Decision Support for Allocating Scarce Drugs

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The 1998 settlement of a lawsuit by the state of California required 19 pharmaceutical firms to provide $150 million worth of drugs free of charge to more than 150 California clinics and hospitals over a three-year period. We developed a decision-support system that utilizes a multiobjective optimization model and a heuristic solution taking into account the efficiency, effectiveness, and equity of the drug-allocation process. As of November 2002, 90 percent of the total budget has been distributed to the various clinics with the help of this decision-support system.

(Health care: pharmaceutical. Decision analysis: systems.)

The 1998 settlement of a lawsuit by the state of California required 19 pharmaceutical firms to provide $150 million worth of drugs free of charge to more than 150 California clinics and hospitals over a three-year period. In July 2001, six other pharmaceutical firms were added to this lawsuit, and they brought an additional $20 million worth of products to distribute. The Public Health Institute (www.phi.org), an independent, nonprofit organization that promotes health for people throughout California and the United States, was given the responsibility of distributing these drugs in a fair and equitable manner. They called this large-scale project the Drug Distribution Project (DDP).

The DDP was to distribute 125 drugs in more than 20 drug categories. These drugs ranged from routine antibiotic drugs, such as Amoxil and Ceftin, to complex drugs, such as Artane (for the nervous system) and Epivir (for HIV). Further, each pharmaceutical firm restricted the wholesale value (in total, by drug category, and by individual drug) of the products it could provide in a given period. The pharmaceutical firms imposed additional restrictions on the clinics in terms of minimum order amounts per period, in dollars as well as drug quantities.

Over 150 clinics and hospitals have participated in the project through October 2002. These medical facilities range in size from very large clinics and hospital networks, such as that of Los Angeles County, to very small specialized clinics. The clinics also vary in their expertise: some are general purpose, while others are specialized, such as mental health clinics. Based on the settlement, each clinic was assigned a maximum dollar value of drugs that it could receive. Because all the drugs were to be provided free of charge, the Public Health Institute expected the clinics to order primarily the most critical drugs and to order up to their maximum dollar-value allocations. This posed a problem in allocation: because there was an overall dollar-value cap on the amount of each drug available for distribution, there was every likelihood that orders for certain critical drugs would exceed the quantities available. To resolve those situations, the Public Health Institute sought my help in developing an allocation heuristic that could be used in a decision-support system to allocate the drugs equitably to the clinics.

Several previous research studies in health-care management and optimization models for nonprofit environments are related to this work. Folland et al. (1997) discuss the prevalence and the relevance of...
nonprofit organizations in the health sector. They also highlight the difficulties associated with managing activities with a nonprofit purpose. Several other researchers have adopted and adapted optimization models for the public-sector environment (Bodily 1978, Heiner et al. 1981, and Mandell 1991). Brill (1979) discusses the limitations of using traditional optimization models for generating alternatives and facilitating their evaluation. He also discusses the role of heuristics and computer algorithms. Savas (1978) identifies three measures of performance in a nonprofit environment—efficiency, effectiveness, and equity. He defines efficiency as the ratio of service inputs to service outputs; effectiveness as how well the need for services is satisfied and adverse consequences are prevented; and equity as the fairness, impartiality, or equality of services rendered.

**Problem Description**

The three-year drug distribution was subdivided into several ordering intervals (that occurred typically once every three months). The sequence of operations for ordering drugs in any order period was as follows. First, the DDP provided the clinics their drug budgets for the order period. The state of California had established rules for determining the drug budgets for clinics based on such characteristics as location, population served, and financial condition. Clinics then had a two-week window in which to place their drug orders with the DDP. Using the online decision-support tool, the DDP also provided the clinics with information on drug availability for the ordering period. A clinic could work on its order, finalize it, and then send it at any point during this time. Once the clinic sent the order to the DDP, it was considered final and could not be changed. After the two-week order-placement window expired, the decision-support system considered all orders as final and began the allocation process. We (I along with the DDP managers) decided that it was reasonable to optimize drug allocation for only one ordering period at a time. This allocation process will be repeated once every quarter over three years.

The decision-support tool took into account efficiency, effectiveness, and equity while allocating drugs in the following manner.

Savas (1978) defines efficiency as the ratio of service outputs to service inputs. In this case, service inputs are fixed and are based on the dollar amount and quantity of drugs that the pharmaceutical firms are obligated to donate. Therefore, allocational efficiency is measured by the extent to which the DDP distributes all resources available. Our objective to minimize the total dollar value of the drug budget left over, which is equivalent to maximizing the dollar value of drugs allocated in a given period, accomplishes the goal of efficiency (Appendix).

We measured allocational effectiveness by the extent to which clinics receive the drugs they needed. To maximize effectiveness, we used a weight matrix \( w_{ik} \) that determines the importance of drug \( k \) for clinic \( i \). The DDP managers based the weight matrix on several factors. For example, it gives mental-health clinics higher priority for mental-health drugs; it gives small clinics priority over large clinics in certain cases; and it gives high priority to clinics participating for the first time in Public Health Institute programs to encourage further participation. To some extent, effectiveness is also affected by the budget limit that the state determines for each clinic.

We measured equity of allocation by the extent to which allocations are fair with respect to clinic characteristics, such as financial resources, geographical remoteness, and medical specialization. Folland et al. (1997) discuss the notion of need-based allocation and the ambiguities involved in discerning the need of constituents that makes this measure difficult to optimize. In this case, for example, large organizations and hospitals need more drugs but also have more resources with which to buy those drugs in the open market. Similarly, specialized clinics need drugs related to their specialties more acutely than other clinics. Along the same lines, some of the small clinics need drugs allocated to them even though their orders may be for very small amounts. To achieve equity, we designed
the allocation heuristic such that each clinic gets a fraction that is proportional to the requested amount (weighted by \( p_{ik} \)) of any drug in short supply.

For example, if clinic 1 has a weight of 5 for drug \( i \) and clinic 2 has a weight of 4 and their orders are 50 and 100 respectively and the budget for this drug is 100, we allocate $38.46 to clinic 1 and $61.54 to clinic 2 to make an overall allocation that reflects the order sizes and the clinics’ respective weights

\[
\frac{5 \times 50}{4 \times 100} = \frac{38.46}{61.54}
\]

Such an allocation may not be always feasible because of limits on minimum orders. However, it is an attempt to equalize the distribution of drugs among the clinics, taking into account their orders and their priorities for the drugs. The ability to change allocations based on the priority weights allows the DDP managers to study the impact of priority weights on allocations made by the decision-support system.

Given the above measures, we formulate the problem (Appendix) with two objectives, to minimize the leftover budget (Objective \( O_1 \), the equivalent of maximizing the dollar value of the drug allocation) in any given period, which addresses efficiency, and to minimize the difference between the ratio of the allocations and the weighted orders from the clinics, thereby addressing the effectiveness and equity measures (Objective \( O_2 \)). The allocation also needed to take into account the following constraints on participating clinics and pharmaceutical firms (Appendix).

The DDP allocated each clinic a budget \( B_i \) for the total dollar value of drugs it could obtain per order period based on a formula determined by the state of California. This clinic constraint (1) ensures that clinics do not exceed their budgets. Although the budget alone should normally be sufficient to restrict the amount allocated to a clinic, we added a constraint that limits orders as well to discourage clinics from ordering irrational or exorbitant amounts.

Each pharmaceutical firm categorizes its products. For example, they put all mental-health drugs together. Constraints (2), (3), and (4) ensure that in any given period the dollar values of total disbursement, category-level disbursement, and drug-level disbursement, do not exceed the limits set in the settlement agreement.

The allocation constraints relate the requested quantities and the allocated quantities. Constraint (5) ensures that the dollar value of a drug delivered to a clinic is less than or equal to the requested amount. Constraint (6) ensures that the DDP meets the minimum order quantity for each drug and clinic. Constraint (7) is the nonnegativity constraint.

The allocation problem has multiple objectives, is nonlinear, and has integer variables. The size of the real problem (more than 150 clinics, 25 pharmaceutical firms, 125 drugs, and 20 drug categories) makes optimal allocation very difficult to achieve in a reasonable time. Because the optimization model was to be used regularly for decision support and the priority weight matrix would be somewhat subjective, we focused our attention on developing a fast heuristic solution that addressed the efficiency, effectiveness, and equity measures.

### Allocation Heuristic

The allocation heuristic develops a solution that takes care of the performance measures and keeps the procedure simple enough for the DDP managers to use it for decision support. The basic idea in the allocation heuristic is to identify scarce drugs (those for which demand is greater than supply) and to find a fair allocation among clinics while taking all the constraints into account (Appendix).

The heuristic starts by allocating to all clinics the quantities requested for all the drugs (Step 0). Then, it checks whether any constraints have been violated; if not, the heuristic ends. Otherwise, the heuristic identifies all the scarce drugs, the drug categories, and the pharmaceutical firms (Steps 1–3). The heuristic then starts at the scarce drug level and identifies the maximum number of clinics that can be served subject to the minimum-order constraint (Step 3a). If this number turns out to be less than the number of clinics that placed an order for a given drug, the heuristic selects
the clinics with the highest priority for the drug (Step 3b). This step addresses the effectiveness criterion. Next, the heuristic allocates the existing budget for this scarce drug to the high-priority clinics to optimize objective $O_2$ for the drug (Steps 3c–3f). Doing this means splitting the existing budget to allocate drugs to clinics based on their order size and their priority. These steps in the heuristic are introduced to achieve equity of allocation. Whenever a surplus is created, the heuristic immediately redistributes it among the clinics to maintain the efficiency of the allocation. The heuristic also adjusts the amount going to the different clinics to satisfy the minimum-order constraint (Steps 3g–3i). This adjustment may take several iterations and occasionally may even lead to a reduction in the number of clinics that are allocated the scarce drug (Step 3i). This is repeated for each of the scarce drugs in the category (Step 3i).

Once the heuristic allocates all the drugs in a category, it evaluates the constraint for the drug category (Step 2a). If the constraint has been violated, the heuristic reduces the budget for all the drugs in that category to meet the constraint (Step 2b). While there are alternative ways to artificially reduce the budgets associated with the different drugs, we used this method to treat all the drugs equally. With the budgets for the drugs in the category reallocated, the heuristic begins the entire process of drug allocation again with the scarce drugs in the category. The heuristic repeats this procedure for all the scarce drug categories for a given manufacturer (Step 2c). Once the heuristic has allocated all the drugs in all the categories established by one manufacturer, it checks the manufacturer's overall budget constraint (Step 1a); if that constraint has been violated, the heuristic reduces the budgets for all of that manufacturer's drugs by an equal fraction and repeats the whole process of allocation (Step 1b). The heuristic ends when allocations have been computed for all the manufacturers (Step 1c).

By maximizing the allocation of drugs, the heuristic accomplishes the first objective $O_1$. Minimum orders, however, introduce inefficiency into the system, and a small amount of money is often left over after allocation. By explicitly incorporating objective $O_2$, the heuristic produces an equitable allocation. Like the performance of any decision-support algorithm, the performance of this heuristic depends to a great extent on the priority weight matrix $\pi_{ik}$ provided by the decision maker.

**Illustrative Example**

Let us consider two firms that manufacture five drugs ($N_1, N_2, N_3, MH, HIV$) which are distributed to five clinics ($C_1, C_2, C_3, C_{MH}, C_{HIV}$) that differ in size and speciality. Each drug comes in only one package, and firms have no separate budgets for various categories. The input parameters are $D_r$, the set of drugs manufactured by firm $j$; $B_r$, the budget allocation for clinic $i$; $dt_r$, the maximum disbursement from firm $j$; $dd_{ijk}$, the maximum disbursement for drug $k$; and $e_{ik}$, the minimum order for drug $k$.

$$D_1 = \{HIV, N_1\}, \quad D_2 = \{N_2, N_3, MH\}.$$ $$B_1 = 1,200, \quad B_2 = 800, \quad B_3 = 300, \quad B_4 = 600, \quad B_5 = 600.$$ $$dt_1 = 1,800, \quad dt_2 = 2,500.$$ $$dd_{11} = 680, \quad dd_{22} = 600, \quad dd_{23} = 200, \quad dd_{24} = 1,700, \quad dd_{15} = 1,020.$$ $$e_1 = 25, \quad e_2 = 35, \quad e_3 = 20, \quad e_4 = 70, \quad e_5 = 60.$$ 

Table 1 shows the priority weight matrix for the example, and Table 2 shows the orders from the five clinics for the five drugs.

Table 1 shows how the heuristic determines the allocation:

**Step 0:** is to allocate $R_{ik} = O_{ik}$ for all $i, k$. Because $N_3$ is a scarce drug (demand is 400 while the drug budget is fixed at 200), we go to Step 1.

**Step 1:** is to identify the manufacturer of the scarce drug; manufacturer 2 produces drug $N_3$.

**Step 3:** for scarce drug $N_3$:

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_{MH}$</th>
<th>$C_{HIV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$N_2$</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$N_3$</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$MH$</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$HIV$</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: This is the priority weight matrix $\pi_{ik}$ in the illustrative example.

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Table 2: These are the dollar values of orders from the clinics for the different drugs ($O_i$) in the illustrative example.

<table>
<thead>
<tr>
<th>Clinic</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_{CHIV}$</th>
<th>$C_{MH}$</th>
<th>$C_{HIV}$</th>
<th>Drug Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$</td>
<td>150</td>
<td>100</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>325</td>
<td></td>
</tr>
<tr>
<td>$N_2$</td>
<td>200</td>
<td>150</td>
<td>40</td>
<td>35</td>
<td>35</td>
<td>460</td>
<td></td>
</tr>
<tr>
<td>$N_3$</td>
<td>32.1052</td>
<td>250</td>
<td>80</td>
<td>80</td>
<td>42.6316</td>
<td>200</td>
<td>905</td>
</tr>
<tr>
<td>MH</td>
<td>300</td>
<td>250</td>
<td>80</td>
<td>42.6316</td>
<td>280</td>
<td>1,020</td>
<td></td>
</tr>
<tr>
<td>HIV</td>
<td>300</td>
<td>250</td>
<td>80</td>
<td>280</td>
<td>110</td>
<td>1,020</td>
<td></td>
</tr>
<tr>
<td>Clinic Total</td>
<td>1,020</td>
<td>780</td>
<td>260</td>
<td>500</td>
<td>550</td>
<td>1,020</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: This is the dollar value allocation of drugs for the clinics ($R_i$) in the illustrative example.

<table>
<thead>
<tr>
<th>Clinic</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_{CHIV}$</th>
<th>$C_{MH}$</th>
<th>$C_{HIV}$</th>
<th>Drug Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$</td>
<td>150</td>
<td>100</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>325</td>
<td></td>
</tr>
<tr>
<td>$N_2$</td>
<td>200</td>
<td>150</td>
<td>40</td>
<td>35</td>
<td>35</td>
<td>460</td>
<td></td>
</tr>
<tr>
<td>$N_3$</td>
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<td>250</td>
<td>80</td>
<td>80</td>
<td>42.6316</td>
<td>200</td>
<td>905</td>
</tr>
<tr>
<td>MH</td>
<td>300</td>
<td>250</td>
<td>80</td>
<td>42.6316</td>
<td>280</td>
<td>1,020</td>
<td></td>
</tr>
<tr>
<td>HIV</td>
<td>300</td>
<td>250</td>
<td>80</td>
<td>280</td>
<td>110</td>
<td>1,020</td>
<td></td>
</tr>
<tr>
<td>Clinic Total</td>
<td>932.1052</td>
<td>742.6316</td>
<td>260</td>
<td>462.6316</td>
<td>512.6316</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

—Step 3a: Determine the maximum number of clinics whose demand can be satisfied; $Z = \min\{dd_j/\epsilon_j, I\} = 5$.
—Step 3b: Because $Z = 5$, fill the orders for all five clinics.
—Step 3c: Because the sum of clinic orders exceeds the maximum disbursement ($\sum_{i=1}^{I} R_{ik} > dd_j$), go to Step 3d.
—Step 3d: Compute the total weight of the orders across all the remaining clinics: $W = \sum_{k} \pi_{ik}O_{ik} = 380$.
—Step 3e: Allocate drug $N_3$ to each clinic based on its fraction: $R_{13} = 31.5789$; $R_{23} = R_{33} = R_{43} = R_{53} = 42.1053$.
—Step 3f: Because that allocation gives $C_3$ more than requested, set $R_{33} = 40$ and add 2.1053 to the surplus.
—Step 3g: Because all the clinics have been allocated more than the minimum quantity, split the surplus equally among clinics $C_1$, $C_2$, $C_{MH}$, $C_{HIV}$. So, $R_{13} = 32.1052$, $R_{23} = R_{33} = R_{43} = R_{53} = 42.6316$, $R_{33} = 40$.

Table 3 shows the final allocation as determined by the heuristic.

Implementation

Beginning in May 2000, the DDP implemented the allocation heuristic in a real-time decision-support system environment, namely the DDP’s custom-designed, online order system. A software firm called Choice Systems implemented the DDP online order system. About 166 safety-net clinics and county hospitals in California used this system to place orders for $40.6 million of pharmaceutical products in May 2000 for the first ordering period. The online ordering and distribution system consisted of three main components: the order-entry system, the order-processing system, and the order-distribution system (Figure 1).

The order-entry system allowed clinics to enter their orders using the Internet. Each clinic was provided with a user name and a password. Clinics could prepare, modify, and submit orders during each ordering period. Further, the system automatically checked that clinics met minimum-order constraints and did not exceed their drug budgets. While placing their orders, the clinics saw a running total of their orders and the remaining budget, and the system accepted only orders that were within the budget and greater than or equal to the minimum order quantity. The clinics could sort drugs by category and by pharmaceutical firm. Educating clinic employees about the ordering process was challenging because many were unfamiliar with computer-based ordering using the Internet. Also, the DDP managers had to make them understand that they might not get exactly what they ordered because of scarcity.

At the end of each ordering period, the order-processing system took all the orders and computed the allocations based on the above heuristic using the priority weight matrix $\pi_{ik}$ provided by the DDP managers. This system also produced reports on scarce
Figure 1: These architectural details reflect the product and information flow in the online drug-ordering and distribution system.
drugs, including degree of scarcity, and the clinics that might be responsible for creating the scarcity.

The third component of the system, order distribution, was not under the control of the Public Health Institute. Once the DDP managers were comfortable with all the allocations, they sent an electronic data interchange (EDI) or fax message to the pharmaceutical firms with the orders to be distributed to the different clinics and updated the ordering system to reflect the allocation of drugs so that the clinics knew what to expect from the different firms. The pharmaceutical firms contacted the clinics and informed them about the actual deliveries of the drugs.

A number of issues arose during implementation that prompted modifications to the original heuristic and priority weight matrices. One issue was that some drugs were available in several package sizes. For example, Monopril could be ordered in packages of 30 or 90. Clinics placed orders for different sized packages. The decision-support system computed their total cost based on wholesale prices for the packages. After determining the clinic’s overall allocation using the allocation heuristic, the decision-support tool had to calculate how many packages of the different types the clinic would receive. We determined package-level allocations using another heuristic called the package heuristic (Appendix). Once the dollar amount $R_k$ of drug $k$ that was to be allocated to clinic $i$ was determined, the package heuristic had to determine how many of each package size to give to a clinic so as to match the allocation as closely as possible to the request, and at the same time distribute the maximum amount of drugs. This problem has characteristics similar to the integer knapsack problem. The package heuristic adopted a greedy approach for each drug by starting from the largest package size, providing the maximum possible amount of the drug in that package and then going down to the next package size, one size at a time (Steps 4–6c).

For example, suppose that a clinic was allocated $802.50 of a drug that was available in three package sizes: (1) 100 capsules for $300.00; (2) 50 capsules for $175.00; and (3) 25 capsules for $90.00. If the clinic ordered three packets of 100 each, the heuristic would first allocate two packets with 100 and then allocate one packet with 50 (Steps 6a–6b). At this point, the remaining budget for this clinic would be $802.50—($600 + $175.00) = $27.50. The heuristic would then note the minimum amount the clinic would have to pay to get one more package (in this case it is $90.00) and add the $27.50 remaining budget to a surplus account (Step 6c). After processing all the orders for the drug in the above manner, the heuristic would distribute the surplus amount among the clinics in order of their priority weight $p_i$ so that each clinic that was short by at least the minimum package amount would get the minimum amount for one more package ($90.00 for the clinic in this example) until the surplus budget was exhausted or dropped below $90.00, the lowest package unit value (Steps 7–8b).

Another important issue that came up during implementation concerned the size of the clinics. The DDP managers had expected to receive orders from both big clinics and small clinics; however, they did not anticipate that some regions would have centralized procurement for the different clinics. Los Angeles County, for example, was one of these regions; it presented orders that were much higher than those presented by other clinics. We changed the priority-weight matrix to prevent such large customers from overshadowing the other clinics in the allocation process. To achieve this, we normalized the base weights for all the clinics with respect to Los Angeles County. For example, if the Los Angeles County hospitals’ budget was 100 times that of a small Native American clinic, we adjusted their base priority weights approximately in the ratio 1:100.

Yet another issue related to the size of the priority-weight matrix ($p_ik$): With over 150 clinics and 125 drugs, the full matrix would contain 18,750 values. To simplify the priority-weight determination, we assigned priority weight for each clinic consisting of a base value (determined by the size of the clinic) and a drug-specific value (which would be added to the base value) for certain crucial drug categories, such as mental health and HIV. For the remaining drugs, we used...
the clinic’s base value as the priority weight. The DDP managers could then input different base values and drug-specific weights to the allocation heuristics to obtain allocations that were reasonable and realistic.

Although we expected a number of glitches to show up in the allocation heuristic during the first period of execution (May 2000), they were very minor, and the order processing went smoothly. As expected, a small percentage of drugs was extremely popular and, as a result, very scarce; however, the allocation heuristic handled the allocation of these scarce drugs well, as it had been designed to do. More details about the actual ordering process can be found at (www.medpin.org/drug_dist/drug_dist.html).

Conclusions
The DDP managers have used the decision-support system since May 2000 when they received the first set of orders. As of November 2002, 90 percent of the total $170 million budget has been distributed to the various clinics with the help of the decision-support system. Despite the need for a few changes to the heuristic, the DDP managers judge the allocation heuristic to be very successful at providing an efficient, effective, and equitable method for determining drug allocations as part of the DDP’s order-processing system. A large number of uninsured patients who would otherwise have had difficult or no access to medication have received their prescribed drugs. Although an accurate count of the number of patients affected is not available, it is estimated that approximately 2.4 million 30-day drug prescriptions have been filled as part of the DDP. Further, many pharmaceutical firms are exploring use of the DDP’s decision-support system to ensure more efficient and equitable operation of the charitable drug programs they now operate.

Appendix
The Model
Indices/Sets
i: index that represents the different clinics (i = 1 . . . I).
j: index that represents the pharmaceutical firms (j = 1 . . . J).
k: index that represents the different drugs (k = 1 . . . K).
S: set of scarce drugs.
Dp: set of drugs manufactured by pharmaceutical firm p.
Cpl: set of drugs manufactured by firm p that are classified under drug category l.

Data
Mk: total number of packages in which drug k is sold.
qm: size of one package of drug k (by quantity contained) in package size m (m = 1 . . . M).
cp: negotiated price of one package of drug k in package size m.
Bp: total budget allowed for clinic p.
dt: maximum disbursement of drugs in the period by firm p (in dollars).
ddp: maximum disbursement of drug k in the period by firm p (in dollars).
dcpl: maximum disbursement of all drugs in the category l in the period by firm p (in dollars).
eik: minimum order for drug k from any clinic.
Oik: dollar value of drug k requested by clinic p.
OPikm: number of packages of size m of drug k ordered by clinic p.
pik: priority weight of drug k for clinic p.
Rik: dollar value of drug k received by clinic p.
Yik: 0–1 variable that represents whether clinic p obtained any allocation of drug k.

Objectives

\[ O1: \text{min} \sum_{j=1}^{J} \left( dt_j - \sum_{i=1}^{I} \sum_{k \in D_j} R_{ik} \right) \]

\[ O2: \text{min} \sum_{k \in S} \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{n=1}^{N} \left( R_{ik} * \pi_{nk} * O_{nk} - R_{ik} * \pi_{ik} * O_{ik} \right)^2 . \]

Constraints

Clinic Constraints
\[ \sum_{j=1}^{J} \sum_{k \in D_j} O_{ik} \leq B_i \quad \forall i. \quad (1) \]

Pharmaceutical Firm Constraints
\[ \sum_{i=1}^{I} \sum_{k \in D_j} R_{ik} \leq dt_j \quad \forall j. \quad (2) \]
Allocating Scarce Drugs

Step 0. Allocate $R_{ik} = O_{ik}$ for each clinic and drug. Check whether all constraints are valid. If yes, then go to Step 4; else go to Step 1.

Step 1. For each scarce manufacturer $j$ (such that $\sum_{l=1}^{L} \sum_{k \in C_{l}} R_{ik} > d_{l}$) or $\exists k \in D_{j}$ s.t. $\sum_{l=1}^{L} R_{ik} > dd_{ik}$ or $\exists l$ s.t. $\sum_{l=1}^{L} \sum_{k \in C_{l}} R_{ik} > dc_{l}$:

- Step 2a. Reduce the budget of all drugs in this category to $O_{ik}$.
- Step 2b. Reduce the budget for all drugs in this category from the first two clinics to bring all clinics with shortfalls up to the minimum order amount, first with the top three clinics, then with the top four clinics, and so on as needed to bring all clinics with shortfalls up to the minimum order. If no more clinics remain from which the surplus was generated and go to Step 3j; else repeat Step 3i(3), first with the top three clinics that have more than the minimum quantity by the ratio of allocation to the next scarce drug in the category.

Step 2. For each scarce drug category $l$ (such that $\sum_{l=1}^{L} \sum_{k \in C_{l}} R_{ik} > dc_{l}$) or $\exists k \in C_{l}$ s.t. $\sum_{l=1}^{L} R_{ik} > dd_{ik}$:

- Step 3a. Determine the maximum number of clinics $Z$ that can be supported with the minimum order requirement, i.e., $Z = \text{largest integer less than or equal to } \min((d_{l}/e_k), l)$.
- Step 3b. Select the top $Z$ clinics (based on $\pi_{ik}$) to fulfill orders and set $R_{ik} = 0$ for the rest.
- Step 3c. Check whether $\sum_{l=1}^{L} R_{ik} \geq dd_{ik}$. If not, go to Step 3d; else go to Step 3j.
- Step 3d. Compute the total weight of orders across all the remaining clinics ($\hat{I}$ where $i \in \hat{I}$ implies that $R_{ik} > 0$) and $W = \sum_{i \in \hat{I}} \pi_{ik} O_{ik}$.
- Step 3e. Allocate drug to each clinic based on its fraction $R_{ik} = ((\pi_{ik} \cdot R_{ik} \cdot dd_{ik})/W)$.
- Step 3f. If any clinic is allocated more than requested ($R_{ik} > O_{ik}$), add this amount to the surplus bank and set $R_{ik}$ to $O_{ik}$.
- Step 3g. Check whether allocations meet minimum requirements, i.e., $R_{ik} > e_k$ for all clinics that have been allocated a positive dollar amount. If yes, allocate any surplus equally among all the clinics, and then go to Step 3j; else go to Step 3h.

- Step 3h. If the clinics can be brought to the minimum level with the surplus money, do so; then exhaust the remaining surplus by dividing it equally and go to Step 3j; else go to Step 3i.

- Step 3i. Go to Step 3i(1).
- • Step 3i(1). Sort the clinics that have more than the minimum quantity by the ratio of allocation to actual order.
- • Step 3i(2). Reduce the allocation of the first clinic (while maintaining at least the minimum order) so that the first and second clinics have the same ratio of allocation to order. Reallocate the extra from the first clinic to the clinics with shortfalls so that they meet the minimum order. If the surplus generated is insufficient to bring all clinics with shortfalls up to the minimum-order amount, go to Step 3i(3); else, if all clinics have met the minimum order, allocate the remaining surplus equally among the clinics from which the surplus was created and go to Step 3j.

\[
\sum_{i=1}^{I} \sum_{k \in C_{l}} R_{ik} \leq dc_{l} \quad \forall j, l, \quad (3)
\]

\[
\sum_{i=1}^{I} R_{ik} \leq dd_{ik} \quad \forall j, k \in D_{j}, \quad (4)
\]

\[
O_{ik} \cdot Y_{ik} - R_{ik} \geq 0 \quad \forall i, k, \quad (5)
\]

\[
R_{ik} - Y_{ik} \cdot e_{k} \geq 0 \quad \forall i, k, \quad (6)
\]

\[
R_{ik} \in R^{+}, \quad Y_{ik} \in \{0, 1\}. \quad (7)
\]
Identify all the scarce drugs in the category; start with the first drug \( k \) in the category, and go to Step 3.

**Step 2c.** If all the drug categories for the manufacturer have been checked, go to Step 1a; else go to Step 2 with the next drug category for the manufacturer.

**Step 1a.** Check whether \( \sum_{k=1}^{n} R_{ik} \leq dt \). If yes, go to Step 1c; else go to Step 1b.

**Step 1b.** Reduce the budget for all drugs by this manufacturer in the following manner:

\[
dd_{ik} = dd_{ik} * \frac{dt_j}{\sum_{k=1}^{n} R_{ik}}.
\]

Identify all the scarce drugs in the category, and go to Step 2.

**Step 1c.** If the drugs from all the manufacturers have been allocated, end; else go to Step 1 with the next manufacturer.

### Package Heuristic

This heuristic is used after the dollar budgets have been allocated only when a drug comes in multiple package sizes. Let \( RP_{ikm} \) be the decision variable that represents the number of packages of \( m \)th size of drug \( k \) that are allocated to clinic \( i \).

**Step 4.** For each scarce drug (identified in the allocation heuristic above) \( k \), set the surplus budget to 0.

**Step 5.** For each clinic \( i \), set the remaining budget (RM) to \( R_{ik} \).

**Step 6.** For package size \( m \), starting with the biggest size and going down:

**Step 6a.** Set

\[
RP_{ikm} = \min\left(\text{integer}\left(\frac{RM}{cP_{km}}, \ OP_{ikm}\right)\right).
\]

where \( \text{integer}(x) \) is the largest integer less than or equal to \( x \).

**Step 6b.** Reduce RM by \( RP_{ikm} * cP_{km} \).

**Step 6c.** If \( RP_{ikm} > 0 \), then go to Step 6a with the next biggest package size; else add RM to the surplus and go to Step 5 with the next clinic. If no clinics remain, go to Step 7.

**Step 7.** Sort the clinics according to \( \pi_{ik} \).

**Step 8.** For each clinic \( i \)

**Step 8a.** If \( R_{ik} - \sum_{m\in\mathcal{M}} RP_{ikm} > \text{Minval} \) (where Minval is the minimum value of a package size, \( cP_{km_{min}} \)) and surplus > Minval, then increase \( RP_{ikm_{\text{min}}} \) by 1 and subtract Minval from surplus.

**Step 8b.** If surplus < Minval and scarce drugs still remain, go to Step 4 with the next drug; else go to Step 8 with the next clinic. If no scarce drugs remain, end.

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### References


Kathryn S. Duke, JD, MPH, Program Director of Medpin, 505 14th Street, Suite 810, Oakland, California 94612, writes: “Under the California litigation settlement, we are distributing nineteen pharmaceutical companies’ products, totaling $148 million in value, to “safety net” clinics and hospitals throughout California over a three year period. In December 1999, we approached Professor Swaminathan to help us with designing a fair and efficient mechanism to distribute these drugs. This problem is very difficult and although we understood the pros and cons of allocating drugs and had a good idea on the constraints on drug budgets that we faced, we had limited experience and knowledge on how to develop a rational model and a...”
decision-support solution that could help us in allocating these drugs.

“Professor Swaminathan modeled the problem capturing all the constraints associated with drug allocation and developed an efficient methodology to determine drug allocations once clinics had placed orders with us. We benefited a great deal from this thorough analysis and solution development. He also mediated with the software provider, ChoiceSystems Inc., the company that created a custom-designed Internet-based ordering system for the DDP, to integrate this allocation approach into the drug distribution software.

“Professor Swaminathan’s analysis and allocation heuristic was very helpful to us in conceptualizing and then operationalizing the unique approach we are taking to implement a litigation settlement. The decision-support tool helped us gain very useful insights on the ordering of drugs and their allocations. When we started our work on DDP we were concerned about our ability to design an allocation system that would meet all of our project’s goals in a cost-efficient manner. We also knew that doing the allocation manually would have been extremely cumbersome if at all possible. We are pleased to report that the basic concept of the DDP continues to work well as we continue to disburse drugs under this project, thanks to the practical experience and conceptual assistance that Professor Swaminathan provided us.”