note that not all retailers in their sample were operating in a duopoly market. In cases where the costs across the three channel are non-identical then the characterization is more complex. The difference between the prices \(p_{N1} - p_{N3}\) is given as follows.

\[p_{N1} - p_{N3} = -\frac{A}{4\beta((-8 + 8\theta - 4\theta^3 + \theta^4)},\]

where

\[A = (-4c_3\beta(4 - 6\theta + 3\theta^2) + c_1\beta(16 - 24\theta + 16\theta^2 - 4\theta^3 + \theta^4) + (-2 + \theta)\theta(-4(-1 + \theta)^2 + c_2\beta(4 - 6\theta + 3\theta^2)))\]

and \((-8 + 8\theta - 4\theta^3 + \theta^4) < 0\) since \(0 < \theta < 1\).

<table>
<thead>
<tr>
<th>(c_1, c_2, c_3)</th>
<th>A</th>
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<tr>
<td>1</td>
<td>&gt; 0</td>
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<td>2</td>
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<tr>
<td>3</td>
<td>&gt; 0</td>
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<td>4</td>
<td>(c_1 &lt; c_2 &lt; c_3) and (c_2 &lt; \frac{16c_1\beta - 8c_1 - 4c_1\beta + 6c_2\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_3 &lt; \frac{16c_1\beta - 8c_1 - 4c_1\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (\frac{16c_1\beta - 8c_1 - 4c_1\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (\frac{16c_1\beta - 8c_1 - 4c_1\beta + 6c_2\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (\frac{16c_1\beta - 8c_1 - 4c_1\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (\frac{16c_1\beta - 8c_1 - 4c_1\beta + 6c_2\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (\frac{16c_1\beta - 8c_1 - 4c_1\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)})</td>
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<td>5</td>
<td>(c_1 &gt; c_2) and (c_2 &lt; \frac{-8(2c_1\beta - 6c_1\beta + 5\theta)(c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_3 &lt; \frac{-8(2c_1\beta - 6c_1\beta + 5\theta)(c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_3 &lt; \frac{-8(2c_1\beta - 6c_1\beta + 5\theta)(c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)})</td>
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<td>6</td>
<td>(c_1 &gt; c_3) and (c_3 &lt; \frac{16c_1\beta - 8c_1 - 4c_1\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_2 &lt; \frac{16c_1\beta - 8c_1 - 4c_1\beta + 6c_2\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_2 &lt; \frac{16c_1\beta - 8c_1 - 4c_1\beta - 4\theta(4c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)})</td>
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<td>7</td>
<td>(c_1 &lt; c_3 &lt; c_2) and (c_2 &lt; \frac{-8(2c_1\beta - 6c_1\beta + 5\theta)(c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_3 &lt; \frac{-8(2c_1\beta - 6c_1\beta + 5\theta)(c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)}) (c_3 &lt; \frac{-8(2c_1\beta - 6c_1\beta + 5\theta)(c_1\beta - 4\theta)}{\beta(-16 + 16\theta + 16\theta^2 - 16\theta^3 + 3\theta^4)})</td>
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If \( A > 0 \) then \( p_{N1} > p_{N3} \). Clearly, when \( c_1 > c_3 \), in cases 1, 2 and 3 above where the incumbent’s internet costs are greater than the Internet-only firms costs, it is beneficial for the incumbent to price higher on the Internet channel. In some cases 5 and 6, even when \( c_1 < c_3 \) and \( c_1 > c_2 \) but not by a huge margin, \( p_{N1} > p_{N3} \). Finally, there are other cases 4 and 7, where \( (c_1 < c_3 \) and \( c_1 < c_2 \)) in which case, whether the Internet price for the incumbent is higher (or lower) depends in addition on the difference between \( c_2 \) and \( c_3 \).

**Identical costs:** In this case, we will assume that \( c_1 = c_2 = c_3 = c \) and \( u_1 = u_2 = u \); \( u_3 = 1 - 2u \).

There are two effects that could determine consumers purchase in a major way – firm loyalty and channel loyalty. If consumers choose products more based on firm than channel, then \( \theta_{12} \geq \theta_{13} \geq \theta_{23} \). On the other hand, under channel loyalty consumers choose products more based on the channel rather than the retail firm, i.e., \( \theta_{13} \geq \theta_{12} \geq \theta_{23} \).

**Proposition 5.** If the costs of the products from the two firms are identical \( (c_1 = c_2 = c_3 = c) \) and the substitutability factors among the channels are \( \theta_{12} > \theta_{13} = \theta_{23} \), the Nash equilibrium price for the Internet channel for the incumbent firm is greater than the price for the Internet-only new entrant firm \( (p_{N3} < p_{N1}) \) when \( \theta_{12} < \frac{2}{\pi - \theta_{12}} \).

It is to be expected that both \( p_{N1} \) and \( p_{N2} \) should decrease with respect to \( \theta_{12} \). The above result indicates that there exists a threshold value for \( \theta_{12} \) (that is dependent on both \( \theta_{13} \) and \( \theta_{23} \)) such that the incumbent firm charges a higher price only when \( \theta_{12} \) is below the threshold. An intuitive explanation of this phenomenon is that when \( \theta_{12} \) becomes high then the price increase on the Internet channel leads to internal cannibalization of demand and hence makes this strategy less attractive for the firm. Thus, when customers choose products based on the firm then \( \theta_{12} \) should be low enough for the incumbent firm to charge higher price on the Internet channel.

**Proposition 6.** If the costs of the products from the two firms are identical \( (c_1 = c_2 = c_3 = c) \) and \( (u_1 = u_2 = u) \) and the substitutability factor \( \theta_{13} > \theta_{12} = \theta_{23} \), the Nash equilibrium price for the Internet channel for the incumbent firm is greater than the price for the Internet-only new entrant firm \( (p_{N3} < p_{N1}) \) when

\[
0 < \theta_{12} < 0.747405 \quad \text{and} \quad \theta_{12} < \theta_{13} < T_1(\theta_{12}) ,
\]

\[
0.747405 < \theta_{12} < 1 \quad \text{and} \quad \theta_{12} < \theta_{13} < T_2(\theta_{12}) .
\]

Based on Figs. 1 and 2, it is clear that \( T_1(\theta_{12}) \) and \( T_2(\theta_{12}) \) are very close to one. Therefore, in almost all cases under the above assumptions, the incumbent firm with two channels has an incentive to price higher on the Internet channel. Intuitively, the incumbent gains more by charging a higher price on the traditional channel since \( \theta_{13} > \theta_{23} \). For the same reason, the Internet-only firm has a greater incentive to price lower than the incumbent on the Internet channel. Further, for the incumbent to go below the Internet-only firm’s price would hurt profits due to internal cannibalization. Thus, in almost all cases, it seems that the incumbent will price the product higher on the Internet channel as compared to the entrant.

Clearly, a change in substitutability across a pair of channels has a significant and complex effect on resulting equilibrium prices. Under firm loyalty settings when \( \theta_{12} \geq \theta_{13} \geq \theta_{23} \) we numerically study how changes in \( \theta_{12} \) affect the prices and profits. In order to illustrate those effects in the following example, we consider two possibilities – (1) the substitutability factor between the two firms (i.e., \( \theta_{13}, \theta_{23} \)) remains unchanged, (2) the substitutability factor between the two firms changes in the same direction as \( \theta_{12} \), i.e., \( \theta_{13}, \theta_{23} \) also increase with \( \theta_{12} \). In the first case, a price drop for the incumbent in one of her two channels could potentially bring new customers from the third channel but at the same time could lead to cannibalization across its two channels. We find (as shown in Fig. 3) that the prices across all channels decrease with \( \theta_{12} \). The prices of the incumbent firm’s two channels decrease slowly at first but once the price is lower than the entrant’s price, there is a rapid drop. One could explain this effect by noting that when the incumbent’s prices are higher than the entrant’s price

![Fig. 1. Constraint for \( \theta_{13} \) and when \( 0 < \theta_{12} < 0.747405 \).](image-url)

the best response to an increase in \( \theta_{12} \) is a drop in the incumbent’s prices so that she could benefit from a larger pool of customers in channel one and two. The price drop is not substantial because there is cannibalization. However when the incumbent’s prices get lower than the entrant’s price, any price drop in addition attracts the entrant’s customer thereby providing greater incentive to decrease the prices. At that point, the profit of the incumbent increase with \( \theta_{12} \) as shown in Fig. 4. Note that the profit of the entrant is decreasing in \( \theta_{12} \). However, if the initial market share of the incumbent is high (as in Figs. 5 and 6), then the profit of the incumbent does not increase even after the price drops below the entrant’s price because the entrant’s market share is not significant. In the case where \( \theta_{13} \) and \( \theta_{23} \) increase with \( \theta_{12} \), a decrease in the
incumbent’s prices enable her to capture the entrant’s market more easily therefore we observe a sharp drop in the prices when \( h_{12} \) increases, as shown in Fig. 7. In this case, since increase in \( h_{12} \) leads to higher substitutability across all channels we find that the profits of the two firms decrease with \( h_{12} \), shown in Fig. 8 (Figs. 9 and 10).

Fig. 5. Price across the two firms where \( \theta_{13} = 0.2, \theta_{23} = 0.1 \) and \( u_1 = u_2 = 0.3, c_1 = c_2 = c_3 = 3, \beta = 0.1 \).

Fig. 6. Profit across the two firms where \( \theta_{13} = 0.2, \theta_{23} = 0.1 \) and \( u_1 = u_2 = 0.3, c_1 = c_2 = c_3 = 3, \beta = 0.1 \).

Fig. 7. Price across the two firms where \( \theta_{12} = \theta_{13} + 0.1 = \theta_{23} + 0.2 \) and \( u_1 = u_2 = 0.2, c_1 = c_2 = c_3 = 3, \beta = 0.1 \).
Under channel loyalty, where $\theta_{13} \geq \theta_{12} \geq \theta_{23}$ we study how changes in $\theta_{13}$ may affect the prices and profits. As above, we consider two possibilities – (1) the other two substitutability factors remain unchanged (i.e., $\theta_{12}, \theta_{23}$), (2) the other two substitutability factors change in the same direction as $\theta_{13}$ (i.e., $\theta_{12}, \theta_{23}$ also increase with $\theta_{13}$). In the first case, we find (as shown in Fig. 11) that the prices across all channels decrease with $\theta_{13}$. Further, as $\theta_{13}$ increases the incumbent drops the

\[ P_{13} = \frac{\theta_{13} + 0.1}{\theta_{23} + 0.2} \]
price on the Internet channel in a much sharper fashion than on its traditional channel. An intuitive explanation is that a price drop for the incumbent on the Internet channel could potentially bring more customers from the third channel with an increase in $\theta_{13}$. However, a price drop on the traditional channel does not have a similar effect since $\theta_{23}$ is held constant. We also find that the profits of the incumbent and the entrant decrease with $\theta_{13}$ as shown in Fig. 12 (Fig. 13).

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Fig. 11. Price across the two firms where $\theta_{12} = 0.2$, $\theta_{23} = 0.1$ and $u_1 = u_2 = 0.2$, $c_1 = c_2 = c_3 = 3$, $\beta = 0.1$.

Fig. 12. Profit across the two firms where $\theta_{12} = 0.2$, $\theta_{23} = 0.1$ and $u_1 = u_2 = 0.2$, $c_1 = c_2 = c_3 = 3$, $\beta = 0.1$.

Fig. 13. Price across the two firms where $\theta_{12} = 0.2$, $\theta_{23} = 0.1$ and $u_1 = u_2 = 0.3$, $c_1 = c_2 = c_3 = 3$, $\beta = 0.1$. 

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In the case where $\theta_{12}$ and $\theta_{23}$ increase with $\theta_{13}$, the incumbent firm can capture the entrant’s market more easily by decreasing the price on both her channels. Therefore we observe a sharp drop in the prices on both the channels when $\theta_{13}$ increases (as shown in Fig. 15). Increase in $\theta_{13}$ leads to higher substitutability across all channels, and as expected we find that the profits of the two firms decrease with $\theta_{13}$, shown in Fig. 16. We also find that the resulting profit of the two firms depends on their market share. For example, when the incumbent’s market share is low (Figs. 16 and 12), then beyond a threshold value for $\theta_{13}$ the incumbent ends up making more profits than the entrant. On the other hand, when the incumbent’s market share is high to begin with (Figs. 14 and 18), the incumbent ends up with a higher profit despite increases in $\theta_{13}$ (Fig. 17).

4.2.2. Change in incumbent profits due to new entrant

In this section, we provide analytical insights on the effect of the new entrant on incumbent profits. In particular, we consider a case where the new entrant brings an additional set of customers to the market. Let $S$ represent the whole market when both firms are present and $S_1$ represent the market when only the first firm is present. Further let the market share of the Internet channel of the incumbent be $u_1$ and that of the traditional channel be $u_2 = hu_1$ and $h > 0$. Then we assume that $S_1 = S(u_1 + hu_1)$. Further we assume that costs across the two channels in the incumbent are the same ($c_1 = c_2$) and the substitutability of the new Internet channel is identical with respect to the existing channels ($\theta_{13} = \theta_{23}$).

**Proposition 7.** If the costs across the two channels in the incumbent are the same ($c_1 = c_2$) and the substitutability of the new Internet channel is identical with respect to the existing channels ($\theta_{13} = \theta_{23}$), the sign of the difference between the profits of the incumbent before and after the entrance of the new entrant is independent of the market share of the Internet channel of the incumbent ($u_1$).

![Fig. 14. Profit across the two firms where $\theta_{12} = 0.2, \theta_{23} = 0.1$ and $u_1 = u_2 = 0.3, c_1 = c_2 = c_3 = 3, \beta = 0.1$.](image)

![Fig. 15. Price across the two firms where $\theta_{13} = \theta_{12} + 0.1 = \theta_{23} + 0.2$ and $u_1 = u_2 = 0.2, c_1 = c_2 = c_3 = 3, \beta = 0.1$.](image)
Proposition 8. When the Internet channel and traditional channel in the incumbent firm has the same market share \( u_1 = u_2 \), the same substitutability factor \( \theta_{13} = \theta_{23} \) and the same cost \( c_1 = c_2 = c \), the gap between the profit of the incumbent firm after and before the entrance of the new entrant \( \text{profit}_{\text{after}} - \text{profit}_{\text{before}} \) is increasing with the cost of the new entrant when \( c < c_3 \).

The above results point out that changes in the existing firm’s profits are not necessarily dependent on the existing market share. Given that a new set of customers are getting added to the market with the entrant, it depends on the cost dif-
ferentials. This illustrates a possibility for the incumbent to have the higher profit with the new entrant when the cost of the new entrant is sufficiently higher than the cost of the incumbent firm. In that case the incumbent may benefit from the new market that the new entrant brings in.

5. A variant: Model with different demand functions

One of the shortcomings of the above general demand function is that when \( u > 1/2 \) and \( p_W > p_T \) then \( \frac{\partial q_T + \partial q_W}{\partial p} > 0 \) as indicated in Choi ([11]). Clearly this is not an intuitive behavior that as substitutability increases the demand should go up. In order to overcome the above shortcoming they consider a modified demand function that we study in this section.

5.1. Single firm: Introduction of a new channel

We adopt the deterministic and linear demand functions similar to one introduced by Choi ([11]) which are given by

\[
q_T = 1 - p_T + \gamma(p_W - p_T),
q_W = 1 - p_W + \gamma(p_T - p_W),
\]

where \( \gamma > 0 \) represents the channel differentiation between the traditional and Internet channels. It can be easily shown that the above is a special case of the demand function in Eq. (3) when \( \mu = 1/2, \mu S = 1, \beta = 1, \) and \( \theta = \frac{1}{1+\gamma} \). Hence, all the results in Section 3 hold correctly for this modified demand function.

5.2. Two firms: Introduction of a new channel

In this setting there are two firms who are competing. When the incumbent firm has a traditional channel (like Section 4.1), the modified demand function is still a special case of the general function as a result, all results hold. The case where the incumbent firm has a mixed channel (as in Section 4.2) is analyzed here. In this case, we can represent the demand as

\[
q_1 = 1 - p_1 + \alpha(p_2 - p_1) + \beta(p_3 - p_1),
q_2 = 1 - p_2 + \alpha(p_1 - p_2) + \beta(p_3 - p_2),
q_3 = 1 - p_3 + \alpha(p_2 - p_3) + \beta(p_3 - p_3),
\]

where \( \alpha > 0 \) represents channel differentiation, and \( \beta > 0 \) represents firm differentiation. It is necessary to impose additional inequality constraints on these parameters in order to guarantee (i) prices exceed marginal costs; (ii) quantities are nonnegative; (iii) industry demand does not increase with increase in price.

In the following passages, we will provide theoretical insights on how changes in costs and substitutability factors affect optimal prices and profits. Under the above conditions, we consider the game beginning with both the incumbent and entrant firms simultaneously choosing prices, where \( p_{N1}, p_{N2} \) are the prices of the two channels of the incumbent firms, and \( p_{N3} \) is the price of the Internet-only entrant firm.

**Proposition 9.** There exists a unique Nash equilibrium in this game under the above assumptions and the resulting prices are given as follows:

\[
p_{N1} = \frac{\alpha(4 + c_2 \beta + 3c_3 \beta) + 2(2 + \alpha(4 + c_1 \beta + c_3 \beta^2) + c_1(4 + \beta)^2 + \alpha(4 + 3\beta))}{2(4 + 8\beta + 3\beta^2 + \alpha(4 + 3\beta))},
\]

\[
p_{N2} = \frac{2(2 + c_3 \beta)(1 + \alpha) + c_2(4 + 8\beta + 3\beta^2 + 4\alpha(1 + \beta)) + 6\beta + c_1 \beta^2 + 2c_3 \beta^2}{2(4 + 8\beta + 3\beta^2 + \alpha(4 + 3\beta))},
\]

\[
p_{N3} = \frac{2 + 3\beta + c_1 \beta + c_1 \beta^2 + 2c_3(1 + \beta)(1 + \alpha + \beta) + \alpha(1 + c_2 + c_2 \beta)}{4 + 8\beta + 3\beta^2 + \alpha(4 + 3\beta)}.
\]

Hence, we get

\[
p_{N1} - p_{N3} = \frac{-2c_3(2 + \beta)(1 + \alpha + \beta) + c_1(4 + 4\alpha + 6\beta + 3\alpha \beta + 2\beta^2) - \alpha(-2 + c_2(2 + \beta))}{2(4 + 8\beta + 3\beta^2 + \alpha(4 + 3\beta))}
\]

\[
= (c_1 - c_3)(4 + 4\alpha + 6\beta + 2\alpha \beta + 2\beta^2) + (c_1 - c_2)\alpha \beta + 2\alpha(1 - c_2). \tag{23}
\]
Proposition 10. Let $c_1, c_2$ be the cost of product on the two channels of the incumbent, firm, and $c_3$ be the cost of the Internet-only entrant firm. $p_{N3} < p_{N1}$ when

\[
\begin{align*}
(i) \quad & c_1 < c_3, c_1 > c_2, c_2 < 1 \quad \text{and} \quad c_3 < \frac{4c_1(1 + x) + 2x(1 - c_2) + 2c_1\beta(3 + \beta) + x\beta(3c_1 - c_2)}{4 + 4x + 6\beta + 2x\beta + 2\beta^2} \quad \text{or} \\
(ii) \quad & c_1 < c_3, c_1 < c_2, c_2 < 1 \quad \text{and} \quad c_3 < \frac{2 + c_1\beta}{2 + \beta}, \quad \text{or} \\
(iii) \quad & c_1 > c_3, c_1 > c_2, c_2 < 1.
\end{align*}
\]

6. Conclusions

In this paper we study the optimal pricing strategies when a product is sold on two channels. We assume a stylized deterministic demand model where the demand on a channel depends on prices, degree of substitution across channels and the overall market potential. We first study four pricing strategies for the monopolistic environment which differ in the degree of autonomy for the Internet channel. We provide theoretical bounds for the pricing strategies. Next, we consider the case where the Internet channel is introduced by a different firm, and show the existence of a unique Nash equilibrium. We derive conditions under which the profits of the incumbent may indeed increase. Next, we study the case where the first firm has both traditional and Internet channels, and faces the competition from a new Internet firm. We show the existence of a unique Nash equilibrium for this situation. We develop conditions under which Internet price of the firm with both channels is greater than the price of the pure e-tailer. Finally, through a limited computational study, we explore the behavior (price and profits) of the above models under different parameters.

Our research has several important managerial insights. Firstly we show that prevalent practices related to providing limited autonomy in terms of pricing on the new Internet channel by firms is not necessarily such a bad idea. In fact, we theoretically prove that such policies restrict the profits by at most 4% in many cases. Secondly, we show that having a new competing channel is not necessarily bad for the incumbent firm particularly in cases where the new channel brings in more customers to the market and the incumbent has a significant cost advantage. In those case, competition becomes a source of opportunity. Finally, through our model we demonstrate that mixed channel firms have a greater likelihood of pricing their products higher on the competing channel while facing competition from a pure player. This validates some of the empirical findings of earlier researchers. The degree of dispersion of prices across the channels depends on the cost differences, relative substitutability and market share of the different channels.

There are several possible extensions of our work. Firstly, we adopt a simple linear demand model for analytical tractability. It would be interesting to test if similar insights could be obtained for more detailed demand models, even computationally. Secondly, extending the analysis to multiple channels and retailers would be another interesting avenue for this research. Finally, analyzing the pricing issue from the manufacturer’s perspective could potentially lead to other insights. These will be the focus of our future research.

7. Uncited reference

Q1 [14].

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Appendix. Proofs

Proof of Theorem 1. In this case, the ratio of the total profit of the firm under pricing strategy II to that under pricing strategy IV is shown to be

\[
R = \frac{(-4u + 4u^2 + \theta^2)A}{16(1 - u)^2u(1 - \theta)(1 + \theta)B},
\]

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The first order derivatives of $R$ with respect to $c_T$ and $c_W$ are given by
\[
\frac{dR}{dc_T} = -\frac{(1 - 2u)\beta(-1 + c_T\beta)(-1 + c_W\beta)(\theta - 2u\theta^2)(2 - c_W\beta + 2u(-1 + c_W\beta) - \theta + c_T\beta)(-4u + 4u^2 + \theta^2)}{(16(1 - u)^2u(1 - \theta)(1 + \theta)B)^2},
\]
\[
\frac{dR}{dc_W} = \frac{(1 - 2u)^2\beta(-1 + c_T\beta)^2\theta^2(2 - c_W\beta + 2u(-1 + c_W\beta) - \theta + c_T\beta\theta)(-4u + 4u^2 + \theta^2)}{(16(1 - u)^2u(1 - \theta)(1 + \theta)B)^2}.
\]

In order to guarantee that the quantity of the Internet channel positive, i.e., $(2 - 2c_W\beta + 2u(-1 + c_W\beta) - \theta + c_T\beta\theta) > 0$, $R$ decreases with $c_T$ and increases with $c_W$, we know that $c_T\beta < 1$ as well. Therefore, ratio is greater than $R(c_T = \frac{1}{2})$ which is shown to be
\[
R^* = R\left(c_T = \frac{1}{2}\right) = \frac{4u - 4u^2 - \theta^2}{4(1 - u)u(1 - \theta)(1 + \theta)}.
\]

The first order derivative of $R^*$ with respect to $\theta$ is given by
\[
\frac{dR^*}{d\theta} = -\frac{(1 - 2u)^2\theta}{2(1 - u)u(-1 + \theta^2)^2} < 0,
\]
then $R^*$ is decreasing with $\theta$. From the constraints, we know that $0 < u < 1/2, 0 \leq \theta \leq \frac{u}{1 - u}$ or $1/2 < u < 1, 0 \leq \theta \leq \frac{1 - u}{u}$, it shows that
\[
R^* \geq \frac{4 - 5u + 2u^2}{4 - 4u} \quad \text{for} \quad 0 < u < 1/2,
\]
\[
R^* \geq \frac{1 + u + 2u^2}{4u} \quad \text{for} \quad 1/2 < u < 1.
\]

We can show that $\frac{4 - 5u + 2u^2}{4 - 4u}$ is convex with respect to $u$ when $0 < u < 1/2$ and $\frac{1 + u + 2u^2}{4u}$ is convex with respect to $u$ when $1/2 < u < 1$. The minimum points are shown to be
\[
\frac{4 - 5u + 2u^2}{4 - 4u} \leq 0.957107 \quad \text{when} \quad u = 0.292893,
\]
\[
\frac{1 + u + 2u^2}{4u} \leq 0.957107 \quad \text{when} \quad u = 0.707107. \quad \square
\]

**Proof of Theorem 2.** Assuming $c_T = c_W$, it is easy to compute the ratio of the total profit of two firms in Cases II and IV. It is computed to be
\[
R = \frac{(-4u + 4u^2 + \theta^2)\beta(-2 + \theta)^2 + 4u\theta^2 - 4u(1 - \theta + \theta^2)}{16(1 - u)^2u(1 - \theta)^2(1 + \theta)}.
\]

The first order derivative of this ratio with respect to $\theta$ is given as follows:
\[
\frac{dR}{d\theta} = \frac{(1 - 2u)^2\theta\left(8 - 8\theta + 40\theta^2 - 9\theta^4 - 4u(4 - \theta + \theta^2) + 4u^2(2 + \theta + \theta^2)\right)}{16(-1 + u)^2u(-1 + \theta)^2(-1 + \theta^2)^2}.
\]

The above derivative is negative, i.e., the ratio decreases with respect to $\theta$, when
\[
0 < u \leq \frac{1}{2} \quad \text{and} \quad 0 \leq \theta \leq \frac{u}{1 - u}
\]
or
\[
\frac{1}{2} \leq u \leq 1 \quad \text{and} \quad 0 < \theta < \text{Root}[4 - 4u - 2x - 2ux + x^2 - 2ux^2 + x^2, 2],
\]
where Root \([f, k]\) represents the \(k\)th root of the polynomial equation \(f(x) = 0\). Let \(\hat{\theta} = \text{Root}[4 - 4u - 2x - 2ux + x^2 - 2ux^2 + x^3, 2]\). The derivative \(\frac{dR}{d\theta}\) becomes positive when

\[
1/2 \leq u \leq 1 \text{ and } \hat{\theta} < \theta < 1.
\]

Therefore,

\[
R \geq \frac{(4 - 5u + 2u^2)(-4 + 8u - 5u^2 + 2u^3)}{16(-1 + u)^3} = R_1 \quad \text{when } 0 \leq u \leq \frac{1}{2}.
\]

\[
R \geq R(\theta = \hat{\theta}) = R_2 \quad \text{when } \frac{1}{2} < u \leq 1.
\]

The first order derivative of \(R_1\) with respect to \(u\) is given by

\[
\frac{dR_1}{du} = \frac{-4 + 32u - 79u^2 + 80u - 40u^4 + 8u^5}{16(-1 + u)^3}
\]

which is negative when \(0 < u < 0.22299\) and positive when \(0.22299 < u < 1/2\). Therefore the minimal point in the first case is

\[
R_{\text{min}} \approx 0.97 \quad \text{when } u^* \approx 0.22299.
\]

The graph of \(R_2\) for \(1/2 < u < 1\) is shown as in Fig. 19. From the graph, we could easily derive that \(R_{\text{min}} \approx 0.99795\) when \(u^* \approx 0.685\).

The above quantities respectively are convex with respect to \(u\). The minimal points in two cases are given as follows:

\[
R_{\text{min}} \approx 0.97 \quad \text{when } u^* \approx 0.22299;
\]

\[
R_{\text{min}} \approx 0.99795 \quad \text{when } u^* \approx 0.685. \quad \square
\]

**Proof of Theorem 3.** Under the assumption of \(c_T = c_W\), the ratio of the total profit of the firm in pricing strategy III and pricing strategy IV is computed as

\[
R = \frac{4u - 4u^2 - \theta^2}{4(1 - u)(1 - \theta)(1 + \theta)}.
\]

The first order derivative of the ratio with respect to \(\theta\) is given as follows:

\[
\frac{dR}{d\theta} = -\frac{(1 - 2u)^2\theta}{2(1 - u)(1 - \theta^2)^2}
\]

which is negative when \(0 < u < 1\).

The second order derivative of the ratio with respect to \(u\) is given as follows:

\[
\frac{d^2R}{du^2} = -\frac{(1 - 3u + 3u^2)\theta^2}{2(1 - u)^2u^2(1 - \theta^2)}.
\]
which is negative when $0 < u < 1$ and $0 < \theta < 1$. From the derivatives of this ratio with respect to $\theta$ and $u$, the ratio decreases with $\theta$ and is concave with respect to $u$. Because the necessary conditions, i.e., $\theta \leq \frac{u}{1-u}$ when $0 \leq u \leq \frac{1}{2}$ and $\theta \leq \frac{1-u}{u}$ when $\frac{1}{2} \leq u \leq 1$,

$$R \geq \frac{4 - 5u + 2u^2}{4 - 4u} \quad \text{when } 0 \leq u \leq \frac{1}{2};$$

$$R \geq \frac{4u^3 - 4u^4 - 1 - u^2 + 2u}{4u(-1 + 3u - 2u^2)} \quad \text{when } \frac{1}{2} \leq u \leq 1. \quad (38)$$

In both cases, the above quantities are convex with respect to $u$. We may compute the minimal points in the two cases as

$$R_{\min} = \frac{1}{4} + \frac{1}{\sqrt{2}} \simeq 0.96 \quad \text{when } u^* = \frac{2 - \sqrt{2}}{2} \simeq 0.2929;$$

$$R_{\min} = \frac{5 - 4\sqrt{2}}{-12 + 8\sqrt{2}} \simeq 0.96 \quad \text{when } u^* = \frac{1}{\sqrt{2}} \simeq 0.7071. \quad \Box$$

**Proof of Proposition 9.**

The total profits of the two channels of the incumbent firm and of the Internet-only entrant firm are given by

$$g_1 + g_2 = q_1(p_1 - c_1) + q_2(p_2 - c_2) \quad \text{and} \quad g_3 = q_3(p_3 - c_3). \quad (39)$$

The second order derivative of the profit with respect to the prices are as follows.

$$\frac{\partial^2(g_1 + g_2)}{\partial p_1^2} = \frac{\partial^2(g_1 + g_2)}{\partial p_2^2} = -2(1 + \alpha + \beta) < 0,$$

$$\frac{\partial^2(g_1 + g_2)}{\partial p_1^2} \frac{\partial^2(g_1 + g_2)}{\partial p_2^2} - \left(\frac{\partial^2(g_1 + g_2)}{\partial p_1 \partial p_2}\right)^2 = 4(1 + \alpha + \beta)^2 - 4\alpha^2 > 0,$$

$$\frac{\partial^2 g_3}{\partial p_3^2} = -2(1 + \alpha + \beta) < 0.$$

Hence, the profit functions are concave with respect to the prices. We derive the unique Nash equilibrium prices from the first order conditions, and the optimal prices are given by

$$p_{N1} = \frac{2(4 + c_1\beta + 3c_2\beta + 2(2 + (3 + c_1)\beta + c_2\beta^2) + c_1(4(1 + \beta)^2 + x(4 + 3\beta))}{2(4 + 8\beta + 3\beta^2 + x(4 + 3\beta))},$$

$$p_{N2} = \frac{2(2 + c_3\beta)(1 + x) + c_2(4 + 8\beta + 3\beta^2 + 4x(1 + \beta)) + 6\beta + c_1\beta^2 + 2c_3\beta^2}{2(4 + 8\beta + 3\beta^2 + x(4 + 3\beta))},$$

$$p_{N3} = \frac{2 + 3\beta + c_1\beta + c_1\beta^2 + 2c_1(1 + \beta)(1 + x + \beta) + x(1 + c_2 + c_2\beta)}{4 + 8\beta + 3\beta^2 + x(4 + 3\beta)}.$$

\[\Box\]

**Proof of Proposition 10.**

(i) $c_1 < c_3$, $c_1 > c_2$, $c_2 < 1$.

We show that $p_{N1} - p_{N3} > 0$, when

$$c_3 < \frac{4c_1(1 + x) + 2\alpha(1 - c_2) + 2c_1\beta(3 + \beta) + x\beta(3c_1 - c_2)}{4 + 4x + 6\beta + 2\alpha\beta + 2\beta^2} \quad (40)$$

and $p_{N1} - c_i > 0$, $i = 1, 2, 3$.

(ii) $c_1 < c_3$, $c_1 < c_2 < c_2 < 1$.

We show that $p_{N1} - p_{N3} > 0$, when

$$c_2 < \frac{2 + c_1\beta}{2 + \beta} \quad \text{and} \quad c_3 < \frac{4c_1(1 + x) + 2\alpha(1 - c_2) + 2c_1\beta(3 + \beta) + x\beta(3c_1 - c_2)}{4 + 4x + 6\beta + 2\alpha\beta + 2\beta^2}. \quad (41)$$
Under the above conditions, \( p_{N1} > c_1, p_{N2} > c_2, \) and \( p_{N3} > c_3. \)

(iii) \( c_1 > c_3, c_1 > c_2, c_2 < 1, \)

Under this condition, \( p_{N1} > p_{N3}. \)

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