Managing New and Differentiated Remanufactured Products

Geraldo Ferrer¹
Graduate School of Business and Public Policy, Naval Postgraduate School
Monterey, CA 93943-5103

Jayashankar M. Swaminathan
Kenan-Flagler Business School, The University of North Carolina at Chapel Hill
Chapel Hill, NC 27599-3490

Abstract

We study a firm that makes new products in the first period and uses returned cores to make remanufactured products (along with new products) in future periods. The remanufactured product is differentiated from the new product, so the firm needs to choose differentiated prices. We analyze the monopoly environment in two-period, multi-period (three, four and five) and infinite planning horizons, and characterize the optimal remanufacturing and pricing strategy for the firm. In the process, we identify remanufacturing savings thresholds that determine the production and pricing strategy for the firm. Among other results, we find—counter to intuition—that in a finite-horizon, multi-period setting, the optimal policy is not necessarily monotone in remanufacturing savings.

Key words: Remanufacturing; differentiated products; monopoly; product portfolio; multi-period horizon.

1 Introduction

Remanufacturing is a process in which used products are disassembled, and their parts are repaired and used in the production of new products. Remanufactured products often serve entry-level customers that are attracted by the brand, but do not wish to pay the price of a new product—as with used cars. A successful remanufacturing operation often adopts high quality standards that allow it to offer products that enhance brand equity and keep customers loyal. Often, the company expands its market coverage by offering remanufactured products at a low price, side-by-side with the new products. A well-designed product line that includes remanufactured and new products may increase market share while sustaining a high profit margin. An organization will find it most economical to remanufacture equipment that satisfies the following conditions: it is owned in large quantities (economies-of-scale requirement); all units have the same configuration (learning curve requirement); all units can be brought to current state of technology (non-obsolescence requirement). The US Department of Defense consistently remanufactures most of its valuable assets (propulsion units, vehicles, radar systems) precisely because they satisfy these conditions. In fact, remanufacturing has been recognized in many government reports as an economical way to maintain all fleets at desirable levels (US DoD 2005).

¹ Corresponding author. Email: gerrer@nps.edu. Tel: +1-831-656-3290.
This study is concerned with the remanufacturing operation from the supplier’s viewpoint. We analyze a monopoly model in which the remanufactured and the original products are clearly distinguishable. We develop models for several planning horizons, such that the manufacturer produces just the new product in the first period, but has the option of making new and remanufactured products in subsequent periods. Pricing decisions impact the dynamics across periods in such cases. For example, if the price is high in the first period, profits in the first period might increase; but, the number of reusable products available in the second period decreases, thereby reducing second-period profit potential. However, if the price is low in the first period, initial profits might decrease, but the firm has better remanufacturing opportunity in future periods.

We start by deriving the optimal quantities and prices for such an operation, and characterize the optimal conditions for a monopolist that offers both product types, unconstrained by the availability of cores, characterizing the strategic regions of operation. Then, we analyze models constrained by core availability. We introduce the infinite-horizon model, assuming steady behavior in the second period and beyond. Finally, we introduce the multi-period problem with limited planning horizon and discuss the initialization and the end-gaming behavior. We provide analytical insights for all cases.

The rest of the paper is organized as follows: In Section 2, we discuss the related literature and our contributions. In Section 3, we present our model and results. In Section 3.1, we present the infinite horizon, and in Section 3.2, we analyze the two-period planning horizon. In Section 3.3, we study the multi-period planning horizon—focusing in particular on three-, four- and five-period horizons. We conclude in Section 4.

2 Related Research

Buy-backs, trade-ins and leasing schemes provide market benefits to the manufacturer that are not trivial. These benefits have raised the question: what is the optimal sales strategy of a company making a durable product to improve its potential profits? How should buy-back and leases be considered? In an early study, Levinthal and Purohit (1989) provide a two-period model that describes a monopolist company selling a durable product for which it may subsequently introduce an improved version. In a situation like this, the customers will expect a forthcoming product, and as a result, will lower the price they are willing to pay for the current product due to its expected loss in value. A buy-back policy is found to be more profitable for large improvements, whereas the policy to phase-out sales of the old product is optimal for modest levels of improvement. Purohit (1992) examines the situation in which technology changes rapidly, and the new versions of a product make earlier versions obsolete. However, when the products are durable, there is the possibility of secondary markets for used products as well as for outdated products. These examples can be found everywhere today—particularly in computer and high-tech
industries. Purohit develops a model to explore the relationship between primary and secondary markets for automobiles. The results suggest that the depreciation of old models is influenced strongly by the types of changes in new models. In related research, Hendel and Lizzeri (1999) propose a model in which consumers have heterogeneous valuations for quality; thus, the used-good markets play an allocative role to address the interference introduced by the first market on the secondary market of a monopolist company. Market-related issues in remanufacturing are related to discussions of other secondary distribution/segmentation channels, as found in Purohit and Staelin (1994). They provide different policies to increase the total manufacturer’s profit in a two-period model that compares buy-back and lease schemes.

There is also great interest in supply chain coordination to maximize multi-period profits. Moorthy and Png (1992), Kim and Chhajed (2002), and Krishnan and Zhu (2006) develop quality-based models for new product development with multiple market segments under different marketing and manufacturing considerations. Desai, Koenigsberg and Purohit (2002) study the coordination problem between manufacturer and retailer of durable products which arises from the potential competition from a secondary market in future periods. In another two-period model, Desai, Koenigsberg and Purohit (2007) evaluate how a manufacturer decides first-period production level under stochastic demand, with excess production carry-over to second period.

The literature on the economics of remanufacturing has seen important contributions in the study of supply-chain coordination (Corbett & Savaskan 2002; Savaskan, Bhattacharya, & Van Wassenhove 2004; Savaskan & Van Wassenhove 2006), collection and leasing (Guide, Teunter, & Van Wassenhove 2003; Ray, Boyaci, & Aras 2003; Heese, Cattani, Ferrer, Gilland, & Roth 2005; Qu & Williams 2008; Liang, Pokharel & Lim 2009). The research in this field is rapidly evolving, as witnessed by the many special issues and technical books (Corbett & Kleindorfer 2001; Dekker, Fleischmann, Inderfurth, & Van Wassenhove 2004; Flapper, Van Nunen, & Van Wassenhove 2005). For an extensive review of the reverse logistics literature, an interested reader may refer to Fleischmann et al. (1997) and Guide, Jayaraman, Srivastava, & Benton (2000).

Debo, Toktay, and Van Wassenhove (2005) develop a multi-period, infinite-horizon model to price remanufactured goods and to determine the product technology a priori to maximize the profitability of the market segmentation. Majumder and Groenevelt (2001) describe a two-period model where the original-equipment manufacturer (OEM) may choose to remanufacture in the second-period or not. The reverse logistics process is based on the “shell allocation mechanism” observed in the respective market. Four of these mechanisms are considered: whether one or the other player (the OEM and the independent operator) can or cannot use the cores that are not utilized by the other company. Ferrer and Swaminathan (2006) expand on the above model and characterize the optimal strategies (production
quantities and prices) in monopoly and duopoly environments for two-period, multi-period and infinite-horizon settings. One of the main findings is that if that once the profit margin in remanufacturing reaches defined threshold, then the firm reduces the price in the first period in order to sell more units and increase the number of available cores in the following periods. They also prove that, if the savings of one party from remanufacturing is high enough compared other parties in the case of competition, the original organization remanufactures all available cores that it collects. Furthermore, they show that for most practical environments, the optimal strategies obtained for the two-period problem are quite similar to the results of multi-period problems. Ferguson and Toktay (2006) formulate a two-period problem in order to examine the recovery strategy of the OEM in the face of a competitor. In the first part of the paper, they show the cannibalization effect of the remanufactured products on the original products. In the second part, they present two entry-deterrent strategies that the OEM may follow in order to keep the remanufacturer away from the market. The results show that the OEM may remanufacture (after collecting the cores) or collect the cores, but not remanufacture (preemptive collection) based on factors such as collection or remanufacturing cost.

It is worth mentioning the increasing literature on closed-loop supply chain that is generally concerned managing the inventory of used cores to meet the needs of the remanufacturing process, either in quantities, quality or both. Recent examples include Choi, Hwang and Koh (2007), Konstantaras and Papachristos (2007), Teunter, Kaparis and Tang (2008), Visich, Li and Khumawala (2007), Zikopoulos and Tagaras (2007) and Zikopoulos and Tagaras (2008). The tutorial by Souza summarizes some of the key components of these models (Souza, 2008).

In most of the above literature, it is assumed that remanufactured and original products are not distinguishable, and in those that study differentiated products, the analysis is restricted to two periods. However, the remanufactured products are often offered as an alternative to the original products with lower price and/or quality. For example, there are a number of industries such as computer systems, auto components and office equipment in which the reconditioned product is priced lower than original products in order to capture the demand in different markets (Ferrer 1997; Ayres, Ferrer, van Leynseele 1997). Robotis, Bhattacharya, and Van Wassenhove (2005) analyze the case of a reseller who procures cores which have an older technology and then either resells a fraction of these cores in “as is” condition (in a developing market) or remanufactures the cores and then sells them at a higher price. They show that the number of collected cores decreases when the reseller utilizes remanufacturing and, depending on the cost structure, it might be always more profitable to remanufacture the collected cores. Incorporating the distinguished (or quality differentiated) nature of remanufactured products complicates the problem substantially, since there is one more lever (related to price differentiation) that needs to be considered.
In this paper, we study a firm that makes new products in the first period and uses returned cores to make remanufactured products (along with new products) in future periods. The remanufactured product is differentiated from the new product, so the firm needs to choose differentiated prices. We present the multi-period (three, four and five) planning horizons and show their relationship with past analysis with the two-period and the infinite planning horizon. In all cases, we characterize the remanufacturing savings thresholds that define the optimal remanufacturing quantity and price strategy for the firm. Among other results, we find (counter to intuition) that in a finite-horizon, multi-period setting, the optimal policy is not necessarily monotone in remanufacturing savings, alternating periods of high and low production of new products in order to exploit the greater profits that can be obtained with a remanufactured product.

3 Monopolist with Differentiated Products

Customers distinguish the remanufactured from the new products made by the monopolist. Hence, the market is better served under price discrimination that allows the customer to choose the product that generates the greater utility. This situation represents, for instance, the tire retreading industry. Although the industry is generally characterized by intense competition, some specialty tires have few suppliers (aircraft landing gears, off-road construction rigs, race cars and other specialty uses) that face almost monopolistic privileges. We model this environment, describing a monopolist that makes just the new product in period 1 and both new and remanufactured products in subsequent periods. We use the following notation:

Variables and Parameters

\( Q \) Size of the potential market, constant every period.

\( p_{ij}, q_{ij} \) Price charged \((p)\) and quantity demanded \((q)\) for product \(j\) in period \(i\). The subscripts are \(i = 1, 2, \ldots\) and \(j = N\) (new) or \(R\) (remanufactured). In the first period, only the new product is made, and \(j\) is usually omitted \((p_1 = p_{1N} \text{ and } q_1 = q_{1N})\). The demand function is assumed to be linearly decreasing in price.

\( c, s \) Marginal cost to make a new product \((c)\) and cost savings per remanufactured product \((s)\). The marginal cost to make a remanufactured product is \(c - s\).

\( \gamma \) Core collection yield \((\gamma \leq 1)\), defined as the fraction of new products made in period \(i\) that is available for remanufacturing in period \(i+1\). Hence, \(q_{iR} \leq \gamma (q_{i-1,N})\).

\( \beta \) Discount factor per period \((\beta \leq 1)\). The discount rate increases as \(\beta\) decreases. E.g., if the discount rate = 5%, than \(\beta = 0.95\).

\( \alpha \) Customers’ tolerance for the remanufactured product \((\alpha \leq 1)\), defined as a fraction of the utility provided by the new product.
Two of these parameters, remanufacturing savings ($s$) and collection yield ($\gamma$), characterize the firm’s ability to perform key remanufacturing activities. The cost parameter ($c$) characterizes the firm’s ability to make new products. The discount factor ($\beta$) encompasses both the value of the money and the time between two cycles. It is closely related to the product’s technological obsolescence. We select the remanufacturing savings ($s$) as the key parameter to describe the strategy space in each scenario that we analyze, because it seems to be the parameter over which managers can have the greatest impact through strategic management of the facility’s resources.

In what follows, we present a useful lemma that describes the self-selection quantities associated with the prices of the new and remanufactured products.

**Lemma 1**: Suppose that two competing products, new (N) and remanufactured (R), are in the market. Let there be $Q$ potential consumers, characterized by the variable $z \in (0, Q)$ uniformly distributed in this domain, according to their valuation of the new product. Let $\alpha \in (0, 1)$ indicate the customers’ tolerance for the remanufactured product. Large values of $\alpha$ indicate that customers accept the remanufactured product better than if $\alpha$ is small. The utility that a consumer of type $z$ enjoys when buying the product is $U_N(z) = z - p_N$ (new) or $U_R(z) = \alpha z - p_R$ (remanufactured). If $\alpha \in \left[p_R/p_N, 1 - (p_N - p_R)/Q\right]$, the number of consumers buying each product type is:

$$q_N(p_N, p_R) = \frac{(1-\alpha)Q - p_N + p_R}{1-\alpha},$$  \hspace{1cm} (1)$$

$$q_R(p_N, p_R) = \frac{\alpha p_N - p_R}{\alpha(1-\alpha)},$$  \hspace{1cm} (2)$$

If $\alpha < p_R/p_N$, $q_R = 0$ and $q_N = Q - p_N$. If $\alpha > 1 - (p_N - p_R)/Q$, $q_R = Q - p_R/\alpha$ and $q_N = 0$.


Equations (1) and (2) are similar to the demand functions in a Bertrand duopoly: the demand for new products increases when their price decreases or when the price of remanufactured products increases, and vice-versa. Lemma 1 provides the coefficients for the Bertrand demand function, identifying how customers select either product. Using this result, we can express the quantities sold as functions of the behavior-inducing prices. Since the monopolist would only consider prices that would lead to non-negative demand, we may disregard prices that would cause $\alpha \notin \left[p_R/p_N, 1 - (p_N - p_R)/Q\right]$.

Based on Equations (1) and (2), we are able to define the optimal policy of a monopolist offering a portfolio of a new and a remanufactured product for a single period, unconstrained by an existing supply of cores. The profit function would be:

$$\max_{p_N, p_R} \left( (p_N - c) q_N + (p_R - c + s) q_R \right)$$
subject to

\[ q_N \geq 0, \quad \text{and} \]

\[ q_R \geq 0, \]

where \( q_N \) and \( q_R \) are given by Equations (1) and (2), respectively. It is trivial to show that the monopolist should choose one of the following quantities, depending on the operating parameters:

\[
\begin{cases}
q_N = \frac{Q - c}{2} & q_R = 0 \quad \text{if } s \leq c(1-\alpha) \\
q_N = \frac{Q(1-\alpha) - s}{2(1-\alpha)} & q_R = \frac{s - c(1-\alpha)}{2(1-\alpha)\alpha} \quad \text{if } c(1-\alpha) < s \leq Q(1-\alpha) \\
q_N = 0 & q_R = \frac{Q(1-\alpha) - c + s}{2\alpha} \quad \text{if } Q(1-\alpha) < s
\end{cases}
\]

This simple result may be useful in environments where myopic decisions are sufficient. For example, the supply of cores may be very large compared to the potential demand, so it is safe to treat the supply of cores as “unconstrained” and adopt the policy above. However, an important aspect of remanufacturing is that the production in one period is dependent of the output in previous periods, so myopic decisions may be inefficient. In those cases, the objective function of the remanufacturing company must recognize that the production of remanufactured products is constrained by the availability of cores at the beginning of each period.

We restrict the analysis to the cases in which only cores derived from new product sales can be remanufactured. In other words, a given item can only have two lives: once as a new product and once as a remanufactured product. While one could remanufacture cores extracted from remanufactured products, it is usually not justifiable on economic terms alone: cores obtained from remanufactured products were usually built on previous technology, which renders their design obsolete. Electronic components may be obsolete; components subject to mechanical stress (tires, engine blocks and axles, turbine axles) start showing the consequences of fatigue, and so on. So, it is reasonable to focus the analysis on products that are remanufactured just once. This restriction reflects the fast obsolescence rate of most industrial products because, although remanufacturing older products might be technically feasible, the economic value is low, and the process loses much of its interest.

In the following three subsections, we analyze models that consider an infinite horizon made of identical periods (Section 3.1), a two-period horizon that considers end-of-period effects (Section 3.2) and finite horizons with three, four and five periods (Section 3.3). Mathematical proofs are included in the Appendix.
3.1 Infinite Planning Horizon

If the planning horizon is very long, we may approximate the problem with the infinite planning horizon model. The infinite horizon case was discussed in Debo et al. (2005). We describe period 1 as the initialization period when the monopolist makes just new products. Starting in period 2, the firm adopts a constant policy that may include a remanufactured item to serve the low end of the market, as follows:

\[
\text{Max} \left( p_1 - c \right) q_1 + \frac{\beta}{1-\beta} \left( p_N - c \right) q_N + \left( p_R - c + s \right) q_R
\]

subject to

\[
q_1 = Q - p_1
\]
\[
\gamma q_1 \geq q_R
\]
\[
\gamma q_N \geq q_R
\]

Moreover, \(q_N\) and \(q_R\) satisfy Equations (1) and (2) respectively, and all variables satisfy the non-negativity constraint. We present the result without proof:

**Theorem 1:** There are three critical values, \(s_1 < s_2 < s_3\), defining four scenarios:

D. If \(s \leq c(1-\alpha) = s_1\), the remanufacturing process generates too little savings. Hence, no remanufactured product is made, and the case defaults to the standard monopoly policy:

\[
p_1 = p_N = \frac{Q + c}{2} \quad \text{and} \quad q_1 = q_N = \frac{Q - c}{2}.
\]

D. If \(s_1 < s \leq \frac{(1-\alpha)(Q\alpha\gamma + c)}{1+\alpha\gamma} = s_2\), the remanufacturing process uses just some of the cores available in each period. The quantities made are:

\[
q_1 = \frac{Q - c}{2}, \quad q_N = \frac{Q(1-\alpha) - s}{2(1-\alpha)} \quad \text{and} \quad q_R = \frac{s - c(1-\alpha)}{2(1-\alpha)^2}.
\]

D. If \(s_2 < s \leq Q\alpha(1+\gamma) + c\left(1-\alpha\left(2+\gamma\right)\right) = s_3\), the remanufacturing process uses just some of the cores collected in period 1, and all cores collected in subsequent periods, making these quantities:

\[
q_1 = \frac{Q - c}{2}, \quad q_N = \frac{Q(1+\alpha\gamma) - c(1+\gamma) + s\gamma}{2 + 2\alpha\gamma(2+\gamma)} \quad \text{and} \quad q_R = \gamma q_N.
\]

D. When \(s_3 < s\), the remanufacturing process uses all cores collected every period and makes these quantities:

\[
q_1 = q_N = \frac{Q(1+\alpha\beta\gamma) - c(1+\beta\gamma) + s\beta\gamma}{2 + 2\alpha\beta\gamma(2+\gamma)} \quad \text{and} \quad q_R = \gamma q_N.
\]
The following example illustrates these policies as the remanufacturing savings \( s \) vary in its domain \((0, c)\). The potential demand is 1000 units/period, customers are uniformly distributed from 0 to 1000, and the cost to make the new product is 600. Hence, \( Q = 1000, c = 600 \). Moreover, let \( \alpha = 0.80, \beta = 0.95, \gamma = 87\% \). Figure 1 illustrates this case, with critical values separating the four scenarios at \( s_1 = 120 < s_2 = 152.8 < s_3 = 718.4 \). As in other examples in this article, scenario D3 is null because \( s_3 > c \).

A counter-intuitive observation is that the graphs are quite angular, often not monotonic.

![Figure 1. Optimal Policy for Infinite Planning Horizon](image)

Here is an interpretation of these charts: As the remanufacturing savings increase from 0 to 600 (the largest meaningful value of \( s \)), the prices generally decrease, and the total quantity increases. However, the number of new units made decreases as \( s \) changes from 120 to 152.8, and it increases again for \( s > 152.8 \). Here is the intuition: when \( 120 < s < 152.8 \), the savings in the remanufacturing process are limited. Hence, the price charged is not much lower than the new product’s price, which leads to limited demand for remanufactured products. In this range, not all cores are used, and the firm can afford to charge full monopoly price for the new product. Customers migrate from new to remanufactured products, shown in the graph as \( q_N \) decreases and \( q_R \) increases. Once the remanufacturing savings reach the second threshold \( (s = 152.8) \) the firm is able to offer the remanufactured product at a price that creates demand for all cores. As the savings increase, the remanufactured product becomes more profitable to the firm. To take advantage of the greater savings, the firm lowers the price of the new product to boost its sales, leading to more cores available for remanufacturing in subsequent periods. Since all cores generated are remanufactured, the sales of new and remanufactured products are directly related in this range, shown in the graph as \( q_N \) and \( q_R \) increasing in seemingly parallel fashion.
### 3.2 Two-period Planning Horizon

If the company has a two-period planning horizon, one important effect that was absent in the infinite-horizon model is introduced: the end-game behavior. Since the monopolist does not consider the supply or demand beyond the second period, the remanufacturing potential of the new products made in period 2 is not recognized. A variation of this problem was addressed in Ferguson and Toktay (2006). This simplified approach follows the model presented in Ferrer and Swaminathan (2006):

\[
\begin{align*}
\text{Max} & \quad \left( \frac{p_1 - c}{p_1} q_1 + \beta \left( \frac{p_{2N} - c}{p_{2N}} q_{2N} + \left( \frac{p_{2R} - c + s}{p_{2R}} q_{2R} \right) \right) \right),
\end{align*}
\]

subject to

\[
\begin{align*}
q_1 &= Q - p_1, \\
q_{2R} &\geq q_{2N}.
\end{align*}
\]

Moreover, \( q_{2N} \) and \( q_{2R} \) satisfy Equations (1) and (2) respectively, and all variables satisfy the non-negativity constraint. This leads to the following result:

**Theorem 2**: If the monopolist has a planning horizon of 2 periods \((M = 2)\), there are three critical values, \( s_{21} < s_{22} < s_{23} \), which define four scenarios representing the optimal policy:

- **D_{20}**: When the remanufacturing savings satisfies \( s \leq c(1 - \alpha) = s_{21} \), no remanufactured product is made, and the case defaults to the standard monopoly policy:
  \[ p_1 = p_{2N} = \frac{Q + c}{2} \quad \text{and} \quad q_1 = q_{2N} = \frac{Q - c}{2}. \]

- **D_{21}**: When the remanufacturing savings satisfies \( s_{21} < s \leq (Q \alpha \gamma + c(1 - \alpha \gamma))(1 - \alpha) = s_{22} \), the monopolist makes both product types—albeit not using all cores derived from period 1 sales. The quantities produced are:
  \[ q_1 = \frac{Q - c}{2}, \quad q_{2N} = \frac{Q(1 - \alpha) - s}{2(1 - \alpha)}, \quad \text{and} \quad q_{2R} = \frac{s - c(1 - \alpha)}{2(1 - \alpha)\lambda}. \]

- **D_{22}**: When \( s_{22} < s \leq \frac{Q(1 - \alpha \gamma^2(1 - \beta \gamma + \alpha \beta \gamma)) - c(1 - \alpha \gamma)}{\alpha \beta \gamma^2} = s_{23} \), the firm makes both product types, using all cores available at the beginning of period 2. The quantities produced are:
  \[ q_1 = \frac{Q - c + \gamma \lambda}{2}, \quad q_{2N} = \frac{Q(1 - \alpha) - s + \lambda}{2(1 - \alpha)}, \quad \text{and} \quad q_{2R} = \frac{s - c(1 - \alpha) - \lambda}{2(1 - \alpha)\lambda}, \]

where \( \lambda = \frac{s - c(1 - \alpha) - \alpha \gamma(1 - \alpha)(Q - c)}{1 + \alpha \beta \gamma^2(1 - \alpha)}. \)
When the remanufacturing savings satisfies $s_{23} < s$, the monopolist uses all cores derived from sales in period 1, but does not make new products in period 2. The quantities produced are:

$$q_1 = \frac{Q - c + \lambda}{2}, \quad q_{2N} = 0$$

and

$$q_{2R} = \frac{Q - c + s - \lambda/\beta}{2\alpha},$$

where

$$\lambda = \frac{(s - c(1-\alpha\gamma)+\alpha Q(1-\gamma)/\beta}{1+\alpha\beta y^2}.$$

**Proof:** See Appendix.

We use the same market as in the previous example to illustrate. This time, however, the monopolist adopts a two-period planning horizon, seen in Figure 2. While the prices and quantities represent the operational decisions that the firm makes on a regular basis, the remanufacturing savings parameter is a technological choice made at the time the product was designed, as proposed by Debo et al (2005). Hence, it is useful to see how it affects the optimal policy. The critical values separate the scenarios at $s_{21} = 120 < s_{22} = 175.7 < s_{23} = 411.4$. The first graph shows the three quantities produced as a function of the remanufacturing savings parameter, and the second shows the respective prices.

![Figure 2. Optimal Policy for 2-period Planning Horizon](image-url)

As the remanufacturing savings increase, prices decrease, and total quantity increases. When $120 < s < 175.7$, the remanufacturing savings are not sufficient to generate a strong demand for the remanufactured product, so not all cores are used. Hence, the firm maintains the standard monopoly price for the new product, and customers gradually migrate from new to remanufactured products in the second period. When $s \geq 175.7$, all cores are used, and the firm lowers the price in the first period to generate more cores for the second period. This allows an increase in the demand of new product in first period and of remanufactured product in second period, shown in the graph as $q_1$ and $q_{2R}$ increasing together. To maintain demand, price for the remanufactured product gradually decreases. This drives away the demand for new product in second period, until eventually it is not offered.
The end-gaming behavior and the initialization process are intertwined here: when remanufacturing is very profitable, production in first period can be very large to enable a large remanufacturing volume in the second period—a process that displaces the new product from the market in period 2. Notice that the new product price in second period is independent of the savings parameter. Likewise, the price of the remanufactured product changes very little when \( s > s_{22} \). This implies that the company has much to gain in improving its remanufacturing capability (increasing \( s \)), because \textit{those gains directly affect the bottom line}. When \( s > s_{23} \), the new product is not made, and the price of the remanufactured product reduces to help increase its sales volume. The following corollaries, shown without proof, are derived from direct observation of the structure of the optimal policies:

**Corollary 2.1:**

i. Lower manufacturing costs (\( \downarrow c \)) or higher remanufacturing savings (\( \uparrow s \)) reduce prices.

ii. Higher customer tolerance for remanufactured products (\( \uparrow \alpha \)) increases the price for the remanufactured product.

**Corollary 2.2:**

iii. Higher customer tolerance (\( \uparrow \alpha \)), higher collection rate (\( \uparrow \gamma \)) or shorter replacement cycle (\( \uparrow \beta \)) increases the proportion of remanufactured products in the second period.

iv. Higher manufacturing cost (\( \uparrow c \)) decreases the proportion of new products in the second period.

### 3.3 Multi-period Planning Horizon

Section 3.2 showed the end-gaming behavior that characterizes the optimal policy when the planning horizon is only two periods and known in advance. This section expands the analysis to longer planning horizons, to reveal a pattern of behavior in the optimal policy. If the firm has a multi-period planning horizon, the objective function is:

\[
\max_{p_1, p_i, N, p_i, R} \pi_M = \left( p_1 - c \right) q_{11} + \sum_{i=2}^{M} \beta^{-1} \left( p_{i,N} - c \right) q_{i,N} + \left( p_{i,R} - c + s \right) q_{i,R}
\]

subject to

\[
q_1 = Q - p_1 \\
q_{1,R} \geq q_{1,N} \\
q_{i+1,N} \geq q_{i,R}, \quad i = 3, \ldots, M
\]

All variables must satisfy the non-negativity constraint. Consider the interior solution of this problem, in which the constraints are not binding. Since \( q_{i,N} \) and \( q_{i,R} \) must satisfy Equations (1) and (2), respectively, the first order conditions of the maximization objective are:
This system of equations leads to the optimal policy for the interior case:

\[ q_i = \frac{Q - c}{2}, \quad (3) \]

\[ q_{i,R} = \frac{s - c(1 - \alpha)}{2(1 - \alpha)}, \quad i = 2, \ldots, M, \quad (4) \]

\[ q_{i,N} = \frac{Q(1 - \alpha)^{i-1}s}{2(1 - \alpha)}, \quad i = 2, \ldots, M. \quad (5) \]

Equations (3)-(5) describe the interior solution for any planning horizon \( M \). This implies in a constant policy by which the number of units sold is the same in every period \( i > 1 \), a behavior consistent with scenarios \( D_1 \) in the infinite horizon and \( D_{21} \) in the two-period horizon models. Now, define the slack variable \( \delta_i \) as the number of cores collected at the end of each period and not used in the remanufacturing process of the following period. So, \( \delta_i = q_{i,N} - q_{i+1,I} \geq 0 \). In the interior case, it can be described as:

\[ \delta_i = \frac{Q(1 - \alpha)^{i-1}s + c(1 - \alpha)\gamma(1 - \alpha)^{-s}}{2(1 - \alpha)}, \quad i = 2, \ldots, M - 1. \quad (6) \]

Based on Equations (3)-(7), we obtain the following result:

**Theorem 3:** If the monopolist has a planning horizon greater than 2 periods \((M > 2)\), then:

a. The optimal policy includes remanufactured products if and only if \( s > c(1 - \alpha) \).

b. The optimal policy is an interior solution if and only if \( c(1 - \alpha)^{-s} < s < \frac{(1 - \alpha)(Q\alpha \gamma + c)}{1 + \alpha \gamma} \).

**Proof:** The inequality in part (a) ensures that the remanufacturing quantity in Equation (4) is positive. The optimization has an interior solution if and only if the slack values are non-negative, as well as \( q_{i,R} > 0 \) and \( q_{i,N} > 0 \). The inequality in part (b) ensures that \( q_{i,R} > 0 \) and \( \delta_i > 0 \). It is simple to show that the values of \( s \) that satisfy \( \delta > 0 \) will also satisfy \( \delta_i > 0 \) and \( q_{i,N} > 0 \). \( QED \)
The values of $s$ that define the interior solution are bounded by the non-negativity of Equations (4) and (7). This theorem provides the condition under which remanufacturing is economically viable in a monopoly environment, regardless of the duration of the planning horizon. Notice that this result is consistent with Theorems (1) and (2). Consequently, if $s$ satisfies the interior solution condition, shown in Theorem 3b, then the policy is the same regardless of the length of the planning horizon. In addition, we have the following results, shown without proof:

**Corollary 3.1:** If $M > 2$ and $s > \frac{(1-\alpha)(Q\alpha\gamma+c)}{1+\alpha\gamma}$, then $\forall i \geq 2$, $q_{i+1,R} = q_{iN}^i$

**Corollary 3.2:** If $M > 2$ and $s < \frac{(1-\alpha)(Q\alpha\gamma+c)}{1+\alpha\gamma}$, the monopolist does not display end-gaming behavior.

Corollary 3.1 says that if $s$ is greater than that threshold, the remanufacturing process in period 3 (or greater) uses all cores that it collects. However, it says nothing about period 2, since the slack variable $\delta_i$ may still be positive when $\delta_i = 0$. The corollary defines the remanufacturing savings values for which some constraints in the optimal policy are binding.

Notice that the periods in the life-cycle of a product may be subject to supply constraint (from earlier sales), demand constraint (from future sales), or both. The first period, present in all models, has just the demand constraint. The last period, absent in the infinite horizon model, has just the supply constraint. The interim period, absent in the two-period model, is subject to both the supply and the demand constraints. The existence of each period type shapes the model, and that is what makes the infinite and the two-period horizon models unique.

To find the optimal policy for larger values of the remanufacturing savings $s$, we write the lagrangean of this optimization problem, using $\lambda_i$ and $\mu_i$ as the multipliers corresponding to the slack variables and to the new product quantities in each period, respectively.

$$L_M = \pi_M + \lambda_i\left(q_{1i} - q_{2i,R}\right) + \sum_{i=2}^{M-1}\left(\lambda_i\left(q_{i,N} - q_{i+1,R}\right) + \mu_i q_{i,N}\right) + \mu_M q_{M,N}$$ (8)

If the planning horizon is $M$, there are $2(M-1)$ lagrangean multipliers. Since each of them may be either zero or positive, there could be as many as $2^{2(M-1)}$ possible solutions to the optimization; e.g., if $M = 5$, there may be 256 solution sets to the optimization. However, only a small number of these solutions satisfy the non-negativity constraints, leading to the following conjecture, which has been proven for planning horizons of 3, 4 and 5 periods:
If the monopolist has a planning horizon greater than 2 periods \((M \geq 3)\), and the profit maximization is characterized by the lagrangean in (8), there are four critical values, \(s_{M0} < s_{M1} < s_{M2} < s_{M3}\), defining five scenarios representing the optimal policy, summarized in Table 1. The scenarios are:

**D\(_{M0}\).** If \(s \leq s_{M1}\), the remanufacturing process generates too little savings. Hence, no remanufactured product is made, and the case defaults to the standard monopoly situation:

\[ q_1 = q_{2N} = \ldots = q_{MN} = \frac{Q - c}{2}. \]

**D\(_{M1}\).** If \(s_{M1} < s \leq s_{M2}\), the monopolist makes both product types, always using fewer cores than what is collected each period. The quantities made then:

\[ q_1 = \frac{Q - c}{2}, \quad q_{2N} = \ldots = q_{MN} = \frac{Q(1 - \alpha) - s}{2(1 - \alpha)} \quad \text{and} \quad q_{2R} = \ldots = q_{3R} = \frac{s - c(1 - \alpha)}{2(1 - \alpha)} \]

**D\(_{M2}\).** If \(s_{M2} < s \leq s_{M3}\), the firm makes both product types and uses all cores collected in every period, except in period 1. Hence, \(q_{i+1,R} = q_{iN}, \ \forall i \in \{2, \ldots, M-1\}\).

**D\(_{M3}\).** If \(s_{M3} < s \leq s_{M4}\), the firm makes both product types and uses all cores collected in every period. Hence, \(q_{i+1,R} = q_{iN}, \ \forall i \in \{1, \ldots, M-1\}\).

**D\(_{M4}\).** When \(s_{M4} < s\), the monopolist remanufactures all cores collected in every period, but it does not make new products in the last period. Hence, \(q_{i+1,R} = q_{iN}, \ \forall i \in \{1, \ldots, M-1\}\) and \(q_{MN} = 0\).

---

**Table 1: Scenarios Characterizing the Optimal Policy for an M-period Planning Horizon**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Range</th>
<th>Lagrangean Multipliers</th>
<th>New Production</th>
<th>Slack variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(_{M0})</td>
<td>(s &lt; s_{M1})</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>D(_{M1})</td>
<td>(s_{M1} &lt; s &lt; s_{M2})</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>D(_{M2})</td>
<td>(s_{M2} &lt; s &lt; s_{M3})</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>D(_{M3})</td>
<td>(s_{M3} &lt; s &lt; s_{M4})</td>
<td>(0)</td>
<td>(0)</td>
<td>(&gt;0)</td>
</tr>
<tr>
<td>D(_{M4})</td>
<td>(s_{M4} &lt; s)</td>
<td>(0)</td>
<td>(&gt;0)</td>
<td>(&gt;0)</td>
</tr>
</tbody>
</table>

Scenarios D\(_{M0}\) and D\(_{M1}\) (with respective thresholds) have been described in Theorem 3. The existence of scenario D\(_{M2}\) is postulated in Corollary 3.1, which identifies the lower-bound \(s_{M2}\). Figure 3 shows the relative position of each of these scenarios, for any combination of values of \(s\) and \(c\) with respect to \(Q\). Notice that the scenarios are bound by the inequality \(s < c < Q\). Moreover, the successive boundaries between the five scenarios intersect at the point \(\{c = Q, s = (1 - \alpha) Q\}\).
In what follows, we discuss the production quantities for scenarios $D_{M2}$, $D_{M3}$ and $D_{M4}$ when the planning horizon is 3, 4 and 5 periods.

**Theorem 4**: If the monopolist has a planning horizon of 3, 4 or 5 periods ($M = 3, 4$ or 5), the critical values defining the non-interior scenarios are given by the values in Error! Reference source not found. (See appendix). Moreover, the optimal policies in these scenarios are:

- **$D_{M2}$**: When $s_{M2} < s < s_{M3}$, the firm always makes both product types, but does not use all cores in period 2 ($q_{iR} < q_{i1}$):

$$q_{i} = \frac{Q-c}{2},$$

$$q_{iR} = \frac{(s-c(1-\alpha)) + (\lambda_{i-1} + \alpha \gamma \lambda_{i}) / \beta^{i-1}}{2(1-\alpha)}, \forall i \in \{2, \ldots, M-1\},$$

$$q_{iN} = \frac{Q(1-\alpha) - s + (\lambda_{i-1} + \gamma \lambda_{i}) / \beta^{i-1}}{2(1-\alpha)}, \forall i \in \{2, \ldots, M-1\},$$

and

$$q_{MN} = \frac{Q(1-\alpha) - s + \lambda_{M-1} / \beta^{M-1}}{2(1-\alpha)}.$$
When $s_{M3} < s \leq s_{M4}$, the firm always uses all cores available to make remanufactured products ($i > 1 \Rightarrow q_{i,R} = \gamma q_{i-1,N}$):

$$q_i = \frac{Q - c + \gamma \lambda_i}{2},$$

$$q_{i,R} = \frac{(s - c(1 - \alpha)) + (\lambda_{i-1} + \alpha \gamma \lambda_i) / \beta^{i-1}}{2(1 - \alpha)}, \forall i \in \{2, \ldots, M - 1\},$$

$$q_{i,N} = \frac{(Q(1 - \alpha) - s) + (\lambda_{i-1} + \gamma \lambda_i) / \beta^{i-1}}{2(1 - \alpha)}, \forall i \in \{2, \ldots, M - 1\},$$

and $q_{MN} = \frac{(Q(1 - \alpha) - s + \lambda_{M-1} / \beta^{M-1}}{2(1 - \alpha)}$.

When $s_{M4} < s$, the monopolist uses all cores available ($i > 1 \Rightarrow q_{i,R} = \gamma q_{i-1,N}$), but does not make new products in period M:

$$q_i = \frac{Q - c + \gamma \lambda_i}{2},$$

$$q_{i,R} = \frac{(s - c(1 - \alpha)) + (\lambda_{i-1} + \alpha \gamma \lambda_i) / \beta^{i-1}}{2(1 - \alpha)}, \forall i \in \{2, \ldots, M - 1\},$$

$$q_{i,N} = \frac{(Q(1 - \alpha) - s) + (\lambda_{i-1} + \gamma \lambda_i) / \beta^{i-1}}{2(1 - \alpha)}, \forall i \in \{2, \ldots, M - 1\},$$

and $q_{MN} = 0$.

**Proof:** See Appendix.

The multipliers ($\lambda_i$) differ in each scenario and in each planning horizon, affecting the quantities produced with each situation. Let the monopolist in the previous examples operate with a 3-period planning horizon. Hence, $Q = 1000, c = 600, \alpha = 0.8, \beta = 0.95, \gamma = 87\%$. The example in Figure 4 illustrates. The critical values separate the five scenarios at $s_{M1} = 120 < s_{M2} = 152 < s_{M3} = 232 < s_{M4} = 914$.

The two lower ranges of remanufacturing savings are similar to the examples shown so far: the remanufactured product is not offered when $s \leq 120$, and when $120 < s < 152$, some of the core is remanufactured, with gradual migration of customers from the new product to the remanufactured product in second and third period. When $s \geq 152$, all cores are remanufactured in third period, shown in the graph by the similar increases of $q_{2N}$ and $q_{3R}$, but not all cores are remanufactured in second period. As the remanufacturing savings increase from 152 to 232, the remanufactured quantity in second period continues to increase. This leads to a reduction of new products in second period, and fewer cores in third
period. Once \( s \geq 232 \), all cores in second period are remanufactured. As the remanufacturing savings increase past this threshold, the monopolist increases the number of new products made in first and second period to have more cores to remanufacture in second and third period. This leads to a drop in demand of new products in third period. We can recognize in the graph that when \( s > s_{33} \), the remanufactured quantities in one period follow closely the new product quantities in the previous period, reflecting the result \( q_{i-1,N} = q_{iR} \). Notice that since \( s_{34} > c \) with these parameters, scenario \( D_{34} \) is null, because the remanufacturing savings cannot be greater than the cost to make the new product.

![Graph](image_url)

**Figure 4. Optimal Policy for the 3-Period Planning Horizon**

The illustration in Figure 5 shows the 4-period planning horizon. We use the same scenario as in the previous examples (\( Q = 1000; \ c = 600; \ \alpha = 0.80; \ \beta = 0.95 \) and \( \gamma = 87\% \)), and \( M = 4 \). These parameters define the critical values for the optimal policy: \( s_{41} = 120, \ s_{42} = 153, \ s_{43} = 199, \ s_{44} = 529 \). For better visibility, we separate the result in three graphs as follows: the optimal policy in period 1 appears as the upper-envelope in all three graphs. Graphs a-c show the policy in periods 2-4, respectively. In all of them, dashed lines represent remanufactured items; continuous lines represent new product quantities.

Unlike the 3-period planning horizon, all critical values are in the range \( (0, c) \), with this parameter set. It is surprising to see how the remanufacturing savings affect the optimal policy, depending on its relative position with the cutoff values, as illustrated in Figure 5. For high values of \( s \), the production of remanufactured products increases with \( s \) in periods 2 and 4, but it is not very sensitive to variations in \( s \) in period 3. The unusual shape of these graphs can be easily explained: When \( 120 < s < 153 \), not all cores are remanufactured in any period. Hence, as the remanufacturing savings increase in this range, the demand in all periods gradually migrates from new to remanufactured products. When \( s \geq 153 \), all cores in period 3 and 4 are remanufactured. As the remanufacturing savings increase from 153 to 199, the demand for new products continues to migrate from new to remanufactured products, gradually
consuming all cores in period 2. This reduces the availability of cores for remanufacturing in period 3, leaving room for more new products in period 3 (seen in Figure 5b, as $q_{3R}$ decreases and $q_{3N}$ increases). This generates more cores to remanufacture in period 4, reducing the demand for new products then. When $s \geq 199$, all cores are remanufactured in all periods. The price in period 1 gradually reduces (not shown) to induce greater demand then, and more cores to remanufacture in period 2. This reduces the demand for new products in period 2 and the core availability in period 3, allowing for more new product demand in the third period, leading to more cores to remanufacture in period 4. Ultimately, when remanufacturing savings are high, the firm alternates periods of high and low remanufacturing volumes, finishing with a low volume of new products.

![Graphs](a) (b) (c)

**Figure 5. Optimal Policy in a 4-Period Planning Horizon**

Another way to interpret this demand oscillation is to look at the last period. In the last period, the firm plans to sell a low volume of new products, which do not generate additional income through remanufacturing, and to meet most of last period’s demand with remanufactured products. To enable this
decision, a high volume of new products must be made in the period before last to generate the respective cores for the last period, so forth and so on.

The optimal policy in the 5-period remanufacturing monopoly is shown in Figure 6, using the same parameters as in all other examples \((Q = 1000, c = 600, \alpha = 0.8, \beta = 0.95, \gamma = 87\%\)). In this case, the critical values are \(s_{31} = 120, s_{52} = 153, s_{53} = 217.4, s_{54} = 701\). Again, we separate the result in four charts for better visibility, and the optimal policy in period 1 appears as the upper-envelope in all graphs. Graphs a-d show the policy in periods 2-5, respectively. In all of them, dashed lines represent remanufactured quantities, and continuous lines represent new product quantities.

![Figure 6. Optimal Policy in a 5-Period Planning Horizon](image)

When \(s \geq 153\), all cores are remanufactured in periods 3-5, but the cores in period 2 are not fully consumed until \(s \geq 217.4\). Hence, as the remanufacturing savings increase from 153 to 217.4, the demand in period 2 continues to migrate from new products to remanufactured products, without ever using all cores. Lower production of new products in period 2 affects the availability of remanufactured products, which are replaced by additional new products in period 3, shown in graph b as a decrease of \(q_{3R}\) while...
\( q_{SN} \) is increasing. Once \( s \geq 217.4 \), all cores are consumed in all periods, so the producer increases the production of remanufactured products each period by making more new products in the previous period.

The production of remanufactured products in one period is directly related to the production of new products in the previous period, but inversely related to the production of new products in the same period, a consequence of the quantities defined by Equations (1) and (2). Moreover, the new products made in the last period do not generate additional revenue through remanufacturing. Hence, the optimal policy recommends a drop in the production of new products in the last period as the remanufacturing savings increase, inducing larger production of remanufactured products to meet the existing demand. This raises the demand for new products in the period before last and lowers the demand for remanufactured products in that period. This affects the production of new products in period 3, and so on. The product line oscillation, alternating high and low production of new products, is also present in the 5-period planning horizon, dampened by the remanufacturing yield (substantially lower than 100%), which requires the production of new products to replace the cores lost each period.

4 Conclusions and Future Research

Many companies have organized their product line based on remanufacturing capabilities (e.g., makers of printer cartridges, single-use cameras, tires, hospital beds, military equipment and many other products.) Often, they operate in a monopoly or quasi-monopoly environment. This paper analyzes a monopoly in which the new and remanufactured products are clearly differentiated. We characterize the optimal pricing and production strategy under finite (two, three, four and five periods) and infinite planning horizons of a remanufacturing monopolist, to identify patterns in the optimal policy.

An over-arching observation is that, as the marginal cost of remanufacturing decreases, the value of making new products in the first period increases, and the value of making new products in future periods decreases. In other words, if remanufacturing is very profitable, the firm forgoes some of the early period margins by supplying additional units in the first period to increase the number of cores available for remanufacturing later. So, the optimal policy for the finite problem is dynamic: each period, the firm should produce a different quantity of new and a different quantity of remanufacturing products. This quantity is driven by the need for cores (used products) in the following period. However, there is another effect that comes into play, related to the amount of remanufactured products in the interim periods: If more remanufactured products are sold in the current period, a partial cannibalization puts pressure on the amount of new products that can be sold at the same time. This, in turn, impacts the number of remanufactured savings that can be realized in the next period, leading to situations in which it is optimal not to remanufacture more in the interim periods—even though remanufacturing savings
increase – to enable a higher number of cores available for remanufacturing in the last period of the life-cycle.

Our study shows the impact of the planning horizon on the firm’s optimal policy. We illustrated this result with 5 examples that were identical in every aspect, except regarding their planning horizons. As seen in the figures, the optimal policy for a given period is different for each planning horizon (M). In our example, if the remanufacturing savings is $300, the optimal number of new items to make in period 2 can be as low as 29 (M = 2) and as high as 139 (M = ∞). See Table 2. This is another evidence that proper management of remanufacturing systems require careful long term planning.

Table 2: Optimal q_N in period 2 for different planning horizons

<table>
<thead>
<tr>
<th>Remanufacturing savings (s)</th>
<th>Planning Horizon (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>200</td>
<td>55</td>
</tr>
<tr>
<td>300</td>
<td>29</td>
</tr>
<tr>
<td>400</td>
<td>3</td>
</tr>
</tbody>
</table>

This article helps understanding the life cycle of a product, starting from an offering of just all-new products to a mixed product line, in which remanufactured and new products coexist. By analyzing the two-, three-, four- and five-period problems and characterizing the optimal policy, we are able to conjecture the optimal policy for a larger M-period, finite-horizon problem. If the remanufacturing savings is greater than the internal solution threshold, the product line oscillates from period to period: the segmentation in the last period has more remanufactured products and fewer new products, the period before that has the opposite segmentation, and so on.

As this area of research expands, it is important to understand the complete lifecycle of the remanufactured product line. For example, the impact of demand variability deserves attention. What would happen to the optimal policy if the stream of demands throughout the life-cycle were stochastic? What if it were known, but variable? Considering that our model does not allow the firm to save the cores to remanufacture in future periods, the stochastic problem would have components typical of the Newsboy model. Possibly, new product manufacture will show the same oscillating pattern that we already saw in our study, with a correction that will be affected by the relative size of the underage and the overage costs, and the remanufactured production will follow. The case of the variable and known demand would add a new set of constraints to our current model, which will probably limit the analysis to a few periods. We plan to explore those issues in a future study.
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6 References


