Supplier diversification: effect of discrete demand

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Abstract

Diversification under supply uncertainty has been adopted by manufacturers in order to improve performance. In the presence of multiple suppliers, it is essential to develop an operational policy in order to utilize the services of different suppliers effectively. Anupindi and Akella (Manage. Sci. 39 (8) (1993) 944-963) consider two suppliers differing in cost and reliability and develop the optimal inventory policy for the manufacturer under continuous demand distribution. The policy obtained by them indicates that the manufacturer should never order products from the more expensive supplier alone. In this paper, we consider the case where demand is discrete and provide examples to show that ordering products from the more expensive (and more reliable) supplier alone is optimal. We also provide sufficient conditions under which it is optimal to order a larger share from the more expensive supplier when demand is discrete. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Supplier diversification is employed by manufacturers to reduce the risk and dependencies that arise due to reliance on a single supplier. Usage of more than one supplier (often two) for critical components is common in electronic and auto industry (McMillan [4]). Best practices in industry advocate a total cost basis (incorporating on time delivery, quality and other aspects) rather than a price basis for doing business with suppliers (Chapman et al. [2]). In the presence of multiple suppliers it is essential for the manufacturer to develop an efficient ordering policy. Comparative analysis of diversification strategies for the manufacturer under supply uncertainty is discussed in Moinzadeh and Nahmias [5] and Lau and Lau [3]. Anupindi and Akella [1] consider a model where the manufacturer has two alternative suppliers and consider three different versions which capture different possible delivery processes from suppliers. They analyze single- and multiple-period models in each version and develop the optimal ordering policy for the manufacturer. The policy is characterized by three regions of ordering based on the current inventory level. The manufacturer places orders with both suppliers when the inventory is very low, chooses the less expensive supplier in the next region, and does not place any orders when the inventory is greater than a threshold level.

Literature in inventory management typically has utilized continuous demand distributions in order to
obtain optimal policies and derive analytical insights. A continuous and differentiable demand distribution often simplifies the analysis and enables one to derive simple and intuitive policies which are then adapted for discrete demand situations found in the real world. One such example is the traditional newsboy problem. However, there are problems where the optimal policies for discrete and continuous demand distributions may be totally different. Our aim in this paper is to analyze the supply diversification problem studied in [1] to illustrate and explain the difference in policy structure under discrete and continuous demand distributions. In this paper, we assume a discrete demand and analyze one of the models of supplier reliability considered by [1], where suppliers deliver either the whole quantity or nothing with a probability. We have restricted our attention to only one model of supplier uncertainty because our aim is to demonstrate the differences in an optimal policy that may arise when the demand is described by a discrete rather than continuous probability distribution. We first provide examples for single-period and two-period problems where it is optimal to order exclusively from the more expensive supplier under discrete demand. Subsequently, for a single period problem we analytically justify the absence of such a region when the demand distribution is continuous. Further, we approximate the discrete demand distribution by a continuous demand distribution and compare the differences in the ordering policy. Our analysis indicates that for the most part, the region where orders are placed with both suppliers, bulk of the order is placed with the more reliable and more expensive supplier and very little is ordered from the cheaper supplier. As a result, the structure of the optimal policy derived in [1], although technically correct, is misleading because it emphasizes that it is never optimal to place an order with the more expensive supplier alone. Finally, we provide sufficient conditions, for both single- and multi-period models in the discrete demand case, that show when it is optimal to order from the more expensive supplier. The results presented in this paper provide further reasons for doing business with suppliers based on total cost rather than on price.

The rest of the paper is organized as follows. In Section 2, we present the model description and examples for single and two period problems when it is optimal to order exclusively from the more expensive supplier. In Section 3, we consider the analytical model for a single period case in greater detail and focus our attention on the region in which orders are placed only with the more expensive supplier. We provide justifications for the absence of that region in the optimal ordering policy. We also provide a continuous approximation to the discrete distribution and compare the optimal policy with the optimal policy when demand is described by a discrete random variable. In Section 4, we provide sufficient conditions for multi-period and single period problems where it is optimal to order exclusively or mostly from the more expensive supplier.

2. Model and example

In this section, we first introduce one of the models developed in [1]. We call this model M1 in the remainder of this paper. We then discuss their important result, which stipulates the structure of the optimal ordering policy. Subsequently, we present simple examples where that policy structure is invalid when a discrete distribution describes the demand process.

2.1. Ordering policy

We consider a manufacturer who orders products from two alternative suppliers. Suppliers deliver all of the ordered quantity or nothing with a specified probability. The suppliers differ in terms of the cost charged for the components as well as the probability that the supplier delivers the product. The demand for the final product is stochastic. In addition, the model assumes that all the components that are received are converted into final products. We first consider the above problem in a single-period setting. We use the following notation:

- \( w_i \) quantity ordered from supplier \( i \);
- \( y \) on-hand inventory initially;
- \( c_i \) per unit ordering cost from supplier \( i \);
- \( h \) holding cost per unit per period for finished product;
- \( \pi \) penalty cost per unit per period for finished product;
• \( \beta_i \) probability that the quantity ordered from supplier \( i \) will be delivered, and \( \overline{\beta}_i = 1 - \beta_i \); 
• \( \xi \) random demand; 
• \( f(.) \) density of demand; 
• \( F(.) \) distribution function of demand; 
• \( M(x, w_1, w_2) \) single-period expected cost.

The objective of the manufacturer is to minimize the single-period cost function \( M(y, w_1, w_2) \) given by

\[
M(y, w_1, w_2) = c_1 w_1 \beta_1 + c_2 w_2 \beta_2 + \pi \beta_1 \beta_2 \int_{y-w_1}^{\infty} (\xi - y - w_1) f(\xi) d\xi + \pi \beta_1 \beta_2 \int_{y-w_1-w_2}^{\infty} (\xi - y - w_1 - w_2) f(\xi) d\xi + \pi \beta_1 \beta_2 \int_{y}^{\infty} (\xi - y) f(\xi) d\xi + h \beta_1 \beta_2 \int_{0}^{y+w_1} (y + w_1 - \xi) f(\xi) d\xi + h \beta_1 \beta_2 \int_{0}^{y+w_1+w_2} (y + w_1 + w_2 - \xi) f(\xi) d\xi + h \beta_1 \beta_2 \int_{0}^{y} (y - \xi) f(\xi) d\xi + h \beta_1 \beta_2 \int_{0}^{y+w_1} (y + w_1 - \xi) f(\xi) d\xi + h \beta_1 \beta_2 \int_{0}^{y+w_1+w_2} (y + w_2 - \xi) f(\xi) d\xi.
\]

This single-period cost function is convex in \((w_1, w_2, y)\) (refer Lemma 3.1 in [1]). The optimal policy derived in [1] is as follows.

**Theorem 1.** Assume that \( c_1 < c_2 < \pi \) and \( \beta_1, \beta_2 > 0 \). Then there exist two constants \( \overline{\pi} \) and \( \overline{\pi} \) such that the optimal ordering policy is:

\[
(w_1^*, w_2^*) = \begin{cases} 
(w_1^*, w_2^*) & \text{if } y < \overline{\pi}, \\
(w_1^*, 0) & \text{if } \overline{\pi} > y \geq \overline{\pi}, \\
(0, 0) & \text{if } y \geq \overline{\pi},
\end{cases}
\]

where

\[
w_1^* > 0, \quad \overline{\pi} = \frac{\pi - c_1}{\pi + h}
\]

and

\[
\overline{\pi} = \frac{(\pi - c_2) - \beta_1 (\pi - c_1)}{\beta_1 (\pi + h)} \quad \text{if and only if } \beta_1 \leq \frac{\pi - c_2}{\pi - c_1}.
\]

The optimal ordering policy has three distinct regions: Region-I where nothing is ordered, which is the case when \( y \geq \overline{\pi} \); Region-II where \( \overline{\pi} < y \leq \overline{\pi} \) and only the less expensive supplier is used; Region-III where \( y \leq \overline{\pi} \) and orders are placed with both the suppliers. Thus, there is no region where orders are placed with the more expensive supplier alone even though that supplier might be more reliable (the case when \( \beta_2 > \beta_1 \)). In the proof for the above theorem, it is also shown that if \( \beta_1 > (\pi - c_2)/(\pi - c_1) \), then the more expensive supplier is never used (i.e., Region-III does not exist).

For the multi-period discounted case in [1], the authors assume that if the suppliers are unable to deliver in a period, then delivery is made before the beginning of the next period. In addition, they assume that all unmet demand is backlogged. They derive the following ordering policy for the multi-period case.

**Theorem 2.** Assume that \((1 + \beta_2 - \beta_1)c_1 c_2 < \pi \) and \( \beta_1, \beta_2 > 0 \). Then there exist two constants \( \overline{\pi} \) and \( \overline{\pi} \), such that the optimal ordering policy is

\[
(w_1^{m*}, w_2^{m*}) = \begin{cases} 
(w_1^*, w_2^*) & \text{if } y \leq \overline{\pi}, \\
(w_1^*, 0) & \text{if } \overline{\pi} > y \geq \overline{\pi}, \\
(0, 0) & \text{if } y \geq \overline{\pi},
\end{cases}
\]

Here, the quantities \( w_1^{m*} \) and \( w_2^{m*} \) represent the quantities that are ordered with \( n \) periods to go. When \((1 + \beta_2 - \beta_1)c_1 c_2 \) they show that it is never optimal to order from the more expensive supplier alone. In the following sections, we provide counter examples for the single and two period problems, respectively, when demand is described by a discrete random variable. In particular, our examples show that it is optimal for the manufacturer to order exclusively from the more expensive supplier.
2.2. Single-period example

Let us consider an example with the following parameters. \( c_1 = 0.4; \ c_2 = 0.5; \ \pi = 2; \ \beta_1 = 0.2; \ \beta_2 = 0.8; \ h = 0.3 \). Further, assume that the starting inventory \( y = 0 \) and the demand has the following distribution \( f(\xi = 0) = 0.5 \) and \( f(\xi = 2) = 0.5 \). We consider a situation where the manufacturer orders only integer quantities. So there are nine feasible choices for the \((w_1, w_2)\) pair that need to be considered. Other feasible pairs are clearly more expensive. The expected cost in each case is written along with the choices given below:

\[(0, 0): \ \pi = 2 \]
\[(1, 0): \ c_1 \beta_1 + 0.5(\beta_1 \pi + \beta_1 h) + \overline{\beta}_1 \pi = 1.72 \]
\[(0, 1): \ c_2 \beta_2 + 0.5(\beta_2 \pi + \beta_2 h) + \overline{\beta}_2 \pi = 1.91 \]
\[(1, 1): \ c_1 \beta_1 + c_2 \beta_2 + \beta_1 \beta_2 h + \overline{\beta}_1 \overline{\beta}_2 \pi + 0.5(\overline{\beta}_1 \beta_2 + \beta_1 \overline{\beta}_2)(\pi + h) = 1.63 \]
\[(2, 0): \ 2c_1 \beta_1 + \beta_1 h + \overline{\beta}_1 \pi = 1.82 \]
\[(2, 1): \ 2c_1 \beta_1 + c_2 \beta_2 + 2\beta_1 \beta_2 h + \overline{\beta}_1 \overline{\beta}_2 \pi + 0.5(\overline{\beta}_1 \beta_2 + \beta_1 \overline{\beta}_2)(\pi + h) = 1.724 \]
\[(1, 2): \ c_1 \beta_1 + 2c_2 \beta_2 + 2\beta_1 \beta_2 h + \overline{\beta}_1 \overline{\beta}_2 \pi + 0.5\overline{\beta}_2 \beta_1 (\pi + h) + \beta_2 \overline{\beta}_1 h = 1.534 \]
\[(0, 2): \ 2c_2 \beta_2 + \beta_2 h + \overline{\beta}_2 \pi = 1.44 \]
\[(2, 2): \ 2c_1 \beta_1 + 2c_2 \beta_2 + 3\beta_1 \beta_2 h + \overline{\beta}_1 \overline{\beta}_2 \pi + (\overline{\beta}_2 \beta_2 + \beta_2 \overline{\beta}_1) h = 1.628 \].

The minimum cost is incurred when the manufacturer buys two units from the more expensive supplier and buys nothing from the less expensive supplier. This is an ordering structure which according to the optimal policy derived in [1] should never be utilized. The only difference between the example and their assumptions is that demand is described by a discrete random variable in the example.

2.3. Two-period example

Let us consider an example for the multi-period problem (two periods in this case) with the following parameters. \( c_1 = 1; \ c_2 = 2; \ \pi = 10; \ \beta_1 = 0.01; \ \beta_2 = 0.99; \ h = 0.8; \ \alpha = 0.45 \). Note that \((1 + \beta_2 - \beta_1)c_1 < c_2 < \pi \). Let us further assume that the starting inventory \( y = 1 \), and the demand has the following distribution: \( f(\xi = 1) = 0.6 \), \( f(\xi = 2) = 0.4 \). Assume that whatever is not delivered by a supplier in the current period is delivered by the beginning of the next period, and that all unmet demand in the current period is backlogged. We consider a situation where the manufacturer orders only integral quantities.

**Observation 1.** The following policy is optimal for the manufacturer for the last period when the inventory at the beginning of that period is \( y \).

\[(w_1^{*}, w_2^{*}) = \begin{cases} (0, 0) & \text{if } y \geq 2, \\ (0, 2 - y) & \text{if } 2 > y \geq -1. \end{cases} \]

**Observation 2.** The optimal ordering policy for the manufacturer in the first period with starting inventory \( y = 1 \) is \((w_1^{*}, w_2^{*}) = (0, 1)\).

Thus, in this problem it is optimal for the manufacturer to order products exclusively from the more expensive supplier. Note that a nonzero starting inventory, no point mass at zero for the demand distribution, and multi-period setting do not change the structure shown by the single-period example.

3. Model revisited

In this section, we revisit the single period model M1 and try to understand the absence of the region where \( w_1 = 0; w_2 > 0 \). Since this case is not explicitly considered in [1], we focus our attention on that region and show that such a region is not optimal when the demand distribution is continuous.

**Lemma 1.** When the demand distribution is continuous, it is never optimal to order from the more expensive supplier alone i.e. \((w_1 = 0; w_2 > 0)\) under model M1.

**Proof.** Consider the region where only the more expensive supplier is utilized. The partial derivatives of
the cost function with respect to \( w_1 \) and \( w_2 \) are
\[
\frac{\partial M(y,w_1,w_2)}{\partial w_1} = \beta_1(c_1 - \pi) + \beta_1(h + \pi)
\times [\beta_2 F(w_1 + w_2 + y) + \beta_2 F(w_1 + y)]
\]
and
\[
\frac{\partial M(y,w_1,w_2)}{\partial w_2} = \beta_2(c_2 - \pi) + \beta_2(h + \pi)
\times [\beta_1 F(w_1 + w_2 + y) + \beta_1 F(w_2 + y)].
\]

If \( w_1 = 0 \) and \( w_2 > 0 \), then
\[
\frac{\partial M(y,w_1,w_2)}{\partial w_2} = 0 \quad \text{and} \quad \frac{\partial M(y,w_1,w_2)}{\partial w_1} \geq 0.
\]
This implies that \( F(w_2 + y) = (\pi - c_2)/(\pi + h) \) and
\[
\beta_1(c_1 - \pi) + \beta_1(h + \pi)\beta_2 F(w_2 + y) + \beta_2 F(y) \geq 0,
\]
or
\[
F(w_2 + y) = (\pi - c_2)/(\pi + h) \quad \text{and} \quad \beta_1 F(w_1 + w_2 + y) + \beta_1 F(w_2 + y) \geq (\pi - c_1)/(\pi + h).
\]

Since \((\pi - c_1)/(\pi + h) > (\pi - c_2)/(\pi + h)\), so \( w_1 = 0 \) and \( w_2 > 0 \) need not be considered.

However, as shown by examples in Sections 2.2 and 2.3, it may be optimal to order only from the more expensive supplier when the demand distribution is discrete. Next, we establish the ordering decision suggested by the optimal policy when the discrete demand is approximated by a continuous demand distribution. Consider the single-period example in Section 2.2 where \( y = 0 \). The values of \( \bar{\nu}^1 \) and \( \bar{\nu}^2 \) are given by \( F(\bar{\nu}^1) = 0.69 \) and \( F(\bar{\nu}^2) = 0.64 \). Thus, \( y < \bar{\nu}^1 \) and the policy suggests that both suppliers be used. The quantities ordered from the two suppliers are determined by setting the partial derivatives with respect to \( w_1 \) and \( w_2 \) to zero. The two equations are given by
\[
\beta_1(c_1 - \pi) + \beta_1(h + \pi)(\beta_2 F(w_1 + w_2 + y) + \beta_2 F(w_1 + y)] = 0
\]
and
\[
\beta_2(c_2 - \pi) + \beta_2(h + \pi)(\beta_1 F(w_1 + w_2 + y) + \beta_1 F(w_2 + y)] = 0.
\]

We approximate the discrete demand by a uniform continuous distribution between \( 2 - \varepsilon \) and 2 where \( 0 < \varepsilon < 1 \) and a point mass of 0.5 at zero (see Fig. 1). On solving the above equations for this distribution, we get the optimal quantities \( w_1^* = 0.22\varepsilon; w_2^* = 2 - 0.74\varepsilon \).

Thus, the optimal policy orders a negligible quantity from the less expensive supplier when the demand distribution is very small the ordering policy does nearly give the same solution as the optimal solution. Thus, what we observe to be the structure of the policy at a coarse level may be misleading because the policy suggests that the second supplier is never used alone. Actually, the region given by \( w_1 > 0 \) and \( w_2 > 0 \) could favor large orders from the more expensive supplier if that supplier’s shipments are highly reliable. It is to be noted that our continuous approximation does not satisfy the twice differentiability conditions on demand distribution required in model M1. However, we describe this approximation only to provide intuitive arguments while comparing the effect of discrete demand distributions.

### 4. Ordering from expensive supplier

In this section, we consider a discrete demand distribution and provide sufficient conditions under which it is optimal to order the majority from the more expensive supplier for single and multi-period models. Let

- \( p_i \): be the probability that demand is \( i \).
- \( EC(x) \): be the expected cost incurred in a single (current) period when on-hand inventory is \( x \) after shipments from suppliers have arrived.
- \( M(y,w_1,w_2) \): be the expected cost incurred in a single period when on-hand inventory is \( y \) and \( w_i \) is ordered from supplier \( i \).
- \( J_n(y,w_1,w_2) \): be the expected cost incurred with \( n \) periods remaining where \( y \) is the on-hand inventory and \( w_i \) is the amount ordered from supplier \( i \) and the optimal policy is followed for the remaining \( n - 1 \) periods.
- \( J_2(y) \): be the optimal cost incurred with \( n \) periods remaining when inventory on hand is \( y \).
4.1. Single-period problem

First, we consider the general case when demand lies in an interval \([0, n]\) and derive sufficient conditions for purchasing exclusively from the more expensive supplier. Subsequently, we consider two special cases of the demand distribution when demand is all or nothing and when demand lies in an interval \([a, b]\).

We derive simple sufficient conditions so that ordering is done exclusively from the expensive supplier (all or nothing case) and at least \(a - y\) units are ordered from the more expensive supplier (interval demand case).

4.1.1. Demand between 0 and \(n\)

We assume that demand occurs between 0 and \(n\) and \(p_i \geq 0\) is the probability that the demand is equal to \(i\) with \(\sum_{i=0}^{n} p_i = 1\). Note that a point mass at zero is not a requirement in this general distribution. The expected cost incurred when on-hand inventory (after delivery from suppliers) is \(x\) is given by

\[
EC(x) = \pi \sum_{i=0}^{x} (x-i)p_i + \pi \sum_{i=x+1}^{n} (i-x)p_i.
\]

\[
M(y, w_1, w_2) = c_1 \beta_1 w_1 + c_2 \beta_2 w_2 + \beta_1 \beta_2 EC(y + w_1 + w_2) + \beta_1 \beta_2 EC(y + w_1) + \beta_1 \beta_2 EC(y) + \beta_1 \beta_2 EC(y + w_2).
\]

**Proposition 1.** If the starting inventory is \(y\) and \(w_1(c_1 \beta_1 - c_2 \beta_2) > \beta_2 \beta_1 (EC(y + w_2) - EC(y + w_2 - w_1)) + \beta_1 \beta_2 (EC(y) - EC(y + w_1))\) for \(0 < w_1 < w_2 < n - y\), then \(M(y, 0, w_2) < M(y, w_1, w_2 - w_1)\).

**Proof.** \(w_1(c_1 \beta_1 - c_2 \beta_2) > \beta_2 \beta_1 (EC(y + w_2) - EC(y + w_2 - w_1)) + \beta_1 \beta_2 (EC(y) - EC(y + w_1))\) implies

\[
w_1(c_1 \beta_1 - c_2 \beta_2) > \beta_2 (1 - \beta_1) (EC(y + w_2) - EC(y + w_1)) + \beta_1 (1 - \beta_2) (EC(y) - EC(y + w_1)).
\]

By rearranging terms,

\[
\beta_2 EC(y + w_2) + \beta_2 EC(y) < -c_2 \beta_2 w_1 + c_1 \beta_1 w_1 + \beta_1 \beta_2 EC(y + w_2) + \beta_1 \beta_2 EC(y + w_1) + \beta_1 \beta_2 EC(y + w_2) + \beta_1 \beta_2 EC(y + w_1) + \beta_1 \beta_2 EC(y).
\]

Adding \(c_2 \beta_2 w_2\) to both sides we get,

\[
\beta_2 EC(y + w_2) + \beta_2 EC(y) + c_2 \beta_2 w_2 < c_2 \beta_2 (y + w_2 - w_1) + c_1 \beta_1 w_1 + \beta_1 \beta_2 EC(y + w_2) + \beta_1 \beta_2 EC(y + w_1) + \beta_1 \beta_2 EC(y + w_2) + \beta_1 \beta_2 EC(y + w_1) + \beta_1 \beta_2 EC(y),
\]

which implies that

\[
M(y, 0, w_2) < M(y, w_1, w_2 - w_1).
\]

The sufficient condition implies that for any quantity \(w_1\), it is better to order from the more expensive supplier if the expected increase in ordering cost is smaller than the expected decrease in holding and stock-out costs. Thus, when the combined order is for less than \(n\) units, it is optimal to order exclusively from the more expensive supplier. In addition to conditions in Proposition 1, if there exists \(w'_2 < n - y\) such that \(M(y, w'_1, w'_2) \geq M(y, 0, w'_2) \forall w_1 + w_2 > n - y\), then it
is always optimal to place orders only with the more expensive supplier when the inventory is \( y \).

### 4.1.2. Special cases

In this section we consider special cases of the demand distribution and derive simple and easily verifiable sufficient conditions under which it is optimal to order exclusively from the more expensive supplier or to provide a lower bound on the amount ordered from that supplier.

#### 4.1.2.1. All or nothing demand.

In this case, there is either a demand of \( n \) units with probability \( p_n \) or there is no demand with probability \( p_0 \). Given an on-hand inventory (after receipt of shipments) of \( x \), the expected cost incurred is given by

\[
EC(x) = hxp_0 + \pi(n-x)p_n \quad x \leq n.
\]

**Proposition 2.** There exists an optimal solution which orders either 0 or \( n-y \) from any supplier where \( y \leq n \) is the existing inventory.

**Proof.** Let us assume that the optimal procurement is such that \( w_1 = \alpha \), \( w_2 = \gamma \) where \( \alpha, \gamma > 0 \). The expected cost incurred is given by

\[
M(y, \alpha, \gamma) = \beta_1c_1\alpha + \beta_2c_2\gamma + \beta_1\beta_2EC(y + \alpha + \gamma) + \beta_3\beta_4EC(y + \alpha + \gamma) + \beta_1\beta_4EC(y).
\]

It is clear that both \( \alpha \) and \( \gamma \) are less than or equal to \( n-y \). Let us assume \( \alpha + \gamma > n-y \). We now show how to generate a solution which incurs lower cost while ordering all or nothing from the suppliers.

The first differential of the cost with respect to \( \alpha \) and \( \gamma \) are given by

\[
\delta_\alpha = \beta_1[\beta_2h + c_1 + \beta_2(hp_0 - \pi p_n)], \quad \delta_\gamma = \beta_2[\beta_1h + c_2 + \beta_1(hp_0 - \pi p_n)].
\]

If both \( \delta_\alpha \leq 0 \) and \( \delta_\gamma \leq 0 \) then \( M(y, n-y, n-y) \leq M(y, x, y) \).

If \( \delta_\alpha > 0 \), \( \delta_\gamma \leq 0 \) then \( M(y, n-y, 0) \leq M(y, x, y) \).

Similarly, \( M(y, n-y, 0) \leq M(y, x, y) \) if \( \delta_\alpha \leq 0 \), \( \delta_\gamma > 0 \). If both first differentials are positive then decrease the values of \( \alpha \) and \( \gamma \) so that \( \alpha + \gamma = n-y - 1 \). At this point,

\[
\delta_\alpha = \beta_1[c_1 + hp_0 - \pi p_n], \quad \delta_\gamma = \beta_2[c_2 + hp_0 - \pi p_n].
\]

If \( \delta_\alpha < \delta_\gamma \leq 0 \), then \( M(y, n-y, 0) \leq M(y, x, \gamma) \). If \( \delta_\alpha \leq \delta_\gamma \leq 0 \), then \( M(y, x, y) \leq M(y, x, \gamma) \). If \( \delta_\gamma > 0 \), \( \delta_\gamma \leq 0 \) then \( M(y, n-y, 0) \leq M(y, x, \gamma) \).

Similarly, \( M(y, n-y, 0) \leq M(y, x, \gamma) \) if \( \delta_\gamma \leq 0 \), \( \delta_\gamma > 0 \).

Finally if \( \delta_\gamma > 0 \), \( \delta_\gamma \leq 0 \) then \( M(y, 0, 0) \leq M(y, x, \gamma) \).

For any solution where the ordering is not all or nothing from suppliers, it is possible to generate an all or nothing order which incurs the same or lower expected cost. Therefore, it is sufficient to consider policies which order all or nothing from the suppliers. \( \square \)

**Proposition 3.** Under the following three conditions it is optimal to order exclusively from the more reliable and expensive supplier - (i) \( c_2 < \pi p_n - hp_0 \); (ii) \( c_1 > \beta_2(\pi p_n - hp_0) - h\beta_2 \); (iii) \( c_2\beta_2 - c_1\beta_1 < (\beta_2 - \beta_1)(\pi p_n - hp_0) \).

**Proof.** Based on Proposition 2, it is sufficient to consider cases where ordering is done all or nothing from the suppliers. Let us analyze each condition one at a time. Let \( y \) be the initial inventory.

(i) \( c_2 < \pi p_n - hp_0 \).

Multiplying by \( \beta_2(n-y) \) and rearranging the terms we get \( c_2\beta_2(n-y) + \beta_2(n-y)(hp_0 - \pi p_n) < 0 \). This implies that \( M(y, 0, n-y) < M(y, 0, 0) \).

(ii) \( c_1 > \beta_2(\pi p_n - hp_0) - h\beta_2 \).

Multiplying by \( \beta_1(n-y) \) yields \( \beta_1\beta_2(n-y)(\pi p_n - hp_0) - c_1\beta_1(n-y) - \beta_1\beta_2(h(n-y) - c_1\beta_1 < (\beta_2 - \beta_1)(\pi p_n - hp_0) \).

Thus, if (i)–(iii) are satisfied, then \( M(y, 0, n-y) \) incurs the minimum cost among the all or nothing ordering policies and thus is optimal. \( \square \)

Each of the above conditions provides intuitive arguments. Condition (i) implies that ordering a unit from the more expensive supplier is better than not ordering that unit provided it is the only supplier used. Condition (ii) implies that the benefits of ordering a unit from the first supplier, given that \( n-y \) units are
ordered from the second supplier is less than the cost of ordering that unit. Condition (iii) implies that given a choice of buying only one unit from either of the suppliers, it is better to buy it from the more expensive supplier. The three conditions together ensure that ordering is done exclusively from the second supplier.

4.1.2.2. Interval demand. Suppose demand is known to occur in an interval \([a, b]\). In such cases, \(a\) might be a large number and the difference \(b - a\) determines the accuracy with which the demand is known. We provide sufficient conditions under which it is optimal to order at least \(a - y\) units from the more expensive supplier when the starting inventory is \(y \leq a\). The expected cost incurred when on-hand inventory is \(x\) after the receipt of shipments, is given by

\[
EC(x) = h \sum_{i=a}^{x} (x - i) p_i \\
+ \pi \sum_{i=x+1}^{b} (i - x) p_i \quad \text{for } a \leq x \leq b.
\]

**Proposition 4.** If \(\beta_2(\pi - c_2) > \beta_1(\pi - c_1)\), then it is optimal to order at least \(a - y\) units from the more expensive supplier when the starting inventory is \(y \leq a\).

**Proof.** It is clear that if the on-hand starting inventory is \(y\), then at least \(a - y\) units will have to be ordered from both suppliers together. In that region the expected reduction in cost per unit ordered from each of the suppliers is given as follows:

First supplier:
- expected cost reduction = \(\beta_1(\pi - c_1)\).

Second supplier:
- expected cost reduction = \(\beta_2(\pi - c_2)\).

As a result, if \(\beta_2(\pi - c_2) > \beta_1(\pi - c_1)\), then all the \(a - y\) units will be ordered from the more expensive supplier. Thus, an order of at least \(a - y\) units is placed with the more expensive supplier. \(\square\)

The above result could be interpreted in terms of marginal benefit per unit of product ordered from either supplier. Since \(a - y\) units need to be bought in any case, they are ordered from the supplier that exhibits higher marginal benefit.

4.2. Multi-period model

In the multi-period model, we assume that if suppliers cannot deliver in this period, then they deliver it before the beginning of the next period. Assume demand occurs from 0 through \(n\), and \(p_i\) is the probability that the demand is equal to \(i\), where \(p_i \geq 0\) and \(\sum_{i=0}^{n} p_i = 1\).

The expected cost incurred in the current period when on-hand inventory (after delivery from suppliers) is \(x\) is given by

\[
EC(x) = h \sum_{i=0}^{x} (x - i) p_i + \pi \sum_{i=x+1}^{n} (i - x) p_i.
\]

As a result

\[
J_{\pi}(y;w_1, w_2) = c_1 w_1 + c_2 w_2 + \beta_1 \beta_2 EC(y + w_1 + w_2) + \beta_1 \beta_2 EC(y + w_1) + \beta_1 \beta_2 EC(y) + E_j J_{\pi-1}(y + w_1 + w_2 - \xi).
\]

It is evident that the manufacturer should not place an order for more than \(2n - y\) from a single supplier in any period because orders placed in a period, even if not delivered in the current period, are delivered before the beginning of the next period.

**Proposition 5.** If \(w_1(c_1 - c_2) > \beta_2 \beta_1 (EC(y + w_2) - EC(y + w_2 - 1)) + \beta_1 \beta_2 (EC(y) - EC(y + w_1))\) for \(0 < w_1 \leq w_2 \leq 2n - y\) and the combined order to both suppliers is less than \(2n - y\), then it is optimal to order exclusively from the more expensive supplier.

**Proof.** The expected costs \(J_k(y;0, w_2)\) and \(J_k(y; w_1, w_2 - w_1)\) are given by

\[
J_k(y;0, w_2) = c_2 w_2 + \beta_2 EC(y + w_2) + \beta_2 EC(y) + E_j J_{k-1}(y + w_2 - \xi)
\]

and

\[
J_k(y; w_1, w_2 - w_1) = c_2(w_2 - w_1) + c_1 w_1 + \beta_1 \beta_2 EC(y + w_2) + \beta_1 \beta_2 EC(y) + \beta_1 \beta_2 EC(y + w_2 - 1) + \beta_1 \beta_2 EC(y) + E_j J_{k-1}(y + w_2 - \xi).
\]

If \(w_1(c_1 - c_2) > \beta_2 \beta_1 (EC(y + w_2) - EC(y + w_2 - 1)) + \beta_1 \beta_2 (EC(y) - EC(y + w_1))\), then \(w_1(c_1 - c_2) > \beta_2 (1 - \beta_1) (EC(y + w_2) - EC(y + w_2 - 1)) + \beta_1 (1 - \beta_2) (EC(y) - EC(y + w_1))\).
Upon rearranging terms,
\[
\beta_2 EC(y + w_2) + \tilde{\beta}_2 EC(y) < -c_2 w_1 + c_1 w_1 \\
+ \beta_1 \beta_2 EC(y + w_2) + \tilde{\beta}_2 \beta_1 EC(y + w_1) \\
+ \tilde{\beta}_1 \beta_2 EC(y + w_2 - w_1) + \tilde{\beta}_1 \tilde{\beta}_2 EC(y).
\]
By adding \(c_2 w_2\) to both sides we get
\[
\beta_2 EC(y + w_2) + \tilde{\beta}_2 EC(y) + c_2 w_2 < c_2(w_2 - w_1) \\
+ c_1 w_1 + \beta_1 \beta_2 EC(y + w_2) + \tilde{\beta}_2 \beta_1 EC(y + w_1) \\
+ \tilde{\beta}_1 \beta_2 EC(y + w_2 - w_1) + \tilde{\beta}_1 \tilde{\beta}_2 EC(y),
\]
which implies
\[
J_k(y; 0, w_2) < J_k(y; w_1, w_2 - w_1).
\]
Thus, under the above conditions, whenever \(y + w_2 \leq 2n\), orders are placed only with the more expensive supplier. \(\square\)

In addition to the above condition, if there exists \(w'_2 \leq 2n - y\) such that \(J_k(y, w_1, w_2) > J_k(y, 0, w'_2) \quad \forall w_1 + w_2 > 2n - y\), then orders are always exclusively placed with the more expensive supplier when \(k\) periods remain and the starting inventory is \(y\). Note that the sufficient condition in Proposition 5 for the multi-period case is very similar to the condition given in Proposition 1 for the single period case. The only difference being that the cost of buying one unit from a supplier is \(c_i\) in the multi-period case (since delivery will occur in the next period for sure) as opposed to \(\beta c_i\) in the single period case.

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References